# Doubly charmed baryon masses and quark wave functions in baryons

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We compute the masses of the ground state baryons and their spin excited states discussing quark wave functions in baryons. Doubly charmed baryon masses are given in the expected accuracy of $\pm 10$ MeV.

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For a decade, we have been waiting for the experimental discoveries of $su$ and $sd$ flavor symmetric charmed states $\Xi_c^+(c\{su\})$ and $\Xi_c^0(c\{sd\})$, which may decay to $\Xi_c^+(c[su])\gamma$ and $\Xi_c^0(c[sd])\gamma$, respectively [1], where $[su]$ and $[sd]$ represent flavor antisymmetric states. Recently, the CLEO Collaboration [2] has reported the observation of them. The measured masses of $\Xi_c^+(c\{su\})=2573.4\pm 3.3$ MeV and $\Xi_c^0(c\{sd\})=2577.3\pm 3.4$ MeV are in wonderful agreement with our computed values [1] of $\Xi_c^+(c\{su\})$ $=2579.6$ MeV and $\Xi_c^0(c\{sd\})=2579.3$ MeV, respectively. Also other charmed baryons have been observed in this decade. They are $\Omega_c^0(c\{ss\})$, $\Sigma_c^{*++}(\{cuu\})$, $\Sigma_c^{*0}(\{cdd\})$, $\Xi_c^{*+}(\{csu\})$, and $\Xi_c^{*0}(\{csd\})$. All the measured masses [3] are in good agreement with our computed values [1].

Though the agreement is good enough, the measured masses [3] of $\Sigma_c^{*++}(\{cuu\})=2519.4\pm 1.5$ MeV and $\Sigma_c^{*0}(\{cdd\})=2517.5\pm 1.4$ MeV are lower than the computed values [1] by 12.5 and 12.7 MeV, respectively. These differences are a little bit large compared with the accuracy of the computation. So it is a good time to compute again the baryon masses including these new data. The results are given in Table I. The experimental values $M(\text{expt})$ in Table I are those of the Particle Data Group [3] except for $\Xi_c^+(c\{su\})$ and $\Xi_c^0(c\{sd\})$ [2]. Now, the computed values agree within $\pm 8.5$ MeV with experimental values for all well-known ground state baryons and their spin excited states.

Our successful fit is due to the discussion about quark distances. The stronger the attractive force is between quarks, the closer the quarks may be. Representing this, we assume a relation for modified quark distances $r_{ij}$,

$$r_{ij}=R_{ij}-\kappa\frac{\partial}{\partial R_{ij}}\left\langle\sum_{l<m}U_{lm}\right\rangle, \tag{1}$$

where $U_{lm}$ are the Fermi-Breit terms

$$\begin{aligned}U_{lm}=&\left(\alpha Q_lQ_m-\frac{2}{3}\alpha_s\right)\left[\frac{1}{|\mathbf{r}_{lm}|}-\frac{1}{2m_lm_m}\right.\\&\times\left(\frac{\mathbf{p}_l\cdot\mathbf{p}_m}{|\mathbf{r}_{lm}|}+\frac{\mathbf{r}_{lm}\cdot(\mathbf{r}_{lm}\cdot\mathbf{p}_l)\mathbf{p}_m}{|\mathbf{r}_{lm}|^3}\right)\\&\left.-\frac{\pi}{2}\delta(\mathbf{r}_{lm})\left(\frac{1}{m_l^2}+\frac{1}{m_m^2}+\frac{16\mathbf{s}_l\cdot\mathbf{s}_m}{3m_lm_m}\right)\right],\end{aligned} \tag{2}$$

and $\kappa$ is a parameter which represents reductions of quark distances. The $R_{ij}$ is the quark distance between $i$th and $j$th quarks defined by

$$\frac{1}{R_{ij}}=\left\langle\Psi\left|\frac{1}{|\mathbf{r}_{ij}|}\right|\Psi\right\rangle, \tag{3}$$

where $\mathbf{r}_{ij}=\mathbf{r}_i-\mathbf{r}_j$. The Gaussian wave function $\Psi$, which is the exact solution of the three-body Schrödinger equation of the harmonic-oscillator potential

$$V=\frac{1}{2}K[(\mathbf{r}_1-\mathbf{r}_2)^2+(\mathbf{r}_2-\mathbf{r}_3)^2+(\mathbf{r}_3-\mathbf{r}_1)^2], \tag{4}$$

where $K$ is the spring constant, is

$$\begin{aligned}\Psi=&\left[\frac{2}{\pi}\right]^{3/2}(a_{12}a_{23}+a_{23}a_{31}+a_{31}a_{12})^{3/4}\\&\times\exp[-a_{12}(\mathbf{r}_1-\mathbf{r}_2)^2-a_{23}(\mathbf{r}_2-\mathbf{r}_3)^2-a_{31}(\mathbf{r}_3-\mathbf{r}_1)^2],\end{aligned} \tag{5}$$

with

$$\begin{aligned}a_{ij}=&\frac{1}{4M}\left[\frac{m_im_jK}{m_k}\right]^{1/2}\left\{(\lambda_0+\lambda_1)^{1/2}\left[1+\frac{m_k^2-m_im_j}{\lambda_1}\right]\right.\\&\left.+(\lambda_0-\lambda_1)^{1/2}\left[1-\frac{m_k^2-m_im_j}{\lambda_1}\right]\right\},\end{aligned} \tag{6}$$

 

TABLE I. Baryon masses, computed and experimental, in units of MeV. Quarks in { } and [ ] are flavor symmetric and antisymmetric, respectively. The values of the parameters are $m_u=435.91$ MeV, $m_d=441.55$ MeV, $m_s=686.58$ MeV, $m_c=1952.97$ MeV, $\alpha_s=0.445547$, $K=4.87722\times10^7$ MeV$^3$, $\kappa=2.81091\times10^{-9}$ MeV$^{-3}$, and $E_0=-1241.40$ MeV.

| | $p$ | $n$ | $\Lambda$ | $\Sigma^+$ | $\Sigma^0$ | $\Sigma^-$ | $\Xi^0$ | $\Xi^-$ |
|---|---|---|---|---|---|---|---|---|
| Quarks | $d\{uu\}$ | $u\{dd\}$ | $s[du]$ | $s\{uu\}$ | $s\{du\}$ | $s\{dd\}$ | $u\{ss\}$ | $d\{ss\}$ |
| $M$(comp) | 944.8 | 946.0 | 1112.4 | 1188.5 | 1191.6 | 1197.3 | 1322.1 | 1329.8 |
| $M$(expt) | 938.27231 | 939.56563 | 1115.683 | 1189.37 | 1192.642 | 1197.449 | 1314.9 | 1321.32 |
| | $\pm0.00028$ | $\pm0.00028$ | $\pm0.006$ | $\pm0.06$ | $\pm0.024$ | $\pm0.030$ | $\pm0.6$ | $\pm0.13$ |
| | $\Delta^{++}$ | $\Delta^+$ | $\Delta^0$ | $\Delta^-$ | $\Sigma^{*+}$ | $\Sigma^{*0}$ | $\Sigma^{*-}$ | $\Xi^{*0}$ |
| Quarks | $\{uuu\}$ | $\{duu\}$ | $\{udd\}$ | $\{ddd\}$ | $\{suu\}$ | $\{sdu\}$ | $\{sdd\}$ | $\{uss\}$ |
| $M$(comp) | 1232.2 | 1230.9 | 1232.4 | 1236.9 | 1385.8 | 1387.1 | 1391.3 | 1531.4 |
| $M$(expt) | 1230.9 | 1231.6 | 1233.3 | | 1382.8 | 1383.7 | 1387.2 | 1531.80 |
| | $\pm0.3$ | | $\pm0.4$ | | $\pm0.4$ | $\pm1.0$ | $\pm0.5$ | $\pm0.32$ |
| | $\Xi^{*-}$ | $\Omega^-$ | $\Lambda_c^+$ | $\Sigma_c^{++}$ | $\Sigma_c^+$ | $\Sigma_c^0$ | $\Xi_c^+$ | $\Xi_c^0$ |
| Quarks | $\{dss\}$ | $\{sss\}$ | $c[du]$ | $c\{uu\}$ | $c\{du\}$ | $c\{dd\}$ | $c[su]$ | $c[sd]$ |
| $M$(comp) | 1535.2 | 1668.0 | 2276.4 | 2452.2 | 2449.4 | 2449.3 | 2473.1 | 2476.0 |
| $M$(expt) | 1535.0 | 1672.45 | 2284.9 | 2452.8 | 2453.6 | 2452.2 | 2465.6 | 2470.3 |
| | $\pm0.6$ | $\pm0.29$ | $\pm$ 0.6 | $\pm0.6$ | $\pm0.9$ | $\pm0.6$ | $\pm1.4$ | $\pm1.8$ |
| | $\Xi_c^+$ | $\Xi_c^0$ | $\Omega_c^0$ | $\Sigma_c^{*++}$ | $\Sigma_c^{*+}$ | $\Sigma_c^{*0}$ | $\Xi_c^{*+}$ | $\Xi_c^{*0}$ |
| Quarks | $c\{su\}$ | $c\{sd\}$ | $c\{ss\}$ | $\{cuu\}$ | $\{cdu\}$ | $\{cdd\}$ | $\{csu\}$ | $\{csd\}$ |
| $M$(comp) | 2579.1 | 2578.7 | 2696.8 | 2526.5 | 2524.3 | 2524.8 | 2650.1 | 2650.3 |
| $M$(expt) | 2573.4 | 2577.3 | 2704 | 2519.4 | | 2517.5 | 2644.6 | 2643.8 |
| | $\pm3.3$ | $\pm3.4$ | $\pm4$ | $\pm1.5$ | | $\pm1.4$ | $\pm2.1$ | $\pm1.8$ |
| | $\Omega_c^0$ | $\Xi_{cc}^{++}$ | $\Xi_{cc}^+$ | $\Omega_{cc}^+$ | $\Xi_{cc}^{*++}$ | $\Xi_{cc}^{*+}$ | $\Omega_{cc}^+$ | $\Omega_{ccc}^{++}$ |
| Quarks | $\{css\}$ | $u\{cc\}$ | $d\{cc\}$ | $s\{cc\}$ | $\{ucc\}$ | $\{dcc\}$ | $\{scc\}$ | $\{ccc\}$ |
| $M$(comp) | 2764.3 | 3649.2 | 3644.5 | 3749.2 | 3734.6 | 3731.2 | 3825.7 | 4847.6 |
| $M$(expt) | | | | | | | | |

$$\lambda_0=m_1m_2+m_2m_3+m_3m_1, \quad \lambda_1=(\lambda_0^2-3m_1m_2m_3M)^{1/2},$$

$$M=m_1+m_2+m_3,$$

where $i,j,k$ are 1, 2, 3 and their cyclic permutations.

With the Gaussian wave function Eq. (5), the expectation value of a function $f(|\mathbf{r}_{ij}|)$ is given by

$$\langle f(|\mathbf{r}_{ij}|)\rangle=\frac{4}{\pi^2}\left(\frac{2\pi(a_{ij}a_{jk}+a_{jk}a_{ki}+a_{ki}a_{ij})}{a_{jk}+a_{ki}}\right)^{3/2}\times\int_0^\infty \exp\left[-\frac{2(a_{ij}a_{jk}+a_{jk}a_{ki}+a_{ki}a_{ij})}{a_{jk}+a_{ki}}r^2\right]\times f(r)r^2dr. \tag{7}$$

Then from Eq. (3), we get

$$\frac{1}{R_{ij}}=2\left[\frac{2(a_{ij}a_{jk}+a_{jk}a_{ki}+a_{ki}a_{ij})}{\pi(a_{jk}+a_{ki})}\right]^{1/2}. \tag{8}$$

We can calculate the expectation values of the other terms in $U_{lm}$ as a function of the quark distances $R_{lm}$ using Eq. (3.4) of Ref. [1] and get

$$\langle U_{lm}\rangle=\left(\alpha Q_lQ_m-\frac{2}{3}\alpha_s\right)\left[\frac{1}{R_{lm}}+\frac{2a_{lm}}{m_lm_mR_{lm}}-\frac{\pi}{16R_{lm}^3}\left(\frac{1}{m_l^2}+\frac{1}{m_m^2}+\frac{16\langle\mathbf{s}_l\cdot\mathbf{s}_m\rangle}{3m_lm_m}\right)\right], \tag{9}$$

where we can write $a_{lm}$ in terms of $R_{lm}$, $R_{mn}$, and $R_{nl}$ from Eq. (8),

$$a_{lm}=\frac{\pi}{4}\frac{(-R_{lm}^2+R_{mn}^2+R_{nl}^2)}{2(R_{lm}^2R_{mn}^2+R_{mn}^2R_{nl}^2+R_{nl}^2R_{lm}^2)-R_{lm}^4-R_{mn}^4-R_{nl}^4}. \tag{10}$$

As for the spin part of the wave functions, we use SU(6) symmetric one. So we get $\langle\mathbf{s}_l\cdot\mathbf{s}_m\rangle=1/4$ and $\langle\mathbf{s}_l\cdot\mathbf{s}_m\rangle=-3/4$ for spin triplet and singlet pair of quarks, respectively, $\langle\mathbf{s}_1\cdot\mathbf{s}_2\rangle=\langle\mathbf{s}_1\cdot\mathbf{s}_3\rangle=-1/2$ for $\Sigma^0(s\{du\})$ type baryons in

TABLE II. Wave-function parameters $a_{ij}$ in units of $\mathrm{MeV}^2$ in Eq. (5).

| | | Exact solutions of HO's | | | Modified by Eq. (1) | | |
|---|---|---|---|---|---|---|---|
| | Quarks | $a_{12}$ | $a_{23}$ | $a_{31}$ | $a_{12}$ | $a_{23}$ | $a_{31}$ |
| $p$ | $d\{uu\}$ | 42271.7 | 42001.3 | 42271.7 | 63449.0 | 36093.9 | 63449.0 |
| $n$ | $u\{dd\}$ | 42181.2 | 42453.4 | 42181.2 | 63189.1 | 36808.9 | 63189.1 |
| $\Lambda$ | $s[du]$ | 48292.7 | 39110.5 | 48625.4 | 51295.3 | 69420.6 | 51678.7 |
| $\Sigma^0$ | $s\{du\}$ | 48292.7 | 39110.5 | 48625.4 | 69364.0 | 34148.9 | 69527.2 |
| $\Xi^-$ | $d\{ss\}$ | 45132.2 | 56671.6 | 45132.2 | 62960.4 | 58375.7 | 62960.4 |
| $\Sigma^{*0}$ | $\{sdu\}$ | 48292.7 | 39110.5 | 48625.4 | 52449.6 | 42554.4 | 52820.4 |
| $\Lambda_c^+$ | $c[du]$ | 60523.7 | 32963.2 | 60984.2 | 56595.9 | 69859.2 | 57910.4 |
| $\Sigma_c^{++}$ | $c\{uu\}$ | 60619.0 | 32827.6 | 60619.0 | 70682.6 | 35159.9 | 70682.6 |
| $\Xi_c^+$ | $c[su]$ | 74337.2 | 37727.2 | 57130.3 | 87200.8 | 71015.4 | 50138.0 |
| $\Xi_c^+$ | $c\{su\}$ | 74337.2 | 37727.2 | 57130.3 | 100162.5 | 38337.0 | 67513.6 |
| $\Xi_{cc}^+$ | $d\{cc\}$ | 49178.2 | 109050.3 | 49178.2 | 57591.9 | 159400.1 | 57591.9 |
| $\Delta^-$ | $\{ddd\}$ | 42362.7 | 42362.7 | 42362.7 | 45636.3 | 45636.3 | 45636.3 |
| $\Omega^-$ | $\{sss\}$ | 52825.1 | 52825.1 | 52825.1 | 60139.6 | 60139.6 | 60139.6 |
| $\Omega_{ccc}^{++}$ | $\{ccc\}$ | 89092.9 | 89092.9 | 89092.9 | 125725.8 | 125725.8 | 125725.8 |

which $\langle \mathbf{s}_2\cdot\mathbf{s}_3\rangle = 1/4$ [4], and $\langle \mathbf{s}_1\cdot\mathbf{s}_2\rangle = \langle \mathbf{s}_1\cdot\mathbf{s}_3\rangle = 0$ for $\Lambda(s[du])$ type baryons in which $\langle \mathbf{s}_2\cdot\mathbf{s}_3\rangle = -3/4$.

Then we can get modified quark distances $r_{ij}$ by Eq. (1), and modified Gaussian wave functions by substituting

$$a_{ij} = \frac{\pi}{4}\,\frac{(-r_{ij}^2 + r_{jk}^2 + r_{ki}^2)}{2(r_{ij}^2 r_{jk}^2 + r_{jk}^2 r_{ki}^2 + r_{ki}^2 r_{ij}^2) - r_{ij}^4 - r_{jk}^4 - r_{ki}^4} \tag{11}$$

for $a_{ij}$ in Eq. (5), where we can get Eq. (11) from Eq. (10) replacing $R_{lm}$ by $r_{ij}$ and so on. We compute the baryon masses as the expectation values of the Hamiltonian by these modified Gaussian wave functions. The Hamiltonian is given by

$$H = \sum (m_l^2 + \mathbf{p}_l^2)^{1/2} + \sum_{l<m} \frac{1}{2} K \mathbf{r}_{lm}^2 + \sum_{l<m} U_{lm} + H_0, \tag{12}$$

where $H_0$ represents everything else, but we approximate the expectation value $\langle H_0\rangle$ by a constant $E_0$. The expectation value of the Fermi-Breit terms $\langle U_{lm}\rangle$ of the modified wave function is obtained by substituting $r_{lm}$ for $R_{lm}$ in Eq. (9). Our parameters are $m_u, m_d, m_s, m_c, \alpha_s, K, \kappa$, and $E_0$. We fit these parameters to suit the experimental values of the baryon masses in Table I [5].

In Table II, we give some of the wave-function parameters $a_{ij}$ of baryons. One can use the wave functions of proton and neutron in discussions of nuclear structures. From the values of parameters in Table I, we obtain

$$\kappa K = 0.137. \tag{13}$$

This value agrees with

$$\kappa K = \frac{\pi}{24} = 0.131, \tag{14}$$

which is given in Ref. [1]. In the case of mesons, we can show under some conditions that Eq. (14) holds for any perturbation $U$ with the spring constant $K$ of the confining harmonic-oscillator potential [6]. The new values of parameters are better than those of Ref. [1] in the following points. (1) The largest differences between the computed and the experimental values were $\pm 9.0$ MeV, but now they are $\pm 8.5$ MeV. (2) The fit was for 22 particles, but now it is for 30 particles. (3) The value of $\kappa K$ was 0.095, but now it is 0.137 which is closer to the expected value of $\pi/24$.

Most of singly charmed baryons being discovered, it is time to discuss doubly charmed baryons. The low-lying doubly charmed baryons have masses smaller than the bottom baryon $\Lambda_b = 5624 \pm 9$ MeV, as can be seen in Table I. Among them, $\Xi_{cc}^+(d\{cc\})$ has the smallest mass, rather than $\Xi_{cc}^{++}(u\{cc\})$. This is due to the repulsive Coulomb forces between $2e/3$ charged quarks in $\Xi_{cc}^{++}(u\{cc\})$. The mass difference of $\Xi_{cc}^{*++}(\{ucc\}) - \Xi_{cc}^{*+}(\{dcc\})$ is smaller than that of $\Xi_{cc}^{++}(u\{cc\}) - \Xi_{cc}^{+}(d\{cc\})$ due to the larger quark distances. We predicted in Ref. [1] that $\Sigma_c^{++}(c\{uu\}) > \Sigma_c^0(c\{dd\})$ and $\Sigma_c^+(c\{du\}) > \Sigma_c^0(c\{dd\})$. At that time, the experimental values were suggesting $\Sigma_c^{++} < \Sigma_c^0$ and $\Sigma_c^+ < \Sigma_c^0$. But now, the experimental data [3] show $\Sigma_c^{++} > \Sigma_c^0$ and $\Sigma_c^+ > \Sigma_c^0$ as we expect. Of course the recent values are more reliable than those of a decade ago, as can be seen from their experimental errors. Some models give $\Sigma_c^{++} < \Sigma_c^0$ and/or $\Sigma_c^+ < \Sigma_c^0$ as Genovese *et al.* [7] have given in the table. Also note that we predicted $\Sigma_c^{*++}(\{cuu\}) > \Sigma_c^{*0}(\{cdd\})$, but there were not any experimental data about it. Now, the experimental data show $\Sigma_c^{*++}(\{cuu\}) > \Sigma_c^{*0}(\{cdd\})$, too.

Spin-3/2 doubly charmed baryons $\Xi_{cc}^{*++}(\{ucc\})$ and $\Xi_{cc}^{*+}(\{dcc\})$ have smaller masses than spin-1/2 doubly charmed baryons $\Xi_{cc}$ plus $\pi$, so they may decay as

$$\Xi_{cc}^{*++}(\{ucc\}3735) \to \Xi_{cc}^{++}(u\{cc\}3649)\,\gamma,$$

$$\Xi_{cc}^{*+}(\{dcc\}3731) \to \Xi_{cc}^{+}(d\{cc\}3645)\gamma.$$

Also $\Omega_c^0(\{css\})$ and $\Omega_{cc}^+(\{scc\})$ may decay as

$$\Omega_c^0(\{css\}2764) \to \Omega_c^0(c\{ss\}2704)\gamma,$$

$$\Omega_{cc}^+(\{scc\}3826) \to \Omega_{cc}^+(s\{cc\}3749)\gamma,$$

respectively, because $\Omega_c^0(\{css\})$ and $\Omega_{cc}^+(\{scc\})$ have smaller masses than $\Omega_c^0(c\{ss\})\pi$ and $\Omega_{cc}^+(s\{cc\})\pi$, respectively.

We expect all the doubly charmed baryons in Table I to be discovered within $\pm 10$ MeV of the computed values. As for the triply charmed baryon $\Omega_{ccc}^{++}$, the new result is 10.6 MeV higher than the value predicted in Ref. [1], so its parameter dependence is stronger than that of doubly charmed baryons. But we expect $\Omega_{ccc}^{++}$ to be discovered within $\pm 15$ MeV of the computed value.

It was difficult to compare isomultiplet splittings of charmed baryons with experiments a decade ago, because the experimental errors were very large. But now we can do, partly, as discussed above and can be seen in Table I, thanks to experiments. It is difficult to compare the result of the splittings of $\Xi_c^+(c\{su\}) - \Xi_c^0(c\{sd\})$ and $\Xi_c^{*+}(\{csu\}) - \Xi_c^{*0}(\{csd\})$ with experiments now, because the experimental errors of them are still large. We welcome the prospect of an improved measurement of $\Xi^0$ mass by the KTeV experimental program at Fermilab [8], and expect that we can compare our result with experiment more clearly in the next millennium.


[1] C. Itoh, T. Minamikawa, K. Miura, and T. Watanabe, Phys. Rev. D **40**, 3660 (1989); ''Σ'' in Eq. (3.2) of this paper must be deleted.

[2] CLEO Collaboration, C.P. Jessop *et al.*, Phys. Rev. Lett. **82**, 492 (1999).

[3] Particle Data Group, C. Caso *et al.*, Eur. Phys. J. C **3**, 1 (1998).

[4] C. Itoh, T. Minamikawa, K. Miura, and T. Watanabe, Prog. Theor. Phys. **61**, 548 (1979).

[5] We will release the computer program baryon2000.c used in this work on the internet http://www.meijigalenin.ac.jp/~citoh/. A work station or a recent personal computer with C may be enough for it. If we put as the beginning value of $\kappa$ between $0.007\times 10^{-9}$ MeV$^{-3}$ and $2.81\times 10^{-9}$ MeV$^{-3}$, it may become $2.81\times 10^{-9}$ MeV$^{-3}$ soon, not zero. This means that our Eq. (1) is working very well.

[6] C. Itoh, T. Minamikawa, K. Miura, and T. Watanabe, Nuovo Cimento A **105**, 1539 (1992); **107**, 263 (1994).

[7] M. Genovese, J.-M. Richard, B. Silvestre-Brac, and K. Varga, Phys. Rev. D **59**, 014012 (1999); also J. Franklin, *ibid.* **59**, 117502 (1999).

[8] J.L. Rosner, Phys. Rev. D **57**, 4310 (1998); KTeV Collaboration, Fermilab Experiment Nos. E-799 and E-832 (unpublished).