Mode coupling in rotating gravitational collapse: Gravitational and electromagnetic perturbations

Shahar Hod
The Racah Institute for Physics, The Hebrew University, Jerusalem 91904, Israel
(Received 1 February 1999; published 25 February 2000)

We consider the late-time evolution of *gravitational* and electromagnetic perturbations in realistic *rotating* Kerr spacetimes. We give a detailed analysis of the mode-coupling phenomena in rotating gravitational collapse. A consequence of this phenomena is that the late-time tail is dominated by modes which, in general, may have an angular distribution different from the original one. In addition, we show that different types of fields have *different* decaying rates. This result turns over the traditional belief (which has been widely accepted during the last three decades) that the late-time tail of gravitational collapse is universal.

PACS number(s): 04.70.Bw, 04.20.Ex, 04.20.Ha

I. INTRODUCTION

The *no-hair conjecture*, introduced by Ruffini and Wheeler in the early 1970s [1], states that the external field of a black hole relaxes to a Kerr-Newman field characterized solely by the black-hole mass, charge and angular momentum.

Price [2] was the first to analyze the mechanism by which the spacetime outside a (nearly *spherical*) star divests itself of all radiative multipole moments, and leaves behind a Schwarzschild black hole; it was demonstrated that all radiative perturbations decay asymptotically as an inverse power of time, the power indices equal $2l+3$ (in absolute value), where l is the multipole order of the perturbation. These inverse power-law tails are a direct physical consequence of the backscattering of waves off the effective curvature potential at asymptotically far regions [3,2]. Leaver [4] demonstrated that the late-time tail can be associated mathematically with the existence of a branch cut in the Green's function for the wave propagation problem.

The analysis of Price has been extended by many authors. We shall not attempt to review the numerous works that have been written addressing the problem of the late-time evolution of gravitational collapse. For a partial list of references, see, e.g., [5–16].

The above-mentioned analyses were restricted, however, to *spherically* symmetric backgrounds. It is well known, however, that realistic stellar objects generally rotate about their axis, and are therefore not spherical. Thus, the nature of the physical process of stellar core collapse to form a black hole is essentially *non*-spheric, and an astrophysically realistic model must take into account the angular momentum of the background geometry.

The corresponding problem of wave dynamics in realistic *rotating* Kerr spacetimes is much more complicated due to the lack of spherical symmetry. A first progress has been achieved only recently [17–22]. Evidently, the most interesting situation from a physical point of view is the dynamics of *gravitational* waves in *rotating* Kerr spacetimes. Recently, we have begun an analytic study of this fascinating problem [23]. This was done by analyzing the asymptotic late-time solutions of Teukolsky's master equation [24,25], which governs the evolution of massless perturbations fields in Kerr spacetimes. In this paper we give a detailed analysis of the problem. In particular, we give a full account of the phenomenon of mode coupling in rotating spacetimes (this phenomenon has been observed in numerical solutions of Teukolsky's equation [17,18]).

The plan of the paper is as follows. In Sec. II we give a short description of the physical system and summarize the main analytical results presented in Ref. [23]. In Sec. III we discuss the effects of rotation and the mathematical tools needed for the physical analysis are derived. In Sec. IV we analyze the active coupling of different gravitational and electromagnetic modes during a rotating gravitational collapse, with pure initial data. In Sec. V we consider the late-time evolution of realistic rotating gravitational collapse, with generic initial data. We conclude in Sec. VI with a summary of our analytical results and their physical implications.

II. REVIEW OF RECENT ANALYTICAL RESULTS

The dynamics of massless perturbations outside a realistic rotating Kerr black hole is governed by Teukolsky's master equation [24,25]

$$\left[\frac{(r^2+a^2)^2}{\Delta}-a^2\sin^2\theta\right]\frac{\partial^2\psi}{\partial t^2}+\frac{4Mar}{\Delta}\frac{\partial^2\psi}{\partial t\partial\varphi}+\left[\frac{a^2}{\Delta}-\frac{1}{\sin^2\theta}\right]\frac{\partial^2\psi}{\partial\varphi^2}-\Delta^{-s}\frac{\partial}{\partial r}\left(\Delta^{s+1}\frac{\partial\psi}{\partial r}\right)-\frac{1}{\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial\psi}{\partial\theta}\right)-2s\left[\frac{a(r-m)}{\Delta}+\frac{i\cos\theta}{\sin^2\theta}\right]\frac{\partial\psi}{\partial\varphi}-2s\left[\frac{M(r^2-a^2)}{\Delta}-r-ia\cos\theta\right]\frac{\partial\psi}{\partial t}+(s^2\cot^2\theta-s)\psi=0, \tag{1}$$

where M and a are the mass and angular momentum per unit mass of the black hole, and $\Delta=r^2-2Mr+a^2$. (We use gravitational units in which $G=c=1$.) The parameter s is called the spin weight of the field. For gravitational perturbations $s=\pm 2$, while for electromagnetic perturbations $s=\pm 1$. The field quantities ψ which satisfy Teukolsky's equation are given in [25].

Resolving the field in the form

$$\psi=\Delta^{-s/2}(r^2+a^2)^{-1/2}\sum_{m=-\infty}^{\infty}\Psi^m e^{im\varphi}, \tag{2}$$

where m is the azimuthal number, one obtains a wave equation for each value of m (we suppress the index m):

$$D\Psi\equiv\left[B_1\frac{\partial^2}{\partial t^2}+B_2\frac{\partial}{\partial t}-\frac{\partial^2}{\partial y^2}+B_3 -\frac{\Delta}{(r^2+a^2)^2}\frac{1}{\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial}{\partial\theta}\right)\right]\Psi=0, \tag{3}$$

where the tortoise radial coordinate y is defined by $dy=[(r^2+a^2)/\Delta]dr$. The coefficients $B_i(r,\theta)$ are given by

$$B_1(r,\theta)=1-\frac{\Delta a^2\sin^2\theta}{(r^2+a^2)^2} \tag{4}$$

and

$$B_2(r,\theta)=\left\{\frac{4iMmar}{\Delta}-2s\left[\frac{M(r^2-a^2)}{\Delta}-r-ia\cos\theta\right]\right\}\frac{\Delta}{(r^2+a^2)^2}. \tag{5}$$

[The explicit expression of $B_3(r,\theta)$ is not important for the analysis.]

The time evolution of a wave field described by Eq. (3) is given by

$$\Psi(z,t)=2\pi\int\int_0^{\pi}\{B_1(z')[G(z,z';t)\Psi_t(z',0)+G_t(z,z';t)\Psi(z',0)]+B_2(z')G(z,z';t)\Psi(z',0)\}\sin\theta' d\theta' dy', \tag{6}$$

for $t>0$, where z stands for (y,θ). The (retarded) Green's function $G(z,z';t)$ is defined by $DG(z,z';t)=\delta(t)\delta(y-y')\delta(\theta-\theta')/2\pi\sin\theta$, with $G=0$ for $t<0$. We express the Green's function in terms of the the Fourier transform $\tilde{G}_l(y,y';\omega)$:

$$G(z,z';t)=\frac{1}{(2\pi)^2}\sum_{l=l_0}^{\infty}\int_{-\infty+ic}^{\infty+ic}\tilde{G}_l(y,y';\omega)\times {}_sS_l^m(\theta,a\omega)\,{}_sS_l^m(\theta',a\omega)e^{-i\omega t}d\omega, \tag{7}$$

where c is some positive constant and $l_0=\max(|m|,|s|)$. The functions ${}_sS_l^m(\theta,a\omega)$ are the spin-weighted spheroidal harmonics which are solutions to the angular equation [25]

$$\left[\frac{1}{\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial}{\partial\theta}\right)+a^2\omega^2\cos^2\theta-\frac{m^2}{\sin^2\theta}-2a\omega s\cos\theta-\frac{2ms\cos\theta}{\sin^2\theta}-s^2\cot^2\theta+s+{}_sA_l^m\right]{}_sS_l^m=0. \tag{8}$$

For the $a\omega=0$ case, the eigenfunctions ${}_sS_l^m(\theta,a\omega)$ reduce to the spin-weighted spherical harmonics ${}_sY_l^m(\theta,\phi)={}_sS_l^m(\theta)e^{im\varphi}$, and the separation constants ${}_sA_l^m(a\omega)$ are simply ${}_sA_l^m=(l-s)(l+s+1)$ [26].

The Fourier transform is analytic in the upper half w plane and it satisfies the equation [25]

$$\tilde{D}(\omega)\tilde{G}_l\equiv\left\{\frac{d^2}{dy^2}+\left[\frac{K^2-2is(r-M)K+\Delta(4ir\omega s-\lambda)}{(r^2+a^2)^2}-H^2-\frac{dH}{dy}\right]\right\}\tilde{G}_l(y,y';\omega)=\delta(y-y'), \tag{9}$$

where $K=(r^2+a^2)\omega-am$, $\lambda=A+a^2\omega^2-2am\omega$, and $H=s(r-M)/(r^2+a^2)+r\Delta/(r^2+a^2)^2$.

Define two auxiliary functions $\tilde{\Psi}_1$ and $\tilde{\Psi}_2$ as solutions to the homogeneous equation $\tilde{D}(\omega)\tilde{\Psi}_1=\tilde{D}(\omega)\tilde{\Psi}_2=0$, with the physical boundary conditions of purely ingoing waves crossing the event horizon and purely outgoing waves at spatial infinity, respectively. In terms of $\tilde{\Psi}_1$ and $\tilde{\Psi}_2$, and henceforth assuming $y'<y$,

$$\tilde{G}_l(y,y';\omega)=-\tilde{\Psi}_1(y',\omega)\tilde{\Psi}_2(y,\omega)/W(\omega),$$

where we have used the Wronskian relation $W(\omega)=W(\tilde{\Psi}_1,\tilde{\Psi}_2)=\tilde{\Psi}_1\tilde{\Psi}_{2,y}-\tilde{\Psi}_2\tilde{\Psi}_{1,y}$.

It is well known that the late-time behavior of massless perturbations fields is determined by the backscattering from asymptotically *far* regions [3,2]. Thus, the late-time behavior is dominated by the *low*-frequency contribution to the Green's function, for only low frequencies will be backscattered by the small effective curvature potential (at $r\gg M$). Therefore, a *small*-ω approximation [or equivalently, a large-r approximation of Eq. (9)] is sufficient in order to study the asymptotic *late-time* behavior of the fields [12]. With this approximation, the two basic solutions required in order to build the Fourier transform are $\tilde{\Psi}_1=r^{l+1}e^{i\omega r}M(l+s+1-2i\omega M,2l+2,-2i\omega r)$ and $\tilde{\Psi}_2=r^{l+1}e^{i\omega r}U(l+s+1-2iwM,2l+2,-2i\omega r)$, where $M(a,b,z)$ and $U(a,b,z)$ are the two standard solutions to the confluent hypergeometric equation [27]. Then

$$W(\tilde{\Psi}_1,\tilde{\Psi}_2)=i(-1)^{l+1}(2l+1)!(2\omega)^{-(2l+1)}/(l+s)!. \tag{10}$$

In order to calculate $G(z,z';t)$ using Eq. (7), one may close the contour of integration into the lower half of the complex frequency plane. Then, one identifies three distinct contributions to $G(z,z';t)$ [4]: prompt contribution, quasinormal modes, and tail contribution. The late-time tail is associated with the existence of a branch cut (in $\tilde{\Psi}_2$) in the complex frequency plane [4] (usually placed along the negative imaginary ω axis). A little arithmetic leads to [23]

$$\tilde{G}^C_l(y,y';\omega)=\left[\frac{\tilde{\Psi}_2(y,\omega e^{2\pi i})}{W(\omega e^{2\pi i})}-\frac{\tilde{\Psi}_2(y,\omega)}{W(\omega)}\right]\tilde{\Psi}_1(y',\omega)$$
$$=\frac{(-1)^{l-s}4\pi M\omega(l-s)!}{(2l+1)!}\frac{\tilde{\Psi}_1(y,\omega)\tilde{\Psi}_1(y',\omega)}{W(\omega)}. \tag{11}$$

Taking cognizance of Eq. (7), we obtain

$$G^C(z,z';t)=\sum_{l=l_0}^{\infty}\frac{iM(-1)^s 2^{2l+1}(l+s)!(l-s)!}{\pi[(2l+1)!]^2}\times\int_0^{-i\infty}\tilde{\Psi}_1(y,\omega)\tilde{\Psi}_1(y',\omega)\,{}_sS_l(\theta,a\omega)\times{}_sS_l(\theta',a\omega)\omega^{2l+2}e^{-i\omega t}d\omega. \tag{12}$$

III. ROTATION EFFECTS: THE COUPLING OF DIFFERENT MODES

The *rotational* dragging of reference frames, caused by the rotation of the black hole (or star), produces an active *coupling* between modes of *different* l (but the same m). Mathematically, it is the θ dependence of the spin-weighted spheroidal wave functions ${}_sS^m_l(\theta,a\omega)$ and of the coefficients $B_1(r,\theta)$ and $B_2(r,\theta)$ which is responsible for the interaction between different modes; *no* coupling occurs in the nonrotating $(a=0)$ case.

The angular equation (8) is amenable to a perturbation treatment for small aw [28,29]; we write it in the form $(L^0+L^1)_sS^m_l=-{}_sA^m_l\,{}_sS^m_l$, where $L^0(\theta)$ is the ω-*independent* part of Eq. (8), and

$$L^1(\theta,a\omega)=(a\omega)^2\cos^2\theta-2a\omega s\cos\theta, \tag{13}$$

and we use the spin-weighted spherical functions ${}_sY^m_l$ as a representation. They satisfy $L^0{}_sY_l=-{}_sA^{(0)}_l{}_sY_l$ with ${}_sA^{(0)}_l=(l-s)(l+s+1)$ (we suppress the index m on ${}_sA_l$ and ${}_sY_l$). For small $a\omega$ a standard perturbation theory yields (see, for example, [30])

$${}_sS_l(\theta,a\omega)=\sum_{k=l_0}^{\infty}C_{lk}(a\omega)^{|l-k|}{}_sY_k(\theta), \tag{14}$$

where, to leading order in $a\omega$, the coefficients $C_{lk}(a\omega)$ are $\omega-$*independent* [21,29]. Equation (14) implies that the black-hole rotation *mixes* (and ignites) different spin-weighted spherical harmonics.

The coefficients $B_1(r,\theta)$ and $B_2(r,\theta)$ appearing in the time-evolution equation (6) depend explicitly on the angular variable θ through the *rotation* of the black hole (no such dependence exist in the non-rotating $a=0$ case). Therefore, in order to elucidate the coupling between different modes we should evaluate the integrals $\langle slm|skm\rangle$, $\langle slm|\sin^2\theta|skm\rangle$, and $\langle slm|\cos\theta|skm\rangle$, where $\langle slm|F(\theta)|skm\rangle\equiv\int{}_sY^{m*}_l F(\theta){}_sY^m_k d\Omega$ [see Eqs. (4) and (5) for the definition of the $B_i(r,\theta)$ coefficients]. In addition, the values of the coefficients C_{lk} depend on the integrals [21,29] $\langle slm|\cos^2\theta|skm\rangle$ and $\langle slm|\cos\theta|skm\rangle$ [see Eq. (13) for the definition of the perturbation term $L^1(\theta,a\omega)$, which is responsible for the mixing of modes in rotating backgrounds].

The spin-weighted spherical harmonics are related to the rotation matrix elements of quantum mechanics [31]. Hence, standard formulas are available for integrating the product of three such functions (these are given in terms of the Clebsch-Gordan coefficients [28,21,29]). In particular, the integrals $\langle sl0|\sin^2\theta|sk0\rangle$ and $\langle sl0|\cos^2\theta|sk0\rangle$ vanish unless $l=k,k\pm2$, while the integral $\langle sl0|\cos\theta|sk0\rangle$ vanishes unless $l=k\pm1$. For non-axially symmetric $(m\neq0)$ modes, $\langle slm|\sin^2\theta|skm\rangle\neq0$ for $l=k,k\pm1,k\pm2$ (the same holds for the integral $\langle sl0|\cos^2\theta|sk0\rangle$), and $\langle slm|\cos\theta|skm\rangle\neq0$ for $l=k,\ k\pm1$ (all other matrix elements vanish). Note also that the *complex* coefficient B_2 couples the real and imaginary parts of Ψ^m.

We are now in a position to evaluate the late-time evolution of realistic rotating gravitational collapse. We shall consider two kinds of initial data: pure initial data, which corresponds to the assumption that the initial angular distribution is characterized by a pure spin-weighted spherical harmonic function ${}_sY^m_{l*}$, and generic initial data where the initial pulse consists of all allowed modes (all spherical harmonics functions with $l\geqslant l_0$).

IV. PURE INITIAL DATA

A. Asymptotic behavior at timelike infinity

As explained, the late-time behavior of the fields should follow from the *low*-frequency contribution to the Green's function. Actually, it is easy to verify that the effective contribution to the integral in Eq. (12) should come from $|\omega|=O(1/t)$. Thus, we may use the $|\omega|r\ll1$ limit of $\tilde{\Psi}_1(r,\omega)$ in order to obtain the asymptotic behavior of the fields at *timelike infinity* (where $y,y'\ll t$). Using Eq. 13.5.5 of [27] one finds $\tilde{\Psi}_1(r,\omega)\simeq Ar^{l+1}$. Substituting this into Eq. (12), and using the representation, Eq. (14), for the spin-weighted spheroidal wave functions ${}_sS_l$, together with the cited properties of the angular integrals [of the form $\langle slm|F(\theta)|skm\rangle$], we find that the asymptotic late-time behavior of the l mode (where $l\geqslant l_0$) is dominated by the following effective Green's function:

$$G_l^C(z,z';t)=\sum_{k=l_0}^{L}\frac{M(-1)^{(l^*+l+2-q-2s)/2}2^{2k+1}(k+s)!(k-s)!(l^*+l+2-q)!}{\pi[(2k+1)!]^2}(yy')^{k+1}C_{kl}C_{kl^*-qs}Y_l(\theta)_sY^*_{l^*-q}(\theta')\times a^{l^*+l-2k-q}t^{-(l^*+l+3-q)}, \tag{15}$$

where $q=\min(l^*-l_0,2)$. Here, $L=l^*-q$ for $l\geqslant l^*-1$ modes, and $L=l$ for $l\leqslant l^*-2$ modes. Thus, the late-time behavior of the gravitational and electromagnetic fields at the asymptotic region of timelike infinity i_+ is dominated by the lowest allowed mode, i.e., by the $l=l_0$ mode. The corresponding damping exponent is $-(l^*+l_0+3-q)$.

B. Asymptotic behavior at future null infinity

We further consider the behavior of the fields at the asymptotic region of future null infinity $scri_+$. It is easy to verify that for this case the effective frequencies contributing to the integral in Eq. (12) are of order $O(1/u)$. Thus, for $y-y'\ll t\ll 2y-y'$ one may use the $|\omega|y'\ll 1$ asymptotic limit of $\tilde{\Psi}_1(y',\omega)$ and the $M\ll|\omega|^{-1}\ll y$ ($\mathrm{Im}\,\omega<0$) asymptotic limit of $\tilde{\Psi}_1(y,\omega)$. Thus, $\tilde{\Psi}_1(y',\omega)\simeq Ay'^{l+1}$, and $\tilde{\Psi}_1(y,\omega)\simeq e^{i\omega y}(2l+1)!e^{-i\pi(l+s+1)/2}(2\omega)^{-(l+s+1)}y^{-s}/(l-s)!$, where we have used Eqs. 13.5.5 and 13.5.1 of [27], respectively. Substituting this into Eq. (12), and using the representation, Eq. (14), for the spin-weighted spheroidal wave functions ${}_sS_l$, together with the cited properties of the angular integrals, one finds that the behavior of the l mode (where $l\geqslant l_0$) along the asymptotic region of null infinity $scri_+$ is dominated by the following effective Green's functions:

$$G_l^C(z,z';t)=\sum_{k=l^*-q_1}^{l^*+q_2}\times\frac{M(-1)^{(l+k-2s+2)/2}2^k(k+s)!(l-s+1)!}{\pi(2k+1)!}\times y'^{k+1}v^{-s}C_{kls}Y_l(\theta)_sY^*_k(\theta')a^{l-k}u^{-(l-s+2)}, \tag{16}$$

for $l\geqslant l^*-1$ modes, where $q_1=\min(l^*-l_0,2)$ and $q_2=\min(l-l^*,2)$, and

$$G_l^C(z,z';t)=\frac{M(-1)^{(l^*+l-2s)/2}2^l(l+s)!(l^*-s-1)!}{\pi(2l+1)!}\times y'^{l+1}v^{-s}C_{ll^*-2s}Y_l(\theta)_sY^*_{l^*-2}(\theta')\times a^{l^*-l-2}u^{-(l^*-s)}, \tag{17}$$

for $l\leqslant l^*-2$ modes. The dominant modes at null infinity and the corresponding damping exponents are given in Table I.

C. Asymptotic behavior along the black-hole outer horizon

The asymptotic solution to the homogeneous equation $\tilde{D}(\omega)\tilde{\Psi}_1(y,\omega)=0$ at the black-hole outer horizon H_+ ($y\to-\infty$) is [25] $\tilde{\Psi}_1(y,\omega)=C(\omega)\Delta^{-s/2}e^{-i(\omega-m\omega_+)y}$, where $\omega_+=a/(2Mr_+)$ [$r_+=M+(M^2-a^2)^{1/2}$ is the location of the black-hole outer horizon]. In addition, we use $\tilde{\Psi}_1(y',\omega)\simeq Ay'^{l+1}$. Regularity of the solution requires C to be an analytic function of ω. We thus expand $C(\omega)=C_0+C_1\omega+\cdots$ for small ω (as already explained, the late-time behavior of the field is dominated by the *low*-frequency contribution to the Green's function).

Substituting this into Eq. (12), and using the representation Eq. (14) for the spin-weighted spheroidal wave functions ${}_sS_l$, we find that the asymptotic behavior of the l mode (where $l\geqslant l_0$) along the black-hole outer horizon H_+ is dominated by the following effective Green's function:

TABLE I. Dominant modes and asymptotic damping exponents for gravitational and electromagnetic fields: pure initial data. $l_0=\max(|m|,|s|)$, and l^* is the initial mode of the perturbation.

Asymptotic region	l^*	Dominant mode(s)	Damping exponent
Timelike infinity	$l_0\leqslant l^*\leqslant l_0+1$	l_0	$-(2l_0+3)$
	$l_0+2\leqslant l^*$	l_0	$-(l^*+l_0+1)$
Null infinity	$l_0\leqslant l^*\leqslant l_0+1$	l_0	$-(l_0-s+2)$
	$l_0+2\leqslant l^*$	$l_0\leqslant l\leqslant l^*-2$	$-(l^*-s)$
Outer horizon	$l_0\leqslant l^*\leqslant l_0+1$	l_0	$-(2l_0+3+b)$
	$l_0+2\leqslant l^*$	l_0	$-(l^*+l_0+1+b)$

$$G_l^C(z,z';t)=\sum_{k=l_0}^{L}{}_s\Gamma_k\frac{M(-1)^{(l^*+l+2-q-2s)/2}2^{2k+1}(k+s)!(k-s)!(l^*+l+2-q)!}{\pi[(2k+1)!]^2}\Delta^{-s/2}y'^{k+1}C_{kl}C_{kl^*-qs}Y_l(\theta)$$
$$\times{}_sY^*_{l^*-q}(\theta')a^{l^*+l-2k-q}e^{imw_+y}v^{-(l^*+l+3-q+b)}, \qquad (18)$$

where q and L are defined as before, ${}_s\Gamma_k$ are constants, and $b=0$ *generically*, except for the unique case $m=0$ with $s>0$, in which $b=1$ [32]. Hence, the late-time behavior of the gravitational and electromagnetic fields along the black-hole outer horizon is dominated by the lowest allowed mode, i.e. by the $l=l_0$ mode. The corresponding damping exponent is $-(l^*+l_0+3-q+b)$.

V. GENERIC INITIAL DATA

So far we have assumed that the initial pulse is made of *pure* data, characterized by one particular spherical harmonic function ${}_sY^m_{l^*}$. In this section we consider the generic case. That is, we assume that the initial pulse consists of all the allowed ($l\geqslant l_0$) modes (see also the most recent analysis of Barack [33]).

The analysis here is very similar to the one presented in Sec. IV: Using Eq. (12), together with the appropriate asymptotic forms of $\tilde{\Psi}_1(y,\omega)$ and $\tilde{\Psi}_1(y',\omega)$ (as given in Sec. IV for the various asymptotic regions), and the representation Eq. (14) for the spheroidal wave functions, we find that the asymptotic late-time behavior of the l mode (where $l\geqslant l_0$) is dominated by the following effective Green's functions:

$$G_l^C(z,z';t)=MF_1(yy')^{l_0+1}{}_sY_l(\theta){}_sY^*_{l_0}(\theta')a^{l-l_0}t^{-(l+l_0+3)}, \qquad (19)$$

at *timelike infinity* i_+, where

$$F_1=F_1(l,l_0,m,s)=(-1)^{(l+l_0+2s+2)/2}2^{2l_0+1}\times(l+l_0+2)!(l_0+s)!(l_0-s)!C_{l_0l}/\pi[(2l_0+1)!]^2,$$

$$G_l^C(z,z';t)=\sum_{k=l_0}^{l}MF_2y'^{k+1}v^{-s}{}_sY_l(\theta){}_sY^*_k(\theta')\times a^{l-k}u^{-(l-s+2)}, \qquad (20)$$

at future null infinity $scri_+$, where $F_2=F_2(l,k,m,s)=(-1)^{(l+k+2s+2)/2}2^k(k+s)!(l-s+1)!C_{kl}/\pi(2k+1)!$, and

$$G_l^C(z,z';t)={}_s\Gamma'_lMF_1\Delta^{-s/2}y'^{l_0+1}{}_sY_l(\theta){}_sY^*_{l_0}(\theta')\times a^{l-l_0}e^{im\omega_+y}v^{-(l+l_0+3+b)}, \qquad (21)$$

at the black-hole outer horizon H_+, where ${}_s\Gamma'_l$ are constants.

VI. SUMMARY AND PHYSICAL IMPLICATIONS

We have analyzed the dynamics of *gravitational* (physically, the most interesting case) and electromagnetic fields in realistic *rotating* black-hole spacetimes. The main results and their physical implications are as follows:

(1) We have shown that the late-time evolution of realistic rotating gravitational collapse is characterized by inverse power-law decaying tails at the three asymptotic regions: timelike infinity i_+, future null infinity $scri_+$, and the black-hole outer horizon H_+ (where the power-law behavior is multiplied by an oscillatory term, caused by the dragging of reference frames at the event horizon). The relaxation of the fields is in accordance with the *no-hair* conjecture [1]. This work reveals the *dynamical* physical mechanism behind this conjecture in the context of rotating gravitational collapse.

The dominant modes at asymptotic late-times and the values of the corresponding damping exponents are summarized in Table I (for pure initial data) and Table II (for generic initial data). For reference we also include in Table III the results for the scalar field toy model with pure initial data (the $s=0$ case) [21,22] (the results for generic initial data coincide with those of gravitational and electromagnetic perturbations). In these tables, l is the multipole order of the perturbation, $l_0=\max(|m|,|s|)$, and l^* is the initial mode of the perturbation (for pure initial data). For the scalar field case ($s=0$), we have $p=0$ if $l-|m|$ is even, and $p=1$ otherwise. Note that for pure initial data, the pulse with $l^*=l_0,l_0+1$ differs from initial data with $l_0+2\leqslant l^*$. This is caused by the fact that the l_0 mode is not ignited (not coupled) to modes with smaller values of l.

The somewhat different character of the scalar field case can be traced back to Eq. (5) for $B_2(r,\theta)$ and Eq. (13) for $L^1(\theta,a\omega)$; it turns out that B_2 is θ *independent* in the $s=0$ case, and thus this term cannot couple different modes. To this we should add the fact that for the scalar field case, $L^1(\theta,a\omega)$ is proportional to $(a\omega)^2$ (the term proportional to aws vanishes), and thus the coefficients C_{lk} in Eq. (14) vanish if $|l-k|$ is odd [21].

The damping exponents for generic initial data derived in this paper agree with those derived most recently by Barack

TABLE II. Dominant modes and asymptotic damping exponents: generic initial data.

Asymptotic region	Dominant mode	Damping exponent
Timelike infinity	l_0	$-(2l_0+3)$
Null infinity	l_0	$-(l_0-s+2)$
Outer horizon	l_0	$-(2l_0+3+b)$

TABLE III. Dominant modes and asymptotic damping exponents for scalar fields: pure initial data. $p=0$ if $l-|m|$ is even, and $p=1$ otherwise.

Asymptotic region	l^*	Dominant mode(s)	Damping exponent
Timelike infinity	$l_0\leqslant l^*\leqslant l_0+1$	l^*	$-(2l^*+3)$
	$l_0+2\leqslant l^*$	l_0+p	$-(l^*+l_0+p+1)$
Null infinity	$l_0\leqslant l^*\leqslant l_0+1$	l^*	$-(l^*+2)$
	$l_0+2\leqslant l^*$	$l_0+p\leqslant l\leqslant l^*-2$	$-l^*$
Outer horizon	$l_0\leqslant l^*\leqslant l_0+1$	l^*	$-(2l^*+3)$
	$l_0+2\leqslant l^*$	l_0+p	$-(l^*+l_0+p+1)$

[33] using an independent analysis. Note, however, that Barack's analysis cannot yield the values of the damping exponents for pure initial data.

(2) The *unique* and important feature of *rotating* gravitational collapse (besides the oscillatory behavior along the black-hole horizon) is the active *coupling* of different modes. Physically, this phenomenon is caused by the dragging of reference frames, due to the black-hole (or star's) rotation (this phenomenon is absent in the non-rotating $a=0$ case). As a consequence, the late-time evolution of realistic rotating gravitational collapse has an angular distribution which is generically different from the original angular distribution (in the initial pulse).

(3) We emphasize that the power indices at a fixed radius in *rotating* Kerr spacetimes ($l+l_0+3$ for generic initial data and $l^*+l+3-q$ for pure initial data) are generically *smaller* than the corresponding power indices (the well-known $2l+3$) in *spherically* symmetric Schwarzschild spacetimes. (For generic initial data there is an equality only for the lowest allowed mode $l=l_0$, while for pure initial data there is an equality only for the $l=l_0$ mode provided it characterizes the initial pulse.) This implies a *slower* decay of perturbations in rotating Kerr spacetimes. Stated in a more pictorial way, a rotating Kerr black hole becomes ''bald'' slower than a spherically symmetric Schwarzschild black hole. From Eq. (19) it is easy to see that the time scale t_c at which the late-time tail of rotating gravitational collapse is considerably different from the corresponding tail of non-rotating collapse (for $l>l_0$ modes) is $t_c=yy'/a$, where y' is roughly the average location of the initial pulse.

(4) It has been widely accepted that the late-time tail of gravitational collapse is *universal* in the sense that it is *independent* of the type of the massless field considered (e.g., scalar, neutrino, electromagnetic, and gravitational). This belief was based on *spherically* symmetric analyses. Our analysis, however, turns over this point of view. In particular, the power indices $l+l_0+3$ at a fixed radius found in our analysis are generically *different* from those obtained in the scalar field toy model [21,22] $l+|m|+p+3$ (where $p=0$ if $l-|m|$ is even, and $p=1$ otherwise). Thus, different types of fields have *different* decaying rates. This is a rather surprising conclusion, which has been overlooked in the last three decades.

ACKNOWLEDGMENTS

I thank Tsvi Piran for discussions. This research was supported by a grant from the Israel Science Foundation.

[1] R. Ruffini and J. A. Wheeler, Phys. Today **24**, 30 (1971); C. W. Misner, K. S. Thorne and J. A. Wheeler, *Gravitation* (Freeman, San Francisco, 1973).
[2] R. H. Price, Phys. Rev. D **5**, 2419 (1972).
[3] K. S. Thorne, in *Magic without Magic: John Archibald Wheeler*, edited by J. Klauder (Freeman, San Francisco, 1972), p. 231.
[4] E. W. Leaver, Phys. Rev. D **34**, 384 (1986).
[5] J. Bičák, Gen. Relativ. Gravit. **3**, 331 (1972).
[6] C. Gundlach, R. H. Price, and J. Pullin, Phys. Rev. D **49**, 883 (1994).
[7] E. S. C. Ching, P. T. Leung, W. M. Suen, and K. Young, Phys. Rev. D **52**, 2118 (1995); Phys. Rev. Lett. **74**, 2414 (1995).
[8] S. Hod and T. Piran, Phys. Rev. D **58**, 024017 (1998); **58**, 024018 (1998); **58**, 024019 (1998).
[9] P. R. Brady, C. M. Chambers, and W. Krivan, Phys. Rev. D **55**, 7538 (1997).
[10] P. R. Brady, C. M. Chambers, W. G. Laarakkers, and E. Poisson, Phys. Rev. D **60**, 064003 (1999).
[11] S. Hod and T. Piran, Phys. Rev. D **58**, 044018 (1998).
[12] N. Andersson, Phys. Rev. D **55**, 468 (1997).
[13] L. Barack, Phys. Rev. D **59**, 044017 (1999).
[14] S. Hod, Phys. Rev. D **60**, 104053 (1999).
[15] C. Gundlach, R. H. Price, and J. Pullin, Phys. Rev. D **49**, 890 (1994).
[16] L. M. Burko and A. Ori, Phys. Rev. D **56**, 7820 (1997).
[17] W. Krivan, P. Laguna, P. Papadopoulos, Phys. Rev. D **54**, 4728 (1996).
[18] W. Krivan, P. Laguna, P. Papadopoulos, and N. Andersson, Phys. Rev. D **56**, 3395 (1997).
[19] A. Ori, Gen. Relativ. Gravit. **29**, 881 (1997).
[20] L. Barack, in *Internal Structure of Black Holes and Spacetime Singularities*, Volume XIII of the Israel Physical Society, edited by L. M. Burko and A. Ori (Institute of Physics, Bristol, 1997).
[21] S. Hod, Phys. Rev. D. **61**, 024033 (2000).

[22] L. Barack and A. Ori, Phys. Rev. Lett. **82**, 4388 (1999).
[23] S. Hod, Phys. Rev. D **58**, 104022 (1998).
[24] S. A. Teukolsky, Phys. Rev. Lett. **29**, 1114 (1972).
[25] S. A. Teukolsky, Astrophys. J. **185**, 635 (1973).
[26] J. N. Goldberg, A. J. Macfarlane, E. T. Newman, F. Rohrlich, and E. C. G. Sudarshan, J. Math. Phys. **8**, 2155 (1967).
[27] *Handbook of Mathematical Functions*, edited by M. Abramowitz and I.A. Stegun (Dover, New York, 1970).
[28] Many of the technical details here are analogous to those discussed in [21] for the scalar field case. We therefore give here only the details which are unique to the $s \neq 0$ case. Full details can be found in [29].
[29] S. Hod, gr-qc/9902073.
[30] L. Schiff, *Quantum Mechanics*, 3rd ed. (McGraw-Hill, New York, 1968).
[31] W. B. Campbell and T. Morgan, Physica (Amsterdam) **53**, 264 (1971).
[32] Barack and Ori have recently shown, Phys. Rev. D **60**, 124005 (1999), that C_0 vanishes in the particular case $am=0$ with $s>0$. Thus, $b=1$ in this non-generic case, whereas $b=0$ in all other cases.
[33] L. Barack, Phys. Rev. D **61**, 024026 (2000).