Large *CP* violation, large mixings of neutrinos, and a democratic-type neutrino mass matrix

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We propose a democratic-type neutrino mass matrix based on Z_3 symmetry. This mass matrix predicts the CP violation phase $\delta=\pi/2$ and the mixing angle between the mass eigenstates ν_2 and ν_3, $\sin^2\theta_{23}=\cos^2\theta_{23}=1/2$ which is essential for the large atmospheric neutrino mixing between ν_μ and ν_τ. In this model, the large CP violation effect may be expected.

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I. INTRODUCTION

The recent data on the atmospheric neutrino by Super-Kamiokande (Super-K) [1] show that the origin of the zenith angle dependence of the neutrino flux is due to the oscillation between ν_μ and ν_τ. The possibility of the ν_μ oscillation to the sterile neutrino ν_s is almost excluded [1]. Also, the possibility of ν_μ to ν_e oscillation is small [1] in accordance with the CHOOZ data [2]. The Super-K data are strengthened by the other data from the MACRO [3] and Soudan 2 [4] experiments. The preferable values of mass and mixing parameters are

$$\sin^2 2\theta_{\rm atm}=1.0, \quad \Delta m^2_{\rm atm}=3.5\times10^{-3}\mathrm{eV}^2. \tag{1}$$

At 90% confidence level, the allowed region is $2\times10^{-3}\mathrm{eV}^2<\Delta m^2_{\rm atm}<6\times10^{-3}\mathrm{eV}^2$ and $\sin^2 2\theta_{\rm atm}>0.85$.

The situation of the solar neutrino problem is more involved. There are various solutions that explain the absolute flux deficits by the Homestake [5], the Super-K [6], the GALLEX [7] and the SAGE [8] data, the small-angle Mikheyev-Smirnov-Wolfenstein (MSW) solution ($\Delta m^2_{\rm solar}=\text{a few}\times10^{-5}\mathrm{eV}^2$), the large-angle MSW solution ($10^{-5}\mathrm{eV}^2<\Delta m^2_{\rm solar}<10^{-4}\mathrm{eV}^2$), the large angle low mass solution ($\Delta m^2_{\rm solar}\sim10^{-7}\mathrm{eV}^2$) and the Just-so solution ($10^{-11}\mathrm{eV}^2<\Delta m^2_{\rm solar}<10^{-10}\mathrm{eV}^2$). In order to discriminate these solutions, the Super-K made the extensive study on the flux-independent analysis [6] by observing the day-night flux difference, the energy spectrum distortion of the recoil electron, and the seasonal variation. Although the statistic is not sufficient, there is a tendency that the large mixing angle solutions are preferable. If the flux of the hep neutrino is taken free, the large angle MSW and the large angle low mass solutions have the advantage [6]. These are evidence that support that the solar neutrino calls for large mixing between ν_e and ν_μ.

At present, three typical mixing schemes to realize large mixing both for the atmospheric neutrino and the solar neutrino mixings are known, the trimaximal mixing [9], the bimaximal mixing [10], and the democratic mixing [11]. Among them, the bimaximal mixing and the democratic mixing matrix contain no CP violation phase. The reason is due to the absence of the mixing between the first and the third mass eigenstates. In contrast, the trimaximal mixing predicts the maximal CP violation, which is the inevitable consequence of its structure.

In view of the interest in the structure to give the large mixing between ν_μ to ν_τ and the maximal CP violation in the trimaximal mixing, which are derived from a democratic mass matrix as we see later, we propose a democratic-type neutrino mass matrix based on Z_3 symmetry. We expected that this mass matrix interpolates the trimaximal mixing scheme and the bimaximal mixing scheme. Surprisingly, we found that this mass matrix predicts that $\cos^2\theta_{23}=\sin^2\theta_{23}=1/2$. Here, we used θ_{ij} for the mixing angle between mass eigenstates, ν_i and ν_j. This relation is mostly needed to realize the large atmospheric neutrino mixing. We also found that this model predicts the CP violation phase, $\delta=\pi/2$. In our model, the mixing angle between ν_1 and ν_2, θ_{12}, and the mixing angle between ν_1 and ν_3, θ_{13}, are left free. In order to examine the CP violation effect, we calculated the Jarlskog parameter and found that it takes about half of its maximal value if the large angle solar neutrino solutions are taken.

In Sec. II, we give the democratic-type neutrino mass matrix. In Sec. III, the mixing matrix which is predicted by the mass matrix is derived and the physical implication is discussed. The possible derivation of the democratic-type neutrino mass matrix is presented based on Z_3 symmetry in Sec. V. In Sec. VI, the summary is given.

II. DEMOCRATIC-TYPE NEUTRINO MASS MATRIX

Throughout of this paper, we consider the neutrino mass matrix in the diagonal mass basis of charged leptons. The name ‘‘democratic-type’’ for the mass matrix is used so that the mass matrix includes the democratic forms of matrices and their deformations.

A. Democratic mass matrix

We first define the democratic forms of the matrices:

$$S_1=\frac{1}{3}\begin{pmatrix}1 & \omega^2 & \omega\\ \omega^2 & \omega & 1\\ \omega & 1 & \omega^2\end{pmatrix},\quad S_2=\frac{1}{3}\begin{pmatrix}1 & \omega & \omega^2\\ \omega & \omega^2 & 1\\ \omega^2 & 1 & \omega\end{pmatrix},$$

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$$S_3=\frac{1}{3}\begin{pmatrix}1&1&1\\1&1&1\\1&1&1\end{pmatrix},\tag{2}$$

where $\omega=\exp(i2\pi/3)$ or $\exp(i4\pi/3)$, which satisfies $\omega^3=1$ and $1+\omega+\omega^2=0$. The matrix S_3 is commonly referred to as a democratic form [11], but we consider that the other two have the same right to be called democratic forms, because these matrices are related to each other by the phase transformation as

$$PS_1P=S_2,\quad PS_2P=S_3,\quad PS_3P=S_1,\tag{3}$$

where

$$P=\begin{pmatrix}1&0&0\\0&\omega^2&0\\0&0&\omega\end{pmatrix},\tag{4}$$

and thus S_1 and S_2 are derived from S_3 by the phase transformation. It may be worthwhile to note that the phase matrix P^* transforms S_i in the reverse cyclic direction as $P^*S_2P^*=S_1$.

We define the democratic mass matrix by the linear combination of these three democratic matrices as

$$m_{\nu,\mathrm{demo}}=m_1^0S_1+m_2^0S_2+m_3^0S_3.\tag{5}$$

Here we consider that mass parameters $|m_i^0|$ are quantities of the same order of magnitude, following the spirit of the democratic form.

B. The deformation from the democratic mass matrix

The deformation from the democratic form can be achieved by using the following three matrices:

$$T_1=\begin{pmatrix}1&0&0\\0&\omega&0\\0&0&\omega^2\end{pmatrix},\quad T_2=\begin{pmatrix}1&0&0\\0&\omega^2&0\\0&0&\omega\end{pmatrix},$$

$$T_3=\begin{pmatrix}1&0&0\\0&1&0\\0&0&1\end{pmatrix}.\tag{6}$$

Other symmetric mass matrices are formed by the linear combinations of S_i and T_i. Thus, the general mass matrix is given by

$$m_\nu=m_{\nu,\mathrm{demo}}+\tilde{m}_1T_1+\tilde{m}_2T_2+\tilde{m}_3T_3=\frac{1}{3}\begin{pmatrix}\overline{m}_1+\overline{m}_2+\overline{m}_3 & m_1^0\omega^2+m_2^0\omega+m_3^0 & m_1^0\omega+m_2^0\omega^2+m_3^0\\ m_1^0\omega^2+m_2^0\omega+m_3^0 & \overline{m}_1\omega+\overline{m}_2\omega^2+\overline{m}_3 & m_1^0+m_2^0+m_3^0\\ m_1^0\omega+m_2^0\omega^2+m_3^0 & m_1^0+m_2^0+m_3^0 & \overline{m}_1\omega^2+\overline{m}_2\omega+\overline{m}_3\end{pmatrix},\tag{7}$$

where $\overline{m}_i=m_i^0+3\tilde{m}_i$. In the following, we call m_i^0 (or $\overline{m}_i$) and $\tilde{m}_i$ mass parameters. We call this mass matrix the democratic-type mass matrix.

III. NEUTRINO MIXING MATRIX

The democratic-type mass matrix contains six complex parameters and thus it is a general matrix. In order to reduce the degree of freedom, we assume that ‘‘*all mass parameters, m_i^0 and $\tilde{m}_i$ are real.*’’

With this assumption, the mass matrix contains six real freedoms which correspond to neutrino masses and mixing angles. Thus, in general the CP violation phases are predicted once neutrino masses and mixing angles are given.

This assumption is one of the cases of the rather mild ansatz ‘‘*mass parameters are proportional to either one of three quantities, 1, ω and ω^2.*’’ Two other possibilities along this ansatz are discussed in Appendix B.

In our model there are two cases, $\omega=e^{i2\pi/3}$ and $e^{i4\pi/3}$ which is the complex conjugate to $e^{i2\pi/3}$. The mass matrix m_ν with real mass parameters has the following property:

$$m_\nu(\omega=e^{i2\pi/3})=m_\nu^*(\omega=e^{i4\pi/3}).\tag{8}$$

The neutrino mixing matrix V is defined by $V^Tm_\nu V=D_\nu$ where $D_\nu=\mathrm{diag}(m_1,m_2,m_3)$. If V is the unitary matrix to diagonalize $m_\nu(\omega=e^{i2\pi/3})$, then V^* is the one for $m_\nu(\omega=e^{i4\pi/3})$. Below, we discuss the neutrino mixing matrix V for $\omega=e^{i2\pi/3}$, by keeping in mind that V^* is also allowed in our model.

A. The neutrino mixing matrix

We consider m_ν for $\omega=e^{i2\pi/3}$. We first transform the mass matrix by using the trimaximal mixing matrix V_T as $V_T^Tm_\nu V_T$, where

$$V_T=\frac{1}{\sqrt{3}}\begin{pmatrix}1&1&1\\\omega&\omega^2&1\\\omega^2&\omega&1\end{pmatrix}.\tag{9}$$

Surprisingly, we find that the transformed mass matrix is a real symmetric matrix:

$$\tilde{m}_\nu = V_T^T m_\nu V_T = \begin{pmatrix} m_1^0+\tilde{m}_1 & \tilde{m}_3 & \tilde{m}_2 \\ \tilde{m}_3 & m_2^0+\tilde{m}_2 & \tilde{m}_1 \\ \tilde{m}_2 & \tilde{m}_1 & m_3^0+\tilde{m}_3 \end{pmatrix}. \tag{10}$$

Then, the matrix $\tilde{m}_\nu$ is diagonalized by an orthogonal matrix O.

Now, the unitary matrix V which diagonalizes m_ν is expressed by

$$V = V_T O = \frac{1}{\sqrt{3}} \begin{pmatrix} O_{11}+O_{21}+O_{31} & O_{12}+O_{22}+O_{32} & O_{13}+O_{23}+O_{33} \\ \omega O_{11}+\omega^2 O_{21}+O_{31} & \omega O_{12}+\omega^2 O_{22}+O_{32} & \omega O_{13}+\omega^2 O_{23}+O_{33} \\ \omega^2 O_{11}+\omega O_{21}+O_{31} & \omega^2 O_{12}+\omega O_{22}+O_{32} & \omega^2 O_{13}+\omega O_{23}+O_{33} \end{pmatrix}. \tag{11}$$

This unitary matrix is the neutrino mixing matrix because we consider the neutrino mass matrix in the diagonal mass basis of charged leptons. This mixing matrix seems to have a complex form, but it has the outstanding property that $V_{2i}=V_{3i}^*$ for $i=1,2,3$. This property restricts the neutrino mixings tightly. Since it is hard to treat this mixing matrix directly, we attack it from a slightly different point of view.

We first observe that by the phase transformation of charged leptons and neutrinos, V can be made into the standard form V_{SF} as given in the particle data [12].

$$V_{\mathrm{SF}} = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23}-c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23}-s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23}-c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23}-s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{pmatrix}, \tag{12}$$

where $s_{ij}=\sin\theta_{ij}$, $c_{ij}=\cos\theta_{ij}$, and θ_{ij} is the mixing angle which mixes mass eigenstates ν_i and ν_j. That is, we can write $V=PV_{\mathrm{SF}}P'$, where P and P' are diagonal phase matrices.

The restrictions $V_{2i}=V_{3i}^*$ for $i=1,2,3$ lead to the constraints $|(V_{\mathrm{SF}})_{2i}|=|(V_{\mathrm{SF}})_{3i}|$ for $i=1,2,3$, which are expressed by

$$|-s_{12}c_{23}-c_{12}s_{23}s_{13}e^{i\delta}| = |s_{12}s_{23}-c_{12}c_{23}s_{13}e^{i\delta}|,$$

$$|c_{12}c_{23}-s_{12}s_{23}s_{13}e^{i\delta}| = |-c_{12}s_{23}-s_{12}c_{23}s_{13}e^{i\delta}|,$$

$$|s_{23}c_{13}| = |c_{23}c_{13}|. \tag{13}$$

By solving these equations, we find

$$c_{23}^2 = s_{23}^2, \quad \cos\delta = 0, \tag{14}$$

by omitting the uninteresting possibility $c_{13}=0$. It is amazing that our model predicts the *CP* violation phase, $\delta=\pi/2$, and $c_{23}^2=s_{23}^2=1/2$ which is quite important to explain the almost full mixing between ν_μ and ν_τ in the two mixing limit. The most interesting point is that the mixing angle θ_{23} and the *CP* violation phase δ are fixed independently of mass parameters.

B. General form of neutrino mixing matrix

We take $s_{23}=-c_{23}=-1/\sqrt{2}$. Then, the diagonal phase matrices P and P' are determined such that the matrix $V_T^\dagger P V_{\mathrm{SF}} P'$ becomes a real orthogonal matrix. In this way, we found

$$V = \begin{pmatrix} 1 & 0 & 0 \\ 0 & e^{i\rho} & 0 \\ 0 & 0 & e^{-i\rho} \end{pmatrix} \times \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & -is_{13} \\ -\dfrac{s_{12}-ic_{12}s_{13}}{\sqrt{2}} & \dfrac{c_{12}+is_{12}s_{13}}{\sqrt{2}} & -\dfrac{c_{13}}{\sqrt{2}} \\ -\dfrac{s_{12}+ic_{12}s_{13}}{\sqrt{2}} & \dfrac{c_{12}-is_{12}s_{13}}{\sqrt{2}} & \dfrac{c_{13}}{\sqrt{2}} \end{pmatrix} \times \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & i \end{pmatrix}. \tag{15}$$

In addition to interesting predictions for θ_{23} and δ, our neutrino mass matrix predicts the Majorana phase matrix $\mathrm{diag}(1,1,i)$ [13,14], which shows no *CP* violation intrinsic to Majorana system. The other phase matrix $\mathrm{diag}(1,e^{i\rho},e^{-i\rho})$ does not have any physical effect, because this phase is absorbed by charged leptons. Our mass matrix contains six real parameters which are converted to three neutrino masses, two mixing angles, θ_{12} and θ_{13}, and one unphysical phase ρ.

The other case $s_{23}=c_{23}=1/\sqrt{2}$ reduces to the case of $\delta=-\pi/2$, which is included in the mixing matrix V^*.

Below, we discuss that our mixing reduces to two well-known typical large mixing matrices, the trimaximal mixing and the bimaximal mixing by imposing simple conditions on mass parameters.

C. Trimaximal and bimaximal mixing limits

By taking the mass parameters in some special values, our model reduces to models to reproduce the trimaximal mixing and the bimaximal mixing.

1. The trimaximal mixing limit

By taking the mixing angles and phase matrices as $s_{12}=-1/\sqrt{2}$, $c_{12}=1/\sqrt{2}$, $s_{13}=1/\sqrt{3}$, $c_{13}=\sqrt{2/3}$, $\rho=\pi/2$, the matrix V reduces to the trimaximal mixing matrix

$$V=V_T\begin{pmatrix} 1 & 0 & 0\\ 0 & -1 & 0\\ 0 & 0 & 1\end{pmatrix}, \tag{16}$$

where the phase matrix diag$(1,-1,1)$ does not have any physical meaning.

From Eq. (A2) in the Appendix, the mass parameters are now restricted by

$$m_1^0=m_1,\quad m_2^0=m_2,\quad m_3^0=m_3,$$
$$\tilde{m}_1=\tilde{m}_2=\tilde{m}_3=0. \tag{17}$$

Now we see the mass matrix m_ν which is reduced to the $m_{\nu,\mathrm{demo}}$ as

$$m_\nu=m_1^0S_1+m_2^0S_2+m_3^0S_3. \tag{18}$$

The democratic mass matrix $m_{\nu,\mathrm{demo}}$ has various interesting properties which are discussed in Appendix A.

2. The bimaximal mixing limit

By taking the mixing angles and phase matrices as $s_{12}=-1/\sqrt{2}$, $c_{12}=1/\sqrt{2}$, $s_{13}=0$, $c_{13}=1$, $\rho=0$, the matrix V reduces to the bimaximal mixing matrix

$$V=\begin{pmatrix} 1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 1\end{pmatrix}O_B\begin{pmatrix} 1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & i\end{pmatrix}, \tag{19}$$

where O_B is the bimaximal mixing matrix defined by

$$O_B=\begin{pmatrix} \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} & 0\\ \frac{1}{2} & \frac{1}{2} & -\frac{1}{\sqrt{2}}\\ \frac{1}{2} & \frac{1}{2} & \frac{1}{\sqrt{2}}\end{pmatrix}. \tag{20}$$

From Eq. (A2) in the Appendix, the mass parameters are now restricted by

$$m_1^0=m_2^0,\quad \tilde{m}_1=\tilde{m}_2, \tag{21}$$

and in this case the mass matrix becomes

$$m_{\nu,B}=\frac{1}{3}\begin{pmatrix} \overline{m}_3+2\overline{m}_1 & m_3^0-m_1^0 & m_3^0-m_1^0\\ m_3^0-m_1^0 & \overline{m}_3-\overline{m}_1 & m_3^0+2m_1^0\\ m_3^0-m_1^0 & m_3^0+2m_1^0 & \overline{m}_3-\overline{m}_1\end{pmatrix}. \tag{22}$$

This matrix satisfies the condition that all elements are real, $(m_{\nu,B})_{22}=(m_{\nu,B})_{33}$ and $(m_{\nu,B})_{12}=(m_{\nu,B})_{13}$.

The mass parameters are expressed by neutrino masses and mixing angles as

$$m_1^0=m_2^0=\frac{1}{4}(2m_3+m_2+m_1)+\frac{1}{2\sqrt{2}}(m_2-m_1),$$

$$m_3^0=\frac{1}{4}(2m_3+m_2+m_1)-\frac{1}{\sqrt{2}}(m_2-m_1),$$

$$\tilde{m}_1=\tilde{m}_2=-\frac{1}{6\sqrt{2}}(m_2-m_1),$$

$$\tilde{m}_3=-\frac{1}{4}(2m_3-m_2-m_1)+\frac{1}{3\sqrt{2}}(m_2-m_1). \tag{23}$$

It is interesting to observe that our model connects the trimaximal mixing and the bimaximal mixing by keeping the CP violation phase, $\delta=\pi/2$. In our model, the absence of the CP violation in the bimaximal limit is solely due to $\sin\theta_{13}=0$ and any deviation from it recovers $\delta=\pi/2$. Since the restriction $\sin^2\theta_{23}=\cos^2\theta_{23}=1/2$ is the most advantageous situation to realize large mixing angle $\sin^2 2\theta_{\mathrm{atm}}$ by deviating $\sin\theta_{13}$ from zero, this model provides the most advantageous case for the CP violation.

IV. ANALYSIS OF OUR MIXING SCHEME

We consider the hierarchy of neutrino masses as

$$\Delta_{\mathrm{atm}}\equiv\Delta_{32}\simeq\Delta_{31}\simeq3\times10^{-3}\mathrm{eV}^2,$$
$$\Delta_{\mathrm{solar}}\equiv\Delta_{21}\ll\Delta_{\mathrm{atm}}. \tag{24}$$

A. Vacuum oscillations

We first derive the probabilities of neutrino oscillations in the vacuum. We use the abbreviation, $P(l\to l')$ for $P(\nu_l\to\nu_{l'})$. We find

$$P(\tau\to\tau)=P(\mu\to\mu),\quad P(e\to\tau)=P(\mu\to e),$$
$$P(\tau\to e)=P(e\to\mu) \tag{25}$$

and

$$P(e\to e)=1-4s_{12}^2c_{12}^2c_{13}^4\sin^2\left(\frac{\Delta_{21}}{4E}L\right)-4c_{12}^2s_{13}^2c_{13}^2\times\sin^2\left(\frac{\Delta_{31}}{4E}L\right)-4s_{12}^2s_{13}^2c_{13}^2\sin^2\left(\frac{\Delta_{32}}{4E}L\right),$$

$$P(\mu\to\mu)=1-A^2B^2\sin^2\left(\frac{\Delta_{21}}{4E}L\right)-c_{13}^2A^2\sin^2\left(\frac{\Delta_{31}}{4E}L\right)-c_{13}^2B^2\sin^2\left(\frac{\Delta_{32}}{4E}L\right),$$

$$\begin{aligned}P(e\to\mu)=&\frac{1}{2}c_{13}^2[s_{13}+c_{12}A+s_{12}B]^2\\&-2s_{12}c_{12}c_{13}^2AB\,\sin^2\left(\frac{\Delta_{21}}{4E}L+\frac{\delta_1}{2}+\frac{\delta_2}{2}\right)\\&-2c_{12}s_{13}c_{13}^2A\,\sin^2\left(\frac{\Delta_{31}}{4E}L+\frac{\delta_1}{2}-\frac{\pi}{4}\right)\\&-2s_{12}s_{13}c_{13}^2B\sin^2\left(\frac{\Delta_{32}}{4E}L-\frac{\delta_2}{2}-\frac{\pi}{4}\right),\end{aligned}$$

$$\begin{aligned}P(\mu\to e)=&\frac{1}{2}c_{13}^2[s_{13}+c_{12}A+s_{12}B]^2\\&-2s_{12}c_{12}c_{13}^2AB\,\sin^2\left(\frac{\Delta_{21}}{4E}L-\frac{\delta_1}{2}-\frac{\delta_2}{2}\right)\\&-2c_{12}s_{13}c_{13}^2A\,\sin^2\left(\frac{\Delta_{31}}{4E}L-\frac{\delta_1}{2}+\frac{\pi}{4}\right)\\&-2s_{12}s_{13}c_{13}^2B\sin^2\left(\frac{\Delta_{32}}{4E}L+\frac{\delta_2}{2}+\frac{\pi}{4}\right),\end{aligned}$$

$$\begin{aligned}P(\mu\to\tau)=&1-A^2B^2\sin^2\left(\frac{\Delta_{21}}{4E}L-\delta_1-\delta_2\right)\\&-c_{13}^2A^2\sin^2\left(\frac{\Delta_{31}}{4E}L-\delta_1-\frac{\pi}{2}\right)\\&-c_{13}^2B^2\sin^2\left(\frac{\Delta_{32}}{4E}L+\delta_2-\frac{\pi}{2}\right),\end{aligned}$$

$$\begin{aligned}P(\tau\to\mu)=&1-A^2B^2\sin^2\left(\frac{\Delta_{21}}{4E}L+\delta_1+\delta_2\right)\\&-c_{13}^2A^2\sin^2\left(\frac{\Delta_{31}}{4E}L+\delta_1-\frac{\pi}{2}\right)\\&-c_{13}^2B^2\sin^2\left(\frac{\Delta_{32}}{4E}L-\delta_2-\frac{\pi}{2}\right),\end{aligned}\tag{26}$$

where

$$A=\sqrt{s_{12}^2+c_{12}^2s_{13}^2},\quad B=\sqrt{c_{12}^2+s_{12}^2s_{13}^2},$$

$$\delta_1=\tan^{-1}\left(\frac{c_{12}s_{13}}{s_{12}}\right),\quad \delta_2=\tan^{-1}\left(\frac{s_{12}s_{13}}{c_{12}}\right),\tag{27}$$

and $\Delta_{ij}=m_i^2-m_j^2$. These are the general formula and a simpler form of the oscillation probability is obtained once the distance L is specified.

B. The analysis

We start from the CHOOZ data which restrict $|V_{e3}|^2<0.05$ which leads to

$$s_{13}^2<0.05.\tag{28}$$

Next, the probability of ν_μ to ν_e and ν_τ at the atmospheric range are simply expressed by

$$P(\mu\to e)\simeq 2s_{13}^2c_{13}^2\sin^2\frac{\Delta_{\rm atm}}{4E}L,$$

$$P(\mu\to\tau)\simeq c_{13}^4\sin^2\frac{\Delta_{\rm atm}}{4E}L.\tag{29}$$

Therefore, by combining our model and the CHOOZ data we predict that the probability for ν_μ to ν_e is small, $P(\mu\to e)<0.1$, and the effective mixing angle between ν_μ and ν_τ is

$$\sin^2 2\theta_{\rm atm}=c_{13}^4>0.90.\tag{30}$$

As for the solar neutrino problem, we assume $10^{-11}\text{eV}^2<\Delta_{\rm solar}<10^{-4}\text{eV}^2$. In the vacuum, we find

$$P(\nu_e\to\nu_e)\simeq 1-2s_{13}^2c_{13}^2-4s_{12}^2c_{12}^2c_{13}^4\sin^2\frac{\Delta_{\rm solar}}{4E}L,$$

$$P(\nu_e\to\nu_\mu)\simeq s_{13}^2c_{13}^2+2s_{12}^2c_{12}^2c_{13}^4\sin^2\frac{\Delta_{\rm solar}}{4E}L+s_{12}c_{12}s_{13}c_{13}^2\sin\frac{\Delta_{\rm solar}}{2E}L,$$

$$P(\nu_e\to\nu_\tau)\simeq s_{13}^2c_{13}^2+2s_{12}^2c_{12}^2c_{13}^4\sin^2\frac{\Delta_{\rm solar}}{4E}L-s_{12}c_{12}s_{13}c_{13}^2\sin\frac{\Delta_{\rm solar}}{2E}L.\tag{31}$$

Thus, we find that

$$\sin^2 2\theta_{\rm solar}\simeq\sin^2 2\theta_{12}c_{13}^4>0.90\sin^2 2\theta_{12}.\tag{32}$$

Thus, our model can accommodate all four solutions, the small-angle MSW, the large-angle MSW, the low mass, and the Just-so solutions.

C. CP violation

In order to see the size of the CP violation, we consider the Jarlskog parameter that is defined by [15]

$$J_{CP} \equiv \mathrm{Im}(V_{e1}V_{e2}^{*}V_{\mu 1}^{*}V_{\mu 2}) = s_{12}c_{12}s_{23}c_{23}s_{13}c_{13}^{2}\sin\delta \leqslant 1/6\sqrt{3}. \tag{33}$$

Our predicted values, $c_{23} = -s_{23} = 1/\sqrt{2}$ and $\sin\delta = 1$ give the most advantageous case to obtain large J_{CP} concerning θ_{23} and δ,

$$(J_{CP})_{\text{our model}} = -\frac{1}{2}s_{12}c_{12}s_{13}c_{13}^{2}. \tag{34}$$

The prediction of J_{CP} depends on θ_{12} and θ_{13}. If we take the value $s_{13}^2 = 0.05$, we have $J_{CP} = -0.053\sin 2\theta_{12}$. If the solar neutrino mixing turns out to be one of large-angle solutions, $\sin^2 2\theta_{12} \sim 0.8$, we find $J_{CP} = -0.047$ which is about half of the maximal value $(J_{CP})_{\max} \simeq 0.096$. For the small-angle case, we obtain about a 10 times smaller value than the large-angle case.

V. SOME DERIVATIONS OF THE NEUTRINO MASS MATRIX

The neutrino mass matrix that we discussed in the former section may be derived by the following considerations in the basis where charged leptons are mass eigenstates.

A. Neutrino mass term and S_3 symmetry with Z_3 phases

We consider the following three types of transformations;

$$\text{(I)}\quad \nu_e \to \omega^2\nu_\mu,\quad \nu_\mu \to \omega^2\nu_\tau,\quad \nu_\tau \to \omega^2\nu_e,$$

$$\text{(II)}\quad \nu_e \to \omega\nu_\mu,\quad \nu_\mu \to \omega\nu_\tau,\quad \nu_\tau \to \omega\nu_e,$$

$$\text{(III)}\quad \nu_e \to \nu_\mu,\quad \nu_\mu \to \nu_\tau,\quad \nu_\tau \to \nu_e, \tag{35}$$

where $\omega = \exp(i2\pi/3)$ or $\exp(i4\pi/3)$. They are considered to be S_3 transformations with Z_3 phases.

The Majorana mass matrix for left-handed neutrinos which is invariant under one of these transformation is expressed by

$$M_i = m_i^0 S_i + \tilde{m}_i T_i, \tag{36}$$

where $i = 1,2,3$, S_i, and T_i are defined in Eqs. (2) and (6). The mass matrix M_1 is derived by imposing the transformation (I) and so on.

Since there is no principle to discriminate these three matrices M_i, we assume that the neutrino mass matrix m_ν is expressed by the sum of these three mass matrices, although there is no good reason to explain this. Then, we obtain the neutrino mass matrix m_ν in Eq. (7).

B. Z_3 invariant Lagrangian

Another reason to introduce the mass matrix in Eq. (7) may be given by imposing the Z_3 symmetry on Yukawa interaction. The left-handed doublet leptons can be arranged in eigenstates of Z_3 symmetry as

$$\Psi_1 = \frac{\omega^2 l_e + \omega l_\mu + l_\tau}{\sqrt{3}},$$

$$\Psi_2 = \frac{\omega l_e + \omega^2 l_\mu + l_\tau}{\sqrt{3}},$$

$$\Psi_3 = \frac{l_e + l_\mu + l_\tau}{\sqrt{3}}, \tag{37}$$

where $l_e^T = (\nu_{eL}, e_L)$ and so on. Under the S_3 transformation, $l_e \to l_\tau$ and $l_\mu \to l_e$, $l_\tau \to l_\mu$, they are transformed as

$$\Psi_1 \to \omega^2\Psi_1,\quad \Psi_2 \to \omega\Psi_2,\quad \Psi_3 \to \Psi_3. \tag{38}$$

Then, we introduce three kinds of triplet Higgs which transform as $\Delta_1 \to \omega^2\Delta_1$, $\Delta_2 \to \omega\Delta_2$, and $\Delta_3 \to \Delta_3$. Then, the invariant Yukawa interaction terms among two doublets and a triplet are

$$\begin{aligned}
\mathcal{L}_{\mathrm{y}} = &-\Bigg((m_1^0 + \tilde{m}_1)\omega^2\overline{(\Psi_1)^C} i\tau_2 \frac{\Delta_1}{v_1}\Psi_1 + (m_2^0 + \tilde{m}_2) \\
&\times \omega\overline{(\Psi_2)^C} i\tau_2 \frac{\Delta_2}{v_2}\Psi_2 + (m_3^0 + \tilde{m}_3)\overline{(\Psi_3)^C} i\tau_2 \frac{\Delta_3}{v_3}\Psi_3 \Bigg) \\
&-2\Bigg(\tilde{m}_1\omega^2\overline{(\Psi_2)^C} i\tau_2 \frac{\Delta_1}{v_1}\Psi_3 + \tilde{m}_2\omega\overline{(\Psi_3)^C} i\tau_2 \frac{\Delta_2}{v_2}\Psi_1 \\
&+ \tilde{m}_3\overline{(\Psi_1)^C} i\tau_2 \frac{\Delta_3}{v_3}\Psi_2 \Bigg),
\end{aligned} \tag{39}$$

where v_i are vacuum expectation values of Δ_i. When vacuum expectation values of triplet Higgs are generated, the Majorana-type mass term given in Eq. (7) is generated for neutrinos. We argue that in order to acquire small vacuum expectation values of triplet Higgs bosons, the seesaw suppression mechanism [16] should be adopted.

C. Nonrenormalizable interaction

The triplet representation can be composed of the two doublet representation. We can explicitly construct the Higgs triplet, Δ_i by the combinations of two Higgs doublets, H_j which transform as

$$H_1 \to \omega H_1,\quad H_2 \to H_2. \tag{40}$$

The symmetric combinations H_1H_1, H_1H_2, and H_2H_2 transform as Δ_1, Δ_2, and Δ_3. Thus, we obtain the Lagrangian as

$$\mathcal{L}_{\rm y}=-\Bigg((m_1^0+\tilde{m}_1)\omega^2\overline{(\Psi_1)^C}\Psi_1\frac{H_1H_1}{u_1^2}+(m_2^0+\tilde{m}_2)\times\omega\overline{(\Psi_2)^C}\Psi_2\frac{H_1H_2}{u_1u_2}+(m_3^0+\tilde{m}_3)\overline{(\Psi_3)^C}\Psi_3\frac{H_2H_2}{u_2^2}\Bigg)-2\Bigg(\tilde{m}_1\omega^2\overline{(\Psi_2)^C}\Psi_3\frac{H_1H_1}{u_1^2}+\tilde{m}_2\omega\overline{(\Psi_1)^C}\Psi_3\frac{H_1H_2}{u_1u_2}+\tilde{m}_3\overline{(\Psi_1)^C}\Psi_2\frac{H_2H_2}{u_2^2}\Bigg), \tag{41}$$

where u_i is the vacuum expectation value of the neutral component of H_i. After the symmetry breaking, the neutrino mass matrix in Eq. (7) is obtained.

VI. DISCUSSIONS

We introduced the democratic-type neutrino mass matrix by extending the democratic mass matrix and found that one angle θ_{23} and the CP violation phase intrinsic to the Dirac system are predicted to be $\theta_{23}=-\pi/4$ and $\delta=\pi/2$. As a consequence, the mixing matrix is expressed by two angles, θ_{12} and θ_{13}, as shown in Eq. (15). If the solar neutrino problem turns out to be solved by the large angle solutions, the large CP violation effect is expected. In this situation, our model predicts that the Jarlskog parameter is about half of the maximal value, $J_{CP}=-0.047$ with $\sin^2 2\theta_{\rm solar}=0.8$. This could be explored by the future long-baseline experiments.

Our model predicts no CP violation intrinsic to the Majorana neutrino system [13,14]. The phase i in the Majorana phase matrix diag$(1,1,i)$ in Eq. (15) relates to the CP signs of mass eigenstate neutrinos [17] in addition to the relative signs of neutrino masses. The phase matrix diag$(1,e^{i\rho},e^{-i\rho})$ in Eq. (15) are absorbed by charged leptons.

The effect for the neutrinoless double β decay is given by [18]

$$|\langle m_\nu\rangle|\equiv\left|\sum_j{}' \; U_{ej}^2 m_j\right|=|(m_1c_{12}^2+m_2s_{12}^2)c_{13}^2+m_3s_{13}^2|, \tag{42}$$

where the dash in the sum means that j extends to light neutrinos. The mixing matrix U is the matrix including the Majorana phase matrix, $U=V_{\rm SF}{\rm diag}(1,1,i)$. The effective mass $|\langle m_\nu\rangle|$ depends on the relative signs among m_1, m_2, and m_3, which corresponds to CP signs of mass eigenstate neutrinos [17]. Here we take $m_1>0$. In case that $|m_1|\simeq|m_2|$, we find

$$|\langle m_\nu\rangle|=\begin{cases}|m_1c_{13}^2+m_3s_{13}^2| & (m_2>0)\\ |m_1\cos 2\theta_{12}c_{13}^2+m_3s_{13}^2| & (m_2<0)\end{cases}. \tag{43}$$

There are three typical cases.

(1) *The similar mass case* $|m_2|\sim|m_3|\sim m_1$. In this case, $\Delta_{\rm solar}$ and $\Delta_{\rm atm}$ do not constrain the neutrino mass themselves. The effective mass $|\langle m_\nu\rangle|\sim m_1$ or $m_1|\cos 2\theta_{12}|$ could be as large as the sensitivity of the neutrinoless double β decay experiment. It may be worthwhile to comment that $\cos 2\theta_{12}\sim 1$ for the small-angle solution and ~ 0.44 for the large-angle solutions, such as $\sin^2 2\theta_{\rm solar}\simeq 0.8$ for the solar neutrino problem.

(2) *The hierarchical case.* (a) $|m_3|\gg m_1\simeq|m_2|$. We expect that $|m_3|\sim\sqrt{\Delta_{\rm atm}}\sim 0.05$ eV. Then, we expect $|\langle m_\nu\rangle|\ll|m_3|\sim 0.05$ eV, which may be hard to detect. (b) $m_1\simeq|m_2|\gg|m_3|$. We expect that $m_1\sim\sqrt{\Delta_{\rm atm}}\sim 0.05$ eV. Then, we expect $|\langle m_\nu\rangle|\sim m_1$ or $|\cos 2\theta_{12}|m_1$ which is about the order 0.05 eV, which may be within the reach of future experiments.

Note added. After submitting our paper, we were informed by H. Fritzsch and Z.Z. Xing [19] that they discussed another possibility to obtain the large CP violation.

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APPENDIX A: EXPLICIT EXPRESSIONS OF MASS PARAMETERS AND THE INTERESTING PROPERTY OF THE DEMOCRATIC MASS MATRIX

1. Mass parameters

Mass parameters m_i^0 and $\overline{m_i}=m_i^0+3\tilde{m}_i$ are explicitly expressed in terms of neutrino masses and mixing angles, θ_{12} and θ_{13}, and the unphysical phase ρ which is eaten by the phase redefinition of charged leptons. This is achieved by examining

$$m_\nu=V^*\begin{pmatrix}m_1 & 0 & 0\\ 0 & m_2 & 0\\ 0 & 0 & m_3\end{pmatrix}V^\dagger. \tag{A1}$$

We find for $\omega=e^{i2\pi/3}$

$$m_3^0=\left\{\frac{1}{2}(s_{12}^2+c_{12}^2s_{13}^2)-\sqrt{2}c_{12}c_{13}(s_{12}\cos\rho+c_{12}s_{13}\sin\rho)\right\}m_1+\left\{\frac{1}{2}(c_{12}^2+s_{12}^2s_{13}^2)+\sqrt{2}s_{12}c_{13}(c_{12}\cos\rho-s_{12}s_{13}\sin\rho)\right\}m_2+\left\{\frac{1}{2}c_{13}^2+\sqrt{2}s_{13}c_{13}\sin\rho\right\}m_3,$$

$$m_2^0=\left\{\frac{1}{2}(s_{12}^2+c_{12}^2s_{13}^2)-\sqrt{2}c_{12}c_{13}\left[s_{12}\cos\left(\rho+\frac{2\pi}{3}\right)+c_{12}s_{13}\sin\left(\rho+\frac{2\pi}{3}\right)\right]\right\}m_1+\left\{\frac{1}{2}(c_{12}^2+s_{12}^2s_{13}^2)+\sqrt{2}s_{12}c_{13}\left[c_{12}\cos\left(\rho+\frac{2\pi}{3}\right)-s_{12}s_{13}\sin\left(\rho\right.\right.\right.$$

$$+\frac{2\pi}{3}\Big)\Big]\Big\}m_2+\Big\{\frac{1}{2}c_{13}^2+\sqrt{2}s_{13}c_{13}\sin\Big(\rho+\frac{2\pi}{3}\Big)\Big\}m_3,$$

$$m_1^0=\Big\{\frac{1}{2}(s_{12}^2+c_{12}^2s_{13}^2)-\sqrt{2}c_{12}c_{13}\Big[s_{12}\cos\Big(\rho-\frac{2\pi}{3}\Big)+c_{12}s_{13}\sin\Big(\rho-\frac{2\pi}{3}\Big)\Big]\Big\}m_1+\Big\{\frac{1}{2}(c_{12}^2+s_{12}^2s_{13}^2)+\sqrt{2}s_{12}c_{13}\Big[c_{12}\cos\Big(\rho-\frac{2\pi}{3}\Big)-s_{12}s_{13}\sin\Big(\rho-\frac{2\pi}{3}\Big)\Big]\Big\}m_2+\Big\{\frac{1}{2}c_{13}^2+\sqrt{2}s_{13}c_{13}\sin\Big(\rho-\frac{2\pi}{3}\Big)\Big\}m_3,$$

$$\overline{m}_3=\{c_{12}^2c_{13}^2+(s_{12}^2-c_{12}^2s_{13}^2)\cos2\rho+2s_{12}c_{12}s_{13}\sin2\rho\}m_1+\{s_{12}^2c_{13}^2+(c_{12}^2-s_{12}^2s_{13}^2)\cos2\rho-2s_{12}c_{12}s_{13}\sin2\rho\}m_2+\{s_{13}^2-c_{13}^2\cos2\rho\}m_3,$$

$$\overline{m}_2=\Big\{c_{12}^2c_{13}^2+(s_{12}^2-c_{12}^2s_{13}^2)\cos\Big(2\rho-\frac{2\pi}{3}\Big)+2s_{12}c_{12}s_{13}\sin\Big(2\rho-\frac{2\pi}{3}\Big)\Big\}m_1+\Big\{s_{12}^2c_{13}^2+(c_{12}^2-s_{12}^2s_{13}^2)\cos\Big(2\rho-\frac{2\pi}{3}\Big)-2s_{12}c_{12}s_{13}\sin\Big(2\rho-\frac{2\pi}{3}\Big)\Big\}m_2+\Big\{s_{13}^2-c_{13}^2\cos\Big(2\rho-\frac{2\pi}{3}\Big)\Big\}m_3,$$

$$\overline{m}_1=\Big\{c_{12}^2c_{13}^2+(s_{12}^2-c_{12}^2s_{13}^2)\cos\Big(2\rho+\frac{2\pi}{3}\Big)+2s_{12}c_{12}s_{13}\sin\Big(2\rho+\frac{2\pi}{3}\Big)\Big\}m_1+\Big\{s_{12}^2c_{13}^2+(c_{12}^2-s_{12}^2s_{13}^2)\cos\Big(2\rho+\frac{2\pi}{3}\Big)-2s_{12}c_{12}s_{13}\sin\Big(2\rho+\frac{2\pi}{3}\Big)\Big\}m_2+\Big\{s_{13}^2-c_{13}^2\cos\Big(2\rho+\frac{2\pi}{3}\Big)\Big\}m_3. \tag{A2}$$

2. Interesting properties of the democratic mass matrix

The democratic mass matrix $m_{\nu,\mathrm{demo}}$ defined in Eq. (5) consists of matrices S_i which are rank 1 and have a special property that they are diagonalized simultaneously by the bilinear transformation $V_T^T S_i V_T$ with the unitary matrix V_T. The condition that symmetric matrices A and B are diagonalized simultaneously by this transformation is $A^*B=B^*A$ and matrices S_i satisfy $S_i^*S_j=0$ for $i\neq j$, so that they satisfy the condition trivially. In fact, we find

$$V_T^T S_i V_T=D_i, \tag{A3}$$

where D_i are diagonal matrix as $D_1=\mathrm{diag}(1,0,0)$, $D_2=\mathrm{diag}(0,1,0)$, $D_3=\mathrm{diag}(0,0,1)$, and

$$V_T=\frac{1}{\sqrt{3}}\begin{pmatrix}1&1&1\\ \omega&\omega^2&1\\ \omega^2&\omega&1\end{pmatrix}. \tag{A4}$$

By using V_T, the democratic neutrino mass matrix $m_{\nu,\mathrm{demo}}$ in Eq. (5) is diagonalized as

$$V_T^T m_{\nu,\mathrm{demo}} V_T=\begin{pmatrix}m_1^0&0&0\\ 0&m_2^0&0\\ 0&0&m_3^0\end{pmatrix}. \tag{A5}$$

Thus, in this limit m_i^0 are interpreted to be masses of neutrinos.

The unitary matrix V_T is nothing but the trimaximal mixing matrix. The matrix V_T is transformed into the standard form as

$$\begin{pmatrix}1&0&0\\ 0&i&0\\ 0&0&i\end{pmatrix}V_t\begin{pmatrix}1&0&0\\ 0&1&0\\ 0&0&-i\end{pmatrix}=\frac{1}{\sqrt{3}}\begin{pmatrix}1&1&-i\\ i\omega&i\omega^2&1\\ i\omega^2&i\omega&1\end{pmatrix}. \tag{A6}$$

Therefore, the CP violation phase intrinsic to a Dirac neutrino system is $\delta=\pi/2$, i.e., the maximal CP violation. There are two other phases that are intrinsic to the Majorana neutrino system which is the same as the general case given in Eq. (15).

APPENDIX B: OTHER ANSATZ ABOUT MASS PARAMETERS

In the text, we considered the model which predicts $V_{2j}=V_{3j}^*$. Here we consider other such possibilities.

1. The mass matrix which predicts $V_{1j}=V_{3j}^*$

We consider the case where m_1^0 and $\tilde{m}_1$ are proportional to ω^2, m_2^0 and $\tilde{m}_2$ to ω, and m_3^0 and $\tilde{m}_3$ to 1. When this mass matrix is transformed by V_T, we obtain the mass matrix $\tilde{m}_\nu$ given in Eq. (10) which is a complex symmetric matrix. However, these complex phases are removed by the phase transformation by phase matrix $\mathrm{diag}(\omega^2,\omega,1)$ and $\tilde{m}_\nu$ can be transformed into a real symmetric matrix. That is, by the trimaximal mixing matrix

$$V_T'=V_T\begin{pmatrix}\omega^2&0&0\\ 0&\omega&0\\ 0&0&1\end{pmatrix}, \tag{B1}$$

the mass matrix m_ν is transformed by a real symmetric mass matrix $\tilde{m}_\nu'$

$$\tilde{m}_\nu'=V'^{T}_{T}m_\nu V_T'. \tag{B2}$$

This mass matrix is diagonalized by an orthogonal matrix O'.

Thus, the mixing matrix is given by

$$V = V_T' O'. \tag{B3}$$

This mixing matrix has the property that $V_{1j} = V_{3j}^*$ for $j = 1,2,3$. As we discussed in the text, this condition implies that $|(V_{\rm SF})_{1j}| = |(V_{\rm SF})_{3j}|$ for $j = 1,2,3$. By solving these equations, we find

$$c_{23}^2 = \frac{s_{13}^2}{c_{13}^2}, \quad \cos\delta = -\frac{s_{23}}{c_{23}s_{13}} \cot 2\theta_{12}. \tag{B4}$$

Since the CHOOZ data gives the severe constraints, $s_{13}^2 < 0.05$ and $c_{23}^2 \simeq s_{13}^2$, we cannot predict the large mixing between ν_μ and ν_τ. Thus, unfortunately this model cannot explain the atmospheric data and the CHOOZ data simultaneously.

2. The mass matrix which predicts $V_{1j} = V_{2j}^*$

We consider the case that m_1^0 and $\tilde{m}_1$ are proportional to ω, m_2^0 and $\tilde{m}_2$ to ω^2, and m_3^0 and $\tilde{m}_3$ to 1. By repeating the same discussion for the previous case, we find that m_ν can be transformed into the real symmetric mass matrix $\tilde{m}_\nu$ by the trimaximal mixing matrix as

$$\tilde{m}_\nu = V_T'' m_\nu V_T'', \tag{B5}$$

where

$$V_T'' = V_T \begin{pmatrix} \omega & 0 & 0 \\ 0 & \omega^2 & 0 \\ 0 & 0 & 1 \end{pmatrix}. \tag{B6}$$

Then, we find that $V_{1j} = V_{2j}^*$ for $j = 1,2,3$, which implies that $|(V_{\rm SF})_{1j}| = |(V_{\rm SF})_{2j}|$ for $j = 1,2,3$. We find

$$s_{23}^2 = \frac{s_{13}^2}{c_{13}^2}, \quad \cos\delta = \frac{c_{23}}{s_{23}s_{13}} \cot 2\theta_{12}. \tag{B7}$$

Since s_{23}^2 should be very small to explain the CHOOZ data, this model cannot explain the atmospheric data.

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