Primordial hypermagnetic knots

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Topologically nontrivial configurations of the hypermagnetic flux lines lead to the formation of hypermagnetic knots (HKs) whose decay might seed the baryon asymmetry of the universe. HKs can be dynamically generated provided a topologically trivial (i.e., stochastic) distribution of flux lines is already present in the symmetric phase of the electroweak (EW) theory. In spite of the mechanism generating the HKs, their typical size must exceed the diffusivity length scale. In the minimal standard model (but not necessarily in its supersymmetric extension) HKs are washed out. A classical hypermagnetic background in the symmetric phase of the EW theory can produce interesting amounts of gravitational radiation.

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Topologically nontrivial configurations of the magnetic field lines are allowed in terrestrial tokamaks and astrophysical plasmas [1]. The presence of hypermagnetic knots (HKs) in the symmetric phase of the electroweak (EW) theory (i.e., $T>T_c$) cannot be excluded. Since the conductivity σ_c of the EW plasma is typically large, in analogy with the electromagnetic case, we can expect that the topological structure of the hypermagnetic flux lines will be approximately conserved (up to corrections of order $1/\sigma_c$) for sufficiently large scales. The importance of the topological properties of long range (Abelian) hypercharge magnetic fields has been stressed in the past [2,3]. In [4] it was argued that if the spectrum of hypermagnetic fields is dominated by parity noninvariant Chern-Simons (CS) condensates, the baryon asymmetry of the universe (BAU) could be the result of their decay. Most of the mechanisms often invoked for the origin of large scale magnetic fields in the early Universe seem to imply the production of topologically trivial (i.e., stochastic) configurations of magnetic fields [5].

The purpose of this paper is to connect the topological properties of the HKs to the generation of the BAU. We show that HKs can be dynamically generated and can seed the BAU only if the correlation scale of the knot is larger than the diffusivity scale. We exclude this possibility in the minimal standard model (MSM). Since hypermagnetic fields present in the symmetric phase of the EW theory can radiate gravitational waves (GWs), we propose possible phenomenological tests of our generation mechanism.

Suppose that the EW plasma is filled, for $T>T_c$ with topologically trivial hypermagnetic fields $\vec{\mathcal{H}}_Y$, which can be physically pictured as a collection of flux tubes (closed because of the transversality of the field lines) evolving independently without breaking or intersecting with each other. If the field distribution is topologically trivial (i.e., $\langle\vec{\mathcal{H}}_Y\cdot\vec{\nabla}\times\vec{\mathcal{H}}_Y\rangle=0$), parity is a good symmetry of the plasma and the field can be completely homogeneous. We name hypermagnetic knots those CS condensates carrying a nonvanishing (averaged) hypermagnetic helicity (i.e., $\langle\vec{\mathcal{H}}_Y\cdot\vec{\nabla}\times\vec{\mathcal{H}}_Y\rangle\neq 0$). If $\langle\vec{\mathcal{H}}_Y\cdot\vec{\nabla}\times\vec{\mathcal{H}}_Y\rangle\neq 0$ parity is broken for scales comparable with the size of the HKs, the flux lines are knotted and the field $\vec{\mathcal{H}}_Y$ cannot be completely homogeneous.

In order to seed the BAU a network of HKs should be present at high temperatures [4,6]. In fact for temperatures larger than T_c the fermionic number is stored both in HKs and in real fermions. For $T<T_c$, the HKs should release real fermions since the ordinary magnetic fields (present *after* EW symmetry breaking) do not carry fermionic number. If the EW phase transition (EWPT) is strongly first order, the decay of the HKs can offer some seeds for the BAU generation [4]. This last condition can be met in the minimal supersymmetric standard model (MSSM) [7,8].

Under these hypotheses the integration of the $U(1)_Y$ anomaly equation [4] gives the CS number density carried by the HKs which is in turn related to the density of baryonic number n_B for the case of n_f fermionic generations [6]:

$$\frac{n_B}{s}(t_c)=\frac{\alpha'}{2\pi\sigma_c}\frac{n_f}{s}\frac{\langle\vec{\mathcal{H}}_Y\cdot\vec{\nabla}\times\vec{\mathcal{H}}_Y\rangle}{\Gamma+\Gamma_{\mathcal{H}}}\frac{M_0\Gamma}{T_c^2},\quad \alpha'=\frac{g'^2}{4\pi} \tag{1}$$

[g' is the $U(1)_Y$ coupling and $s=(2/45)\pi^2 N_{eff}T^3$ is the entropy density; N_{eff} is the effective number of massless degrees of freedom at T_c (106.75 in the MSM); $M_0=M_P/1.66\sqrt{N_{eff}}\simeq 7.1\times 10^{17}$ GeV]. In Eq. (1), Γ is the perturbative rate of the right electron chirality flip processes (i.e., scattering of right electrons with the Higgs and gauge bosons and with the top quarks because of their large Yukawa coupling) which are the slowest reactions in the plasma and

$$\Gamma_{\mathcal{H}}=\frac{783}{22}\frac{\alpha'^2}{\sigma_c\pi^2}\frac{|\vec{\mathcal{H}}_Y|^2}{T_c^2} \tag{2}$$

is the rate of right electron dilution induced by the presence of a hypermagnetic field. In the MSM we have that $\Gamma<\Gamma_{\mathcal{H}}$ [9] whereas in the MSSM Γ can naturally be larger than $\Gamma_{\mathcal{H}}$ [6]. Unfortunately, in the MSM a hypermagnetic field can modify the phase diagram of the phase transition but cannot make the phase transition strongly first order for large masses of the Higgs boson [10]. Therefore, we will concentrate on the case $\Gamma>\Gamma_{\mathcal{H}}$ and we will show that in the opposite limit the BAU will be small anyway even if some (presently unknown) mechanism would make the EWPT strongly first order in the MSM. We want to stress that the main reason in order to take into account right electrons is that their equilibration temperature is quite low (of the order of $T_R \sim 80$ TeV). In this sense the right electron chirality flip processes are the slowest ones. Thus, the number of right electrons is perturbatively conserved at temperatures higher than 80 TeV and a chemical potential can be introduced for it. On the other hand this charge is not conserved because of the (singlet) Abelian anomaly and it is then coupled to the hypermagnetic fields. When $T < T_R$ right electrons come in thermal equilibrium and Eq. (1) holds. The right electron number is nonconserved (even if $\Gamma = 0$), because of the Abelian anomaly, if some hypermagnetic knot is present. The limit $\Gamma = 0$ will be analyzed at the end of this paper (see below) but it does not seem phenomenologically interesting for the BAU generation even assuming strongly first order EWPT.

HKs can be dynamically generated. Gauge invariance and transversality of the magnetic fields suggest that perhaps the only way of producing $\langle \vec{\mathcal{H}}_Y \cdot \vec{\nabla} \times \vec{\mathcal{H}}_Y \rangle \neq 0$ is to postulate a time-dependent interaction between the two (physical) polarizations of the hypercharge field Y_α. Having defined the Abelian field strength $Y_{\alpha\beta} = \nabla_{[\alpha} Y_{\beta]}$ and its dual $\tilde{Y}_{\alpha\beta}$ such an interaction can be described, in curved space, by the Lagrangian [11]

$$L_{eff} = \sqrt{-g}\left[-\frac{1}{4} Y_{\alpha\beta} Y^{\alpha\beta} + c\,\frac{\psi}{4M} Y_{\alpha\beta} \tilde{Y}^{\alpha\beta} \right], \tag{3}$$

where $g_{\mu\nu}$ is the metric tensor and g its determinant, c is the coupling constant, and M is a typical scale. This interaction is plausible if the $U(1)_Y$ anomaly is coupled (in the symmetric phase of the EW theory) to dynamical pseudoscalar particles ψ (such as the axial Higgs boson of the MSSM). Thanks to the presence of pseudoscalar particles, the two polarizations of $\vec{\mathcal{H}}_Y$ evolve in a slightly different way, producing, ultimately, inhomogeneous HKs.

Suppose that an inflationary phase with $a(\tau) \sim \tau^{-1}$ is continuously matched, at the transition time τ_1, to a radiation-dominated phase where $a(\tau) \sim \tau$. Consider then a massive pseudoscalar field ψ which oscillates during the last stages of the inflationary evolution with typical amplitude $\psi_0 \sim M$. As a result of the inflationary evolution $|\vec{\nabla}\psi| \ll \psi'$. Consequently, the phase of ψ can get frozen [12]. Provided the pseudoscalar mass m is larger than the inflationary curvature scale $H_i \sim$ const, the ψ oscillations are converted, at the end of the quasi–de Sitter stage, into a net helicity arising as a result of the different evolution of the two (circularly polarized) vector potentials

$$Y''_{\pm} + \sigma Y'_{\pm} + \omega^2_{\pm} Y_{\pm} = 0, \quad \vec{H}_Y = \vec{\nabla} \times \vec{Y},$$
$$\psi \sim a^{-3/2} \psi_0 \sin[m(t - t_1)], \quad \omega^2_{\pm} = k^2 \mp k \frac{c}{M} a \dot{\psi} \tag{4}$$

(where we denoted by $\vec{H}_Y = a^2 \vec{\mathcal{H}}_Y$ the curved space fields and by $\sigma = \sigma_c a$ the rescaled hyperconductivity; the prime denotes derivation with respect to conformal time τ whereas the overdot denotes differentiation with respect to cosmic time t). Since $\omega_+ \neq \omega_-$, the helicity gets amplified according to Eq. (4) and the BAU can be obtained (for $\Gamma > \Gamma_{\mathcal{H}}$) from Eq. (1):

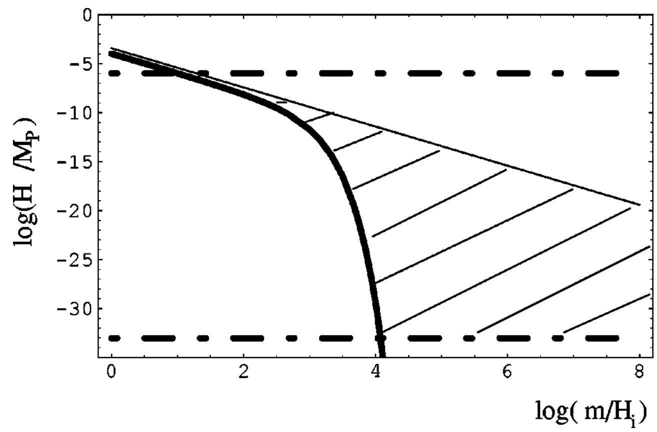

FIG. 1. With the solid thick line we illustrate the bound of Eq. (6) for a fiducial set of parameters ($c = 0.01$, $\sigma_0 \sim 70$, and $N_{eff} = 106.75$). In order to produce a sizable BAU we have to be within the shaded area. The thin line corresponds to the hyperconductivity bound (i.e., $k < k_\sigma$) for Fourier modes amplified during the inflationary epoch and evolving, subsequently, in the radiation phase. Notice that $x = \log r$ and $y = \log n_B/s$.

$$\frac{n_B}{s} = \delta \left(\frac{m}{H_i}\right)^5 \left(\frac{H_i}{M_P}\right)^{5/2} e^{c(m/H_i)(\psi_0/M)} e^{-2(\omega_{\rm m}/\omega_\sigma)^2}$$
$$\delta = \frac{45 g'^2 c^5 n_f N_{eff}^{1/4}}{512 \pi^6 \sigma_0} \frac{M_0}{T_c}, \quad \sigma_0 = \frac{\sigma_c}{T_c}, \tag{5}$$

where $\omega_{\rm m} = k_{\rm m}/a \sim (c/2)(\psi_0/M) m$ is the maximally amplified frequency corresponding to the center of the first (and larger) instability band of the Mathieu-type equation for $Y_\pm$ and $\omega_\sigma(\tau_c) \sim \sigma_0^{1/2} N_{eff}^{1/4} (T_c/M_P)^{1/2} T_c$ is the maximal (hyperconductivity) frequency of the spectrum. The possible oscillations arising in $\langle \vec{\mathcal{H}}_Y \cdot \vec{\nabla} \times \vec{\mathcal{H}}_Y \rangle$ are smeared out as a consequence of the growth of the hyperconductivity which is exactly zero in the inflationary phase but which gets large as soon as the radiation phase is approached.

Without fine-tuning the amplitude of the ψ oscillations we are led to require $\psi_0 \sim M$. If we impose that $n_B/s \gtrsim 10^{-10}$, we get, from Eq. (5) and in the case of three fermionic generations, the condition

$$\log_{10} \frac{H_i}{M_P} \gtrsim -8.5 + \log_{10}\left[\frac{\sigma_0^{2/5}}{c^2 \;\; N_{eff}^{1/10}}\right] - \frac{2c}{5} \frac{m}{H_i} \log_{10} e - 2 \log_{10} \frac{m}{H_i}, \tag{6}$$

illustrated in Fig. 1 with the thick (solid) line. In order to produce a sizable BAU ($\gtrsim 10^{-10}$) we need to be above the thick solid line but also below the thin (solid) line representing the condition $\omega_{\rm m}(t) < \omega_\sigma(t)$. Moreover, in order to be consistent with the (undetected) tensor contribution to the cosmic microwave background anisotropy we are led to require $H_i/M_P \lesssim 10^{-6}$. In order to have inflation prior to the onset of the EW epoch we must impose $H_i/M_P > 10^{-33}$. In Fig. 1 this last requirement corresponds to the shaded region within the two dot-dashed lines. Thus, provided $m/H_i \gtrsim 10^4$ the pseudoscalar oscillations produce sufficient helicity to seed the BAU also for a reasonably small inflationary scale $H_i \sim 10^{-22} M_P$ (see [13] for further details).

During an inflationary stage, $\sigma \to 0$. If the ψ oscillations take place in a radiation-dominated epoch ($\sigma \neq 0$), the evolution of the hypercharge is damped, from the very beginning, thanks to the finite value of the hyperconductivity according to

$$\sigma Y'_{\pm} + \left[k^2 \mp kc \frac{\psi'}{M} \right] Y_{\pm} = 0. \tag{7}$$

More precisely, for $T > T_c$, Eq. (7) should be complemented by the equations of anomalous magnetohydrodynamics (AMHD) accounting for the coupled evolution of ψ, μ_R (the right electrons chemical potential) and of the velocity field $\vec{v}$

$$\frac{(\mu_R a)'}{a} = -\frac{g'^2}{4\pi^2} \frac{783}{88} \frac{\vec{H}_Y \cdot \nabla \times \vec{H}_Y}{\sigma a^3 T^3} - (\Gamma + \Gamma_H)(\mu_R a) + D_R \nabla^2 (\mu_R a), \tag{8}$$

$$\vec{H}_{,Y} = -\frac{4 a \alpha'}{\pi \sigma} \vec{\nabla} \times (\mu_R \vec{H}_Y) - \frac{c}{M} \vec{\nabla} \times [\psi' \vec{H}_Y] + \vec{\nabla} \times (\vec{v} \times \vec{H}_Y) + \frac{1}{\sigma} \nabla^2 \vec{H}_Y, \tag{9}$$

$$\vec{v}' + [\vec{v} \cdot \vec{\nabla}] \vec{v} = \frac{[\vec{H}_Y \cdot \vec{\nabla}] \vec{H}_Y}{[\rho + p]} + \nu \nabla^2 \vec{v}, \tag{10}$$

where $a(\tau) d\tau = dt$ and $H = (\ln a)^{\cdot}$. In Eq. (10) we neglected the Lorentz term which is subleading in the case of maximally helical fields [11] and we also used the incompressible closure (i.e., $\vec{\nabla} \cdot \vec{v} = 0$) of the AMHD equations in the assumption of a perfect fluid with radiationlike equation of state $p = \rho/3$. In Eq. (8) on top of the chirality changing rates we introduced the diffusion coefficient of the right electron chemical potential D_R, leading to a typical diffusion scale $k_D \sim \alpha' (T/M_0)^{1/2} T$. Equations (8)–(10) should be generalized to include, in principle, *all* the processes which are in local thermal equilibrium for $T > T_c$. However, if we want to focus our attention on the generation of HKs right before the EWPT, we are led to consider with special care the right electrons whose equilibration temperature can fall (in the MSM) in the TeV range [9]. If the thermal and hypermagnetic diffusion coefficients are of the same order (i.e., $\nu \sim \sigma$), the solution of Eq. (7) together with Eqs. (8),(9) determines the evolution of the HKs at finite fermionic density. If we insert the result into Eq. (1), we get, in the limit $k \ll k_\sigma$ and $k \ll k_D$, that the BAU is given by

$$\frac{n_B}{s} \simeq \frac{45 n_f}{8 \pi^3 \sigma_0} c \quad \alpha' \frac{\Delta \psi}{M} r, \quad \omega_{\rm m} = \frac{c}{2a} \frac{\Delta \psi}{M} \frac{T_c^2}{M_0}, \tag{11}$$

where $r = |\vec{\mathcal{H}}_Y|^2 / (N_{eff} T_c^4)$ is the critical fraction of energy density stored in the initial (topologically trivial) hypermagnetic distribution for $\omega \sim \omega_{\rm m}$. Notice also that $\omega_{\rm m}$ differs, in the present case, from the maximally amplified frequency defined in the context of the inflationary amplification. If we do not fine-tune the initial amplitude of the oscillations to be much larger than M, we have that $\Delta \psi \sim M$. Concerning Eq. (11) three remarks are in order: (i) it holds provided $\omega_{\rm m} < \omega_\sigma$ [indeed only in this limit Eq. (1) is meaningful [6]], (ii) it holds provided we are in the context MSSM since only in this case can Γ be large enough and EWPT be strongly first order; (iii) it can give a relevant BAU if (and only if) an initial distribution of topologically trivial hypermagnetic fluctuations is postulated; namely, $r(\omega_{\rm m})$ needs to be at least 10^{-3}, whereas in the case of vacuum fluctuations $r(\omega_{\rm m}) \sim N_{eff} (\omega_{\rm m}/T)^4 \sim 10^{-33}$.

We would like to point out the analogies and the differences of our analysis with the one performed in [14] where a large amplification of hypercharge fields was observed for modes $k \simeq T$. In our case $k \ll T$. The reason for this choice is that only for $k \ll T$ (to be more precise $k < \alpha_W^2 T$) does the classical treatment of hypercharge modes seem to be, in our opinion, justified. It is not excluded that a suitable description of the $k \sim T$ case could be achieved by extending the classical description to a fully quantum mechanical regime.

It seems natural, in our scenario, to assume that at the scale where ψ oscillates in the radiation epoch there is a topologically trivial (stochastic) hypermagnetic distribution since it might have been generated, for example, thanks to the breaking of conformal invariance or through some other mechanism [5]. We focus our attention on temperatures in the TeV range where the right electrons can be still out of thermal equilibrium. Inspired by the axial Higgs boson we will be concerned with pseudoscalar masses $m \gtrsim 300$ GeV as required in order to have a MSM Higgs sector not too different from the one of the MSSM [15].

Comparing the BAU obtained from the previous equation with 10^{-10} we obtain a condition on the parameters of the model, namely, by imposing $y \gtrsim -10$

$$-2.4 + \log_{10} c - \log_{10} \sigma_0 + \log_{10} \left(\frac{\Delta \psi}{M} \right) + x \gtrsim -10 \tag{12}$$

[where now $x = \log_{10} r(\omega_{\rm m})$ and $y = \log_{10}(n_B/s)$]. This result is illustrated in Fig. 2 for the accessible region of the parameter space in the case $n_f = 3$. One could also wonder if the MSM physics would be enough to produce HKs. Let us assume, for a moment, the MSM. Then, as argued in [6,13], $\Gamma_{\mathcal{H}} > \Gamma$. Even assuming the EWPT to be strongly first order in the presence of a large hypermagnetic field (which is not the case) from Eq. (1) the BAU will be given by the expectation value of the highly nonlocal operator

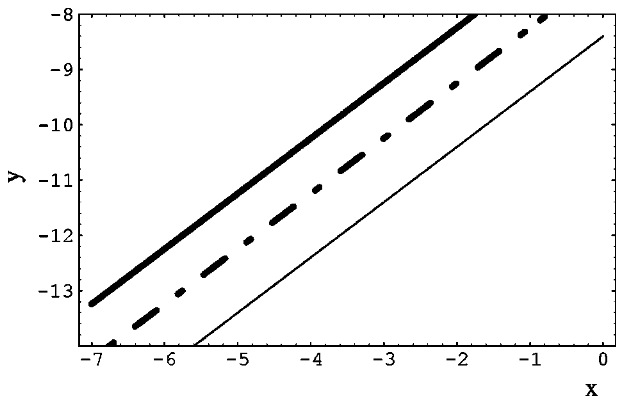

FIG. 2. We illustrate the logarithmic variation of the BAU computed in Eq. (12) for different sets of parameters. More precisely we have $c=0.01$, $\sigma_0=70$ (solid thick line), $c=10^{-3}$, $\sigma_0=70$ (dot-dashed line), $c=10^{-4}$, $\sigma_0=100$ (solid thin line).

$$\left\langle \frac{\vec{\mathcal{H}}_Y \cdot \vec{\nabla} \times \vec{\mathcal{H}}_Y}{|\vec{\mathcal{H}}_Y|^2} \right\rangle \simeq \frac{\langle \vec{\mathcal{H}}_Y \cdot \vec{\nabla} \times \vec{\mathcal{H}}_Y \rangle}{\langle |\vec{\mathcal{H}}_Y|^2 \rangle}, \tag{13}$$

where the last equality can be obtained for sufficiently large scales [6]. Then, using Eq. (7) the BAU turns out to be

$$\frac{n_B}{s} \simeq 0.04\ c\ \left(\frac{\Delta\psi}{M}\right)\left(\frac{T_R}{M_0}\right), \tag{14}$$

where $T_R \sim 80$ TeV is the right-electron equilibration temperature [9]. For the accessible region of the parameter space the BAU is of the order of 10^{-18}.

Our considerations can have direct phenomenological implications. It is amusing to notice that if a hypermagnetic background is present for $T > T_c$, then, as discussed in [16] in the context of ordinary magnetohydrodynamics (MHD), the energy momentum tensor will acquire a small anisotropic component which will source the evolution equation of the tensor fluctuations $h_{\mu\nu}$ of the metric $g_{\mu\nu}$:

$$h''_{ij} + 2\mathcal{H} h'_{ij} - \nabla^2 h_{ij} = -16\pi G \tau^{(T)}_{ij}, \tag{15}$$

where $\tau^{(T)}_{ij}$ is the *tensor* component of the *energy-momentum tensor* [16] of the hypermagnetic fields. Suppose now, as assumed in [10], that $|\vec{\mathcal{H}}|$ has constant amplitude and that it is also homogeneous. Then as argued in [17] we can easily deduce the critical fraction of energy density present today in relic gravitons of EW origin:

$$\Omega_{\rm gw}(t_0) = \frac{\rho_{\rm gw}}{\rho_c} \simeq z_{\rm eq}^{-1} r^2, \quad \rho_c(T_c) \simeq N_{eff} T_c^4 \tag{16}$$

($z_{\rm eq} = 6000$ is the redshift from the time of matter-radiation equality to the present time t_0). Because of the structure of the AMHD equations, stable hypermagnetic fields will be present not only for $\omega_{\rm ew} \sim k_{\rm ew}/a$ but for all the range $\omega_{\rm ew} < \omega < \omega_\sigma$. The (present) value of $\omega_{\rm ew}$ is

$$\omega_{\rm ew}(t_0) = 2.01 \times 10^{-7}\ \left(\frac{T_c}{1\ \ \mathrm{GeV}}\right)\ \left(\frac{N_{eff}}{100}\right)^{1/6}\ \mathrm{Hz}. \tag{17}$$

Thus, $\omega_\sigma(t_0) \sim 10^8 \omega_{\rm ew}$. Suppose now that $T_c \sim 100$ GeV; then we will have that $\omega_{\rm ew}(t_0) \sim 10^{-5}$ Hz and that $\omega_\sigma(t_0) \sim 10^3$ Hz. Suppose now, as assumed in [10], that

$$|\vec{\mathcal{H}}|/T_c^2 \gtrsim 0.3. \tag{18}$$

This requirement imposes $r \simeq 0.1 - 0.01$ and, consequently,

$$h_0^2 \Omega_{\rm GW} \simeq 10^{-7} - 10^{-8}. \tag{19}$$

Notice that this signal would occur in a (present) frequency range between 10^{-5} and 10^3 Hz. This signal satisfies the presently available phenomenological bounds on the graviton backgrounds of primordial origin. The pulsar timing bound (which applies for present frequencies $\omega_P \sim 10^{-8}$ Hz and implies $h_0^2 \Omega_{\rm GW} \lesssim 10^{-8}$) is automatically satisfied since our hypermagnetic background is defined for 10^{-5} Hz$\lesssim \omega \lesssim 10^3$ Hz. The large scale bounds [mainly coming from the Cosmic Microwave Background Explorer (COBE)] satellite would imply $h_0^2 \Omega_{\rm GW} < 7 \times 10^{-11}$ [18] but at a much lower frequency (i.e., 10^{-18} Hz). Our signal is completely absent around 10^{-18} Hz since, in our case, $\omega > \omega_{\rm ew}(t_0) \sim 10^{-5}$ Hz. Our signal is compatible with (but smaller than) the bounds coming from BBN [18,19] and implying $h_0^2 \Omega_{\rm GW} \lesssim 10^{-6}$. Notice finally that the frequency of operation of the interferometric devices aiming at a direct detection of stochastic background of relic gravitons is located in the range between a few Hz and 10 kHz, offering an interesting perspective for our considerations.

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