Scheme independence of $g_1^p(x,Q^2)$

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We work with two general factorization schemes in order to explore the consequences of imposing scheme independence on $g_1^p(x,Q^2)$. We see that although the light quark sector is indifferent to the choice of a particular scheme, the extension of the calculations to the heavy quark sector indicates that a scheme such as the $\overline{\text{MS}}$ is preferable.

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The problem raised by the results of the European Muon Collaboration (EMC) spin experiment [1] was deeply influential on a substantial part of the 1990s research in both theoretical and experimental hadron and particle physics. Their data implied that the quark singlet axial charge measured in a proton target, g_a^0, was compatible with zero, while quark model calculations predicted g_a^0 to lie in the range of 0.6–0.7. After a series of experiments made at CERN, SLAC, and the DESY e^+e^- collider HERA over the past 10 years, it is accepted today that $g_a^0 \approx 0.3$, which is still far from those early theoretical expectations.

In 1974, Ellis and Jaffe proposed a sum rule [2] for the integral in x of $g_1^p(x)$, where they assumed that the sea quarks in the proton are not polarized. This implies that Δs, the helicity carried by the strange quarks, is zero. Experimentally, $\int_0^1 g_1^p(x,Q^2=10\ \text{GeV}^2)dx = 0.120 \pm 0.005 \pm 0.006 \pm 0.014$ [3], while the Ellis-Jaffe sum rule gives $\int_0^1 g_1^p(x,Q^2=10\ \text{GeV}^2)dx = 0.176 \pm 0.006$.

In the parton model, the first moment of $g_1^p(x)$ is given by $\int_0^1 g_1^p(x)dx = \frac{1}{12}g_a + \frac{1}{36}g_a^8 + \frac{1}{9}g_a^0$, with $g_a = \Delta u - \Delta d$ the isotriplet axial charge, $g_a^8 = \Delta u + \Delta d - 2\Delta s$ the octet axial charge, and $g_a^0 = \Delta u + \Delta d + \Delta s$. The parton model structure functions are, actually, QCD structure functions with $\mathcal{O}(\alpha_s^0)$ corrections. Beyond the parton model, the identification of the singlet axial charges with the sum of the quark helicities ceases to be true, because of the clash between a gauge-invariant and a chiral-symmetric renormalization procedure of the axial charge [4,5].

The failure of the Ellis-Jaffe sum rule to agree with the experiments is translated into the nonequivalence between the quark singlet and octet axial charges. From the start there has been a large controversy on the mechanism responsible for $g_a^0 \neq g_a^8$. In the context of the parton model, $\Delta s \neq 0$ settles the question. However, as proposed by Altarelli and Ross [6] and Carlitz, Collins, and Mueller [7], it is still possible to have $\Delta s = 0$ if one takes into account the axial anomaly [8] which appears in the QCD calculations of $g_1^p(x,Q^2)$ at $\mathcal{O}(\alpha_s)$. Later, it became clear that these two different scenarios, $\Delta s \neq 0$ or an anomaly contribution, are simply related by a change of scheme defining the parton distributions and the coefficient functions. This will be, indeed, the main contribution of this work. We will argue that although the appearence of gluons in the first moment of $g_1^p(x,Q^2)$, in the light quark sector, is a matter of scheme preference, the introduction of heavy quarks suggests that a scheme where the gluons do not contribute, such as the modified minimal subtraction scheme ($\overline{\text{MS}}$) scheme, is preferable.

A part of what is discussed in this work has already been addressed in the literature: specifically, the importance in isolating the hard part of the photon-gluon cross section [9–12]. However, some missconceptions still persist, mainly those connected with the heavy quark contribution to $g_1^p(x,Q^2)$, which is one of the motivations for the explicit discussions made here on the ways to calculate a polarized gluon coefficient function which is free of ambiguity in the infrared region.

The choice of a factorization scheme is a reflection of the choice of a regulator and of a subtraction for the soft and collinear divergences appearing in the calculation of the partonic cross sections. In the specific case of the axial anomaly contribution to g_1^p, much has been discussed about the ambiguity in the choice of a quark or of a gluon mass to regulate these divergences [6,7,9,10]. In a satisfactory calculation, the hard part of the partonic cross sections should not present any ambiguity. The infrared singularities are present in the full virtual photon-gluon cross section [13,14], $\tilde{C}_g$, and they appear explicitly when the $Q^2 \to \infty$ limit is taken. As a general rule, the divergent (or soft) part of the cross section can be calculated from the expectation value of the quark singlet axial current between off-shell gluon lines [7,9,10,15]:

$$\Delta q^g(x,m_q^2,P^2,\mu^2) = -2\alpha_s \mu^{-(D-4)} \int \frac{d^{D-2}k_\perp}{(2\pi)^{D-2}} \times \left[\frac{k_\perp^2(1-2x) - m_q^2}{[k_\perp^2 + m_q^2 + x(1-x)P^2]^2} - 2\frac{D-4}{D-2}(1-x)\frac{k_\perp^2}{[k_\perp^2+m_q^2+x(1-x)P^2]^2} \right], \tag{1}$$

where $P^2 = -p^2$ is the gluon virtuality, m_q is the quark mass, and the number of quark flavors was set to 1. For an arbitrary

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number of flavors, Eq. (1) is multiplyed by n_f. The integral can be calculated in D dimensions as it stands, and the use of the $\overline{\mathrm{MS}}$ method to remove the UV divergence of Δq^g will define the coefficient functions and parton distributions in that scheme. A second option is to take from the start the limit $D\to 4$, and use a cutoff μ^2 to regularize the mass divergences. This is a momentum subtraction scheme, and the anomalous gluon contribution to the first moment of $g_1^p(x,Q^2)$ will appear.[1] Explicitly,

$$\Delta q_g^{\overline{\mathrm{MS}}}(x,m_q^2,P^2,\mu^2)=\frac{\alpha_s}{2\pi}\left[(2x-1)\ln\left(\frac{\mu^2}{m_q^2+x(1-x)P^2}\right)-\frac{m_q^2}{m_q^2+x(1-x)P^2}+1\right], \tag{2}$$

$$\Delta q_g^{\mu}(x,m_q^2,P^2,\mu^2)=\frac{\alpha_s}{2\pi}\left[(2x-1)\ln\left(\frac{\mu^2+m_q^2+x(1-x)P^2}{m_q^2+x(1-x)P^2}\right)+(1-x)\frac{\mu^2}{\mu^2+m_q^2+x(1-x)P^2}\,\frac{2m_q^2+x(1-2x)P^2}{m_q^2+x(1-x)P^2}\right]. \tag{3}$$

Both Eqs. (2) and (3) are dependent on the m_q^2/P^2 ratio, which is not a real problem because they are only part of the gluon coefficient function. What configures a problem is the attempt to draw conclusions about the possible anomalous gluon contribution to $g_1^p(x,Q^2)$ from those two equations. The standard procedure is to look at the subtracted partonic cross sections:

$$C_g^{\overline{\mathrm{MS}}}(x,Q^2,\mu^2)=\tilde{C}_g(x,m_q^2,P^2,Q^2)-\Delta q_g^{\overline{\mathrm{MS}}}(x,m_q^2,P^2,\mu^2),$$
$$C_g^{\mu}(x,Q^2,\mu^2)=\tilde{C}_g(x,m_q^2,P^2,Q^2)-\Delta q_g^{\mu}(x,m_q^2,P^2,\mu^2). \tag{4}$$

The hard part of the cross sections should not depend on the m_q^2/P^2 ratio for $Q^2,\mu^2\gg m_q^2,P^2$, which is the reason for the neglect of those two variables on the left hand side (LHS) of Eq. (4). We also use the label $\overline{\mathrm{MS}}$ to denote the fact that the usual $\overline{\mathrm{MS}}$ gluon coefficient function is recovered in the large Q^2 limit. The same for the C_g^{μ} defined in a momentum subtraction scheme.

In the region of low $k_\perp$, the integrands of Eq. (1) and of $\tilde{C}_g$ are equal. Hence, the fact that the RHS of Eqs. (4) turns out to be nonzero is a reflection of the large $k_\perp$ region and of the UV regulator of Eq. (1): the redefinition of the parton distributions, through an absortion of finite parts of the cross section, is a matter of taste. Depending on the regulator chosen, one can also absorb or not the axial anomaly term into the redefinition of the parton distributions. Explicitly, when $Q^2,\mu^2\gg m_q^2,P^2$, we have

$$C_g^{\overline{\mathrm{MS}}}(x,Q^2,\mu^2)=\frac{\alpha_s}{2\pi}\left\{(2x-1)\left[\ln\left(\frac{Q^2}{\mu^2}\right)+\ln\left(\frac{1-x}{x}\right)-1\right]+2(1-x)\right\},$$

$$C_g^{\mu}(x,Q^2,\mu^2)=\frac{\alpha_s}{2\pi}(2x-1)\left[\ln\left(\frac{Q^2}{\mu^2}\right)+\ln\left(\frac{1-x}{x}\right)-1\right]. \tag{5}$$

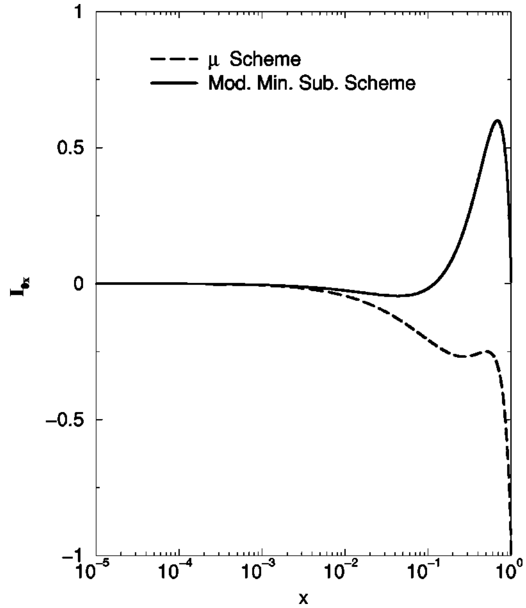

FIG. 1. The integral from 0 to x of the polarized gluon coefficient function as a function of x, for the $\overline{\mathrm{MS}}$ and μ schemes, in units of $\alpha_s/2\pi$.

Contrary to repeated claims in the literature [16–18], the schemes discussed here have a well-defined separation of hard effects in the coefficient functions and soft effects in the parton distribuitions. In principle, the polarized light quark sector is well described by both of them. For a better understanding of both schemes, we show in Fig. 1 the integrals in x of Eqs. (5) as a function of x, denoted by $I_{0x}=\int_0^x C_g^{\mu,\overline{\mathrm{MS}}}(x,Q^2)dx$. As is well known, $\int_0^1 C_g^{\overline{\mathrm{MS}}}(x,Q^2,\mu^2)=0$ and $\int_0^1 C_g^{\mu}(x,Q^2,\mu^2)=-1$, in units of $\alpha_s/2\pi$. The interesting feature is that the main contribution to both integrals comes from the large x region. In fact, $x=0.001$ is already a good zero, while the $x>0.8$ region is essential to give the integrals the value they have.

The physical structure function is indifferent to which scheme is used to define the parton distribution and the coefficient functions. This is expressed as

[1]Schemes such as the AB of Ref. [16] or JET of Ref. [17] belong to this class, and we will denote this class of schemes by μ schemes.

$$g_1^p(x,Q^2)=(\tfrac{1}{12}\,g_a^{\overline{\rm MS}}(x)+\tfrac{1}{36}\,g_a^{8,\overline{\rm MS}}(x))\otimes C_q^{NS,\overline{\rm MS}}(x,Q^2)+\tfrac{1}{9}\,g_a^{0,\overline{\rm MS}}(x)\otimes C_q^{S,\overline{\rm MS}}(x,Q^2)+\tfrac{1}{9}\,\Delta g^{\overline{\rm MS}}(x)\otimes C_g^{\overline{\rm MS}}(x,Q^2)$$
$$=(\tfrac{1}{12}\,g_a^{\mu}(x)+\tfrac{1}{36}\,g_a^{8,\mu}(x))\otimes C_q^{NS,\mu}(x,Q^2)+\tfrac{1}{9}\,g_a^{0,\mu}(x)\otimes C_q^{S,\mu}(x,Q^2)+\tfrac{1}{9}\,\Delta g^{\mu}(x)\otimes C_g^{\mu}(x,Q^2). \quad (6)$$

Although we did not write explicitly the Q^2 dependence of the various distributions, we remind the reader that only the singlet axial charge, in the $\overline{\rm MS}$ scheme, has a Q^2-dependent first moment.

To $\mathcal{O}(\alpha_s)$, $C_q(x,Q^2)\equiv C_q^{NS}(x,Q^2)=C_q^S(x,Q^2)=\delta(x-1)+[\alpha_s(Q^2)/2\pi]C_q^{(1)}(x)$. Using Eqs. (5) and the relation between the singlet axial charges between the two schemes,

$$g_a^{0,\overline{\rm MS}}(x,Q^2)=g_a^{0,\mu}(x,Q^2)-\frac{\alpha_s(Q^2)}{\pi}n_f(1-x)\otimes\Delta g(x,Q^2),$$

the second line of Eq. (6) can be rewritten as

$$g_1^p(x,Q^2)=(\tfrac{1}{12}\,g_a^{\mu}(x)+\tfrac{1}{36}\,g_a^{8,\mu}(x))\otimes C_q^{\mu}(x,Q^2)+\tfrac{1}{9}\,g_a^{0,\overline{\rm MS}}(x)\otimes C_q^{\mu}(x,Q^2)+\tfrac{1}{9}\,\Delta g^{\mu}(x)\otimes C_g^{\overline{\rm MS}}(x,Q^2), \quad (7)$$

where the term $\alpha_s^2/2\pi^2(1-x)\otimes\Delta g(x)\otimes C_q^{(1)}(x)$ was disregarded. We can now relate the remaining distributions and coefficient functions in the two schemes in the following way:

$$C_q^{\mu}(x)=C_q^{\overline{\rm MS}}(x)+\delta C_q(x),\quad \Delta g^{\mu}(x)=\Delta g^{\overline{\rm MS}}(x)+\delta g(x), \quad (8)$$
$$\tfrac{1}{12}\,g_a^{\mu}(x)+\tfrac{1}{36}\,g_a^{8,\mu}(x)=\tfrac{1}{12}\,g_a^{\overline{\rm MS}}(x)+\tfrac{1}{36}\,g_a^{8,\overline{\rm MS}}(x)+\delta q(x),$$

where $\delta C_q(x)$, $\delta g(x)$, and $\delta q(x)$ are some general functions, of $\mathcal{O}(\alpha_s)$. Their specific form is not of interest to us at this given moment. However, the use of Eqs. (8) in Eq. (7), and the requirement that Eq. (6) be satisfied, produces the following consistency relations:

$$(\tfrac{1}{12}\,g_a^{\overline{\rm MS}}(x)+\tfrac{1}{36}\,g_a^{8,\overline{\rm MS}}(x)+\tfrac{1}{9}\,g_a^{0,\overline{\rm MS}}(x)+\delta q(x))\otimes\delta C_q(x)+\delta q(x)\otimes C_q^{\overline{\rm MS}}(x)+\delta g(x)\otimes C_g^{\overline{\rm MS}}(x)=0, \quad (9)$$

$$(\tfrac{1}{12}\,g_a^{\mu}(x)+\tfrac{1}{36}\,g_a^{8,\mu}(x)+\tfrac{1}{9}\,g_a^{0,\mu}(x)-\delta q(x))\otimes\delta C_q(x)+\delta q(x)\otimes C_q^{\mu}(x)+\delta g(x)\otimes\left(C_g^{\mu}(x)-\frac{\alpha_s}{\pi}n_f(1-x)\right)=0. \quad (10)$$

The first moment of Eq. (9) certainly respects the equality, as $\int_0^1 C_g^{\overline{\rm MS}}(x)dx=0$ and $\int_0^1\delta q(x)dx=\int_0^1\delta C_q(x)dx=0$ because of the conservation of the nonsinglet axial vector current.[2] The conservation of the nonsinglet axial current also imposes, from the first moment of Eq. (10), that $\int_0^1\delta g(x)dx=0$, because $\int_0^1 C_g^{\mu}(x)dx=-n_f\alpha_s/2\pi\Delta g$. It follows that the first moment of the polarized gluon distribution is the same in the $\overline{\rm MS}$ and μ schemes, independent of whether Δg contributes or not to the first moment of $g_1^p(x,Q^2)$, up to the $(\alpha_s/2\pi)^2\Delta g$ corrections we neglected before. This result is consistent with the fact that Δg starts contributing to $g_1^p(x,Q^2)$ at $\mathcal{O}(\alpha_s)$ only.

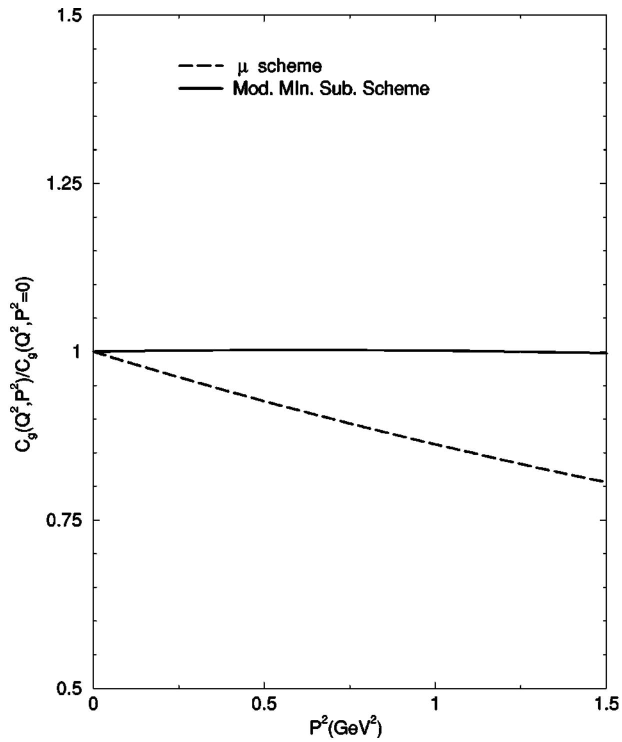

FIG. 2. The integrated (in x) polarized gluon coefficient function in the $\overline{\rm MS}$ and μ schemes, as a function of P^2.

Although both schemes are, in principle, equally good to describe $g_1^p(x,Q^2)$ in the light quark sector, we should also look at their behavior when heavy quarks are introduced. In particular, we do not want the hard part of the cross sections to depend on P^2 once the mass of the heavy quark and the factorization scale are fixed. To investigate that, we calculate Eqs. (4) as a function of P^2 for the charm quark, with $m_c=1.5$ GeV and $Q^2=\mu^2=10$ GeV^2. The resulting curves are shown in Fig. 2, normalized by the coefficient functions calculated with $P^2=0$. It is clear that $C_g^{\overline{\rm MS}}$ is independent of P^2 in the range $0\leq P^2\leq m_c^2$. The same is not true for C_g^{μ}, which shows a strong dependence on P^2.

Numerically, $\int_0^1 C_g^{\overline{\rm MS}}(x,m_q^2=2.25\ {\rm GeV}^2,\ Q^2=10\ {\rm GeV}^2)\,dx\approx 0.4$, in units of $\alpha_s(Q^2)/2\pi$. Of course, this integral

[2]As the change of the coefficient function is dictated by the change of scheme of the anomalous dimension, and the first moment of the nonsinglet anomalous dimension is zero due to current conservation, it follows that $\int_0^1\delta C_q(x)dx=0$.

changes with Q^2, going to zero as $Q^2 \to \infty$, but it is independent of P^2 for fixed Q^2. Hence there is a well-defined contribution from gluons to the first moment of $g_1^p(x,Q^2)$, in the $\overline{\mathrm{MS}}$ scheme, which appears because of the relatively large mass of the charm quark. On the other hand, $\int_0^1 C_g^\mu(x,m_q^2 = 2.25\ \mathrm{GeV}^2, Q^2 = 10\ \mathrm{GeV}^2)dx$ ranges from ~ -0.18, at $P^2=0$, to ~ -0.135, at $P^2 = m_c^2$. Although the difference is not numerically significant $[(0.18-0.135)(\alpha_s/2\pi)\Delta g \sim 0.001\Delta g]$ as long as Δg is not very large, the use of the μ schemes is, to some degree, damaged.

The inclusion of heavy quarks in the framework of the perturbative calculation of structure functions has received great attention in the recent literature [19–24]. These works have focused on the development of shemes that interpolate the pure photon-gluon fussion calculation from the region where $Q^2 \sim m_h^2$, to the usual massless approach (when $Q^2 \gg m_h^2$). A fundamental point of these schemes is that the heavy quark is treated as a massless parton in the Altarelli-Parisi evolution equations, which will have n_f+1 active flavors, while the quark mass dependence is fully kept in the graphs containing the heavy quark lines in the calculation of the coefficient functions. These schemes are generally refered to as interpolating schemes (ISs).

The coefficient functions in Eq. (4) incorporate the full mass corrections, and are reduced to the massless case in the limit of large Q^2. Hence, they are suitable for the calculation of the polarized structure functions for $Q^2 \sim m_h^2$ and $Q^2 \gg m_h^2$, in the spirit of the ISs. As in the ISs the light and the newly introducded heavy quark distributions should be defined in the same scheme, and as the calculation of the C_g^μ for a heavy quark is ambiguous, it follows that, strictly, the $\overline{\mathrm{MS}}$ scheme is formally superior to the μ scheme for the calculation of $g_1^p(x,Q^2)$.

As a last remark, we want to stress that the amount of polarized heavy quark in the proton is not given by the integral in x of Eqs. (2) and (3) or from the integral of $\tilde{C}_g(x,Q^2)$ for a given quark mass. From them, one would conclude that $\int_0^1 \Delta q_g^{\overline{\mathrm{MS}}}(x,m_q^2,P^2,\mu^2)dx = 0$ for $\mu^2 = m_q^2 \gg P^2$, while $\int_0^1 \Delta q_g^{\mu}(x,m_q^2,P^2,\mu^2)dx = \alpha_s/4\pi$ in that same limit. In a framework where heavy quark mass effects are systematically included, one should introduce[3] a polarized heavy quark distribution in the proton $[\Delta h(x,Q^2)]$ at the factorization scale μ^2, with $\Delta h(x,\mu^2)=0$.[4] As we saw here, both $\overline{\mathrm{MS}}$ and μ schemes are suitable for this purpose once Eqs. (4) are given, although, in principle, the $\overline{\mathrm{MS}}$ scheme has the technical advantage of having a P^2-independent gluon coefficient function in the heavy quark sector.

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[3] If the heavy quark contribution to $g_1^p(x,Q^2)$ is calculated through the photon-gluon fussion proccess only, and its higher order corrections, a polarized heavy quark distribution is never introduced.

[4] It is assumed that there is no intrinsic heavy quark polarization in the proton. See Ref. [25] for a different point of view.

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