OSp supergroup manifolds, superparticles, and supertwistors

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We construct simple twistor-like actions describing superparticles propagating on a coset superspace $OSp(1|4)/SO(1,3)$ (containing the $D=4$ anti–de Sitter space as a bosonic subspace), on a supergroup manifold $OSp(1|4)$ and, generically, on $OSp(1|2n)$. Making two different contractions of the superparticle model on the $OSp(1|4)$ supermanifold we get massless superparticles in Minkowski superspace without and with tensorial central charges. Using a suitable parametrization of $OSp(1|2n)$ we obtain even $Sp(2n)$-valued Cartan forms which are quadratic in Grassmann coordinates of $OSp(1|2n)$. This result may simplify the structure of brane actions in super–anti–de Sitter (AdS) backgrounds. For instance, the twistor-like actions constructed with the use of the even $OSp(1|2n)$ Cartan forms as supervielbeins are quadratic in fermionic variables. We also show that the free bosonic twistor particle action describes massless particles propagating in arbitrary space-times with a conformally flat metric, in particular, in Minkowski space and AdS space. Applications of these results to the theory of higher spin fields and to superbranes in AdS superbackgrounds are mentioned.

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I. INTRODUCTION

Conformal (super)symmetry plays an important role in the theory of fundamental interactions based on field-theoretical models as well as on the theory of fundamental extended objects (strings, etc.). Conformal geometrical structure allows one to replace space-time geometry by twistor geometry, where twistors are fundamental conformal spinors [SU(2,2) spinors for $D=4$] and space-time variables become twistor composites [1]. Such a construction allows for a supersymmetric extension [2] where superspace variables are replaced by primary supertwistor coordinates [$SU(2,2|N)$ supertwistors in the $D=4$ case].

In this paper we shall discuss twistors describing anti–de Sitter (AdS) geometry. The isometry of the AdS (super-spaces acts on the (super)AdS boundary as the group of (super)conformal transformations, and, therefore, provides the group-theoretical basis for the AdS/CFT (conformal field theory) correspondence conjecture [3] which attracted a great deal of attention over the last two years (see Ref. [4] for an exhaustive list of references). On the other hand the AdS space is the one where higher spin fields may nontrivially interact with each other [5]. In some aspects the technique developed for the description of the theory of higher spin fields in Minkowski [6] and AdS spaces [7] resembles the (super)twistor approach [8].

In this respect it is tempting to look for possible links between these different manifestations of conformal symmetry, AdS spaces, twistors, and their supersymmetric generalizations. Motivated by the problem of finding a simple form of the action for a superstring propagating in the $AdS_5\times S^5$ superbackground [9], in recent papers [10] a massive bosonic twistor particle model in an AdS_5 space has been proposed and its classical and quantum properties have been considered.[1] In Refs. [12,13] an $OSp(1|8)$ supertwistor model has been proposed for the description of a $D=4$ massless superparticle with the infinite spectrum of quantum states being described by fields of arbitrary integer and half integer spin. The helicity degrees of freedom of the superparticle have been found to be associated with the tensorial

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[1] The authors of Ref. [9] called supertwistors ''quarks.'' It should be mentioned that one of the present authors (J.L.) suggested a long time ago to relate supertwistors (with opposite grading) to quark degrees of freedom [11].

central charge $Z_{[mn]}$ $(m,n=0,1,2,3)$ which extends the $N=1$, $D=4$ super-Poincar'e algebra. One can show that OSp(1|8) is a superextension of the $D=4$ conformal group which contains as a subsupergroup the $N=1$, $D=4$ super-Poincaré algebra enlarged with $Z_{[mn]}$. It is natural to assume that the superparticle with tensorial central charges and the particle on the AdS space are different truncations of the dynamics of a superparticle propagating in the supergroup manifold of the isometries of the corresponding AdS superspace [13].

An aim of this paper is to construct such a model in the supergroup manifold OSp(1|4) and demonstrate how it is related to the $D=4$ twistor superparticle model of higher spins [12,13], and to a superparticle on the AdS_4 superspace OSp(1|4)/SO(1,3). Another motivation for this study has been to find a way of constructing simple worldvolume actions describing the dynamics of superbranes propagating in AdS superbackgrounds, i.e., to make a progress in solving a vital AdS/CFT correspondence problem [14–20].

Using a suitable parametrization of OSp(1|2n) we have found a simple form of the even OSp(1|2n) Cartan forms. They are only quadratic in Grassmann coordinates. This has allowed us to construct simple actions quadratic in fermions for superparticles propagating on OSp(1|4)/SO(1,3), OSp(1|4) and, generically, on OSp(1|2n).

The most interesting examples of the OSp(1|2n) supergroups seem to be OSp(1|32) and OSp(1|64). In Refs. [22–24][2] it has been shown that OSp(1|32) and OSp(1|64) contain the supergroup structures of $D=11$ M theory and $D=10$ superstrings. In particular, OSp(1|32) and OSp(1|64) are extensions of the supergroups SU(2,2|4), OSp(8|4), and OSp(2,6|4) which are isometries of, respectively, $AdS_5\times S^5$, $AdS_4\times S^7$, and $AdS_7\times S^5$ superspaces.[3] Reducing OSp(1|32) and OSp(1|64) down to the AdS supergroups one may hope to get simpler expressions for the Cartan forms of the latter, which might simplify the structure of actions for branes in corresponding AdS superbackgrounds [14–20].[4]

The plan of the paper is as follows. In Sec. II we review properties of the twistor formulation of bosonic particle mechanics and demonstrate that the single twistor particle action generically describes particles propagating in arbitrary space-times which admit a conformally flat metric, such as flat Minkowski space and the AdS space.

In Sec. III A we consider the supertwistor description of a massless superparticle in flat $N=1$, $D=4$ superspace, and in Sec. III B we construct a twistorlike action for the description of the dynamics of a superparticle in the super-AdS space OSp(1|4)/SO(1,3). The action has a simple quadratic form in fermions and, hence, it should not be hard to perform its quantization. However, in contrast to the bosonic AdS_4 superparticle we have not managed to find a complete supertwistor version of this model.

In Sec. IV we construct a twistorlike action for a superparticle on the supergroup manifold OSp(1|4). This action is also quadratic in fermions, and upon an appropriate truncation it reduces to the models of Sec. III.

In Sec. V we describe a superparticle propagating on OSp(1|2n) and show that it preserves $2n-1$ supersymmetries associated with Grassmann generators of OSp(1|2n).

The OSp(1|4) superalgebra and its Cartan forms required for the construction of the actions are given in the Appendix. In particular, we present a simple form of the super-AdS_4 supervielbeins and spin connection which are polynomials of only the second order in Grassmann coordinates. We also present Cartan forms of the supergroup OSp(1|2n) which can be made quadratic in Grassmann variables by an appropriate rescaling of the latter in the Appendix.

II. TWISTOR-LIKE BOSONIC PARTICLES

We start by recalling the form of an action for massless $D=4$ particles which serves as a dynamical basis for the transform from the space-time to the twistor description. The action is

$$S=\int d\tau\,\lambda^A(\sigma_m)_{A\dot A}\bar\lambda^{\dot A}\frac{d}{d\tau}x^m(\tau)=\int \lambda\sigma_m\bar\lambda dx^m(\tau), \tag{2.1}$$

where $x^m(\tau)(m=0,1,2,3)$ is a particle trajectory in $D=4$ Minkowski space, $\lambda^A(\tau)$ is a commuting two-component Weyl spinor, and $\sigma^m_{A\dot A}=\overline{\sigma}^m_{\dot A A}$ are the Pauli matrices.

From Eq. (2.1) we derive that the canonical conjugate momentum of $x^m(\tau)$ is

$$p_m=\lambda\sigma_m\bar\lambda\Rightarrow p_{A\dot A}=\frac{1}{2}p_m\sigma^m_{A\dot A}=\lambda_A\bar\lambda_{\dot A}, \tag{2.2}$$

whose square is identically zero since $\lambda^A\lambda^B\epsilon_{AB}\equiv 0$,[5] i.e.,

$$p_mp^m=0. \tag{2.3}$$

We therefore conclude that the particle is massless.

In the action (2.1) we can make the change of variables by introducing the second commuting spinor

$$\bar\mu_{\dot A}=i\lambda^A(\sigma_m)_{A\dot A}x^m\equiv i\lambda^A x_{A\dot A} \tag{2.4}$$

and its complex conjugate

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[2] An OSp(1|64)-invariant superparticlelike model has been discussed in Ref. [24] (see also Ref. [25] and references therein).

[3] We should remark that the notation Osp(6,2|4) is a somewhat confusing name for the AdS_7 quaternionic supergroup described, in a complex parametrization, by the intersection of two complex supergroups SU(4,4|4) and OSp(8|4;C), with bosonic sectors being, respectively, the spinorial covering of O(6,2) (space-time) and the spinorial covering Sp(2;H)=USp(4;C) of O(5) (the internal sector).

[4] The Cartan forms on supercosets of SU(2,2|N) relevant to the construction of brane actions on AdS superbackgrounds were calculated in the early 1980's [21].

[5] The two-component spinor indices are raised and lowered by the unit antisymmetric matrices $\epsilon^{AB}=\epsilon_{AB}$, $\epsilon^{\dot A\dot B}=\epsilon_{\dot A\dot B}$.

$$\mu_A = -i x_{A\dot{B}} \bar{\lambda}^{\dot{B}}. \tag{2.5}$$

The four-component spinors

$$Z_\alpha = (\lambda_A, \bar{\mu}^{\dot{A}}), \quad \bar{Z}^\alpha = (\mu^A, \bar{\lambda}_{\dot{A}}) \tag{2.6}$$

are called twistors.

In terms of Eq. (2.6) the action (2.1) takes the form

$$S = i \int d\tau [\bar{Z}^\alpha dZ_\alpha + l(\tau) \bar{Z}^\alpha Z_\alpha], \tag{2.7}$$

where we have added the second term with the Lagrange multiplier $l(\tau)$ which produces the constraint

$$\bar{Z}^\alpha Z_\alpha = 0. \tag{2.8}$$

This constraint implies that μ_A and $\bar{\mu}_{\dot{A}}$ are determined by Eqs. (2.4) and (2.5). But, as we shall see below, this flat space solution of the twistor constraint (2.8) is not the unique one. The AdS_4 space is also admissible, as well as any space with a conformally flat metric.

Passing from the action (2.1) to Eq. (2.7) we have performed the twistor transform, Eqs. (2.2), (2.4), and (2.5) being the basic twistor relations [1]. The action (2.7) is invariant under the conformal SU(2,2)∼SO(2,4) transformations, since the twistors are in the fundamental representation of the conformal group. The choice of twistor variables demonstrates how conformal symmetry appears in the theory of free massless particles.

We can generalize the action (2.1) to describe a massless particle propagating in a curved (gravitational) background. For this purpose we introduce the vierbein one-form $e^a(x) = dx^m e_m^a(x)$ with the index $a = 0,1,2,3$ corresponding to local SO(1,3) transformations in the tangent space of the background. Equation (2.1) takes the form

$$S = \int d\tau \lambda \sigma_a \bar{\lambda} e_m^a \partial_\tau x^m = \int \lambda \sigma_a \bar{\lambda} e^a. \tag{2.9}$$

Note that λ still transforms under a spinor representation of SO(1,3)∼SL(2,C).

In particular, one can consider an AdS_4 space as a background where the particle propagates. Let us mention that a different formulation of *massive* particle mechanics on AdS_4 has been considered in Ref. [26].

The AdS_4 *particle*. To consider a particle in the AdS_4 background we should specify the form of $e_m^a(x)$. A convenient choice of local coordinates and of the form of the metric on AdS_4 is

$$ds^2 = dx^m dx^n e_m^a e_n^b \eta_{ab} = \left(\frac{r}{R}\right)^2 dx^i \eta_{ij} dx^j + \left(\frac{R}{r}\right)^2 dr^2, \tag{2.10}$$

where $x^m = (x^i, r)(i = 0,1,2)$ are coordinates of the AdS_4, and R is the AdS_4 radius, whose inverse square is the constant AdS_4 curvature (or the cosmological constant). The coordinates x^i are associated with the three-dimensional boundary of AdS_4 when the radial coordinate r tends to infinity.

From Eq. (2.10) we find that the components of the vierbein one-form $e^a = dx^m e_m^a$ are

$$e^a = dx^m e_m^a = \left(\frac{r}{R}\right) \delta_i^a dx^i + \left(\frac{R}{r}\right) \delta_3^a dr. \tag{2.11}$$

Note that the coordinates x^i, r transform nonlinearly under the action of the AdS_4 isometry group SO(2,3) which is the conformal symmetry of the boundary x^i

$$\delta x^i = a_\Pi^i + a_M^{ij} x_j + a_D x^i + a_K^i x^j x_j - 2(x_j a_K^j) x^i + R^2 \frac{a_K^i}{r^2},$$
$$\delta r = (2 x_i a_K^i - a_D) r, \tag{2.12}$$

where a_Π^i, a_M^{ij}, a_D, and a_K^j are parameters of, respectively, $D=3$ translations, Lorentz rotations, dilatation, and conformal boosts, with Π_i, M_{ij}, D, and K_j being the corresponding generators of the SO(2,3) algebra $(i,j = 0,1,2)$ (see the Appendix).

We can now substitute Eq. (2.11) into Eq. (2.9). The action takes the form

$$S = \int d\tau \left[\left(\frac{r}{R}\right) \lambda \sigma_i \bar{\lambda} \dot{x}^i + \left(\frac{R}{r}\right) \lambda \sigma_3 \bar{\lambda} \dot{r} \right]. \tag{2.13}$$

Let us try to carry out the twistor transform of this action in a way similar to that considered above for the flat target space. To this end we redefine λ as

$$\hat{\lambda}_A = \left(\frac{r}{R}\right)^{1/2} \lambda_A. \tag{2.14}$$

The action (2.13) takes the form

$$S = \int d\tau \left[\hat{\lambda} \sigma_i \hat{\bar{\lambda}} \partial_\tau x^i + \left(\frac{R}{r}\right)^2 \hat{\lambda} \sigma_3 \hat{\bar{\lambda}} \partial_\tau r \right]. \tag{2.15}$$

In the limit $r \to \infty$ one obtains the twistor-like massless particle in three-dimensional Minkowski space, i.e., on the boundary of AdS_4. The complex Weyl spinor $\hat{\lambda}$ describes a pair of two-component real $D=3$ spinors which turn out to be proportional to each other on the mass shell. In the limit $r \to \infty$ the action (2.15) is still invariant under the $D=3$ conformal group $Sp(4) = O(3,2)/Z_2$ supplemented by O(2) rotations corresponding to the phase transformations of $\hat{\lambda}$.

In what follows we shall, however, keep r finite and make the change of variable

$$\hat{x}^3 = -\frac{R^2}{r}. \tag{2.16}$$

Then the action (2.15) formally becomes the same as Eq. (2.1) in the flat case

$$S = \int d\tau [\hat{\lambda} \sigma_i \hat{\bar{\lambda}} \partial_\tau x^i + \hat{\lambda} \sigma_3 \hat{\bar{\lambda}} \partial_\tau \hat{x}^3] = \int d\tau \hat{\lambda} \sigma_m \hat{\bar{\lambda}} \partial_\tau \hat{x}^m, \tag{2.17}$$

where $\hat{x}^m=(x^i,\hat{x}^3)$. The essential difference is that upon the redefinition (2.14) the SL(2,C) spinors $\hat{\lambda}$ transform *nonlinearly* under the action of the isometry group SO(2,3) via the radial coordinate r.

We can now make the twistor transform of the action (2.17) by introducing

$$\hat{\bar{\mu}}_{\dot{A}}=i\hat{\lambda}^A\hat{x}_{A\dot{A}} \tag{2.18}$$

and combining $\hat{\lambda}$ and $\hat{\bar{\mu}}$ into the twistor

$$\mathcal{Z}_\alpha=(\hat{\lambda}_A,\hat{\bar{\mu}}^{\dot{A}}). \tag{2.19}$$

The pure twistor form of the action (2.17) is the same as Eq. (2.9), and, hence, it is invariant under the group SU(2,2) $\sim$SO(2,4) of the conformal transformations acting *linearly* on the twistor (2.19). The twistor $\mathcal{Z}_\alpha$ is in the fundamental representation of SU(2,2).

As it has been explained in detail in [10] for the particle in AdS_5, the linear conformal SU(2,2) transformations of $\mathcal{Z}_\alpha$ induce nonlinear transformations of the AdS coordinates x^i and r when the twistor components are related to x^i and r through Eqs. (2.14), (2.16), and (2.18)

$$\mathcal{Z}_\alpha=\left(\frac{r}{R}\right)^{1/2}\left(\lambda_A,-ix^i\sigma_i^{\dot{A}B}\lambda_B+i\frac{R^2}{r}\sigma_3^{\dot{A}B}\lambda_B\right). \tag{2.20}$$

In the case under consideration we thus find the nonlinear conformal SO(2,4) transformations of the AdS_4 space coordinates, with the isometry group SO(2,3) [see Eq. (2.12)] being a subgroup of the conformal group. The conformal transformations of $\hat{x}^m=x^i,\hat{x}^3$ [where $\hat{x}^3$ was defined in Eq. (2.16)] are similar to the conformal transformations of the Minkowski space coordinates. They are

$$\delta\hat{x}^m=a_\Pi^m+a_M^{mn}\hat{x}_n+a_D\hat{x}^m+a_K^m\hat{x}^n\hat{x}_n-2(\hat{x}_na_K^n)\hat{x}^m, \tag{2.21}$$

where a_Π^m, a_M^{mn}, a_D, and a_K^m are parameters of, respectively, $D=4$ translations, Lorentz rotations, dilatation, and conformal boosts, with Π_m, M_{mn}, D, and K_m the corresponding generators of the SO(2,4) algebra ($m,n=0,1,2,3$) (see the Appendix).

Substituting the expression (2.16) for $\hat{x}^3$ into Eq. (2.21), one can deduce the explicit form of the conformal transformations of the coordinate r. Then the SO(2,3) isometry transformations (2.12) of the AdS_4 coordinates are obtained by putting to zero all parameters in Eq. (2.21) which carry the index 3, the remaining ones being a_D and all those with three-dimensional indices $i,j=0,1,2$.

The observation that the AdS spaces are conformally transformed according to Eq. (2.21) implies that these manifolds are conformally flat. For instance, the AdS_4 metric (2.10) becomes conformally flat upon the redefinition of the coordinate r just as in Eq. (2.16) (which we made to perform the twistor transform)

$$ds^2=\left(\frac{R}{\hat{x}^3}\right)^2[dx^i\eta_{ij}dx^j+(d\hat{x}^3)^2]=\left(\frac{R}{\hat{x}^3}\right)^2d\hat{x}^md\hat{x}^n\eta_{mn}. \tag{2.22}$$

We have thus shown that the twistor constraint (2.8) has two solutions which correspond to the twistor transform of the flat $D=4$ Minkowski space and of AdS_4, both spaces being conformally flat. This observation allows us to conclude that any other space whose metric is conformally flat should also arise as a corresponding solution of the twistor constraint (2.8). We now turn to the supersymmetrization of the action (2.9).

III. TWISTOR-LIKE $N=1$, $D=4$ SUPERPARTICLES

The form of the action (2.9) is suitable for a straightforward supersymmetric generalization. To this end we should consider e^a as a vector component of the supervielbein one form

$$e^I(z)=dz^Me_M^{\ I}=(e^a,e^A,\overline{e^{\dot{A}}}), \tag{3.1}$$

where $z^M=(x^m,\theta^A,\overline{\theta^{\dot{A}}})$ are coordinates which parametrize a target superspace in which the particle propagates. θ^A and its complex conjugate $\overline{\theta^{\dot{A}}}$ are Grassmann-odd Weyl spinor coordinates.

A. A superparticle in flat superspace

In the case of flat target superspace

$$e^a=dx^a-i\theta\sigma^ad\bar{\theta}+i\,d\theta\,\sigma^a\bar{\theta},\quad e^A=d\theta^A,\quad \overline{e^{\dot{A}}}=d\overline{\theta^{\dot{A}}}. \tag{3.2}$$

Substituting e^a from Eq. (3.2) into the action (2.9) we can transform it into the pure supertwistor action by introducing the supertwistor [2]

$$Z_{\mathcal{A}}=(\lambda_A,\overline{\mu^{\dot{A}}},\chi) \tag{3.3}$$

and its conjugate

$$\overline{Z^{\mathcal{A}}}=(\mu^A,\overline{\lambda_{\dot{A}}},\bar{\chi}), \tag{3.4}$$

where now

$$\overline{\mu_{\dot{A}}}=i\lambda^A(x_{A\dot{A}}-i\theta_A\overline{\theta_{\dot{A}}}) \tag{3.5}$$

and

$$\chi=\theta^A\lambda_A,\quad \bar{\chi}=\overline{\theta^{\dot{A}}}\overline{\lambda_{\dot{A}}}. \tag{3.6}$$

Upon supertwistorization the action (2.9) with the supervielbein (3.2) takes the form similar to Eq. (2.7)

$$S=i\int d\tau[\overline{Z^{\mathcal{A}}}dZ_{\mathcal{A}}+l(\tau)\overline{Z^{\mathcal{A}}}Z_{\mathcal{A}}], \tag{3.7}$$

where now the supertwistor constraint is

$$\overline{Z^{\mathcal{A}}}Z_{\mathcal{A}}=\mu^A\lambda_A-\overline{\mu_{\dot{A}}}\overline{\lambda^{\dot{A}}}+2\bar{\chi}\chi=0. \tag{3.8}$$

For further details on twistor superparticles in flat superspace we refer the reader to the papers [2,27–33] and proceed with constructing an action for a superparticle propagating in the coset superspace OSp(1|4)/SO(1,3) whose bosonic subspace is AdS_4.

B. The superparticle on OSp(1|4)/SO(1,3)

To get an explicit form of the particle action on OSp(1|4)/SO(1,3) we should know an explicit form of the supervielbein (3.1), which is part of the Cartan forms on OSp(1|4). The components of the latter can be computed using the method of nonlinear realizations [34–37].

The Cartan forms of the supergroup OSp(1|4) and corresponding Cartan forms for the supercoset OSp(1|4)/SO(1,3) were calculated in [38–41]. Below we present simpler expressions for the Cartan forms which allow to write down a simple form of the AdS superparticle action, since our choice of the parametrization of the supercoset differs from that used in Refs. [39,41].

To derive the Cartan forms on OSp(1|4)/SO(1,3) we take the supercoset element in the form

$$K(z^M)=B(x)F(\theta)=B(x^m)e^{i(\theta^A Q_A+\bar{\theta}_{\dot{A}}\bar{Q}^{\dot{A}})}, \qquad (3.9)$$

where $B(x^m)$ is the purely bosonic matrix taking its values in the coset SO(2,3)/SO(1,3), i.e., it is associated with the bosonic AdS_4 space locally parametrized by coordinates x^m. The Grassmann coordinates θ^A and $\bar{\theta}^{\dot{A}}$ extend AdS_4 to the coset superspace, and Q_A and $\bar{Q}_{\dot{A}}$ are the odd generators of OSp(1|4) (see the Appendix).

The Cartan form on OSp(1|4)/SO(1,3) is

$$\frac{1}{i}K^{-1}dK=E^a(z)P_a+E^A(z)Q_A+Q_{\dot{A}}\bar{E}^{\dot{A}}(z)+\Omega^{ab}(z)M_{ab}. \qquad (3.10)$$

It takes values in the OSp(1|4) superalgebra.

The one-forms $E^I=(E^a,E^A,\bar{E}^{\dot{A}})$ are the supervielbeins and Ω^{ab} is the SO(1,3) connection on the coset superspace. In the representation (3.9) the Cartan form (3.10) is

$$K^{-1}dK=F^{-1}(B^{-1}dB)F+F^{-1}dF\equiv F^{-1}\mathcal{D}F, \qquad (3.11)$$

where the purely bosonic Cartan form $B^{-1}dB$ takes values in the SO(2,3) algebra and describes a vierbein $e^a(x)$ and a spin connection $\omega^{ab}(x)$ on the bosonic AdS_4 space

$$\frac{1}{i}B^{-1}dB=e^aP_a+\omega^{ab}M_{ab}. \qquad (3.12)$$

Depending on the choice of $B(x)$ one can get different forms of $e^a(x)$ and $\omega^{ab}(x)$. For instance, the coset element $B(x)$ can be chosen in such a way that $e^a(x)$ in Eq. (3.12) is the same as in Eq. (2.11) and the connection $\omega^{ab}(x)$ is

$$\omega^{i3}=\frac{1}{R}e^i, \quad \omega^{ij}=0 \qquad (3.13)$$

(remember that the index 3 corresponds to the radial coordinate r of AdS_4).

We can now substitute Eq. (3.12) into Eq. (3.11) and calculate the explicit form of the supervielbeins E^I and the superconnection Ω^{ab}, using a trick, described, for example, in Ref. [14], or by the method presented in the Appendix. In the Majorana spinor representation the expressions for the Cartan forms are given in the Appendix. In the two-component spinor formalism the supervielbein form (A18) or (A21) of the Appendix can be written as follows:

$$E^a=P(\theta^2,\bar{\theta}^2)[e^a(x)-i\Theta\sigma^a D\bar{\Theta}+iD\Theta\sigma^a\bar{\Theta}], \qquad (3.14)$$

where

$$P(\theta^2,\bar{\theta}^2)=1-\frac{i}{2R}(\theta^2-\bar{\theta}^2)+\frac{1}{3!R^2}\theta^2\bar{\theta}^2, \qquad (3.15)$$

$$\theta^2\equiv\theta^A\theta_A, \quad \bar{\theta}^2\equiv\bar{\theta}_{\dot{A}}\bar{\theta}^{\dot{A}},$$

$$\Theta=\left(\frac{1+\frac{i}{3!R}(\theta^2-\bar{\theta}^2)}{P(\theta^2,\bar{\theta}^2)}\right)^{1/2}\theta, \qquad (3.16)$$

$$D=d+\omega^{bc}\sigma_{bc}, \quad \sigma^{ab}=\frac{1}{4}(\sigma^a\bar{\sigma}^b-\sigma^b\bar{\sigma}^a).$$

To get the action for the superparticle in the super-AdS background we should simply substitute Eq. (3.14) into Eq. (2.9).

$$S=\int \lambda\sigma_a\bar{\lambda}P(\theta^2,\bar{\theta}^2)[e^a-i\Theta\sigma^a D\bar{\Theta}+iD\Theta\sigma^a\bar{\Theta}]. \qquad (3.17)$$

The polynomial $P(\theta^2,\bar{\theta}^2)$ can be absorbed by properly rescaled λ and $\bar{\lambda}$, namely, $\Lambda=\sqrt{P(\theta^2,\bar{\theta}^2)}\lambda$. Then the action takes an even simpler form which is quadratic in fermions

$$\begin{aligned}S&=\int \Lambda\sigma_a\bar{\Lambda}[e^a-i\Theta\sigma^a D\bar{\Theta}+iD\Theta\sigma^a\bar{\Theta}]\\&=\int \Lambda\sigma_a\bar{\Lambda}[e^a-i\Theta\sigma^a d\bar{\Theta}+id\Theta\sigma^a\bar{\Theta}\\&\quad+i\omega^{bc}(x)\Theta(\sigma^a\bar{\sigma}_{bc}+\sigma_{bc}\bar{\sigma}^a)\bar{\Theta}].\end{aligned} \qquad (3.18)$$

If in Eq. (3.18) there were no terms containing the spin connection ω^{bc} the action (3.18) could be completely supertwistorized in the same way as we have done in the case of the AdS_4 particle and of the superparticle in flat superspace. However, the term with ω^{bc} does not allow one to perform the complete supertwistorization of Eq. (3.18) in terms of *free* supertwistors, at least in a straightforward way.

Using the notion of Killing spinors on AdS spaces one can replace in Eq. (3.18) the covariant differential D with the ordinary one. To this end it is convenient to switch to the four-component Majorana spinor formalism

$$\Lambda^{\alpha}=(\lambda_A,\bar{\lambda}^{\dot{A}}),\quad \Theta^{\alpha}=(\Theta_A,\bar{\Theta}^{\dot{A}}).$$

By definition (see, for instance, Ref. [42]) AdS Killing spinors satisfy the condition

$$\mathcal{D}K^{\alpha}{}_{\beta}C^{\beta}=\left(DK^{\alpha}{}_{\beta}+\frac{1}{2R}(e^a\gamma_a)^{\alpha}{}_{\gamma}K^{\gamma}{}_{\beta}\right)C^{\beta}$$
$$=\left(dK^{\alpha}{}_{\beta}+\frac{1}{2}(\omega^{ab}\gamma_{ab})^{\alpha}{}_{\gamma}K^{\gamma}{}_{\beta}+\frac{1}{2R}(e^a\gamma_a)^{\alpha}{}_{\gamma}K^{\gamma}{}_{\beta}\right)C^{\beta}=0, \tag{3.19}$$

where $K^{\alpha}{}_{\beta}(x)$ is a bosonic Killing spinor matrix and C^{β} is an arbitrary constant spinor. If in Eq. (3.20) we replace Θ with $\Theta=K(x)\Theta_K$ (where $\Theta_K\equiv K^{-1}\Theta$) [15] the action (3.18) takes the form

$$S=\frac{1}{2}\int\bar{\Lambda}\gamma_a\Lambda\left[e^a\left(1+\frac{i}{2R}\bar{\Theta}\Theta\right)-i\bar{\Theta}\gamma^aK\,d\Theta_K\right]$$

or (upon an appropriate rescaling of Λ and Θ)

$$S=\frac{1}{2}\int\bar{\Lambda}\gamma_a\Lambda[e^a-i\bar{\Theta}\gamma^aK\,d\Theta_K]. \tag{3.20}$$

Note that in Eq. (3.20) the variable Θ_K is regarded as independent, while $\Theta=K(x)\Theta_K$ is composed from Θ_K and the Killing matrix $K(x)$ whose exact dependence on the AdS_4 coordinates x^m can be found by solving the Killing spinor equation (3.19) [42]. Taking this into account, the term $i\bar{\Theta}\gamma^aK\,d\Theta_K$ in Eq. (3.17) can be rewritten as $i\bar{\Theta}_K\gamma^{\hat{b}\hat{c}}d\Theta_KK^a_{\hat{b}\hat{c}}(x)$, where $K^a_{\hat{b}\hat{c}}(x)$ are SO(2,3) Killing vectors on AdS_4 $(\hat{b},\hat{c}=0,1,2,3,4,\ \gamma^4=\mathbf{1})$. Then the supervielbein

$$E^a=e^a-i\bar{\Theta}_K\gamma^{\hat{b}\hat{c}}d\Theta_KK^a_{\hat{b}\hat{c}} \tag{3.21}$$

takes the form similar to one of the parametrizations considered in Ref. [39].

It would be interesting to understand whether the AdS superparticle action in any of its forms can be completely supertwistorized, i.e., written in the form (3.7), using an appropriate choice of AdS supercoordinates. If it is possible, then the AdS superparticle model would acquire the manifest superconformal SU(2,2|1) symmetry. In any case, the use of commuting spinors, whose bilinears replace the conventional particle momentum, and the suitable choice of the parametrization of the supercoset space OSp(1|4)/SO(1,3) have allowed us to get a simple form of the action for a superparticle propagating in the AdS superbackground, which is bilinear in fermionic variables.

IV. THE SUPERPARTICLE ON OSp(1|4)

We now turn to the construction of the action for a superparticle propagating on the supergroup manifold OSp(1|4) locally parametrized by the supercoset OSp(1|4)/SO(1,3) coordinates x^m and θ, and by six SO(1,3) group coordinates $y^{mn}=-y^{mn}$. This model is intended to produce, upon an appropriate contraction, the superparticles in flat superspace and on super-AdS_4 considered above, as well as the superparticle with tensorial central charges [12,13].

By analogy with Eqs. (2.9) and (3.17), to construct the OSp(1|4) superparticle Lagrangian we take the pullback onto the particle world line of the even Cartan superforms E^a_{OSp} and $\Omega^{ab}_{\mathrm{OSp}}$ given in the Appendix [Eq. (A8)]. These forms comprise the bosonic SO(2,3) part of the supervielbein on OSp(1|4). We contract them with commuting spinor bilinears $\bar{\lambda}\gamma_a\lambda$ and $\bar{\lambda}\gamma_{ab}\lambda$. The OSp(1|4) superparticle action is

$$S_{\mathrm{OSp}}=\frac{1}{2}\int\{E^b(x,\theta)u_b{}^a(y)\bar{\lambda}\gamma_a\lambda+[\Omega^{cd}(x,\theta)u_c{}^au_d{}^a+(u^{-1}du)^{ab}]\bar{\lambda}\gamma_{ab}\lambda\}. \tag{4.1}$$

Using the defining relations for the SO(1,3) matrices $u_b{}^a$ and $v_{\beta}{}^{\alpha}$ (A10) we can make the redefinition

$$u_b{}^a(y)\bar{\lambda}\gamma_a\lambda=\bar{\hat{\lambda}}\gamma_b\hat{\lambda},\quad\text{where}\quad\hat{\lambda}^{\alpha}=\lambda^{\beta}v_{\beta}{}^{\alpha}. \tag{4.2}$$

Then $u_b{}^a(y)$ remains only in one term of the action (4.1), and the latter takes the form

$$S_{\mathrm{OSp}}=\frac{1}{2}\int E^a(x,\theta)\bar{\hat{\lambda}}\gamma_a\hat{\lambda}+\frac{1}{2}\int[\Omega^{ab}(x,\theta)+(duu^{-1})^{ab}]\bar{\hat{\lambda}}\gamma_{ab}\hat{\lambda}. \tag{4.3}$$

We observe that the first integral in Eq. (4.3) is nothing but the action (3.17) for the superparticle on the coset superspace OSp(1|4)/SO(1,3), and the second term contains the spin connection of OSp(1|4)/SO(1,3) extended by the SO(1,3) Cartan form duu^{-1}. In Eq. (4.3) the dependence of the action on the SO(1,3) group manifold coordinates y^{mn} remains only in duu^{-1}.

Since by an appropriate choice of Grassmann coordinates the Cartan forms $E^a(x,\theta)$ and $\Omega^{ab}(x,\theta)$ can be made quadratic in θ [see Eqs. (A21) and (A22)] we see that the OSp(1|4) action (4.3) is *quadratic* in fermions. If we drop the second integral of Eq. (4.3) we get the action for the superparticle considered in Sec. III B, and if we then take the limit when the AdS_4 radius goes to infinity, the action further reduces to the superparticle action in flat $N=1$, $D=4$ superspace. Another way of truncating the action (4.3) is to perform the following contraction of the OSp(1|4) superalgebra (A1). Let us in Eq. (A1) rescale the generators M_{ab} of SO(1,3) as

$$M_{ab}=RZ_{ab}, \tag{4.4}$$

and consider the limit $R\to\infty$ of the algebraic relations (A1)–(A4). Then the generators Z_{ab} become tensorial central charges which commute with all other generators, and the anticommutator of the supercharges becomes

$$\{Q_\alpha,Q_\beta\}=-2(C\gamma^a)_{\alpha\beta}P_a+(C\gamma^{ab})_{\alpha\beta}Z_{ab}. \tag{4.5}$$

The SO(1,3) coordinates y^{mn} become central charge coordinates.

In the limit $R\to\infty$ the supervielbein $E^a(x,\theta)$ reduces to the ''flat'' one-form (3.2)

$$E_Z^a=dx^a-i\bar{\theta}\gamma^a d\theta \tag{4.6}$$

and the superconnection $\Omega_Z^{ab}=R[\Omega^{ab}+(duu^{-1})^{ab}]$ becomes

$$\Omega_Z^{ab}=dy^{ab}+\frac{i}{2}\bar{\theta}\gamma^{ab}d\theta. \tag{4.7}$$

Substituting Eqs. (4.6) and (4.7) into Eq. (4.3) we get the action for a particle with tensorial central charges [12,13]. The quantization of this superparticle model has shown to produce an infinite tower of free massless states with arbitrary integer and half integer spin, with the spin degrees of freedom associated with the central charge coordinates y^{mn}. For a detailed analysis of the model we refer the reader to Refs. [12,13].

Since the higher-spin fields can interact if they do not live in Minkowski space but in an anti–de Sitter space [5], it seems of interest to study the possibility of generalizing the OSp(1|4) superparticle model based on the action (4.3) to include interactions, and then to perform its quantization to check whether such a model can be considered as a classical counterpart of the theory of interacting higher-spin fields.

To conclude this section we demonstrate that OSp(1|4) covariant momenta associated with the OSp(1|4) coordinates x^m, y^{mn}, and θ^α generate the OSp(1|4) superalgebra.[6] Let us rewrite the action (4.3) as

$$S=\frac{1}{2}\int d\tau\,\bar{\hat{\lambda}}\Gamma_I\hat{\lambda}E_M^I(x,\theta,z)\partial_\tau X^M, \tag{4.8}$$

where Γ_I are

$$\Gamma_I=(\gamma_a,\ \ \gamma_{ab}), \tag{4.9}$$

the index I stands for vector a and tensor ab indices, and $X^M\equiv(x^m,z^{mn},\theta^\alpha)$. $E_M^I(x,\theta,z)$ are the OSp(1|4) Cartan form components E^a and Ω^{ab}, which correspond to the bosonic generators P_a and M_{ab} of OSp(1|4) (see the Appendix).

[6]In Ref. [45] similar covariant momenta were used to make the Hamiltonian analysis and the quantization of superparticles propagating in harmonic superspaces.

From Eq. (4.8) we get the canonical momenta conjugate to $X^M\equiv(x^m,z^{mn},\theta^\alpha)$ as

$$\frac{\delta S}{\delta(\partial_\tau X^M)}=P_M=\frac{1}{2}\bar{\hat{\lambda}}\Gamma_I\hat{\lambda}E_M^I. \tag{4.10}$$

Multiplying Eq. (4.10) by the matrix $E_{\hat{I}}^M$ inverse to $E_M^{\hat{I}}$ [where $\hat{I}=(I,\alpha)$] we obtain OSp(1|4) covariant momenta $\mathcal{P}_{\hat{I}}=E_{\hat{I}}^M P_M=(\mathcal{P}_I,\mathcal{P}_\alpha)$ such that

$$\frac{1}{2}\bar{\hat{\lambda}}\Gamma_I\hat{\lambda}=\mathcal{P}_I\equiv E_I^M(X)P_M,\quad \mathcal{P}_\alpha=E_\alpha^M(X)P_M=0. \tag{4.11}$$

Equations (4.10) and (4.11) imply that the expressions for the momenta are constraints on the superparticle phase space variables. For instance, the covariant momentum components $\mathcal{P}_\alpha$ of the Grassmann variable θ^α are zero. These are Grassmann constraints on the dynamics of the OSp(1|4) superparticle, which include first-class constraints generating the κ symmetry of the OSp(1|4) superparticle.

It is well known that, as it occurs for $N=1$, $D=4$ superparticles in an arbitrary supergravity background, the AdS superparticle possesses two-parameter local fermionic κ symmetry, which means that such superparticles preserve half the supersymmetry of a target-space vacuum. In contrast to this, as we shall prove in the next section, the OSp(1|4) superparticle possesses three κ symmetries and, in general, the superparticle propagating on the OSp(1|2n) supergroup manifold has $(2n-1)$ κ symmetries and thus describes Bogomol'nyi-Prasad-Sommerfield (BPS) states with only one broken supersymmetry.

In Ref. [12] the superparticle models with such a symmetry property have been obtained in flat superspaces with additional tensorial central charge coordinates. Here we observe that this unusual feature is also inherent to superparticles propagating in more complicated superspaces.

Because of the Maurer-Cartan equations $(dE-iE\wedge E=0)$ for the Cartan forms $E_M^{\hat{I}}$ the generalized momenta form, under the Poisson brackets, the OSp(1|4) superalgebra, which can be quantized by taking an appropriate ordering of X and P in the definition of Eq. (4.11):

$$[\mathcal{P}_{\hat{I}},\mathcal{P}_{\hat{J}}\}=C_{\hat{I}\hat{J}}{}^{\hat{K}}\mathcal{P}_{\hat{K}}, \tag{4.12}$$

where $C_{\hat{I}\hat{J}}{}^{\hat{K}}$ are OSp(1|4) superalgebra structure constants [see Eq. (A1)].

From Eqs. (4.11) and (4.12) we see that upon the transition to Dirac brackets the bilinears of $\hat{\lambda}_\alpha$ (4.11) become generators of the Sp(4)$\sim$SO(2,3) subalgebra of OSp(1|4), which implies that $\hat{\lambda}_\alpha$ will not commute with respect to the Dirac brackets

$$[\lambda_\alpha,\lambda_\beta]_D=\frac{1}{2R}C_{\alpha\beta}.$$

Note that at $R\to\infty$ λ_α become commuting variables, which correspond to the twistorlike variables of the superparticle model with tensorial central charges [12,13].

From this analysis we conclude that the commutation properties of the superparticle covariant momenta reflect the structure of the global symmetries of the OSp(1|4) superparticle action. To quantize the model one should consider the OSp(1|4) coordinates and momenta as ''generalized'' canonical variables, with graded commutation relations defined by the OSp(1|4) superalgebra (4.12). The detailed study of the model based on the action (4.3) is in progress.

V. THE SUPERPARTICLE ON OSp(1|2N) AS A DYNAMICAL MODEL FOR EXOTIC BPS STATES

We now generalize the OSp(1|4) superparticle action (4.1), (4.3) or (4.8) to the case of the supermanifold OSp(1|2n) whose parametrization we choose to be of the form [see the Appendix for the details on the OSp(1|2n) superalgebra]

$$\mathcal{G}(y,\theta)=B(y)F(\theta)=B(y)e^{i\theta^\alpha Q_\alpha}, \tag{5.1}$$

where $y^{\alpha\beta}=y^{\beta\alpha}$ are coordinates of the Sp(2n) subgroup generated by symmetric operators $M_{\alpha\beta}=M_{\beta\alpha}$, and whose element is denoted as $B(y)$; θ^α are Grassmann coordinates and Q_α are Grassmann generators of OSp(1|2n) transforming under the fundamental representation of Sp(2n), which we call the spinor representation ($\alpha,\beta=1,\dots,2n$).

The OSp(1|2n) Cartan forms are

$$\frac{1}{i}\mathcal{G}^{-1}(y,\theta)d\mathcal{G}(y,\theta)\equiv\frac{1}{i}[F^{-1}(B^{-1}dB)F+F^{-1}dF]$$
$$\equiv F^{-1}\mathcal{D}F=E^\alpha Q_\alpha+\frac{1}{2}\Omega^{\alpha\beta}M_{\alpha\beta}. \tag{5.2}$$

To have the connection with the OSp(1|4) case discussed in Sec. IV and the Appendix we note that for $n=2$ $M_{\alpha\beta}$ can be written in terms of SO(1,3) covariant generators P_a and M_{ab} as follows:

$$M_{\alpha\beta}=-2(C\gamma^a)_{\alpha\beta}P_a+\frac{1}{R}(C\gamma^{ab})_{\alpha\beta}M_{ab}. \tag{5.3}$$

Then the OSp(1|4) Cartan forms presented in Eq. (A8) are related to $\Omega^{\alpha\beta}$ in Eq. (5.2) in the following way:

$$E^a_{\rm OSp}=-(C\gamma^a)_{\alpha\beta}\Omega^{\alpha\beta},\quad \Omega^{ab}_{\rm OSp}=\frac{1}{2R}(C\gamma^{ab})_{\alpha\beta}\Omega^{\alpha\beta}. \tag{5.4}$$

The matrix $C_{\alpha\beta}$ plays the role of the Sp(2n) invariant metric.

The OSp(1|2n) Cartan forms (5.2) computed in the Appendix have the form

$$E^\alpha=\mathcal{D}\theta^\alpha+i\mathcal{D}\theta^{(\alpha}\theta^{\beta)}\theta_\beta P_1(\theta\theta), \tag{5.5}$$

$$\Omega^{\alpha\beta}=\omega^{\alpha\beta}(y)+i\theta^{(\alpha}\mathcal{D}\theta^{\beta)}P_2(\theta\theta), \tag{5.6}$$

where $\omega^{\alpha\beta}(y)$ are Sp(2n) Cartan forms, $P_1(\theta\theta)$ and $P_2(\theta\theta)$ are polynomials in $\theta^\alpha C_{\alpha\beta}\theta^\beta$ [see Eqs. (A40) and (A41)], and $\mathcal{D}$ is the Sp(2n) covariant derivative

$$\mathcal{D}\theta^\alpha=d\theta^\alpha+\frac{\alpha}{2}\omega^\alpha{}_\beta(y)\theta^\beta, \tag{5.7}$$

where α is a dimensional constant factor in the OSp(1|2n) superalgebra (A23), which in the OSp(1|4) case (A1) is $\alpha=4/R$.

The form of Eq. (5.6) prompts us that the polynomial P_2 can be hidden into rescaled $\Theta=\sqrt{P_2}\theta$, then for $\Omega^{\alpha\beta}$ we get the simple expression

$$\Omega^{\alpha\beta}=\omega^{\alpha\beta}(y)+i\Theta^{(\alpha}\mathcal{D}\Theta^{\beta)}. \tag{5.8}$$

The action for a superparticle moving on OSp(1|2n), which generalizes Eq. (4.3), has the form

$$S=\frac{1}{2}\int\lambda_\alpha\lambda_\beta\Omega^{\alpha\beta}\equiv\frac{1}{2}\int d\tau\lambda_\alpha\lambda_\beta\Omega^{\alpha\beta}_\tau, \tag{5.9}$$

where λ_α is an auxiliary bosonic Sp(2n) ''spinor'' variable, and $\Omega^{\alpha\beta}=d\tau\Omega^{\alpha\beta}_\tau$ is the pullback of the even Cartan form (5.6) or (5.8) on the superparticle world line.

Let us now analyze the κ-symmetry properties of the action (5.9) by considering its general variation. A simple way to vary the action (5.9) with respect to OSp(1|2n) coordinates $X^M=(y^{\alpha\beta},\theta^\alpha)$ and the auxiliary variable λ, is to use Maurer-Cartan equations [integrability conditions for Eq. (5.2)] $d(\mathcal{G}^{-1}d\mathcal{G})=\mathcal{G}^{-1}d\mathcal{G}\wedge\mathcal{G}^{-1}d\mathcal{G}$ which imply

$$dE^\alpha+\frac{\alpha}{2}E^\beta\wedge\Omega_\beta{}^\alpha=0, \tag{5.10}$$

$$d\Omega^{\alpha\beta}+\frac{\alpha}{2}\Omega^{\alpha\gamma}\wedge\Omega_\gamma{}^\beta=-iE^\alpha\wedge E^\beta, \tag{5.11}$$

and the expression for the X^M variation of the differential forms

$$\delta\Omega=i_\delta d\Omega+di_\delta\Omega\qquad i_\delta\Omega\equiv\delta X^M\Omega_M. \tag{5.12}$$

Modulo a boundary term the variation of the action (5.9) obtained in this way takes the form

$$\delta S=\int\delta\lambda_\alpha\Omega^{\alpha\beta}\lambda_\beta-\int_{\mathcal{M}^1}\mathcal{D}\lambda_\alpha i_\delta\Omega^{\alpha\beta}\lambda_\beta$$
$$-\frac{i}{2}\int(E^\alpha\lambda_\alpha)(i_\delta E^\beta)\lambda_\beta, \tag{5.13}$$

where the basis in the space of variations is chosen to be $i_\delta\Omega^{\alpha\beta}$ and $i_\delta E^\alpha$ instead of more conventional $\delta y^{\alpha\beta}$ and $\delta\theta^\alpha$.

Note that $i_\delta E^\alpha$ corresponds to the variation of the action with respect to Grassmann coordinates θ^α. Putting $\delta\lambda_\alpha=0$, $i_\delta\Omega^{\alpha\beta}=0$ we thus observe that only one of the $2n$

linearly independent fermionic variations, namely, $i_\delta E^\alpha \lambda_\alpha$, effects the variation of the action. This implies that other $2n-1$ fermionic variations are fermionic κ-symmetries of the dynamical system described by the action (5.9). The κ-symmetry transformations are defined in such a way that $i_\delta E^\alpha \lambda_\alpha$ vanishes (see Refs. [12,13])

$$i_\delta \Omega^{\alpha\beta}=0, \quad \delta\lambda_\alpha=0, \quad i_\delta E^\alpha=\kappa^I \mu_I^\alpha, \quad I=1,\ldots,(2n-1), \tag{5.14}$$

where the μ_I^α are $2n-1$ Sp($2n$) spinors orthogonal to λ_α

$$\mu_I^\alpha \lambda_\alpha=0, \quad I=1,\ldots,(2n-1). \tag{5.15}$$

Thus, we conclude that an unusual property of a twistor-like superparticle with tensorial central charge coordinates [12] to preserve all but one target-space supersymmetries is inherent to the superparticle model on the OSp($1|2n$) supergroup manifold as well.

When the explicit expressions (5.5) and (5.6) for the Cartan forms on OSp($1|2n$) are obtained, one straightforwardly gets the explicit expressions also for the Cartan forms on any coset superspace OSp($1|2n$)/H, where H is a bosonic subgroup of OSp($1|2n$). These expressions are the same as Eqs. (5.5) and (5.6) but with $\omega^{\alpha\beta}$ depending only on the bosonic coordinates of the supercoset [see also Eqs. (A43) and (A44)]. Using the OSp($1|2n$)/H Cartan forms one can construct various types of actions for superparticles and superbranes propagating on the corresponding coset supermanifolds.

VI. CONCLUSION

By taking a suitable parametrization of the supergroup manifold OSp($1|2n$) we have found a simple form of the OSp($1|2n$) Cartan superforms such that the ones which take values in the bosonic subalgebra Sp($2n$) of OSp($1|2n$) are quadratic in Grassmann coordinates. We have used these Cartan forms to construct simple twistorlike actions (which are quadratic in fermions) for describing superparticles propagating on the coset superspace OSp(1|4)/SO(1,3), on the supergroup manifold OSp(1|4), and, in general, on OSp($1|2n$) supermanifolds. The OSp(1|4) superparticle model has been shown to produce (upon a truncation) either the standard massless $D=4$ superparticle or the generalized massless $D=4$ superparticle with tensorial central charges [12,13] whose quantization gives rise to massless free fields of arbitrary (half)integer spin.

We have also shown that the massless particle on AdS_4 $=$SO(2,3)/SO(1,3) can be described (with a particular choice of twistor variables) as a free $D=4$ twistor particle. A direction of further study can be to analyze the OSp(1|4) superparticle model in detail and to look for its role as a classical counterpart in the theory of interacting higher-spin fields [5–8] requiring a finite AdS radius.

Another interesting problem is to generalize the results of this paper to the case of superstrings and superbranes propagating in AdS superbackgrounds with the aim to find a simple form of superbrane actions on AdS. The simple fermionic structure of OSp(1|32) and OSp(1|64) Cartan forms, which we obtained, may be helpful in making a progress in this direction.

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APPENDIX

We use the ''almost plus'' signature $(-,+,\ldots,+)$ of the Minkowski metric η^{ab} $(a,b=0,1,2,3)$.

The OSp(1|4) *superalgebra.*

$$-i[M_{ab},M_{cd}]=\eta_{ad}M_{bc}+\eta_{bc}M_{ad}-\eta_{ac}M_{bd}-\eta_{bd}M_{ac}, \tag{A1}$$

$$-i[M_{ab},P_c]=\eta_{bc}P_a-\eta_{ac}P_b, \tag{A2}$$

$$[P_a,P_b]=\frac{i}{R^2}M_{ab}, \tag{A3}$$

$$\{Q_\alpha,Q_\beta\}=-2(C\gamma^a)_{\alpha\beta}P_a+\frac{1}{R}(C\gamma^{ab})_{\alpha\beta}M_{ab},$$

$$[M_{ab},Q_\alpha]=-\frac{i}{2}Q_\beta(\gamma_{ab})^\beta{}_\alpha, \tag{A4}$$

$$\gamma_{ab}=\frac{1}{2}(\gamma_a\gamma_b-\gamma_b\gamma_a), \tag{A5}$$

$$[P_a,Q_\alpha]=-\frac{i}{2R}Q_\beta(\gamma_a)^\beta{}_\alpha. \tag{A6}$$

The generators M_{ab} form the SO(1,3) subalgebra (A1), and M_{ab} and P_a form the SO(2,3) subalgebra of OSp(1|4). Q_α are four Majorana spinor generators of OSp(1|4). The parameter R is the AdS_4 radius, and $C_{\alpha\beta}$ is the charge conjugation matrix such that

$$\gamma^a_{\alpha\beta}=\gamma^a_{\beta\alpha}\equiv C_{\alpha\gamma}(\gamma^a)^\gamma{}_\beta.$$

The parameters a^i_Π, a^{ij}_M, a_D, and a^i_K (2.12) of SO(2,3) acting as the conformal transformations on the boundary of AdS_4 (associated with the coordinates x^i) correspond to the following linear combinations of M_{ab} and P_a.

Three-dimensional translations

$$a^i_\Pi \to \Pi_i=P_i-M_{i3} \quad i=0,1,2,$$

$$[\Pi_i,\Pi_j]=0.$$

SO(1,2) rotations

$$a_M^{ij}\to M_{ij}.$$

Dilatation

$$a_D\to D=P_3.$$

Special conformal transformations (conformal boosts)

$$a_K^i\to K_i=P_i+M_{i3}.$$

Note that the SO(2,4) algebra has the same structure as SO(2,3) in Eqs. (A1)–(A3) but with indices $a,b,\ldots,$ running from 0 to 4.

The OSp(1|4) *Cartan forms*. We choose the parametrization of an OSp(1|4) group element $G(x,\theta,y)$ as follows:

$$G=K(x,\theta)U(y),\quad K(x,\theta)=B(x)e^{i\theta Q}, \tag{A7}$$

where $K(x,\theta)=B(x)e^{i\theta Q}$ is a group element corresponding to the coset superspace OSp(1|4)/SO(1,3), $B(x)$ is a group element corresponding to the bosonic AdS_4 $=$SO(2,3)/SO(1,3) and $U(y)$ is an element of SO(1,3) generated by M_{ab} with the antisymmetric y^{ab} being six parameters of the SO(1,3) transformations. We do not need to specify the representation of $B(x)$ and $U(y)$.

The OSp(1|4) Cartan forms $G^{-1}dG=E^a_{\rm OSp}P_a+\Omega^{ab}_{\rm OSp}M_{ab}+E^\alpha_{\rm OSp}Q_\alpha$ are

$$E^a_{\rm OSp}=E^b(x,\theta)u_b{}^a(y),$$
$$\Omega^{ab}_{\rm OSp}=\Omega^{cd}(x,\theta)u_c{}^a u_d{}^a+(u^{-1}du)^{ab}, \tag{A8}$$
$$E^\alpha_{\rm OSp}=E^\beta(x,\theta)v_\beta{}^\alpha(y),$$

where $u_b{}^a(y)$ and $v_\beta{}^\alpha(y)$ are matrices of, respectively, the vector and the spinor representation of SO(1,3). They are defined by the relations

$$u_b{}^a(y)P_a=U^{-1}P_bU(y),\quad v_\beta{}^\alpha(y)Q_\alpha=U^{-1}Q_\beta U(y), \tag{A9}$$

and are related to each other by the standard expression

$$\gamma_a u_b{}^a(y)=v(y)\gamma_b v(y). \tag{A10}$$

$E^a(x,\theta)$, $\Omega^{ab}(x,\theta)$, and $E^\alpha(x,\theta)$ are Cartan forms $K^{-1}dK$ corresponding to the coset superspace OSp(1|4)/SO(1,3). The OSp(1|4) Maurer-Cartan equations are

$$dE^\alpha+\frac{2}{R}E^\beta\wedge\Omega_\beta{}^\alpha=0, \tag{A11}$$

$$d\Omega^{\alpha\beta}+\frac{2}{R}\Omega^{\alpha\gamma}\wedge\Omega_\gamma{}^\beta=-iE^\alpha\wedge E^\beta. \tag{A12}$$

The OSp(1|4)/SO(1,3) *supervielbeins and spin connection.* The spinorial supervielbein is

$$E^\alpha=\mathcal{D}\theta^\alpha-\frac{i}{3!R}\bar\theta\gamma^a\mathcal{D}\theta(\gamma_a\theta)^\alpha+\frac{i}{2\times3!R}\bar\theta\gamma^{ab}\mathcal{D}\theta(\gamma_{ab}\theta)^\alpha-\frac{2}{5!R^2}\mathcal{D}\theta^\alpha(\bar\theta\theta)^2 \tag{A13}$$

or by using the Fierz identity

$$C_{\alpha(\beta}C_{\gamma)\delta}=\frac14\gamma^a_{\beta\gamma}(\gamma_a)_{\alpha\delta}-\frac18\gamma^{ab}_{\beta\gamma}(\gamma_{ab})_{\alpha\delta}, \tag{A14}$$

$$E^\alpha=\mathcal{D}\theta^\alpha\left(1+\frac{i}{3R}\bar\theta\theta-\frac{2}{5!R^2}(\bar\theta\theta)^2\right)-\frac{i}{3R}\bar\theta\mathcal{D}\theta\theta^\alpha, \tag{A15}$$

where $\mathcal{D}$ is a covariant differential on the bosonic AdS_4 space defined as

$$\mathcal{D}=d+\frac12\omega^{ab}(x)\gamma_{ab}+\frac{1}{2R}e^a(x)\gamma_a\equiv D+\frac{1}{2R}e^a\gamma_a. \tag{A16}$$

Note that the AdS_4 Killing spinors (3.19) are defined to be covariantly constant with respect to $\mathcal{D}$, i.e., $\mathcal{D}K=0$.

The vector supervielbein is

$$E^a=e^a(x)-i\bar\theta\gamma^a\mathcal{D}\theta-\frac{1}{2\times3!R}\bar\theta\gamma^a\mathcal{D}\theta(\bar\theta\theta)+\frac{1}{4!R}\bar\theta\gamma^{bc}\mathcal{D}\theta(\bar\theta\gamma^a\gamma_{bc}\theta),$$

or [upon applying the Fierz identity (A14)]

$$E^a=e^a(x)-i\theta\gamma^a\mathcal{D}\theta\left(1+\frac{i}{3!R}\bar\theta\theta\right). \tag{A17}$$

Equation (A17) can be further rewritten as

$$E^a=e^a(x)\left(1-\frac{i}{2R}\bar\theta\theta-\frac{1}{2\cdot3!R^2}(\bar\theta\theta)^2\right)-i\theta\gamma^a D\theta\left(1+\frac{i}{3!R}\bar\theta\theta\right), \tag{A18}$$

where $D=d+\frac12\omega^{ab}(x)\gamma_{ab}$. The SO(1,3) connection is

$$\Omega^{ab}=\omega^{ab}(x)+\frac{i}{2R}\bar\theta\gamma^{ab}\mathcal{D}\theta+\frac{1}{4!R^2}(\bar\theta\gamma^c\mathcal{D}\theta)(\bar\theta\gamma^{ab}\gamma_c\theta)-\frac{1}{2\times4!R^2}(\bar\theta\gamma^{cd}\mathcal{D}\theta)(\bar\theta\gamma^{ab}\gamma_{cd}\theta)$$
$$=\omega^{ab}(x)+\frac{i}{2R}\bar\theta\gamma^{ab}\mathcal{D}\theta\left(1+\frac{i}{3!R}\bar\theta\theta\right), \tag{A19}$$

where $e^a(x)$ and $\omega^{ab}(x)$ are the vierbein and the spin connection on AdS_4.

Note that in Eqs. (A17) and (A19) we can make the following change of the Grassmann coordinates:

$$\Theta^\alpha = \left(1 + \frac{i}{3!R}\bar{\theta}\theta\right)^{1/2}\theta^\alpha. \tag{A20}$$

Then, because of the symmetry properties of the Dirac matrices γ^a and γ^{ab}, the Cartan forms become bilinear in Θ

$$E^a = e^a(x) - i\Theta\gamma^a\mathcal{D}\Theta, \tag{A21}$$

$$\Omega^{ab} = \omega^{ab}(x) + \frac{i}{2R}\bar{\Theta}\gamma^{ab}\mathcal{D}\Theta. \tag{A22}$$

The OSp$(1|2n)$ *superalgebra and* OSp$(1|2n)$ *Cartan forms.* The generators of the OSp$(1|2n)$ superalgebra are a symmetric bosonic (spin)tensor $M_{\alpha\beta}=M_{\beta\alpha}$ ($\alpha=1,\ldots,2n$) and a $2n$-component Grassmann spinor Q_α, which satisfy the following (anti)commutation relations:

$$[M_{\alpha\beta},M_{\gamma\delta}] = -i\alpha[C_{\gamma(\alpha}M_{\beta)\delta}+C_{\delta(\alpha}M_{\beta)\gamma}],$$
$$[M_{\alpha\beta},Q_\gamma] = -i\alpha C_{\gamma(\alpha}Q_{\beta)}, \tag{A23}$$
$$\{Q_\alpha,Q_\beta\} = M_{\alpha\beta},$$

where $C_{\alpha\beta}=-C_{\beta\alpha}$ is a constant $2n\times 2n$ antisymmetric matrix (symplectic metric). Note that to have the correspondence with the form of OSp$(1|4)$ superalgebra (A1) the factor α should be chosen to be $\alpha=4/R$.

When $n=2^{k/2}$, C can be regarded as a charge conjugation matrix and Q_α as a spinor representation of a D-dimensional pseudo-rotation group SO$(t,D-t)$ with an appropriately chosen number of dimensions D and timelike dimensions t of space-time. For instance, when $n=16$ the generators Q_α of OSp(1,32) can be associated with SO(1,10) Majorana spinors in $D=11$ or two SO(1,9) Majorana-Weyl spinors of the same or opposite chiralities in $D=10$. This makes the OSp(1,32) supergroup to be related to M theory and superstring theories. OSp(1,32) is a subgroup of OSp$(1|64)$, and the two supergroups are extensions of the isometry supergroups SU$(2,2|4)$, OSp$(8|4)$, and OSp$(2,6|4)$ of $D=10$ and $D=11$ AdS superspaces [22–24].

From a perspective of $D=11$ supergravity and M theory the OSp$(1|32)$ superalgebra contains the SO(1,10) covariant bosonic generators $P_a, M_{ab}=-M_{ba}$ and $M_{a_1\cdots a_5}=M_{[a_1\cdots a_5]}$. A contraction of OSp$(1|32)$ produces the M algebra [43,44] with M_{ab} and $M_{a_1\cdots a_5}$ becoming tensorial central charges.

To compute the OSp$(1|2n)$ Cartan forms we choose the following parametrization of the OSp$(1|2n)$ supergroup element

$$\mathcal{G}(y,\theta) = B(y)F(\theta) = B(y)e^{i\theta^\alpha Q_\alpha}, \tag{A24}$$

where $y^{\alpha\beta}=y^{\beta\alpha}$ are Sp$(2n)$ coordinates. The OSp$(1|2n)$ Cartan forms are

$$\frac{1}{i}\mathcal{G}^{-1}(y,\theta)d\mathcal{G}(y,\theta) \equiv \frac{1}{i}[F^{-1}(B^{-1}dB)F + F^{-1}dF]$$
$$\equiv F^{-1}\mathcal{D}F = E^\alpha Q_\alpha + \frac{1}{2}\Omega^{\alpha\beta}M_{\alpha\beta}. \tag{A25}$$

Let us start with computing the $F^{-1}dF$ term of Eq. (A25).

$$\frac{1}{i}F^{-1}(\theta)dF(\theta) = \Sigma_{n=0}^{\infty}\frac{i^n}{(n+1)!}Ad^n_{\theta Q}(d\theta Q)$$
$$\equiv \mathcal{E}^\alpha Q_\alpha + \frac{1}{2}\Omega_1^{\alpha\beta}M_{\alpha\beta}, \tag{A26}$$

where

$$Ad_B A \equiv [A,B]. \tag{A27}$$

To calculate the forms $\Omega_1^{\alpha\beta}$ and $\mathcal{E}^\alpha$ (A26), note that

$$Ad_{\theta Q}(d\theta Q) \equiv [d\theta Q,\theta Q] = -d\theta^{(\alpha}\theta^{\beta)}M_{\alpha\beta}, \tag{A28}$$

$$Ad^2_{\theta Q}d\theta Q \equiv [[d\theta Q,\theta Q],\theta Q]$$
$$= -i\alpha\, d\theta^{(\beta}\theta^{\alpha)}\theta_\beta Q_\alpha, \tag{A29}$$

$$Ad^3_{\theta Q}(d\theta Q) \equiv [[[d\theta Q,\theta Q],\theta Q],\theta Q]$$
$$= -\left(\frac{i\alpha}{2}\theta^\gamma\theta_\gamma\right)[d\theta Q,\theta Q]$$
$$= -\left(\frac{i\alpha}{2}\theta^\gamma\theta_\gamma\right)Ad_{\theta Q}(d\theta Q),$$

$$Ad^4_{\theta Q}(d\theta Q) \equiv [[[[d\theta Q,\theta Q],\theta Q],\theta Q],\theta Q]$$
$$= -\left(\frac{i\alpha}{2}\theta^\gamma\theta_\gamma\right)[[d\theta Q,\theta Q],\theta Q]$$
$$= -\left(\frac{i\alpha}{2}\theta^\gamma\theta_\gamma\right)Ad^2_{\theta Q}(d\theta Q).$$

Thus we arrive at the recursion relation

$$Ad^{l+2}_{\theta Q}(d\theta Q) = -\left(\frac{i\alpha}{2}\theta^\gamma\theta_\gamma\right)Ad^l_{\theta Q}(d\theta Q) \quad \text{for} \quad l\geqslant 1 \tag{A30}$$

and can express all higher commutators through either Eq. (A28) or Eq. (A29) multiplied by a corresponding power of $(i\alpha/2\ \ \theta^\gamma\theta_\gamma)$.

In such a way we arrive at the generic expression for the forms (A26):

$$\mathcal{E}^\alpha = d\theta^\alpha + i\, d\theta^{(\alpha}\theta^{\beta)}\theta_\beta\Sigma_{l=0}^{n-1}\frac{\alpha}{(2l+3)!}\left(\frac{i\alpha}{2}\theta^\gamma\theta_\gamma\right)^l, \tag{A31}$$

$$\Omega_1^{\alpha\beta}=-i\,d\theta^{(\alpha}\theta^{\beta)}\Sigma_{l=1}^{n-1}\frac{1}{(2l+2)!}\left(\frac{i\alpha}{2}\theta^\gamma\theta_\gamma\right)^l. \quad \text{(A32)}$$

To calculate the first term in Eq. (A25)

$$\frac{1}{i}F^{-1}(B^{-1}dB)F=F^{-1}\left(\frac{1}{2}\omega^{\alpha\beta}M_{\alpha\beta}\right)F\equiv E_0^\alpha+\frac{1}{2}\Omega_0^{\alpha\beta}M_{\alpha\beta} \quad \text{(A33)}$$

we note that because $\omega^{\alpha\beta}(y)$ is symmetric the following relation holds:

$$\theta_\beta\theta^\gamma\omega_\gamma^{(\alpha}\theta^{\beta)}=\frac{1}{2}\theta^\gamma\theta_\gamma(\theta\omega)^\alpha. \quad \text{(A34)}$$

Then one finds

$$Ad_{\theta Q}^{l+2}\frac{1}{2}(\omega M)=-\left(\frac{i\alpha}{2}\theta^\gamma\theta_\gamma\right)Ad_{\theta Q}^{l}\frac{1}{2}(\omega M)\quad \text{for}\quad l\geqslant 1. \quad \text{(A35)}$$

Using Eq. (A35) we get the following expressions for the forms (A33):

$$E_0^\alpha=\frac{\alpha}{2}(\theta\omega)^\alpha\Sigma_{l=0}^{n-1}\frac{1}{(2l+1)!}\left(\frac{i\alpha}{2}\theta^\gamma\theta_\gamma\right)^l, \quad \text{(A36)}$$

$$\Omega_0^{\alpha\beta}=\omega^{\alpha\beta}(y)-\frac{i\alpha}{2}(\theta\omega)^{(\alpha}\theta^{\beta)}\Sigma_{l=0}^{n-1}\frac{1}{(2l+2)!}\left(\frac{i\alpha}{2}\theta^\gamma\theta_\gamma\right)^l. \quad \text{(A37)}$$

Note that in Eqs. (A36) and (A37) the polynomials in $\theta^\gamma\theta_\gamma$ are the same as in Eqs. (A31) and (A32). Thus, inserting Eqs. (A31), (A32), (A36), and (A37) into Eq. (A25) we get the following expressions for the OSp$(1|2n)$ Cartan forms:

$$E^\alpha=\mathcal{D}\theta^\alpha+i\mathcal{D}\theta^{(\alpha}\theta^{\beta)}\theta_\beta P_1(\theta\theta), \quad \text{(A38)}$$

$$\Omega^{\alpha\beta}=\omega^{\alpha\beta}(y)+i\theta^{(\alpha}\mathcal{D}\theta^{\beta)}P_2(\theta\theta), \quad \text{(A39)}$$

where

$$P_1(\theta\theta)=\Sigma_{l=0}^{n}\frac{\alpha}{(2l+3)!}\left(\frac{i\alpha}{2}\theta^\gamma\theta_\gamma\right)^l, \quad \text{(A40)}$$

$$P_2(\theta\theta)=\Sigma_{l=0}^{n}\frac{1}{(2l+2)!}\left(\frac{i\alpha}{2}\theta^\gamma\theta_\gamma\right)^l, \quad \text{(A41)}$$

and

$$\mathcal{D}\theta^\alpha=d\theta^\alpha+\frac{\alpha}{2}\omega^\alpha{}_\beta(y)\theta^\beta. \quad \text{(A42)}$$

The polynomial P_2 (A41) can be hidden into rescaled $\Theta=\sqrt{P_2}\theta$, so that $\Omega^{\alpha\beta}$ become bilinear in Grassmann variables

$$\Omega^{\alpha\beta}=\omega^{\alpha\beta}(y)+i\Theta^{(\alpha}\mathcal{D}\Theta^{\beta)}. \quad \text{(A43)}$$

It is then not hard to verify [using the Maurer-Cartan equations (5.10) and (5.11)] that the odd Cartan forms (A38) take the form

$$E^\alpha=P(\Theta^2)\mathcal{D}\Theta^\alpha-\Theta^\alpha\mathcal{D}P(\Theta^2), \quad \text{(A44)}$$

where

$$P(\Theta^2)=\sqrt{1+\frac{i\alpha}{8}\Theta^\beta\Theta_\beta}.$$

Having in hand the OSp$(1|2n)$ Cartan forms it is straightforward to get the Cartan forms corresponding to any coset superspace OSp$(1|2n)/H$ with H being a bosonic subgroup of OSp$(1|2n)$. To this end in Eqs. (A43) and (A44) one should simply put to zero all parameters $y^{\alpha\beta}$ corresponding to the subgroup H. Then $\omega^{\alpha\beta}$ will depend only on the bosonic coordinates of the supercoset OSp$(1|2n)/H$, and Eq. (A43) will contain the even supervielbeins and the spin connection of OSp$(1|2n)/H$.

[1] R. Penrose, J. Math. Phys. **8**, 345 (1967); Rep. Math. Phys. **12**, 65 (1977); R. Penrose and M. A. H. MacCallum, Phys. Rep. **6**, 241 (1972); R. Penrose and W. Rindler, *Spinors and Space-Time* (Cambridge University Press, Cambridge, England, 1986), Vols. 1 and 2.

[2] A. Ferber, Nucl. Phys. **B132**, 55 (1977).

[3] J. Maldacena, Adv. Theor. Math. Phys. **2**, 231 (1998); S. S. Gubser, I. R. Klebanov, and A. M. Polyakov, Phys. Lett. B **428**, 105 (1998); E. Witten, Adv. Theor. Math. Phys. **2**, 253 (1998).

[4] O. Aharony, S. S. Gubser, J. Maldacena, H. Ooguri, and Y. Oz, ''Large N Field Theories, String Theory and Gravity,'' hep-th/9905111.

[5] E. S. Fradkin and M. A. Vasiliev, Phys. Lett. B **189**, 89 (1987); Nucl. Phys. **B291**, 141 (1987).

[6] M. A. Vasiliev, Sov. J. Nucl. Phys. **32**, 439 (1980).

[7] M. A. Vasiliev, Fortschr. Phys. **35**, 741 (1987); M. A. Vasiliev, Phys. Lett. B **243**, 378 (1992); Class. Quantum Grav. **8**, 1387 (1991); Phys. Lett. B **285**, 225 (1992).

[8] M. A. Vasiliev, Fortschr. Phys. **36**, 33 (1988).

[9] P. Claus, M. Gunaydin, R. Kallosh, J. Rahmfeld, and Y. Zunger, J. High Energy Phys. **05**, 019 (1999).

[10] P. Claus, J. Rahmfeld, and Y. Zunger, ''A Simple Particle Action from a Twistor Parametrization of AdS_5,'' hep-th/9906118; P. Claus, R. Kallosh, and J. Rahmfeld, Phys. Lett. B **462**, 285 (1999).

[11] J. Lukierski, Czech. J. Phys., Sect. B **29**, 44 (1979); J. Math. Phys. **21**, 561 (1980).

[12] I. Bandos and J. Lukierski, Mod. Phys. Lett. A **14**, 1257 (1999).

[13] I. Bandos, J. Lukierski, and D. Sorokin, Phys. Rev. D **61**, 045002 (2000).
[14] R. R. Metsaev and A. A. Tseytlin, Nucl. Phys. **B533**, 109 (1998).
[15] R. Kallosh, J. Rahmfeld, and A. Rajaraman, J. High Energy Phys. **9809**, 002 (1998).
[16] I. Pesando, J. High Energy Phys. **9811**, 002 (1998); Mod. Phys. Lett. A **14**, 343 (1999); J. High Energy Phys. **9902**, 007 (1999); R. Kallosh and J. Rahmfeld, Phys. Lett. B **443**, 143 (1998); R. Kallosh and A. A. Tseytlin, J. High Energy Phys. **9810**, 016 (1998); I. Oda, Phys. Lett. B **444**, 127 (1998); J. High Energy Phys. **9810**, 015 (1998).
[17] R. R. Metsaev and A. A. Tseytlin, Phys. Lett. B **436**, 281 (1998).
[18] B. de Wit, K. Peeters, J. Plefka, and A. Sevrin, Phys. Lett. B **443**, 153 (1998); G. Dall'Agata, D. Fabbri, C. Fraser, P. Fré, P. Termonia, and M. Trigiante, Nucl. Phys. **B542**, 157 (1999).
[19] P. Pasti, D. Sorokin, and M. Tonin, Phys. Lett. B **447**, 251 (1999).
[20] J.-Ge Zhou, ''Super 0-brane and GS Superstring Actions on $AdS_2 \times S^2$,'' hep-th/9906013.
[21] V. P. Akulov, I. A. Bandos, and V. G. Zima, Theor. Math. Phys. **56**, 635 (1983).
[22] B. Craps, J. Gomis, D. Mateos, and A. Van Proeyen, J. High Energy Phys. **9904**, 004 (1999).
[23] S. Ferrara and M. Porrati, Phys. Lett. B **458**, 43 (1998).
[24] I. Bars, C. Deliduman, and D. Minic, Phys. Lett. B **457**, 275 (1999).
[25] I. Bars, C. Deliduman, and D. Minic, ''Strings, Branes and Two-Time Physics,'' hep-th/9906223.
[26] S. M. Kuzenko, S. L. Lyakhovich, A. Yu. Segal, and A. A. Sharapov, Int. J. Mod. Phys. A **11**, 3307 (1996).
[27] T. Shirafuji, Prog. Theor. Phys. **70**, 18 (1983).
[28] A. K. H. Bengtsson, I. Bengtsson, M. Cederwall, and N. Linden, Phys. Rev. D **36**, 1766 (1987); I. Bengtsson and M. Cederwall, Nucl. Phys. **B302**, 104 (1988).
[29] Y. Eisenberg and S. Solomon, Nucl. Phys. **B309**, 709 (1988); Phys. Lett. B **220**, 562 (1989).
[30] D. Sorokin, V. Tkach, and D. V. Volkov, Mod. Phys. Lett. A **4**, 901 (1989).
[31] A. Gumenchuk and D. Sorokin, Sov. J. Nucl. Phys. **51**, 350 (1990); D. Sorokin, Fortschr. Phys. **38**, 923 (1990).
[32] I. A. Bandos, Yad. Fiz. **51**, 1429 [Sov. J. Nucl. Phys. **51**, 906 (1990)]; Pis'ma Zh. Eksp. Teor. Fiz. **52**, 837 (1990) [JETP Lett. **52**, 205 (1990)].
[33] M. S. Plyushchay, Phys. Lett. B **240**, 133 (1990).
[34] S. Coleman, J. Wess, and B. Zumino, Phys. Rev. **177**, 2239 (1969); S. Callan, S. Coleman, J. Wess, and B. Zumino, *ibid.* **177**, 2247 (1969).
[35] D. V. Volkov, Report No. ITP-69-75, Kiev, 1969 (unpublished); D. V. Volkov, Sov. J. Part. Nucl. **4**, 3 (1973).
[36] D. V. Volkov and V. P. Akulov, JETP Lett. **16**, 438 (1972); Phys. Lett. **46B**, 109 (1973).
[37] D. V. Volkov and V. A. Soroka, JETP Lett. **18**, 312 (1973).
[38] W. Keck, J. Phys. A **8**, 1819 (1975).
[39] B. Zumino, Nucl. Phys. **B127**, 189 (1977).
[40] F. Gürsey and L. Marchildon, Phys. Rev. D **17**, 2038 (1978).
[41] E. A. Ivanov and A. S. Sorin, J. Phys. A **13**, 1159 (1980).
[42] H. Lü, C. N. Pope, and P. K. Townsend, Phys. Lett. B **391**, 39 (1997).
[43] J. W. van Holten and A. Van Proeyen, J. Phys. A **15**, 3763 (1982).
[44] P. K. Townsend, ''p-brane democracy,'' hep-th/9507048.
[45] V. Akulov, I. Bandos, and D. Sorokin, Mod. Phys. Lett. A **3**, 1633 (1988).