Renormalization group study of the $(\phi^*\phi)^3$ model coupled to a Chern-Simons field

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We consider the model of a massless charged scalar field, in $2+1$ dimensions, with a self-interaction of the form $\lambda(\phi^*\phi)^3$ and interacting with a Chern-Simons field. We calculate the renormalization group β functions of the coupling constants and the anomalous dimensions γ of the basic fields. We show that the interaction with the Chern-Simons field implies a β_λ which suggests that a dynamical symmetry breakdown occurs. We also study the effect of the Chern-Simons field on the anomalous dimensions of the composite operators $(\phi^*\phi)^n$, obtaining the result that their operator dimensions are lowered.

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I. INTRODUCTION

Self-interacting scalar fields are the simplest nontrivial field theories. Nevertheless they have found numerous applications in many different phenomena. Renormalization group analyses of the model of scalar fields in $2+1$ dimensions with a self-interaction of the form $\lambda\phi^6$ have appeared in the literature [1] in conjunction with other self-interactions, and also in interaction with other fields. On the other side, the Chern-Simons (CS) field theory [2] is known to cause some strange effects in matter fields, the best known being the transmutation of their spins and statistics [3].

Bosons (fermions) interacting with a CS field receive an extra contribution to their spins and statistical phases, changing to anyons and even to fermions (bosons). Studies of the change in the scale behavior of matter fields due to their interaction with the CS field have also been considered [4–6].

In this paper we study the model of a massless charged scalar field with a self-interaction of the form $\lambda(\phi^*\phi)^3$ interacting with an Abelian CS field. Classically it only involves dimensionless parameters and is scale invariant. It is also strictly renormalizable; no induction of terms of the forms $m^2(\phi^*\phi)$ or $g(\phi^*\phi)^2$ occurs. Besides the calculation of the anomalous dimensions of ϕ and A^μ and the β functions related to their coupling constants, we also calculate the anomalous dimensions of composite operators of the form $(\phi^*\phi)^n$. Some of our conclusions agree and others disagree with the previous literature. This will be discussed in Sec. III and in the Conclusions.

To regulate the ultraviolet (UV) behavior we use a simplified version of dimensional regularization, the so-called ‘‘dimensional reduction’’ method. It consists of contracting and simplifying the Lorentz tensors before extending the Feynman integrals out of three dimensions. This procedure, previously used by several authors [4–6], greatly simplifies calculations involving the CS field because it does not require the extension of the Levi-Civita tensor $\varepsilon^{\mu\nu\rho}$ out of three dimensions. In Feynman integrals only involving scalar vertices and propagators, no difference appears between the results obtained by using one or the other method. In graphs involving the CS field and the $\varepsilon^{\mu\nu\rho}$, the differences of this method to a ‘‘full’’ dimensional regularization would only show up [4] in subleading contributions to the Feynman integrals; that is, if D stands for the extended dimension of the spacetime when the Feynman integrals are expanded in the Laurent series in $\epsilon\equiv(D-3)$, no difference in the leading divergent term in $1/\epsilon$ will appear. It is, on the other side, a characteristic of dimensional regularization in $2+1$ dimensions that one-loop graphs are finite and two-loop graphs have at most a single pole divergence in ϵ. As the calculation of the renormalization group parameters only involve the use of the divergent parts of the graphs, no differences to the full dimensional regularization are expected up to two loops in graphs that involve the CS propagator and in any number of loops in graphs only involving the scalar propagator. In this paper we will restrict the calculations to up to two loops in all graphs involving propagators of the CS field, and four loops in graphs involving only the propagator of the scalar field. As we will explicitly show, (at least) to those orders, dimensional reduction is enough to regularize the model and to preserve the gauge symmetry, as expressed by the Ward Identities (WI). We will work in the Landau gauge and, avoiding exceptional momenta, no infrared (IR) divergences will appear.

The plan of the paper is as follows. In Sec. II the model is presented and the divergent UV counterterms, necessary for the renormalization group study, are obtained by calculating the CS two-point function and the scalar field two- and six-point functions. In Sec. III the renormalization group β func-

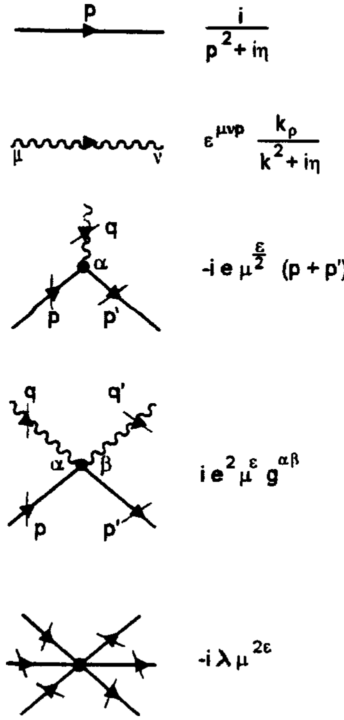

FIG. 1. Feynman rules in the Landau gauge.

tions and anomalous dimensions of the fields are obtained and compared with other calculations. The change in the dynamical behavior of the ϕ field due to the interaction of the CS field is discussed. The influence of the CS in the dimension and the renormalizability of operators of the form $(\phi^*\phi)^n$ is also studied. A summary of the results are presented in the Conclusions. In Appendix A the explicit verification of the WI is given, and in Appendix B some Feynman integrals are calculated as examples.

II. THE MODEL

The model is constituted by a massless charged boson in $2+1$ dimensions represented by a field ϕ with a self-interaction of the form $(\phi^*\phi)^3$ and minimal interaction with a Chern-Simons (CS) field A_μ. Its Lagrangian density is given by

$$\begin{aligned}\mathcal{L}=&\partial_\mu\phi_0^\dagger\partial^\mu\phi_0-ie_0A_0^\mu(\phi_0^\dagger\partial_\mu\phi_0-\partial_\mu\phi_0^\dagger\phi_0)\\&+e_0^2A_0^\mu A_{0\mu}(\phi_0^\dagger\phi_0)\\&-\frac{\lambda_0}{6^2}(\phi_0^\dagger\phi_0)^3+\frac{1}{2}\varepsilon_{\mu\nu\rho}A_0^\mu\partial^\nu A_0^\rho.\end{aligned}\tag{2.1}$$

The metric is $g_{\mu\nu}=(1,-1,-1),\partial_\mu$ stands for $\partial/\partial x^\mu$, $\varepsilon_{\mu\nu\rho}$ is the antisymmetric Levi-Civita tensor with $\varepsilon^{012}=1$, and e_0 and λ_0 are dimensionless coupling constants. The subscript ''0'' means that the corresponding quantity is ''unrenormalized.''

The model is renormalizable; all the UV infinities of the perturbative series can be absorbed in a redefinition of the unrenormalized quantities. It also has a gauge symmetry

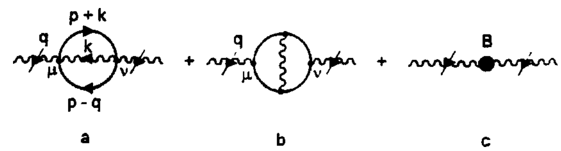

FIG. 2. Divergent diagrams contributing to the CS two-point function.

which suggests the use of dimensional regularization [7]. However, the presence of the Levi-Civita tensor in the CS term makes dimensional regularization cumbersome and the calculations become awkward in more than one loop. So, we will take advantage of some characteristics of $2+1$ dimensions and use a simplified version of dimensional regularization, the so-called dimensional reduction [4,5]. In this procedure, the Lorentz tensor algebra is considered in $2+1$ dimensions and only the remaining scalar Feynman integrals are extended out of $2+1$ dimensions. It was verified in [5], up to two loops, that for the non-Abelian Chern-Simons theory this procedure in fact preserves the Slavnov-Taylor identities. As we will also show below up to two loops, it also preserves the Ward identities in our model and no inconsistencies appear.

To obtain information on the asymptotic behavior of the model, we need to calculate the renormalization group parameters: β functions and anomalous dimensions of the fields. For this task, adopting the renormalization group approach of 't Hooft [8] based on minimal subtraction, we only need to calculate the divergent parts of some vertex functions, more precisely, the residues of the poles in $1/\epsilon$, where $\epsilon=3-D$ and D is the ''extended''dimension of the space-time. In the $2+1$ dimension this means that we must go to at least two-loop calculations because as a characteristic of dimensional regularization, one-loop integrals are finite.

Introducing the renormalized fields ϕ and A_μ and the renormalized coupling constants e and λ through the definitions

$$\phi_0=Z_\phi^{1/2}\phi=(1+A)^{1/2}\phi,\tag{2.2}$$

$$A_0^\mu=Z_A^{1/2}A^\mu=(1+B)^{1/2}A^\mu,\tag{2.3}$$

$$e_0=e\mu^{\epsilon/2}(1+D)/Z_\phi Z_A^{1/2},\tag{2.4}$$

$$e_0^2=e^2\mu^\epsilon(1+E)/Z_\phi Z_A,\tag{2.5}$$

$$\lambda_0=\mu^{2\epsilon}(\lambda+C)/Z_\phi^3,\tag{2.6}$$

where μ is a mass parameter introduced to keep e and λ dimensionless quantities and A to E are the counterterms to be chosen so as to make the renormalized quantities finite in each order of perturbation. As will be seen in the calculations, the renormalization of λ in the presence of the CS field is not multiplicative. By substituting these definitions in Eq. (2.1) we obtain for $\mathcal{L}$

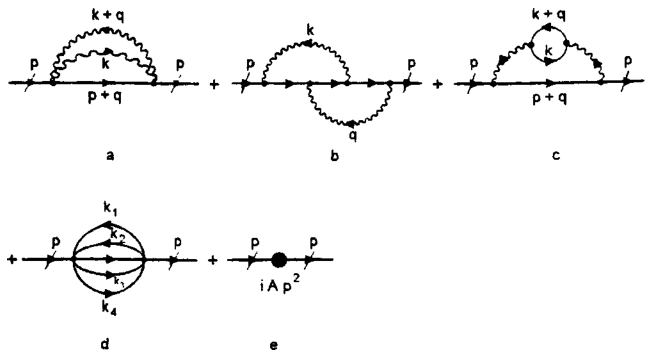

FIG. 3. Divergent diagrams contributing to the scalar field two-point function.

$$\mathcal{L}=\partial_\mu\phi^\dagger\partial^\mu\phi+\frac{1}{2}\varepsilon_{\mu\nu\rho}A^\mu\partial^\nu A^\rho-ie\mu^{\epsilon/2}A^\mu(\phi^\dagger\partial_\mu\phi-\partial_\mu\phi^\dagger\phi)+e^2\mu^\epsilon A^\mu A_\mu(\phi^\dagger\phi)-\frac{\lambda\mu^{2\epsilon}}{6^2}(\phi^\dagger\phi)^3+A\partial_\mu\phi^\dagger\partial^\mu\phi+\frac{B}{2}\varepsilon_{\mu\nu\rho}A^\mu\partial^\nu A^\rho-ie\mu^{\epsilon/2}DA^\mu(\phi^\dagger\partial_\mu\phi-\partial_\mu\phi^\dagger\phi)+e^2\mu^\epsilon EA^\mu A_\mu(\phi^\dagger\phi)-\frac{\mu^{2\epsilon}C}{6^2}(\phi^\dagger\phi)^3. \tag{2.7}$$

The Feynman rules for this Lagrangian in the Landau gauge are depicted in Fig. 1. This gauge can be implemented by adding to the Lagrangian a gauge fixing term $(1/2\xi)(\partial^\mu A_\mu)^2$, inverting the free quadratic part of the A^μ to get the CS propagator and then letting $\xi\to\infty$. The would-be Faddeev-Popov ghost field is completely decoupled of the other fields and does not have any effect. Calling $\Gamma(p)$ the scalar field 1PI two-point function and $\Gamma^\mu(q;p,p')$ and $\Gamma^{\mu\nu}(q,k;p,p')$, respectively, the trilinear and quadrilinear CS scalar field vertices, where q and k represent ''photon'' momenta and p and p' scalar field momenta, we have the WI

$$q^\mu\Gamma_\mu(q;p,p')=-e[\Gamma(p')-\Gamma(p)], \tag{2.8}$$

$$q^\mu\Gamma_{\mu\nu}(q,k;p,p')=-e[\Gamma_\nu(k;p+q,p')-\Gamma_\mu(k;p,p'-q)], \tag{2.9}$$

which require that $E=D=A$, leaving us with only three (we choose A,B,C) counterterms to be fixed. Explicit proof of these WI in two loops is given in Appendix A.

To determine A, B, and C we need to calculate the simple pole part of the two-point function of the CS field, $\Pi_{\mu\nu}(q)$ and the scalar field two- and six-point functions, respectively, Γ_2 and Γ_6. In graphs involving the CS field, we will extend the calculations up to two loops getting at most a simple pole in $1/\epsilon$; in graphs only involving the scalar field we will go up to four loops. So in the tensorial Feynman integrals, in which dimensional reduction could possibly differ from dimensional regularization (in the subleading terms in $1/\epsilon$), no difference between the two methods are expected in the calculation of the counterterms and in the renormalization group parameters.

Let us start with $\Pi_{\mu\nu}$. The only divergent diagrams, up to two loops, are those shown in Fig. 2 (the possible counterterm is also drawn in the figure). Their contributions are given by

$$(2a)=4e^4\int\mathcal{D}q\int\mathcal{D}k\frac{\varepsilon_{\mu\nu\rho}k^\rho}{k^2(q-p)^2(q+k)^2}, \tag{2.10}$$

$$(2b)=e^4\int\mathcal{D}q\int\mathcal{D}k\frac{(2k+q)^\alpha\varepsilon_{\alpha\beta\gamma}q^\gamma(2k+q-2p)^\beta(2k-p)_\mu(2k+2q-p)_\nu}{k^2(k+q)^2q^2(k-p)^2(q+k-p)^2}, \tag{2.11}$$

where $\mathcal{D}q\equiv\mu^\epsilon d^{3-\epsilon}q/(2\pi)^{3-\epsilon}$ and an infinitesimal imaginary part is supposed in every propagator denominator $(p^2\to p^2+i\eta,\eta\ll 1)$. Both integrals are logarithmically divergent. The divergent parts are of the form $\varepsilon_{\mu\nu\rho}p^\rho I$, where I is a scalar integral that can be calculated by the usual dimensional continuation after reducing the denominator to a single monomial through the use of Feynman parameters. The results are

$$(2a)=4\varepsilon_{\mu\nu\rho}p^\rho\left(-\frac{e^4}{96\pi^2}\frac{1}{\epsilon}\right)+\text{finite part}, \tag{2.12}$$

$$(2b)=\varepsilon_{\mu\nu\rho}p^\rho\left(\frac{e^4}{24\pi^2}\frac{1}{\epsilon}\right)+\text{finite part}. \tag{2.13}$$

As can be seen, the divergent parts of the two integrals cancel each other and we are left with only finite contributions to $\Pi_{\mu\nu}$. So the counterterm B can be chosen as $B=0$ and no infinite wave function renormalization of the CS field interacting with massless scalar field is needed. This result extends for massless matter, the result of the Coleman-Hill theorem [9].

Let us now look at the scalar two-point function, $\Gamma_2(p)$. The divergent graphs up to the second order in α and λ are shown in Fig. 3, together with the counterterm. Their contributions are given by

$$(3a)=-2e^4 i\int \mathcal{D}q\,\frac{1}{(p+q)^2}\int \mathcal{D}k\,\varepsilon_{\mu\nu\rho}\frac{k^\rho}{k^2}\varepsilon^{\nu\mu\gamma}\frac{(k+q)_\gamma}{(k+q)^2}, \tag{2.14}$$

$$(3b)=e^4 i^3\int \mathcal{D}q\,\varepsilon_{\alpha\beta\gamma}\frac{q^\gamma}{q^2}\frac{(2p+q)^\beta}{(p+q)^2}\int \mathcal{D}k\,\frac{(2p+k)^\mu(2p+2q+k)^\nu(2k+2p+q)^\alpha}{(p+k)^2(p+q+k)^2}\varepsilon_{\mu\nu\rho}\frac{k^\rho}{k^2}, \tag{2.15}$$

$$(3c)=e^4 i^3\int \mathcal{D}q\,\frac{1}{(p+q)^2}\varepsilon_{\mu\alpha\lambda}\frac{q^\lambda}{q^2}\varepsilon_{\beta\nu\rho}\frac{q^\rho}{q^2}\int \mathcal{D}k\,\frac{(2k+q)^\alpha(2k+q)^\beta(2p+q)^\mu(2p+q)^\nu}{k^2(k+q)^2}, \tag{2.16}$$

$$(3d)=-\frac{\lambda^2}{2^2.3}i^5\int \mathcal{D}k_1\mathcal{D}k_2\mathcal{D}k_3\mathcal{D}k_4\frac{1}{k_1^2k_2^2k_3^2k_4^2(p+k_1+k_2-k_3-k_4)^2}. \tag{2.17}$$

After simplification of the tensor algebra in $2+1$ dimensions we are left with multiple scalar integrals that can be made, one loop at a time, through the reduction of the denominators by successive use of Feynman parameters. The results are

$$(3a)=-2ie^4\left(\frac{p^2}{96\pi^2}\frac{1}{\epsilon}+\cdots\right), \tag{2.18}$$

$$(3b)=-ie^4\left(\frac{p^2}{12\pi^2}\frac{1}{\epsilon}+\cdots\right), \tag{2.19}$$

$$(3c)=-ie^4\left(\frac{p^2}{24\pi^2}\frac{1}{\epsilon}+\cdots\right), \tag{2.20}$$

$$(3d)=-i\frac{\lambda^2}{2^2.3}\left(-\frac{p^2}{3.2^{11}\pi^4}\frac{1}{\epsilon}+\cdots\right). \tag{2.21}$$

For the contribution (iAp^2) of the counterterm to cancel these divergences we must choose

$$A=\left(\frac{7}{48\pi^2}\alpha^2-\frac{1}{3^2.2^{13}\pi^4}\lambda^2\right)\frac{1}{\epsilon}. \tag{2.22}$$

Let us now proceed to the calculation of C, the counterterm of the coupling constant λ. For this task we need to find the divergent parts of $\Gamma_6(p_1,\ldots,p_6)$. After a lengthy analysis of the many graphs involved, we are left with the divergent contributions drawn in Fig. 4. The bullets on the diagrams 4p, 4q, 4r, 4s, and 4t signify the insertion of the counterterm in the corresponding vertex. The calculation of all of these diagrams can be reduced to the calculation of the nine integrals represented in Fig. 5. In Appendix B we show, as examples, the calculation (of the divergent parts) of 5a, 5b, 5d, and 5f. Here we present only the results;

$$\mathcal{G}(p,q)=-\frac{1}{2^5\pi^2}\frac{1}{\epsilon}+\text{finite part}, \tag{2.23}$$

$$\mathcal{H}(p)=\frac{i}{16\pi^2}\frac{1}{\epsilon}+\text{finite part}, \tag{2.24}$$

$$\Delta_3(p)=-\frac{i}{2^5\pi^2}\left[\frac{1}{\epsilon}+\left(\log\frac{4\pi\mu^2}{-p^2}-3-2\gamma-2\log 2\right)+\mathcal{O}(\epsilon)\right], \tag{2.25}$$

$$\mathcal{Y}(p)=-\frac{1}{2^{12}\pi^4}\frac{1}{\epsilon}+\text{finite part}, \tag{2.26}$$

$$\mathcal{Z}(p,q)=\frac{1}{2^{11}\pi^4}\frac{1}{\epsilon}+\text{finite part}, \tag{2.27}$$

$$\mathcal{W}(q,p)=-\frac{1}{2^{11}\pi^4}\left[\frac{1}{\epsilon^2}+\frac{1}{\epsilon}\left(2\log\frac{4\pi\mu^2}{-(p+q)^2}+8-\frac{11}{2}\gamma\right)+\text{finite part}\right], \tag{2.28}$$

$$\mathcal{M}(a,c,d)=\frac{3i}{2^6\pi^2}\frac{1}{\epsilon}+\text{finite part}, \tag{2.29}$$

$$\mathcal{N}(a,c,d)=\frac{i}{2^5\pi^2}\frac{1}{\epsilon}+\text{finite part}, \tag{2.30}$$

and

$$\mathcal{Q}(a,b,c)=\frac{1}{2^5\pi^2}\frac{1}{\epsilon}+\text{finite part}, \tag{2.31}$$

where γ is the Euler constant. In some graphs we will need the result of $\Delta_3^2(p)$, which is

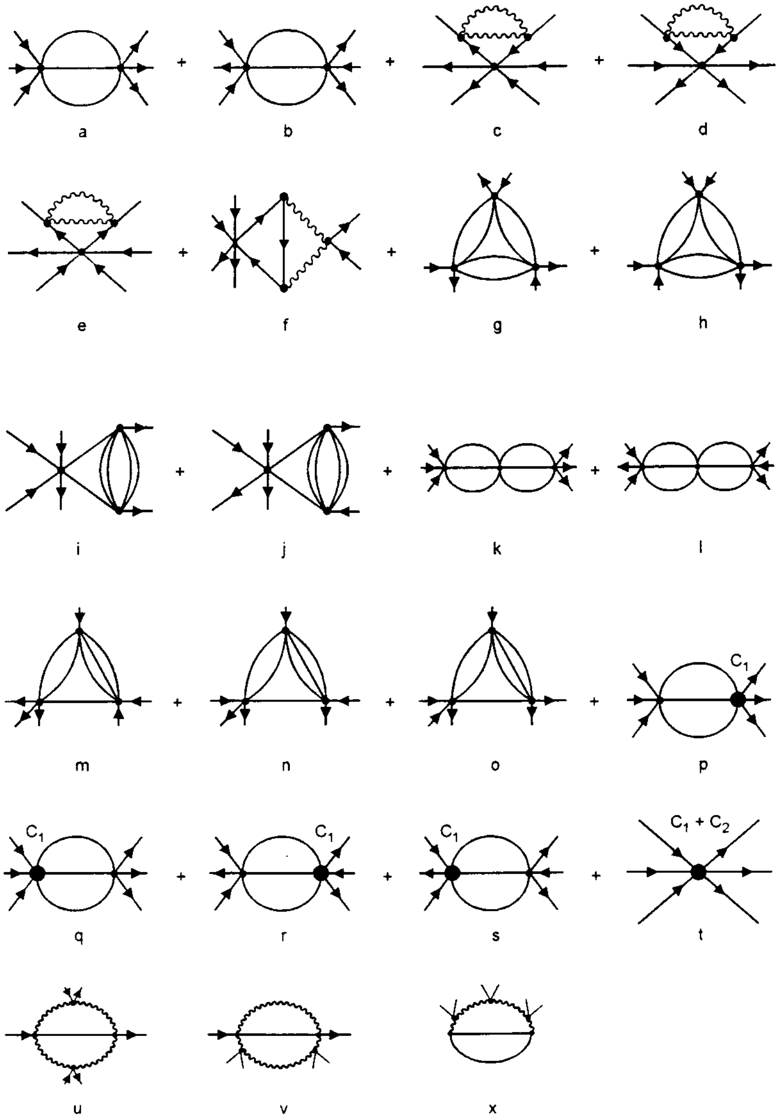

FIG. 4. Divergent contributions to the scalar six-point function. Three others, which are not drawn, but have diagrams similar to (n), (o), and (p) with the sense of all external lines reversed must also be considered.

$$\Delta_3^2(p) = -\frac{1}{2^{10}\pi^4}\left[\frac{1}{\epsilon^2} + \frac{1}{\epsilon}\left(2\log\frac{4\pi\mu^2}{-p^2} - 2(3+2\gamma+2\log 2)\right) + \text{finite part}\right]. \tag{2.32}$$

By collecting all of the contributions of Fig. 4 we can write

$$\Gamma_6(p_1,p_2,p_3,p_1',p_2',p_3')\mu^{-2\epsilon}$$

$$\begin{aligned}
&= -\frac{\lambda^2}{6}\Delta_3(p_1+p_2+p_3) - \frac{\lambda^2}{2}[\Delta_3(p_1+p_2-p_2') + 8 \text{ terms}] \\
&\quad + 2i\lambda\alpha^2[\mathcal{G}(p_1,-p_1') + 8 \text{ terms}] + 2i\lambda\alpha^2[\mathcal{G}(p_1,p_2) + 2 \text{ terms}] \\
&\quad + 2i\lambda\alpha^2[\mathcal{G}(-p_1',-p_2') + 2 \text{ terms}] - 2\lambda\alpha^2[\mathcal{H}(p_1-p_1') + 8 \text{ terms}] \\
&\quad + i\frac{5}{4}\lambda^3[\mathcal{Y}(p_1-p_1',p_2-p_2') + 5 \text{ terms}] + i\frac{3}{4}\lambda^3[\mathcal{Y}(p_1+p_2,-(p_1+p_2')) + 8 \text{ terms}] \\
&\quad + i\frac{1}{4}\lambda^3[\mathcal{Z}(p_1,p_2) + 2 \text{ terms}] - \frac{5}{12}\lambda^2[\mathcal{Z}(p_1,-p_1') + 8 \text{ terms}] + i\frac{1}{4}\lambda^3[\mathcal{Z}(-p_1',-p_2') + 2 \text{ terms}]
\end{aligned}$$

$$+i\frac{1}{36}\lambda^3[\Delta_3^2(p_1+p_2+p_3)]+i\frac{1}{4}\lambda^3[\Delta_3^2(p_1+p_2-p_1')+8\text{ terms}]+i\frac{1}{4}\lambda^3[\mathcal{W}(p_1,p_2+p_3)+2\text{ terms}]$$
$$+i\frac{1}{4}\lambda^3[\mathcal{W}(-p_1',-(p_2'+p_3'))+2\text{ terms}]+i\frac{3}{4}\lambda^3[\mathcal{W}(p_1,p_2-p_1')+17\text{ terms}]+i\frac{3}{4}\lambda^3[\mathcal{W}(-p_1',p_1-p_2')+17\text{ terms}]$$
$$+i\frac{7}{12}\lambda^3[\mathcal{W}(p_1,-(p_1'+p_2'))+8\text{ terms}]+i\frac{7}{12}\lambda^3[\mathcal{W}(-p_1',p_1+p_2)+8\text{ terms}]-\frac{\lambda C}{3}\Delta_3(p_1+p_2+p_3)$$
$$-\lambda C[\Delta_3(p_1+p_2-p_1')+8\text{ terms}]-iC+2^4\alpha^4[\mathcal{M}(p_1,p_2-p_2',p_3-p_3')+17\text{ terms}]$$
$$+2^5\alpha^4[\mathcal{N}(p_1,p_2-p_2',p_3-p_3')+17\text{ terms}]+i2^2\alpha^4[\mathcal{Q}(p_1-p_1',p_2-p_2',p_3-p_3')+35\text{ terms}], \tag{2.33}$$

from which, after imposing that the result be finite, we obtain

$$C=\lambda^2\frac{7}{48\pi^2}\frac{1}{\epsilon}-\lambda\alpha^2\left[\frac{33}{16\pi^2}\right]\frac{1}{\epsilon}+\alpha^4\frac{72}{2\pi^2}\frac{1}{\epsilon}.$$
$$-\lambda^3\left[\frac{582+57\pi^2-1092\gamma}{2^{14}\pi^4}\right]\frac{1}{\epsilon}+\lambda^3\left[\frac{49}{2^8 3^2\pi^4}\right]\frac{1}{\epsilon^2}. \tag{2.34}$$

The term proportional to α^4 in the above expression shows that the renormalization of λ is not multiplicative, a fact that will lead to an interesting effect in the renormalization group equations. In the next section, results (2.22), (2.34), and $B=0$ will be used to determine the renormalization group parameters.

III. RENORMALIZATION GROUP ANALYSES

Let us start by verifying that the CS coupling does not run. Equation (2.5) is

$$\alpha_0=\alpha\mu^{\epsilon}\frac{1+E}{(1+A)(1+B)}. \tag{3.1}$$

As we have seen in the previous section, $B=0$ and, as a consequence of the Ward identities, we also have $E=A$. Thus Eq. (3.1) reduces to

$$\alpha_0=\alpha\mu^{\epsilon}, \tag{3.2}$$

from which, in the way of [8], we obtain

$$0\equiv\mu^{1-\epsilon}\frac{d\alpha_0}{d\mu}=\epsilon\alpha+\mu\frac{d\alpha}{d\mu}, \tag{3.3}$$

and therefore

$$\beta_\alpha=\mu\frac{d\alpha}{d\mu}\bigg|_{\epsilon\to 0}\to 0, \tag{3.4}$$

showing that α does not run under a rescaling of μ or the momenta of the Green function. A similar result was obtained in [5] for a model of a scalar field interacting with a non-Abelian CS field. These results extend to massless matter, the result of the theorem of Coleman-Hill [8].

For calculating β_λ we start with Eq. (2.6),

$$\lambda_0=\mu^{2\epsilon}\frac{\lambda+C}{(1+A)^3}=\mu^{2\epsilon}(\lambda+C-3A+\cdots). \tag{3.5}$$

By substituting Eqs. (2.22) and (2.34) in this equation we obtain

$$\lambda_0=\mu^{2\epsilon}\left(\lambda+\frac{\lambda_1(\alpha,\lambda)}{\epsilon}+\cdots\right), \tag{3.6}$$

where

$$\lambda_1(\alpha,\lambda)=a(\lambda^2-c\alpha^2\lambda+d\alpha^4-b\lambda^3) \tag{3.7}$$

with

$$a=\frac{7}{48\pi^2}=0.01478, \tag{3.8}$$

$$b=\frac{1}{2^{10}7\pi^2}(1744+171\pi^2-3276\gamma)=0.0218, \tag{3.9}$$

$$c=\frac{120}{7}=17.1429, \tag{3.10}$$

and

$$d=\frac{1728}{7}=246.86. \tag{3.11}$$

From Eq. (3.6) we have

$$0=\mu^{1-2\epsilon}\frac{d\lambda_0}{d\mu}$$
$$=2\epsilon\left(\lambda+\frac{\lambda_1}{\epsilon}+\cdots\right)+\left(\mu\frac{\partial\lambda}{\partial\mu}+\mu\frac{\partial\lambda}{\partial\mu}\frac{\partial\lambda_1}{\partial\lambda}\frac{1}{\epsilon}\right.$$
$$\left.+\mu\frac{\partial\alpha}{\partial\mu}\frac{\partial\lambda_1}{\partial\alpha}\frac{1}{\epsilon}+\cdots\right), \tag{3.12}$$

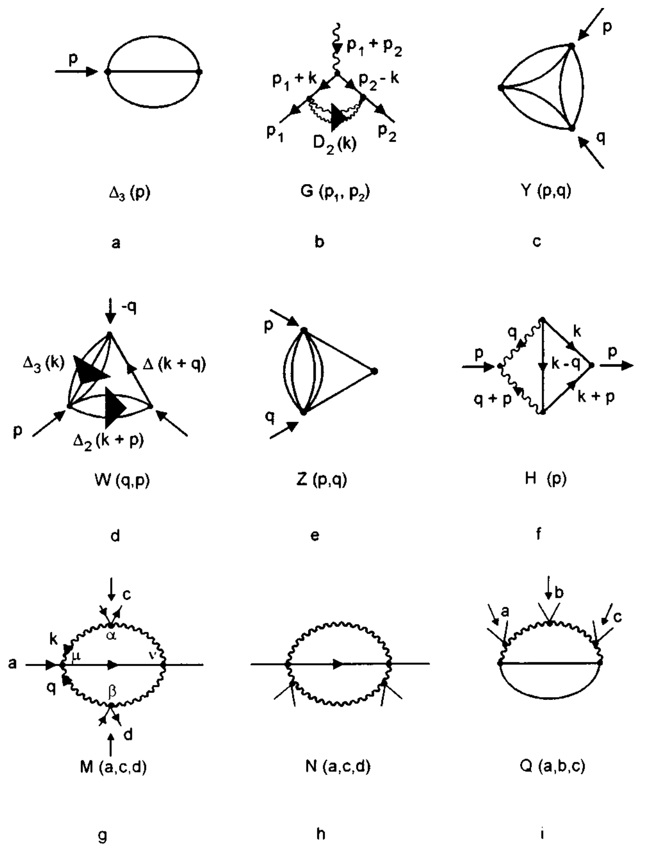

FIG. 5. Representation of the divergent integrals that appear in the diagrams of Fig. 4.

and using Eq. (3.3) we find

$$\beta_\lambda = \mu \frac{\partial \lambda}{\partial \mu}$$

$$= \left(\alpha \frac{\partial}{\partial \alpha} + 2\lambda \frac{\partial}{\partial \lambda} - 2 \right) \lambda_1(\alpha,\lambda) - 2\lambda\epsilon$$

$$= 2a(\lambda^2 - c\lambda\alpha^2 + d\alpha^4 - 2b\lambda^3) \quad (\text{for } \epsilon \to 0). \tag{3.13}$$

Up to two loops (the terms in λ^2, $\lambda\alpha^2$, and α^4) this result qualitatively coincides with that of [4] for this same model. It does not, however, coincide with the result of [14] (we will discuss this fact in the Conclusions). As can be seen from Eq. (3.10), the contribution of the four loops graphs (the term in λ^3) is small and will not qualitatively change the results for β_λ.

Making $\alpha = 0$ we go to the pure $(\phi^\dagger\phi)^3$ model. In this case β starts at zero for $\lambda = 0$ and increases monotonically with λ [1]. The model presents an infrared (IR) fix point at $\lambda = 0$. For $\alpha \neq 0$ a drastic change occurs. In this case β starts at $(4ad\alpha^4) > 0$ for $\lambda = 0$ and never vanishes in the perturbative range of the two coupling constants. A similar behavior of the β function, already in one-loop order, is shown in the Coleman-Weinberg model (CW) [15] in 3+1 dimensions. There, a dynamical symmetry breakdown occurs and masses are generated for the two fields. In [14] the effective potential was calculated in two loops and a breakdown of symmetry was also shown to appear. We would like to stress that our results for Γ^2 and Γ^6 are compatible with that conclusion. The $\Gamma^2(v)$ for the displaced field $\psi = \phi - v$, where v is a constant with dimension $(mass)^{1/2}$, would be written, in terms of the functions that we calculated for ϕ, as a series of the form $\Gamma^2(v) = \Gamma^2 + (v^2/2)\Gamma^4 + (v^4/4!)\Gamma^6 + \cdots$. As can be seen from the graphs proportional to α^4 in Fig. 5, Γ^6 receives a constant (independent of p) finite contribution. As a consequence, $\Gamma^2(v)$ will have a singularity displaced to some non-null value of p^2, compatible with a non-null dynamically generated mass for ϕ.

The anomalous dimensions of the fields A_μ and ϕ are given by

$$\gamma_A = \frac{1}{2} \frac{\mu}{Z_A} \frac{dZ_A}{d\mu}, \tag{3.14}$$

$$\gamma_\phi = \frac{1}{2} \frac{\mu}{Z_\phi} \frac{dZ_\phi}{d\mu}. \tag{3.15}$$

As shown in Sec. II, $Z_A = 1 + B = 1$ and so $\gamma_A = 0$. By writing

$$Z_\phi = 1 + A = 1 + \frac{a_1(\alpha,\lambda)}{\epsilon} + \cdots, \tag{3.16}$$

where a_1 is given in Eq. (2.22), we can write Eq. (3.15) in the form

$$2\left(1 + \frac{a_1}{\epsilon} + \cdots\right)\gamma_\phi = \mu \frac{\partial \lambda}{\partial \mu} \frac{\partial a_1}{\partial \lambda} \frac{1}{\epsilon} + \mu \frac{\partial \alpha}{\partial \mu} \frac{\partial a_1}{\partial \alpha} \frac{1}{\epsilon} + \cdots, \tag{3.17}$$

and using Eqs. (3.3) and (3.13) we obtain

$$\gamma_\phi = -\lambda \frac{\partial a_1}{\partial \lambda} - \frac{\alpha}{2} \frac{\partial a_1}{\partial \alpha}. \tag{3.18}$$

By substituting a_1 from Eq. (2.22), in Eq. (3.18) we have

$$\gamma_\phi = -\frac{7}{48\pi^2}\alpha^2 + \frac{1}{3^2.2^{12}\pi^4}\lambda^2. \tag{3.19}$$

The contribution in α^2 qualitatively agrees with the result of [4]. The term in λ^2 comes from four-loop graphs (not calculated in [4]) and is very small compared to the term in α^2. It can be seen from Eq. (3.18) that the scalar field dimension, $D_\phi = \frac{1}{2} + \gamma_\phi$, decreases with the coupling to the CS field. As is well known, in the nonperturbative approach in quantum mechanics, the coupling of matter fields to a CS field changes the spin and statistics of the matter fields, driving bosons into anyons and also, for strong enough couplings, into fermions. Based on these results, there is a conjecture in the literature [10] that, even in the perturbative quantum field approach (in which the strength $\alpha \ll 1$), the dimension of a boson coupled to a CS should receive an increase in the direction of the fermion dimension $d_\psi = 1$ (for the corresponding problem of fermions, a decrease in the direction of the boson dimension should be expected). As

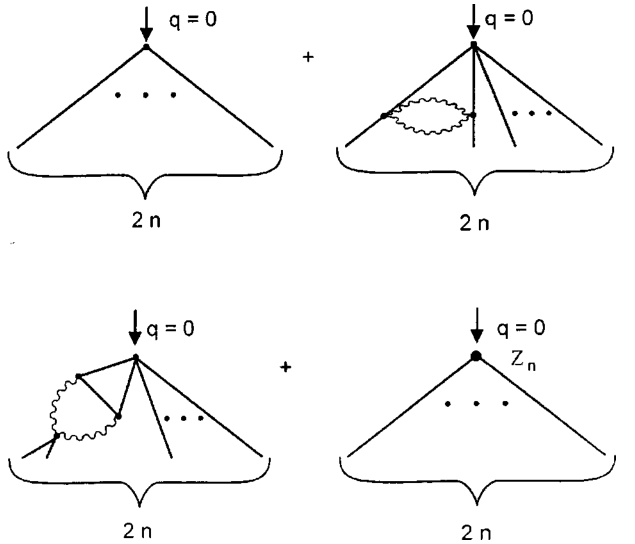

FIG. 6. Divergent contributions to $\Gamma_{[(\phi^*\phi)^n]}(2n)$; that is, the $2n$-point function with one insertion of the composite operator $[(\phi^*\phi)^n]$.

shown in Eq. (3.19) this conjecture is not realized; the coupling to the CS field works in the direction of decreasing the dimension of ϕ.

To go a bit further in testing this conjecture, we have also looked at the anomalous dimensions of the composite operators $[(\phi^\dagger\phi)^n]$, where n is an integer number. As we are mainly interested in the effect of the coupling of the boson to the CS field, to simplify the calculations we will restrict the analysis to $\lambda=0$. In terms of monomials of ϕ this composite operator can be written [12] as

$$[(\phi^\dagger\phi)^n]=Z_n(\phi^\dagger\phi)^n+Z^0_{n-1}(\phi^\dagger\phi)^{n-1}+Z^2_{n-2}(\phi^\dagger\phi)^{n-2}(\phi^\dagger\partial^2\phi)+\cdots. \quad (3.20)$$

The determination of the Z^i_m $(m\leqslant n)$ requires the calculation of the divergent parts of the $2m$ scalar field 1PI vertex functions with the insertion of one integrated composite operator

$$\Gamma_{[(\phi^*\phi)^n]}(x_1,\ldots,y_m)=\int d^3z\langle T[(\phi^\dagger\phi)^n](z)\times\phi(x_1)\cdots\phi(x_m)\phi^\dagger(y_1)\cdots\phi^\dagger(y_m)\rangle \quad (3.21)$$

or, in momentum space, the $\Gamma_{[(\phi^*\phi)^n]}(p_1,\ldots,p_{2m})$ function with zero momentum q entering through the special vertex $[(\phi^\dagger\phi)^n]$. Up to order α^2, the divergent graphs contributing to $\Gamma_{[(\phi^*\phi)^n]}(p_1,\ldots,p_{2n})$ are shown in Fig. 6. In Fig. 7 we draw some of the graphs that could contribute to $\Gamma_{[(\phi^*\phi)^n]}(p_1,\cdots p_{2(n-1)})$. Diagrams in Fig. 7 are in fact all null, which implies that the renormalization parameters Z^i_{n-1} also vanish. The same can be shown to be true for all Z^i_m in which any $m<n$. So the right-hand side of Eq. (3.21) reduces to only the first monomial and $[(\phi^\dagger\phi)^n]$ does not mix with other operators (mixing will, however, appear if we consider $\lambda\neq0$). Its renormalization only requires the calculation of Z_n, which means to calculate the divergent parts of the graphs in Fig 6. The involved Feynman integrals are the $\mathcal{G}(p,q)$ and $\mathcal{H}(p)$ from Fig. 5. By writing $Z_n=1+A_n$ we have

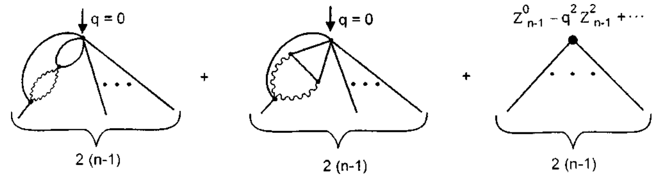

FIG. 7. Some possible contributions to $\Gamma_{[(\phi^*\phi)^n]}[2(n-1)]$ that is the $2(n$-1)-point function with one insertion of the composite operator $[(\phi^*\phi)^n]$.

$$\Gamma_{[(\phi^*\phi)^n]}(p_1,\ldots,p_{2n})=(n!)^2[A_n-(4n^2-2n)\alpha^2\mathcal{G}-2in^2\alpha^2\mathcal{H}]+\text{finite graphs}, \quad (3.22)$$

and we have for A_n

$$A_n=\text{DivPart}\{(4n^2-2n)\alpha^2\mathcal{G}+2in^2\alpha^2\mathcal{H}\}=-\frac{4n^2-n}{16\pi^2}\frac{\alpha^2}{\epsilon}, \quad (3.23)$$

where ''DivPart'' stands for keeping only the divergent part of the following expression.

With these results for Z_m and Eq. (2.22) for Z_ϕ, Eq. (3.20), rewritten in terms of the unrenormalized [see also Eq. (2.2)] field ϕ_0, becomes

$$[(\phi^\dagger\phi)^n]=Z^{-1}_{cn}(\phi^\dagger_0\phi_0)^n, \quad (3.24)$$

where

$$Z_{cn}=(Z_n)^{-1}(Z_\phi)^n=1+\frac{a_{cn}(\alpha)}{\epsilon}+\cdots \quad (3.25)$$

and

$$a_{cn}(\alpha)=\frac{\alpha^2}{4\pi^2}\left(n^2+\frac{n}{3}\right). \quad (3.26)$$

By deriving the two sides of Eq. (3.24) with respect to μ and remembering that ϕ_0 is independent of μ we have

$$\mu\frac{d}{d\mu}[(\phi^\dagger\phi)^n]=-\gamma_{cn}[(\phi^\dagger\phi)^n], \quad (3.27)$$

where

$$\gamma_{cn}=\frac{\mu}{Z_{cn}}\frac{dZ_{cn}}{d\mu} \quad (3.28)$$

is the anomalous dimension of the composite operator. Going through the same steps that lead Eq. (3.16) to Eq. (3.18) we find

$$\gamma_{cn}=-\alpha\frac{da_{cn}}{d\alpha}=-\frac{\alpha^2}{2\pi^2}\left(n^2+\frac{n}{3}\right). \tag{3.29}$$

The dimension of the composite operator $[(\phi^\dagger\phi)^n]$ becomes

$$D_{[(\phi^\dagger\phi)^n]}=n-\frac{\alpha^2}{2\pi^2}\left(n^2+\frac{n}{3}\right). \tag{3.30}$$

This result is in disagreement with [6]. Their calculation seems to miss the contribution of the second graph in Fig. 6. But it is not this fact that makes the major difference. Our counting of the combinatorial factors of the graphs in Fig. 6 gives a term proportional to n^2 (besides the term in n) and different from theirs, which is only proportional to n.

No matter if the composite operator is super-renormalizable ($n<3$), renormalizable ($n=3$), or nonrenormalizable ($n>3$), the effect of the coupling to the CS field is to lower its dimension. Nevertheless, the lowest nonrenormalizable operator, $(\phi^\dagger\phi)^4$, with effective dimension $D_4=4-(52/6\pi^2)\alpha^2$ will never, in the perturbative regime, be driven to be renormalizable. Yet due to the quadratic dependence of the anomalous dimension on n, given any $\alpha\ll 1$, the operators $[(\phi^\dagger\phi)^n]$ with n larger than $n_c\simeq(2\pi^2/\alpha^2)-\frac{10}{3}\gg 1$ have their operator dimensions driven to values smaller than three.

To finish this section, let us look at the renormalization group equations for the $\Gamma_{(2n)}(p,\lambda,\alpha,\mu)$ functions (p is short for the $2n$ external momenta). As the four loops contributions are very small, we will restrict the analysis to two loops. As β_α and γ_A are null we have the renormalization group equation

$$\left(\mu\frac{\partial}{\partial\mu}+\beta_\lambda\frac{\partial}{\partial\lambda}-2n\gamma_\phi\right)\Gamma_{(2n)}(p,\lambda,\alpha,\mu)=0. \tag{3.31}$$

The solution of this equation can be written as

$$\Gamma_{(2n)}(p,\lambda,\alpha,\mu)=\Gamma_{(2n)}(p,\bar{\lambda},\alpha,\mu s_{\bar{\lambda},\lambda})s_{\bar{\lambda},\lambda}^{2n\gamma_\phi}, \tag{3.32}$$

where we used the fact that up to two loops, $\gamma_\phi=-(7/48\pi^2)\alpha^2$ does not change with s. In the above equation, $s_{\bar{\lambda}\lambda}$ stands for the solution of

$$s\frac{d}{ds}\bar{\lambda}=\beta_\lambda(\bar{\lambda})$$

$$=2a(\bar{\lambda}^2-c\alpha^2\bar{\lambda}+d\alpha^4), \tag{3.33}$$

with the condition $\bar{\lambda}(s=1)=\lambda$, that is,

$$s_{\bar{\lambda}\lambda}=\exp\left\{\frac{1}{af\alpha^2}\left[\tan^{-1}\left(\frac{2\bar{\lambda}}{f\alpha^2}-\frac{c}{f}\right)-\tan^{-1}\left(\frac{2\lambda}{f\alpha^2}-\frac{c}{f}\right)\right]\right\}$$

$$\cong\exp\left\{\frac{2.86}{\alpha^2}\left[\tan^{-1}\left(\frac{\bar{\lambda}}{12\alpha^2}-0.71\right)-\tan^{-1}\left(\frac{\lambda}{12\alpha^2}-0.71\right)\right]\right\}, \tag{3.34}$$

where $f=(4d-c^2)^{1/2}$. As β_λ is non-null for $\lambda=0$ (for $\alpha\neq 0$), this equation is well defined if we choose $\lambda=0$. With this choice, in Eq. (3.34) we can write

$$\Gamma_{(2n)}(p,\bar{\lambda},\alpha,\mu)=\Gamma_{(2n)}(p,0,\alpha,\mu s_{\bar{\lambda}0}^{-1})s_{\bar{\lambda}0}^{-2n\gamma_\phi}. \tag{3.35}$$

This equation shows that, up to two loops, the $\Gamma_{(2n)}$ functions of the model defined by the Lagrangian (2.7), can be obtained from the corresponding $\Gamma_{(2n)}$ for the model where only the interaction term with the A_μ field is present or from what is equivalent, from the calculation of the subset of diagrams contributing to $\Gamma_{(2n)}$, which only involves the interaction vertex with the A_μ field. A short inspection of the CW [15] results shows that a similar fact is also true for that model (at least in one loop).

IV. CONCLUSIONS

The coupling to the CS field lowers the dimension of ϕ and of $(\phi^\dagger\phi)^n$. This goes in the opposite direction of the conjecture that the transmutation of the boson into anyon (due to the coupling to the CS field) should be signaled by the dimension of these operators to increase in the direction of the canonical dimension of a fermion field ψ and their composite operators $(\psi^\dagger\psi)^n$, respectively.

In the present paper, as in previous calculations in the literature, the function β_α and the anomalous dimension of the CS field are shown to vanish; the CS coupling constant α does not run with the change of the energy scale. The function β_λ instead shows a drastic change in the presence of the CS field. From an IR trivial fix point for the pure $\lambda(\phi^*\phi)^3$ interaction, the model is driven, to a phase in which no fix point appears for β_λ, in a behavior similar to that of β_λ for the model of Coleman-Weinberg [15].

In [14], the renormalization group functions were calculated up to two loops, although their main aim was to study the effective potential and dynamical symmetry breakdown. The model of [14], defined by their Lagrangian (2.1), can be made to coincide with ours by deleting the $\lambda(\phi^*\phi)^2$ interaction and the $m^2(\phi^*\phi)$ mass term, that is, by making the λ and m zero. Considering also that their coefficient, ν, of the $(\phi^*\phi)^3$ interaction differs from our λ by a factor of 2/5, which also implies in a 2/5 factor of difference in the corresponding β functions, their results [equations (10.7–9) and (11.8)], after being translated to our notation, can be summarized as: (1) $\beta_\alpha=0$ and $\gamma_A=0$. These results are in agreement with Eq. (3.4) and the observation below Eq. (3.15); and (2) $\gamma_\phi=\mathcal{O}(\lambda^2)$ and $\beta_\lambda=2a\lambda^2+\mathcal{O}(\lambda^3)$, both independent of α. Our results (3.13) and (3.19) differ from these last ones by terms dependent on the CS coupling α. Their conclusion is that the model has an IR trivial fix point in λ. Ours instead is that β_λ never vanishes, a result similar to that of CW in a model in which a dynamical symmetry breakdown occurs. A dynamical symmetry breakdown was also seen in [14] for the present model. Our result for β looks so, in accordance with their result on symmetry breakdown.

The discrepancies between our β function and the β function of [14] can be attributed to the different regularization schemes we are using. In [14], the model is regularized through a full-dimensional regularization by extending out of 3D all the tensor structures (including the definition of the $\epsilon_{\mu\nu\rho}$) that appear in the Feynman graphs. As they conclude, in that method the renormalizability of the model is only achieved if an extra regularization, represented by a Maxwell term for the A^μ field (in addition to the CS one), is introduced. Their method requires that this extra regularization be dismissed (their parameter ''a'' taken to zero), only after the continuation back to 3D is made. As can be seen from their results (11.8), some of their β functions become singular when $a\rightarrow 0$, showing that a better understanding of the structure of the renormalization group equation is still lacking in that method. Also, as discussed in their section 10, if a regularization directly in 3D exists and were used, γ_ϕ and β_λ would also be expected to depend on α.

In this paper we used the dimensional reduction regularization scheme, in which all the tensor contractions are first made in 3D and only the remaining scalar Feynman integrals are extended out of 3D. We explicitly verified that this method controls all the UV infinities and preserves the Ward identities and so, the gauge covariance, up to the order of approximation in which we are working (two loops in graphs involving the CS propagator and four loops in graphs only involving the scalar propagator). Although we cannot say that it is a regularization directly in 3D, our results are consistent with the above mentioned discussion in [14].

As a definitive answer to this problem is desirable, we are presently working in a related model, using a direct 3D version of the Bogolubov-Parasiuk-Hepp-Zimmermann (BPHZ) renormalization method. The preliminary results confirm those of the present paper for the renormalization group functions, together with the dynamical symmetry breakdown obtained in [14].

To finalize we would like to summarize the results of two previous papers [11], in which we studied the scale behavior of fermions interacting with a CS field. In the first one, a single fermion with its most general four-fermion nonrenormalizable self-interaction $g(\bar{\psi}\psi)^2$ was considered. We saw that although ψ gets a negative anomalous dimension, making its operator dimension approach that of a boson, no definite pattern of approach to a bosonic scale behavior due to the interaction with the CS field is seen for composite operators; the super-renormalizable operator $\bar{\psi}\psi$ gets a negative anomalous dimension, but the nonrenormalizable operator $(\bar{\psi}\psi)^2$ gets a positive one. In the second paper an extended version of this model with N (small) fermion fields, with their most general four-fermion interaction $g(\bar{\psi}\psi)^2 + h(\bar{\psi}\gamma^\mu\psi)^2$, was considered. We studied operators of canonical dimension four. We showed that one of them has a positive anomalous dimension, the other has a very small negative anomalous dimension, and the third one, more interesting from the renormalization viewpoint, has a negative anomalous dimension, making, through a fine tuning of the coupling constants, its operator dimension as close to three as desired. Nevertheless, no general pattern of approach to a bosonic like behavior (negative anomalous dimension), as advanced by the conjecture in the literature, was seen.

ACKNOWLEDGMENTS

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APPENDIX A: THE WARD IDENTITIES

The two relations among the counterterms A to E can be obtained from the WI among the 1PI four-linear photon-scalar vertex, $\Gamma_{\mu\nu}$, the trilinear photon-scalar vertex, Γ_μ, and the scalar self energy Γ_2. In tree approximation they are given by (see Fig. 1) $\Gamma_{\mu\nu}=2ie^2\mu^\epsilon g_{\mu\nu}$, $\Gamma_\mu=-ie\mu^{\epsilon/2}(p'+p)_\mu$, and $\Gamma_2=iAp^2$. It is easy to see that they satisfy the relations

$$q^\mu\Gamma_\mu(q;p,p')=-e\mu^{\epsilon/2}\{\Gamma_2(p')-\Gamma_2(p)\}, \qquad (A1)$$

$$q^\mu\Gamma_{\mu\nu}(q,q';p,p')=-e\mu^{\epsilon/2}\{\Gamma_\nu(q';p'-q',p') - \Gamma_\nu(q';p,p+q')\}. \qquad (A2)$$

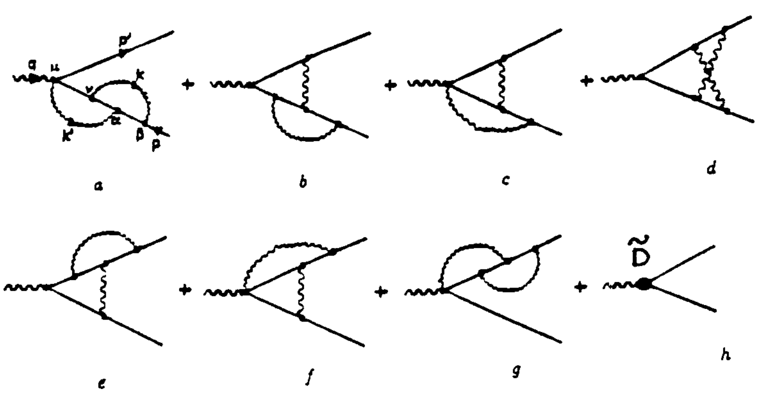

FIG. 8. An example of a family of diagrams contributing to $\Gamma_\mu(q;p,p')$.

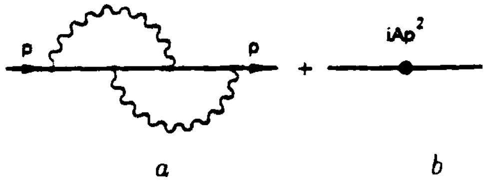

FIG. 9. The diagrams contributing to $\Gamma(p)$, related by the Ward identity, to the family of diagrams in Fig. 8.

As we explicitly verified, these relations are, in fact, valid up to two-loop order. Instead of considering the WI among the sum of all graphs up to two loops contributing to each of the three vertex functions above, we can take advantage of the fact that they can be separated in subclasses to be seen as separately related through the WI (A1) and (A2). As an example consider the graphs 8(a)–8(h) contributing to Γ_μ and 9(a) and 9(b) contributing to Γ_2. Let us call $\tilde{\Gamma}_\mu$ the sum of the contributions of diagrams 8(a)–8(q) and $\tilde{\Gamma}_2$ the graph 9(a). Let $\tilde{D}$ and $\tilde{A}$ be the possible divergent contributions to the counterterms D and A, chosen so as to make the sums of the graphs in Figs. 8 and 9, respectively finite. By using dimensional reduction regularization and explicitly writing all of the Feynman integrands involved, we can verify that

$$q^\mu\{\tilde{\Gamma}_\mu(q;p,p')-ie\mu^{\epsilon/2}(p'+p)_\mu\tilde{D}\}$$
$$=-e\mu^{\epsilon/2}\{[\tilde{\Gamma}_2(p')+ip'^2\tilde{D}]-[\tilde{\Gamma}_2(p)+ip^2\tilde{D}]\}. \quad \text{(A3)}$$

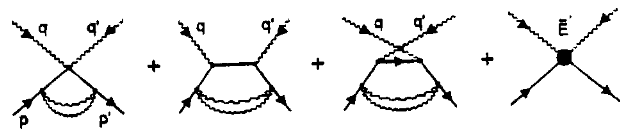

FIG. 10. An example of a family of diagrams contributing to $\Gamma_{\mu\nu}(q,q';p,p')$.

As $\tilde{D}$ is chosen to make the curly bracket in the left-hand side of these equations finite, the right-hand side is also finite, which implies that $ip^2\tilde{D}=-\text{DivPart}\{\tilde{\Gamma}_2(p)\}\equiv ip^2\tilde{A}$; that is, $\tilde{D}=\tilde{A}$. A more direct verification is obtained by explicitly calculating

$$ie\mu^{\epsilon/2}(p'+p)_\mu\tilde{D}=\text{DivPart}\{\tilde{\Gamma}_\mu(q;q,p')\} \quad \text{(A4)}$$

and

$$ip^2\tilde{A}=-\text{DivPart}\{\tilde{\Gamma}_2(p)\}. \quad \text{(A5)}$$

The only really divergent graphs contributing to $\tilde{\Gamma}_\mu$ are 8(a) and 8(g). By going through the calculation of the divergent parts of 8(a) plus 8(g), as exemplified in Appendix B, we obtain

$$ie(p'+p)_\mu\tilde{D}=\text{DivPart}\left\{(-ie)^3(ie^2)(i)^3\frac{12}{3!}\int \mathcal{D}k\int \mathcal{D}k'\,\varepsilon_{\beta\nu\rho}\frac{k^\rho}{k^2}\varepsilon_{\alpha\mu\gamma}\frac{k'^\gamma}{k'^2}\right.$$
$$\left.\times\frac{(2p+k)_\beta(2p+2k+k')_\alpha(2p+2k'+k)_\nu}{(p+k)^2(p+k')^2(p+k+k')^2}+p\leftrightarrow p'\right\}$$
$$=i\frac{e^5}{12\pi^2}\frac{1}{\epsilon}p_\mu+i\frac{e^5}{12\pi^2}\frac{1}{\epsilon}p'_\mu, \quad \text{(A6)}$$

that is, $\tilde{D}=(\alpha^2/12\pi^2)(1/\epsilon)$.

For $\tilde{A}$ we have

$$ip^2\tilde{A}=-\text{DivPart}\{\tilde{\Gamma}(p)\}$$
$$=-\text{DivPart}\{\text{Graph 3(b)}\}$$
$$=i\frac{e^4}{12\pi^2}\frac{1}{\epsilon}p^2, \quad \text{(A7)}$$

as given by Eq. (2.8). So we have $\tilde{D}=\tilde{A}=(\alpha^2/12\pi^2)(1/\epsilon)$.

An example of a subset of graphs that match through the 2^d WI are depicted in Figs. 10 and 11. The identification of $\tilde{E}'=\tilde{D}'$ follows through the same steps as in the example above.

APPENDIX B: FEYNMAN INTEGRALS

To illustrate the method adopted to get the divergent parts of the Feynman integrals that appear in the paper, we will

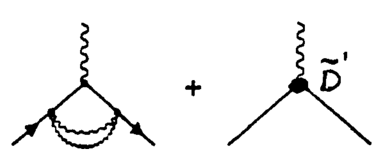

FIG. 11. The diagrams contributing to Γ_μ, related by the Ward identity to the family of diagrams in Fig. 10.

explicitly show as examples the calculation of the diagrams 5(a), 5(b), 5(d), and 5(f). Let us start with 5(a). In the figure, $\Delta(k)=i/(k^2+i\eta)$ as usual and $\Delta_2(k)$ and $\Delta_3(k)$ stand for the subgraphs formed respectively by two and three scalar propagators connecting two vertices, with total momentum k passing through. Its integration can be done successively one loop at time, first finding Δ_2 and then Δ_3. $\Delta_2(p)$ is given by

$$\Delta_2(p)=\int \mathcal{D}k\,\frac{i}{k^2+i\eta}\,\frac{i}{(k+p)^2+i\eta}, \tag{B1}$$

where $\mathcal{D}k=\mu^{\epsilon}d^{3-\epsilon}k/(2\pi)^{3-\epsilon}$. By introducing a Feynman parameter through the use of the identity

$$\frac{1}{A^{\alpha}B^{\beta}}=\frac{\Gamma(\alpha+\beta)}{\Gamma(\alpha)\Gamma(\beta)}\int_0^1 dx\frac{x^{\alpha-1}(1-x)^{\beta-1}}{[Ax+B(1-x)]^{\alpha+\beta}}, \tag{B2}$$

the k integration can be done [12] and then the parametric integration [13] to give

$$\Delta_2(p)=-i\,\frac{(4\pi\mu^2)^{\epsilon/2}}{(4\pi)^{3/2}}\,\frac{\Gamma^2\left(\frac{1}{2}-\frac{\epsilon}{2}\right)\Gamma\left(\frac{1}{2}+\frac{\epsilon}{2}\right)}{\Gamma(1-\epsilon)}\times(-p^2-i\eta)^{-[(1/2)-(\epsilon/2)]}. \tag{B3}$$

$\Delta_3(q)$ can be written as

$$\Delta_3(q)=\int \mathcal{D}p\,\frac{i}{(p+q)^2+i\eta}\Delta_2(p). \tag{B4}$$

This integration can also be done following the same steps as for $\Delta_2(p)$, after explicitly substituting in Eq. (B3) for $\Delta_2(p)$. The result is

$$\Delta_3(q)=-\frac{i}{(4\pi)^3}\,\frac{\Gamma^3\left(\frac{1}{2}-\frac{\epsilon}{2}\right)\Gamma(\epsilon)}{\Gamma\left(\frac{3}{2}-\frac{3\epsilon}{2}\right)}\left(-\frac{4\pi\mu^2}{q^2+i\eta}\right)^{\epsilon}. \tag{B5}$$

For 5(d) we have

$$\mathcal{W}(q,p)=\int \mathcal{D}k\,\frac{i}{(k+q)^2+i\eta}\Delta_2(k+p)\Delta_3(k). \tag{B6}$$

This integral can be found by first reducing the three denominators to a single one, by twice using Eq. (B2) to get a single denominator and then doing the k integration [12]. In terms of the two remaining Feynman parameters it has the form

$$\mathcal{W}(q,p)=-\frac{1}{(4\pi)^6}\,\frac{\Gamma(2\epsilon)\Gamma^5\left(\frac{1}{2}-\frac{\epsilon}{2}\right)}{\Gamma(1-\epsilon)\Gamma\left(\frac{3}{2}-\frac{3\epsilon}{2}\right)}\left(-\frac{4\pi\mu^2}{p^2+i\eta}\right)^{2\epsilon}I_{\epsilon}(q,p), \tag{B7}$$

where $I_{\epsilon}(q,p)$ is given by

$$I_{\epsilon}(q,p)=\int_0^1 dy(1-y)^{\epsilon-1}f_{\epsilon}(y), \tag{B8}$$

and

$$f_{\epsilon}(y)=\int_0^1 dx\,x^{-(1/2)+(\epsilon/2)}y^{(1/2)+(\epsilon/2)}\left\{\frac{q^2}{p^2}[y^2(1-x)^2-y(1-x)]+2\frac{p\cdot q}{p^2}y^2x(1-x)+yx(yx-1)\right\}^{-2\epsilon}. \tag{B9}$$

$I_{\epsilon}(q,p)$ has a single pole in ϵ coming from the integration region in the vicinity of $y=1$. As Eq. (B7) already has a factor $\Gamma(2\epsilon)$, the integral (B6) will present both a single and a double pole in ϵ. To separate their contributions we must calculate the first two terms (single pole and the ϵ independent term) of the Laurent expansion of $I_{\epsilon}(q,p)$. We have

$$I_{\epsilon}(q,p)=I_{1\epsilon}(q,p)+I_{2\epsilon}(q,p)=\frac{A_1}{\epsilon}+(B_1+B_2)+(C_1+C_2)\epsilon+\cdots, \tag{B10}$$

where

$$I_{1\epsilon}(q,p)=\int_0^1 dy(1-y)^{\epsilon-1}f_{\epsilon}(1)=\frac{A_1}{\epsilon}+B_1+C_1\epsilon+\cdots, \tag{B11}$$

$$I_{2\epsilon}(q,p)=\int_0^1 dy(1-y)^{\epsilon-1}f_{\epsilon}(y)-f_{\epsilon}(1)=B_2+C_2\epsilon+\cdots, \tag{B12}$$

where A_1, B_1, and B_2 are still to be determined. B_2 is given by

$$B_2=I_{20}(q,p)=\int_0^1 dy(1-y)^{-1}\int_0^1 dx(y^{1/2}-1)x^{-1/2}$$
$$=4(-1+\log 2). \tag{B13}$$

A_1 and B_1 come from

$$I_{1\epsilon}(p,q)$$
$$=\int_0^1 dy(1-y)^{\epsilon-1}\int_0^1 dx\, x^{(\epsilon/2)-(1/2)}$$
$$\times(x^2-x)^{-2\epsilon}\left\{\frac{(q-p)^2}{(p^2)}\right\}^{-2\epsilon}$$
$$=(-1)^{-2\epsilon}\left(\frac{p^2}{(q-p)^2}\right)^{2\epsilon}\frac{\Gamma\left(\frac{1}{2}-\frac{3\epsilon}{2}\right)\Gamma(1-2\epsilon)\Gamma(\epsilon)}{\Gamma\left(\frac{3}{2}-\frac{7\epsilon}{2}\right)\Gamma(1+\epsilon)}. \tag{B14}$$

The results are $A_1=2$ and

$$B_1=2\left\{5-2\log 2-\frac{7\gamma}{2}+2\log\left(-\frac{p^2}{(p-q)^2}\right)\right\}.$$

Multiplying the Laurent expansion of $I_\epsilon(p,q)$ by the Laurent expansion of the multiplying factor in Eq. (B7) we obtain

$$\mathcal{W}(q,p)=-\frac{1}{2^{11}\pi^4}\frac{1}{\epsilon^2}-\frac{1}{2^{10}\pi^4}\left\{4-\frac{11}{4}\gamma + \log\left(-\frac{4\pi\mu^2}{(p-q)^2}\right)\right\}\frac{1}{\epsilon}+\text{finite part}. \tag{B15}$$

Let us go to (5b). The subdiagram $D_2(k)$ is given by

$$D_2(k)=\int \mathcal{D}q\,\varepsilon^{\mu\nu\lambda}\frac{q_\lambda}{q^2+i\eta}\varepsilon_{\nu\mu\rho}\frac{(q+k)^\rho}{(q+k)^2+i\eta}. \tag{B16}$$

After contracting the tensors in the $2+1$ dimension we are left with the (finite) integral

$$D_2(k)=-2\int \mathcal{D}q\frac{k\cdot q}{[q^2+i\eta][(q+k)^2+i\eta]}$$
$$=-\frac{i}{8}\frac{(4\pi\mu^2)^{(\epsilon/2)}}{(-k^2-i\eta)^{-(1/2)+(\epsilon/2)}}, \tag{B17}$$

where ϵ was made zero whenever possible. Graph (5b) is given by

$$\mathcal{G}(p_1,p_2)=\int \mathcal{D}k\frac{i}{(p_1+k)^2+i\eta}\frac{i}{(p_2+k)^2+i\eta}D_2(k). \tag{B18}$$

This integral is logarithmically divergent and its residue is independent of p_1 and p_2. To get this residue it is sufficient to calculate it for $p_2=-p_1$,

$$\mathcal{G}|_{p_2=-p_1}=\int \mathcal{D}k\frac{i}{[-(p+k)^2-i\eta]^2}\frac{1}{(-k^2-i\eta)^{-(1/2)+(\epsilon/2)}}, \tag{B19}$$

where we have put $\epsilon=0$ whenever possible. After introducing a Feynman parameter through Eq. (B2) and integrating in k we find that

$$\mathcal{G}|_{p_2=-p_1}=-\frac{1}{2^5\pi^2}\Gamma(\epsilon)(-p_1^2)^{-\epsilon}=-\frac{1}{32\pi^2}\frac{1}{\epsilon}+\cdots. \tag{B20}$$

The contribution of diagram (5f) is given by

$$\mathcal{H}(p)=-i\int \mathcal{D}q\mathcal{D}k\,\varepsilon^{\mu\nu\rho}\frac{(p+q)_\rho}{(p+q)^2}\varepsilon^{\alpha\beta\lambda}\frac{q_\lambda}{q^2}\frac{g_{\nu\alpha}(2k+p-q)_\mu(2k-q)_\beta}{(k+p)^2(k-q)^2k^2} \tag{B21}$$

or

$$\mathcal{H}(p)=-2i\varepsilon^{\mu\nu\rho}\varepsilon^{\alpha\beta\lambda}g_{\nu\alpha}\frac{\mu^\epsilon}{(2\pi)^d}\int \mathcal{D}q\frac{1}{(p+q)^2q^2}I_{\beta\mu\lambda\rho}(q), \tag{B22}$$

where

$$I_{\beta\mu\lambda\rho}(q)=\int d^dk\frac{2k_\beta k_\mu(q_\nu p_\rho+q_\nu q_\rho)+k_\beta(q_\nu q_\rho p_\mu-q_\nu q_\mu p_\rho)}{(k+p)^2(k-q)^2k^2}. \tag{B23}$$

Using the identity

$$\frac{1}{ABC}=2\int_0^1 y\,dy\int_0^1 dx\,\frac{1}{[C(1-y)+y(Ax+B(1-x))]^3} \tag{B24}$$

and doing the k integration we obtain

$$\mathcal{H}(p)=-\frac{2\Gamma\left(2-\frac{d}{2}\right)\mu^{\epsilon}}{2^d\pi^{d/2}}\varepsilon^{\mu\nu\rho}\varepsilon^{\alpha\beta\lambda}g_{\nu\alpha}\int_0^1 y\,dy\int_0^1 dx\,\frac{1}{[-a']^{3-(d/2)}}\int\mathcal{D}q\,\frac{1}{(p+q)^2q^2}$$
$$\times\left[\frac{q_\nu q_\mu p_\rho p_\beta xy[(4-d)y(x-1)+1]+q_\nu q_\rho p_\mu p_\beta xy[(4-d)xy-1]}{\left[-q^2-2q\cdot p\frac{b'}{a'}-p^2\frac{c'}{a'}\right]^{3-(d/2)}}+\frac{a'g_{\beta\mu}(q_\nu p_\rho+q_\nu q_\rho)}{\left[-q^2-2q\cdot p\frac{b'}{a'}-p^2\frac{c'}{a'}\right]^{2-(d/2)}}\right], \tag{B25}$$

with $a'=y(x-1)[y(x-1)+1], b'=xy^2(x-1)$, and $c'=xy(xy-1)$. The only divergent term is the last monomial in the square bracket of Eq. (B25). We can write

$$\mathcal{H}(p)=\frac{\Gamma\left(2-\frac{d}{2}\right)\mu^{\epsilon}}{2^{d+1}\pi^{d/2}}\varepsilon^{\mu\nu\rho}\varepsilon^{\alpha\beta\lambda}g_{\nu\alpha}\int_0^1 dy\,y\int_0^1 dx\,\frac{1}{[-a']^{2-(d/2)}}\,\frac{\mu^{\epsilon}}{(2\pi)^d}I_{\text{DivPart}}(p)+\text{fin. parts}, \tag{B26}$$

where

$$I(p)=\int d^dq\frac{q_\nu q_\rho}{(p+q)^2q^2\left[-q^2-2q\cdot p\frac{b'}{a'}-p^2\frac{c'}{a'}\right]^{2-(d/2)}}. \tag{B27}$$

By reducing the denominators through the use of the identity

$$\frac{1}{A^\alpha B^\beta C^\gamma}=\frac{\Gamma(\alpha+\beta+\gamma)}{\Gamma(\alpha)\Gamma(\beta)\Gamma(\gamma)}\int_0^1 dz\int_0^z dt$$
$$\times\frac{t^{\gamma-1}(z-t)^{\beta-\alpha}}{[A+(B-A)z+(C-B)t]^{\alpha+\beta+\gamma}} \tag{B28}$$

and doing the integration in q we obtain for the divergent part

$$I_{\text{DivPart}}(p)=-i\frac{g_{\lambda\rho}}{2^{d+1}\pi^{d/2}[p^2]^{\epsilon}}\frac{\Gamma(\epsilon)}{\Gamma\left(2-\frac{d}{2}\right)}$$
$$\times\int_0^1 dz\int_0^z dt\,t^{1-(d/2)}[a''-b'']^{d-3}, \tag{B29}$$

where $a''=1-z+(b'/a')t$ and $b''=1-z+(c'/a')t$. By inserting Eq. (B29) in Eq. (B27), expanding in ϵ, and doing the parametric integrations, we find that

$$\mathcal{H}=\frac{i}{16\pi^2}\frac{1}{\epsilon}+\text{finite parts}. \tag{B30}$$

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