Brane-world black holes

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Gravitational collapse of matter trapped on a brane will produce a black hole on the brane. We discuss such black holes in the models of Randall and Sundrum where our universe is viewed as a domain wall in five-dimensional anti–de Sitter space. We present evidence that a non-rotating uncharged black hole on the domain wall is described by a ''black cigar'' solution in five dimensions.

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I. INTRODUCTION

There has been much recent interest in the idea that our universe may be a brane embedded in some higher dimensional space. It has been shown that the hierarchy problem can be solved if the higher dimensional Planck scale is low and the extra dimensions large [1,2]. An alternative solution, proposed by Randall and Sundrum (RS), assumes that our universe is a negative tension domain wall separated from a positive tension wall by a slab of anti–de Sitter (AdS) space [3]. This does not require a large extra dimension: the hierarchy problem is solved by the special properties of AdS. The drawback with this model is the necessity of a negative tension object.

In further work [4], RS suggested that it is possible to have an *infinite* extra dimension. In this model, we live on a positive tension domain wall inside anti–de Sitter space. There is a bound state of the graviton confined to the wall as well as a continuum of Kaluza-Klein (KK) states. For non-relativistic processes on the wall, the bound state dominates over the KK states to give an inverse square law if the AdS radius is sufficiently small. It appears therefore that four dimensional gravity is recovered on the domain wall. This conclusion was based on perturbative calculations for zero thickness walls. Supergravity domain walls of finite thickness have recently been considered [5–7] and a non-perturbative proof that the bound state exists for such walls was given in [8]. It is important to examine other non-perturbative gravitational effects in this scenario to see whether the predictions of four dimensional general relativity are recovered on the domain wall.

If matter trapped on a brane undergoes gravitational collapse then a black hole will form. Such a black hole will have a horizon that extends into the dimensions transverse to the brane: it will be a higher dimensional object. Phenomenological properties of such black holes have been discussed in [9] for models with large extra dimensions. In this paper we discuss black holes in the RS models. A natural candidate for such a hole is the Schwarzschild-AdS solution, describing a black hole localized in the fifth dimension. We show in the Appendix that it is not possible to intersect such

a hole with a *vacuum* domain wall so it is unlikely that it could be the final state of gravitational collapse on the brane. A second possibility is that what looks like a black hole on the brane is actually a black string in the higher dimensional space. We give a simple solution describing such a string. The induced metric on the domain wall is simply Schwarzschild, as it has to be if four dimensional general relativity (and therefore Birkhoff's theorem) are recovered on the wall. This means that the usual astrophysical properties of black holes (e.g. perihelion precession, light bending etc.) are recovered in this scenario.

We find that the AdS horizon is singular for this black string solution. This is signalled by scalar curvature invariants diverging if one approaches the horizon along the axis of the string. If one approaches the horizon in a different direction then no scalar curvature invariant diverges. However, in a frame parallelly propagated along a timelike geodesic, some curvature components *do* diverge. Furthermore, the black string is unstable near the AdS horizon — this is the Gregory-Laflamme instability [10]. However, the solution is stable far from the AdS horizon. We will argue that our solution evolves to a ''black cigar'' solution describing an object that looks like the black string far from the AdS horizon (so the metric on the domain wall is Schwarzschild) but has a horizon that closes off before reaching the AdS horizon. In fact, we conjecture that this black cigar solution is the unique stable vacuum solution in five dimensions which describes the endpoint of gravitational collapse on the brane. We suspect that the AdS horizon will be non-singular for the cigar solution.

II. THE RANDALL-SUNDRUM MODELS

Both models considered by RS use five dimensional AdS. In horospherical coordinates the metric is

$$ds^2 = e^{-2y/l}\eta_{ij}dx^i dx^j + dy^2 \qquad (2.1)$$

where $\eta_{\mu\nu}$ is the four dimensional Minkowski metric and l the AdS radius. The global structure of AdS is shown in Fig. 1. Horospherical coordinates break down at the horizon $y = \infty$.

In their first model [3], RS slice AdS along the horospheres at $y = 0$ and $y = y_c > 0$, retain the portion $0 < y < y_c$ and assume Z_2 reflection symmetry at each boundary plane.

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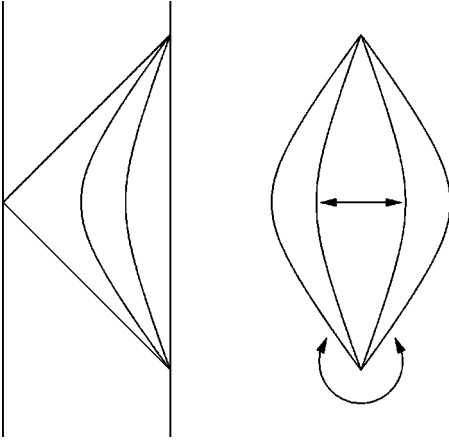‡Email address: H.S.Reall@damtp.cam.ac.uk

FIG. 1. Anti–de Sitter space. Two horospheres and a horizon are shown. The vertical lines represent timelike infinity. (2) Causal structure of Randall-Sundrum model with compact fifth dimension. The arrows denote identifications.

This gives a jump in extrinsic curvature at these planes, yielding two domain walls of equal and opposite tension

$$\sigma = \pm \frac{6}{\kappa^2 l} \tag{2.2}$$

where $\kappa^2 = 8\pi G$ and G is the five dimensional Newton constant. The wall at $y=0$ has positive tension and the wall at $y=y_c$ has negative tension. Mass scales on the negative tension wall are exponentially suppressed relative to those on the positive tension one. This provides a solution of the hierarchy problem provided we live on the negative tension wall. The global structure is shown in Fig. 1.

The second RS model [4] is obtained from the first by taking $y_c \rightarrow \infty$. This makes the negative tension wall approach the AdS horizon, which includes a point at infinity. RS say that their model contains only one wall so presumably the idea is that the negative tension brane is viewed as an auxiliary device to set up boundary conditions. However, if the geometry makes sense then it should be possible to discuss it without reference to this limiting procedure involving negative tension objects. If one simply slices AdS along a positive tension wall at $y=0$ and assumes reflection symmetry then there are several ways to analytically continue the solution across the horizon. These have been discussed in [11–14]. There are two obvious choices of continuation. The first is simply to assume that beyond the horizon, the solution is pure AdS with no domain walls present. This is shown in Fig. 2. An alternative, which seems closer in spirit to the geometry envisaged by RS, is to include further domain walls beyond the horizon, as shown in Fig. 2. In this case, there are infinitely many domain walls present.

III. BLACK STRING IN ADS

Let us first rewrite the AdS metric (2.1) by introducing the coordinate $z = l e^{y/l}$. The metric is then manifestly conformally flat:
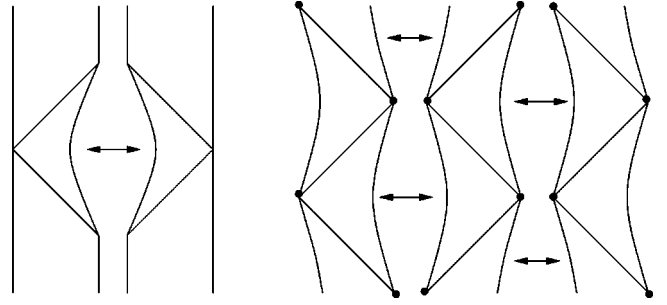

FIG. 2. Possible causal structures for Randall-Sundrum model with non-compact fifth dimension. The dots denote points at infinity.

$$ds^2 = \frac{l^2}{z^2}(dz^2 + \eta_{ij}dx^i dx^j). \tag{3.1}$$

In these coordinates, the horizon lies at $z=\infty$ while the timelike infinity of AdS is at $z=0$. We now note that if the Minkowski metric within the brackets is replaced by *any* Ricci flat metric then the Einstein equations (with negative cosmological constant) are still satisfied.[1] A natural choice for a metric describing a black hole on a domain wall at fixed z is to take this Ricci flat metric to be the Schwarzschild solution:

$$ds^2 = \frac{l^2}{z^2}(-U(r)dt^2 + U(r)^{-1}dr^2$$
$$+ r^2(d\theta^2 + \sin^2\theta d\phi^2) + dz^2) \tag{3.2}$$

where $U(r) = 1 - 2M/r$. This metric describes a black string in AdS. Including a reflection symmetric domain wall in this spacetime is trivial: surfaces of constant z satisfy the Israel equations provided the domain wall tension satisfies Eq. (2.2). For a domain wall at $z=z_0$, introduce the coordinate $w=z-z_0$. The metric on both sides of the wall can then be written

$$ds^2 = \frac{l^2}{(|w|+z_0)^2}(-U(r)dt^2 + U(r)^{-1}dr^2$$
$$+ r^2(d\theta^2 + \sin^2\theta d\phi^2) + dw^2) \tag{3.3}$$

with $-\infty < w < \infty$ and the wall is at $w=0$. It would be straightforward to use the same method to construct a black string solution in the presence of a *thick* domain wall.

The induced metric on a domain wall placed at $z=z_0$ can be brought to the standard Schwarzschild form by rescaling the coordinates t and r. The Arnowitt-Deser-Misner (ADM) mass as measured by an inhabitant of the wall would be $M_* = Ml/z_0$. The proper radius of the horizon in five dimensions is $2M_*$. The AdS length radius l is required to be within a few orders of magnitude of the Planck length [4] so

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[1]This procedure was recently discussed for general p-brane solutions in [15].

black holes of astrophysical mass must have $M/z_0 \gg 1$. If one included a second domain wall with negative tension then the ADM mass on that wall would be exponentially suppressed relative to that on the positive tension wall.

Our solution has an Einstein metric so the Ricci scalar and square of the Ricci tensor are finite everywhere. However the square of the Riemann tensor is

$$R_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma} = \frac{1}{l^4}\left(40 + \frac{48M^2z^4}{r^6}\right), \tag{3.4}$$

which diverges at the AdS horizon $z = \infty$ as well as at the black string singularity at $r = 0$. We shall have more to say about this later.

It is important to examine the behavior of geodesics in this spacetime. Let u denote the velocity along a timelike or null geodesic with respect to an affine parameter λ (taken to be the proper time in the case of a timelike geodesic). The Killing vectors $k = \partial/\partial t$ and $m = \partial/\partial\phi$ give rise to the conserved quantities $E = -k \cdot u$ and $L = m \cdot u$. Rearranging these gives

$$\frac{dt}{d\lambda} = \frac{Ez^2}{U(r)l^2} \tag{3.5}$$

and

$$\frac{d\phi}{d\lambda} = \frac{Lz^2}{r^2l^2}, \tag{3.6}$$

for motion in the equatorial plane ($\theta \equiv \pi/2$). The equation describing motion in the z direction is simply

$$\frac{d}{d\lambda}\left(\frac{1}{z^2}\frac{dz}{d\lambda}\right) = \frac{\sigma}{zl^2}, \tag{3.7}$$

where $\sigma = 0$ for null geodesics and $\sigma = 1$ for timelike geodesics. The solutions for null geodesics are $z = $ const or

$$z = -\frac{z_1 l}{\lambda}, \tag{3.8}$$

The solution for timelike geodesics is

$$z = -z_1 \operatorname{cosec}(\lambda/l). \tag{3.9}$$

In both cases, z_1 is a constant and we have shifted λ so that $z \to \infty$ as $\lambda \to 0-$. The (null) solution $z = $ const is simply a null geodesic of the four dimensional Schwarzschild solution. We are more interested in the other solutions because they appear to reach the singularity at $z = \infty$. The radial motion is given by

$$\left(\frac{dr}{d\lambda}\right)^2 + \frac{z^4}{l^4}\left[\left(\frac{l^2}{z_1^2} + \frac{L^2}{r^2}\right)U(r) - E^2\right] = 0. \tag{3.10}$$

Now introduce a new parameter $\nu = -z_1^2/\lambda$ for null geodesics and $\nu = -(z_1^2/l)\cot(\lambda/l)$ for timelike geodesics. We also define new coordinates $\tilde{r} = z_1 r/l$, $\tilde{t} = z_1 t/l$, and new constants $\tilde{E} = z_1 E/l$, $\tilde{L} = z_1^2 L/l^2$ and $\tilde{M} = z_1 M/l$. The radial equation becomes

$$\left(\frac{d\tilde{r}}{d\nu}\right)^2 + \left(1 + \frac{\tilde{L}^2}{\tilde{r}^2}\right)\left(1 - \frac{2\tilde{M}}{\tilde{r}}\right) = \tilde{E}^2, \tag{3.11}$$

which is the radial equation for a *timelike* geodesic in a four dimensional Schwarzschild solution of mass $\tilde{M}$ [16]. (This is the ADM mass for an observer with $z = z_0 = l^2/z_1$.) Note that ν is the proper time along such a geodesic. It should not be surprising that a null geodesic in five dimensions is equivalent to a timelike geodesic in four dimensions: the non-trivial motion in the fifth dimension gives rise to a mass in four dimensions. What is perhaps surprising is the relationship between the four and five dimensional affine parameters ν and λ.

We are interested in the behavior near the singularity, i.e. as $\lambda \to 0-$. This is equivalent to $\nu \to \infty$ i.e. we need to study the late time behavior of four dimensional timelike geodesics. If such geodesics hit the *Schwarzschild* singularity at $\tilde{r} = 0$ then they do so at finite ν. For infinite ν there are two possibilities [16]. The first is that the geodesic reaches $\tilde{r} = \infty$. The second can occur only if $\tilde{L}^2 > 12\tilde{M}^2$, when it is possible to have bound states (i.e. orbits restricted to a finite range of $\tilde{r}$) outside the Schwarzschild horizon.

The orbits that reach $\tilde{r} = \infty$ have late time behavior $\tilde{r} \sim \nu\sqrt{\tilde{E}^2 - 1}$ and hence

$$r \sim -\frac{z_1 l}{\lambda}\sqrt{\tilde{E}^2 - 1} \tag{3.12}$$

as $\lambda \to 0-$. Along such geodesics, the squared Riemann tensor does *not* diverge. The bound state geodesics behave differently. These remain at finite r and therefore the square of the Riemann tensor *does* diverge as $\lambda \to 0-$. They orbit the black string infinitely many times before hitting the singularity, but do so in finite affine parameter.

It appears that some geodesics encounter a curvature singularity at the AdS horizon whereas others might not because scalar curvature invariants do not diverge along them. It is possible that only part of the surface $z = \infty$ is singular. To decide whether or not this is true, we turn to a calculation of the Riemann tensor in an orthonormal frame parallelly propagated along a timelike geodesic that reaches $z = \infty$ but for which the squared Riemann tensor does not diverge (i.e. a non-bound state geodesics). The tangent vector to such a geodesic (with $L = 0$) can be written

$$u^\mu = \left(\frac{z}{l}\sqrt{\frac{z^2}{z_1^2} - 1}, \frac{Ez^2}{U(r)l^2}, \frac{z^2}{l^2}\sqrt{E^2 - \frac{l^2}{z_1^2}U(r)}, 0, 0\right), \tag{3.13}$$

where we have written the components in the order (z, t, r, θ, ϕ). A unit normal to the geodesic is

$$n^\mu = \left(0, -\frac{zz_1}{l^2 U(r)}\sqrt{E^2 - \frac{l^2}{z_1^2}U(r)}, -\frac{Ez_1 z}{l^2}, 0, 0 \right).$$
(3.14)

It is straightforward to check that this is parallelly propagated along the geodesic i.e. $u \cdot \nabla n^\mu = 0$. These two unit vectors can be supplemented by three other parallelly propagated vectors to form an orthonormal set. However the divergence can be exhibited using just these two vectors. One of the curvature components in this parallelly propagated frame is

$$R_{(u)(n)(u)(n)} \equiv R_{\mu\nu\rho\sigma}u^\mu n^\nu u^\rho n^\sigma = \frac{1}{l^2}\left(1 - \frac{2Mz^4}{z_1^2 r^3} \right),$$
(3.15)

which diverges along the geodesic as $\lambda \to 0$. The black string solution therefore has a curvature singularity at the AdS horizon.

It is well known that black string solutions in asymptotically flat space are unstable to long wavelength perturbations [10]. A black hole is entropically preferred to a sufficiently long segment of string. The string's horizon therefore has a tendency to ''pinch off'' and form a line of black holes. One might think that a similar instability occurs for our solution. However, AdS acts like a confining box which prevents fluctuations with wavelengths much greater than l from developing. If an instability occurs then it must do so at smaller wavelengths.

If the radius of curvature of the string's horizon is sufficiently small then the AdS curvature will be negligible there and the string will behave as if it were in asymptotically flat space. This means that it will be unstable to perturbations with wavelengths of the order of the horizon radius $2M_*$ $= 2Ml/z$. At sufficiently large z, such perturbations will fit into the AdS box, i.e. $2M_* \ll l$, so an instability can occur near the AdS horizon. However for $M/z \gg 1$, the potential instability occurs at wavelengths much greater than l and is therefore not possible in AdS. Therefore the black string solution is unstable near the AdS horizon but stable far from it.

We conclude that, near the AdS horizon, the black string has a tendency to ''pinch off'' but further away it is stable. After pinching off, the string becomes a stable ''black cigar'' which would extend to infinity in AdS space if the domain wall were not present, but not to the AdS horizon. The cigar's horizon acts as if it has a tension which balances the force pulling it towards the center of AdS space. We showed above that if our domain wall is at $z = z_0$ then a black hole of astrophysical mass has $M/z_0 \gg 1$, corresponding to the part of the black cigar far from the AdS horizon. Here, the metric will be well approximated by the black string metric so the induced metric on the wall will be Schwarzschild and the predictions of four dimensional general relativity will be recovered.

IV. DISCUSSION

Any phenomenologically successful theory in which our universe is viewed as a brane must reproduce the large-scale predictions of general relativity on the brane. This implies that gravitational collapse of uncharged non-rotating matter trapped on the brane ultimately settles down to a steady state in which the induced metric on the brane is Schwarzschild. In the higher dimensional theory, such a solution could be a localized black hole or an extended object intersecting the brane. We have investigated these alternatives in the models proposed by Randall and Sundrum (RS). The obvious choice of five dimensional solution in the first case is Schwarzschild-AdS space. However we have shown (in the Appendix) that it is not possible to intersect this with a vacuum domain wall so it cannot be the final state of gravitational collapse on the wall.

We have presented a solution that describes a black string in AdS space. It *is* possible to intersect this solution with a vacuum domain wall and the induced metric is Schwarzschild space. The solution can therefore be interpreted as a black hole on the wall. The AdS horizon is singular. Scalar curvature invariants only diverge if this singularity is approached along the axis of the string. However, curvature components diverge in a frame parallelly propagated along any timelike geodesic that reaches the horizon. This singularity can be removed if we use the first RS model in which there are two domain walls present and we live on a negative tension wall. However if we wish to use the second RS model (with a non-compact fifth dimension) then the singularity will be visible from our domain wall. In [8], it was argued that anything emerging from a singularity at the AdS horizon would be heavily red-shifted before reaching us and that this might ensure that physics on the wall remains predictable in spite of the singularity. However we regard singularities as a pathology of the theory since, in principle, arbitrarily large fluctuations can emerge from the singularity and the red-shift is finite.

Fortunately, it turns out that our solution is unstable near the AdS horizon. We have suggested that it will decay to a stable configuration resembling a cigar that extends out to infinity in AdS space but does not reach the AdS horizon. The solution becomes finite in extent when the gravitational effect of the domain wall is included. Our domain wall is situated far from the AdS horizon so the induced metric on the wall will be very nearly Schwarzschild space. Since the cigar does not extend as far as the AdS horizon, it does not seem likely that there will be a singularity there. Similar behavior was recently found in a non-linear treatment of the RS model [8]. It was shown that pp-waves corresponding to Kaluza-Klein modes are singular at the AdS horizon. These pp-waves are not localized to the domain wall. The only pp-waves regular at the horizon are the ones corresponding to gravitons confined to the wall. We suspect that perturbations of the flat horospheres of AdS space that do not vanish near the horizon will generically give rise to a singularity there.

It seems likely that there are other solutions that give rise to the Schwarzschild solution on the domain wall. For example, the metric outside a star on the wall would be Schwarzschild. If the cigar solution was the only stable solution giving Schwarzschild on the wall then it would have to be possible to intersect it with a non-vacuum domain wall

describing such a star. However, it is then not possible to choose the equation of state for the matter on the wall, for reasons analogous to those discussed in the Appendix. Our solution is therefore not capable of describing generic stars. If this is the case then one might wonder whether there are other solutions describing black holes on the wall. We conjecture that the cigar solution is the unique stable vacuum solution with a regular AdS horizon that describes a non-rotating uncharged black hole on the domain wall.

ACKNOWLEDGMENTS

APPENDIX

One candidate for a black hole formed by gravitational collapse on a domain wall in AdS space is the Schwarzschild-AdS solution, which has metric

$$ds^2 = -U(r)dt^2 + U(r)^{-1}dr^2 + r^2(d\chi^2 + \sin^2\chi \, d\Omega^2),$$
(A1)

where $d\Omega^2$ is the line element on a unit 2-sphere and

$$U(r) = 1 - \frac{2M}{r^2} + \frac{r^2}{l^2}.$$
(A2)

The parameter M is related to the mass of the black hole. We have not yet included the gravitational effect of the wall. We shall focus on the second RS model so we want a single positive tension domain wall with the spacetime reflection symmetric in the wall. Denote the spacetime on the two sides of the wall as $(+)$ and $(-)$. Let n be a unit (spacelike) normal to the wall pointing out of the $(+)$ region. The tensor $h_{\mu\nu} = g_{\mu\nu} - n_\mu n_\nu$ projects vectors onto the wall, and its tangential components give the induced metric on the wall. The extrinsic curvature of the wall is defined by

$$K_{\mu\nu} = h_\mu^\rho h_\nu^\sigma \nabla_\rho n_\sigma$$
(A3)

and its trace is $K = h^{\mu\nu} K_{\mu\nu}$. The energy momentum tensor $t_{\mu\nu}$ of the wall is given by varying its action with respect to the induced metric. The gravitational effect of the domain wall is given by the Israel junction conditions [17], which relate the discontinuity in the extrinsic curvature at the wall to its energy momentum:

$$[K_{\mu\nu} - Kh_{\mu\nu}]_-^+ = \kappa^2 t_{\mu\nu}$$
(A4)

(see [18] for a simple derivation of this equation). Here $\kappa^2 = 8\pi G$ where G is the five dimensional Newton constant. This can be rearranged using reflection symmetry to give

$$K_{\mu\nu} = \frac{\kappa^2}{2}\left(t_{\mu\nu} - \frac{t}{3}h_{\mu\nu}\right),$$
(A5)

where $t = h^{\mu\nu} t_{\mu\nu}$.

Cylindrical symmetry dictates that we should consider a domain wall with position given by $\chi = \chi(r)$. The unit normal to the $(+)$ side can be written

$$n = \frac{\epsilon r}{\sqrt{1 + Ur^2\chi'^2}}(d\chi - \chi' dr),$$
(A6)

where $\epsilon = \pm 1$ and a prime denotes a derivative with respect to r. The timelike tangent to the wall is

$$u = U^{-1/2}\frac{\partial}{\partial t},$$
(A7)

and the spacelike tangents are

$$t = \sqrt{\frac{U}{1 + Ur^2\chi^2}}\left(\chi'\frac{\partial}{\partial\chi} + \frac{\partial}{\partial r}\right),$$
(A8)

$$e_\theta = \frac{1}{r\sin\chi}\frac{\partial}{\partial\theta},$$
(A9)

$$e_\phi = \frac{1}{r\sin\chi\sin\theta}\frac{\partial}{\partial\phi}.$$
(A10)

The non-vanishing components of the extrinsic curvature in this basis are

$$K_{uu} = \frac{\epsilon U' r\chi'}{2\sqrt{1 + Ur^2\chi'^2}},$$
(A11)

$$K_{\theta\theta} = K_{\phi\phi} = \frac{\epsilon}{\sqrt{1 + Ur^2\chi'^2}}\left(\frac{\cot\chi}{r} - U\chi'\right),$$
(A12)

$$K_{tt} = -\frac{\epsilon}{(1 + Ur^2\chi'^2)^{3/2}}(\chi'^3 U^2 r^2 + 2\chi' U + Ur\chi''$$
$$+ U' r\chi'/2).$$
(A13)

A vacuum domain wall has

$$t_{\mu\nu} = -\sigma h_{\mu\nu},$$
(A14)

where σ is the wall's tension. The Israel conditions are

$$K_{\mu\nu} = \frac{\kappa^2}{6}\sigma h_{\mu\nu}.$$
(A15)

These reduce to

$$-K_{uu} = K_{tt} = K_{\theta\theta} = \frac{\kappa^2}{6}\sigma.$$
(A16)

It is straightforward to verify that these equations have no solution. A solution *can* be found for a non-vacuum domain wall with energy-momentum tensor

$$t_{\mu\nu} = \text{diag}(\sigma, p, p, p, 0),$$
(A17)

since then we have three unknown functions $(\sigma(r), p(r), \chi(r))$ and three equations. However this does not allow an equation of state to be specified in advance. We are only interested in *vacuum* solutions since these describe the final state of gravitational collapse on the brane.

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