Determination of the dynamically generated Yukawa coupling in supersymmetric QCD

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We make an attempt to determine the strength of the dynamically generated Yukawa coupling among composite fields. The system of $N=1$ supersymmetric SU(2) gauge theory with massive three flavors is considered as an example. We use the techniques of ''integrating in'' the gluino-gluino bound state in the low energy effective theory and the instanton calculation and Shifman-Vainshtein-Zakharov sum rule (QCD sum rule) in the fundamental theory. The obtained value of the Yukawa coupling among massive composite fields is of the order of unity. This value does not necessarily coincide with the value of the Yukawa coupling among massless composite fields, since the approximation breaks down in the limit of massless flavors.

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I. INTRODUCTION

The recent development of the techniques for analyzing supersymmetric gauge theories [1] has spurred the revival of the investigation of supersymmetric composite models [2–8]. One of the reasons for this revival is that the techniques allow us to obtain not only the particle content at low energy, but also the dynamically generated interactions among composite particles. In many models the dynamically generated Yukawa interactions are identified with or related to the Yukawa interactions among Higgs bosons and quarks or leptons in the standard model. However, the strength of the interactions is not satisfactorily determined yet. In many cases one assumes that it is of the order of unity, but, on the other hand, there is a claim that it must be of the order of 4π [9]. Some explicit calculations on the dynamics are required to determine the strength, since it includes the information of the Kähler potential which cannot be determined only by the symmetry and holomorphy.

Naive dimensional analysis (NDA) of Ref. [9] is the first attempt to determine the coupling constants in the low energy effective theories of supersymmetric gauge theories. The strength of coupling constants, especially for Yukawa couplings, is determined by the renormalization from the Seiberg's effective fields to the canonically normalized effective fields. In NDA the renormalization factor is determined by assuming that the magnitude of the one-loop correction in the effective theory is comparable with the tree-level contribution, and the Yukawa coupling of the order of 4π is obtained. This criterion is effective in the chiral Lagrangian for real QCD. In fact the NDA value of the pion-nucleon Yukawa coupling, 4π, is close to the experimental value, 13.5 [10].

In this paper we make an attempt to determine the strength of the dynamically generated Yukawa coupling among composite fields by doing an explicit calculation in the fundamental gauge theory. We consider $N=1$ supersymmetric SU($N_c=2$) gauge theory with $N_f=3$ massive flavors as an example. In the next section the relation between the dynamically generated Yukawa coupling and the normalization of the effective field is discussed. The argument is almost the same as what has been given in Ref. [9]. We calculate the squark pair condensate as a function of Λ and g_Y, the scale of dynamics in the effective theory and the Yukawa coupling, respectively, and compare it with the result given by the instanton calculation in the fundamental theory. Since the result of the instanton calculation is described by the scale of dynamics in the fundamental theory $\Lambda_{N_c,N_f}=\Lambda_{2,3}$, the Yukawa coupling, g_Y, is described by the ratio of $\Lambda/\Lambda_{2,3}$. In Sec. III the chiral superfield of the gluino-gluino bound state is introduced in the effective theory using the technique of ''integrating in'' [11], and the mass of the bound state is calculated. In Sec. IV a condition which the mass of the bound state follows is obtained using Shifman-Vainshtein-Zakharov (SVZ) sum rule (QCD sum rule) [12] in the fundamental theory. Then we estimate the ratio of $\Lambda/\Lambda_{2,3}$ using the result of the previous section, and obtain a numerical value of the Yukawa coupling. The resulting value is $g_Y\simeq 0.5\sim 1$. This is the value of the Yukawa coupling among massive composite fields which does not necessarily coincide with the one among massless composite fields, since the approximation breaks down in the limit of massless flavors. In the last section we give a summary and conclude.

II. DYNAMICALLY GENERATED YUKAWA COUPLING

The Lagrangian of the fundamental theory, $N=1$ supersymmetric SU(2) gauge theory with massive three flavors, is written as follows:

$$\mathcal{L}=-\int d^4\theta\, Q^{\dagger i}e^{-2g_0\mathcal{V}}Q_i+\int d^2\theta\frac{1}{2}m_0J^{ij}\epsilon_{\alpha\beta}Q_i^{\alpha}Q_j^{\beta}+\text{H.c.}+\frac{1}{4}\int d^2\theta\, W^{a\dot{\alpha}}W^a_{\dot{\alpha}}+\text{H.c.} \tag{1}$$

Here Q_i^{α} is the quark chiral superfield, $\mathcal{V}$ is the gluon vector superfield, $W^{a\dot{\alpha}}$ is the gluon field strength chiral superfield, g_0 is the bare gauge coupling constant, and m_0 is the bare quark mass (flavor independent). The indices $\alpha,\beta=1,2$ and $a=1,2,3$ are of the fundamental and adjoint representations for SU(2) gauge group, respectively, $i,j=1,2,\ldots,6$ are the flavor indices, and $J=\text{diag}(\epsilon,\epsilon,\epsilon)$ is the Sp(3) invariant matrix. See the Appendix for notations. The confinement is expected at low energy, and the effective field

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$$V_{ij} \sim \epsilon_{\alpha\beta} Q_i^\alpha Q_j^\beta \tag{2}$$

is expected to describe the lightest bound state by 't Hooft anomaly matching conditions [13], where V is the canonically normalized field with dimension one. Moreover, it is well known that the effective field follows the superpotential

$$\tilde{W}_{eff} = -\frac{1}{\Lambda_S^3} \mathrm{Pf} \tilde{V} - \frac{1}{2} m \,\mathrm{tr}(J\tilde{V}) \tag{3}$$

in the lowest order in the derivative expansion [1]. Here $\tilde{V}$, which is proportional to the effective field V, is Seiberg's effective field with dimension two and is directly related to the operator $\epsilon_{\alpha\beta} Q_i^\alpha Q_j^\beta$ in the fundamental theory. The renormalization-group invariant quark mass parameter m in the low energy effective theory is proportional to the renormalized quark mass in the fundamental theory. The first term of the above superpotential is the Yukawa interaction.[1]

Although the Kähler potential cannot be determined exactly, we can expect

$$\tilde{K}_{eff} = \frac{a}{\Lambda_S^2} \frac{1}{2} \mathrm{tr}(\tilde{V}^\dagger \tilde{V}) \tag{4}$$

with a positive coefficient a in the lowest order in the derivative expansion by assuming that the effective field $\tilde{V}$ propagates without its vacuum expectation value. The effective action is obtained from the following effective Lagrangian:

$$\mathcal{L}_{eff} = -\int d^4\theta \, \tilde{K}_{eff} + \left(\int d^2\theta \, \tilde{W}_{eff} + \mathrm{H.c.} \right). \tag{5}$$

Since the theory has a unique scale of the dynamics, all the couplings and coefficients in the effective Lagrangian should become of the order of unity, if all dimensionful quantities are scaled appropriately [9]. In fact, if we scale

$$\hat{V} = \left(\frac{\Lambda}{F}\right)^2 \tilde{V}, \quad \hat{\theta} = \theta \Lambda^{1/2}, \quad \hat{\bar{\theta}} = \bar{\theta} \Lambda^{1/2}, \quad \text{and} \quad \hat{m} = \frac{m}{\Lambda}, \tag{6}$$

then the effective Lagrangian becomes

$$\mathcal{L}_{eff} = F^2 \left\{ -\int d^4\hat{\theta} \, \hat{K}_{eff} + \left(\int d^2\hat{\theta} \, \hat{W}_{eff} + \mathrm{H.c.} \right) \right\} \tag{7}$$

with

$$\hat{K}_{eff} = \frac{1}{2} \mathrm{tr}(\hat{V}^\dagger \hat{V}), \tag{8}$$

$$\hat{W}_{eff} = -\mathrm{Pf}\, \hat{V} - \frac{1}{2} \hat{m} \,\mathrm{tr}(J\hat{V}). \tag{9}$$

Here $\Lambda = \Lambda_S / a^2$ and $F = \Lambda_S / a^{5/2}$.

We can determine the canonically normalized effective field by imposing the condition that the coefficient of the kinetic term is unity. Namely,

$$V = \frac{F}{\Lambda} \hat{V} = \frac{\Lambda}{F} \tilde{V} \tag{10}$$

and

$$\mathcal{L}_{eff} = -\int d^4\theta \, K_{eff} + \left(\int d^2\theta \, W_{eff} + \mathrm{H.c.} \right) \tag{11}$$

with

$$K_{eff} = \frac{1}{2} \mathrm{tr}(V^\dagger V), \tag{12}$$

$$W_{eff} = -g_Y \mathrm{Pf}\, V - \frac{1}{2} \frac{\Lambda}{g_Y} m \,\mathrm{tr}(JV), \tag{13}$$

where $g_Y \equiv \Lambda^2 / F = a^{-3/2}$ is nothing but the Yukawa coupling.

Note that the scale Λ in Eq. (6) does not necessarily coincide with Λ_S. If we may set $\Lambda = \Lambda_S$, we have $a = 1$ and $g_Y = 1$. This is the result of the overly strong requirement that all couplings and coefficients should become of the order of unity by the scaling of Eq. (6) with Λ_S instead of Λ.

In NDA the Yukawa coupling g_Y is determined under the requirement that the one-loop quantum effect in the Lagrangian of Eq. (7) is of the same order as the tree-level effect. Namely, when $\hat{m} < 1$ (light matter), the requirement is

$$\frac{\Lambda^4}{(4\pi)^2 F^2} \simeq 1, \tag{14}$$

where $(4\pi)^2 F^2$ is the one-loop suppression factor and Λ is introduced as the ultraviolet cutoff.[2] Then we have $g_Y \simeq 4\pi$ for small $m < \Lambda$.

The squark pair condensate is obtained using the effective Lagrangian of Eq. (11). From the supersymmetric vacuum condition

$$\frac{\partial W_{eff}}{\partial V_{ij}} = 0 \tag{15}$$

and the assumption of $\langle V_{ij} \rangle = v J_{ij}$, we obtain

$$v = \pm \frac{\sqrt{m\Lambda}}{g_Y}. \tag{16}$$

[1]If m is kept finite, it describes the Yukawa interactions among massive composite fields. To have the Yukawa interaction among massless composite fields, we have to set m to zero and introduce some gauge interactions by which the origin of the moduli space is chosen [2,5].

[2]Note that $\Lambda = 1$ in the Lagrangian of Eq. (7), since the unit of the energy is Λ.

Therefore, we have

$$\langle m_0\epsilon_{\alpha\beta}A^{\alpha}_{Q\,i=1}A^{\beta}_{Q\,j=2}\rangle = m\langle\tilde{V}_{12}\rangle = m\left(\frac{F}{\Lambda}v\right) = \pm\frac{\sqrt{m^3\Lambda^3}}{g_Y^2}, \tag{17}$$

where $A^{\alpha}_{Q\,i}$ is the squark field. This is a renormalization-group invariant quantity. The same result is obtained from the condition of $\partial\tilde{W}_{eff}/\partial\tilde{V}=0$. The gluino pair condensate is also obtained through the Konishi anomaly [14],

$$\left\langle\frac{g_0^2}{32\pi^2}\lambda^{a\dot{\alpha}}\lambda^{a}_{\dot{\alpha}}\right\rangle = \langle m_0\epsilon_{\alpha\beta}A^{\alpha}_{Q\,i=1}A^{\beta}_{Q\,j=2}\rangle = \pm\frac{\sqrt{m^3\Lambda^3}}{g_Y^2}, \tag{18}$$

where $\lambda^{a}_{\dot{\alpha}}$ is the gluino field. This is also a renormalization-group invariant quantity.

The gluino pair condensate has already been reliably estimated by the instanton calculation for $N=1$ supersymmetric $SU(N_c)$ gauge theories with N_f flavors [15]:[3]

$$\left\langle\frac{g_0^2}{32\pi^2}\lambda^{a\dot{\alpha}}\lambda^{a}_{\dot{\alpha}}\right\rangle = \left(C_{N_c}(\Lambda^{\text{1-loop}}_{N_c,N_f})^{3N_c-N_f}[1+\mathcal{O}(g(\mu)^4)]\frac{1}{g(\mu)^{2N_c}}\prod_{i=1}^{N_f} m_i(\mu)\right)^{1/N_c} e^{2\pi ik/N_c}, \tag{19}$$

where $k=1,2,\ldots,N_c$, the scale $\Lambda^{\text{1-loop}}_{N_c,N_f}$ is the one where the one-loop running coupling diverges, $g(\mu)$ and $m_i(\mu)$ are the renormalized coupling and mass, respectively, and $C_{N_c}\equiv 2^{2N_c}/(N_c-1)!(3N_c-1)$. This result is obtained by evaluating the one-loop quantum fluctuation around the single instanton background, and the reliability of the approximation is guaranteed by the supersymmetric Ward-Takahashi identities. In the above equation $\mathcal{O}(g(\mu)^4)$ indicates the contribution from the higher-loop quantum fluctuation. We can rewrite this quantity as follows [15]:

$$\begin{aligned}
&(\Lambda^{\text{1-loop}}_{N_c,N_f})^{3N_c-N_f}[1+\mathcal{O}(g(\mu)^4)]\frac{1}{g(\mu)^{2N_c}}\prod_{i=1}^{N_f} m_i(\mu)\\
&=\mu^{3N_c-N_f}\exp\left\{-\frac{8\pi^2}{g(\mu)^2}[1+\mathcal{O}(g(\mu)^2)]\right\}\\
&\quad\times\frac{1}{g(\mu)^{2N_c}}\prod_{i=1}^{N_f} m_i(\mu)\\
&=\mu^{3N_c-N_f}\exp\left(-(3N_c-N_f)\int_g^{g(\mu)}\frac{dg'}{\beta(g')}\right)\\
&\quad\times\exp\left(-N_f\int_g^{g(\mu)}dg'\frac{\gamma_m(g')}{\beta(g')}\right)\prod_{i=1}^{N_f} m_i(\mu)\\
&=(\Lambda_{N_c,N_f})^{3N_c-N_f}\prod_{i=1}^{N_f}[m_i]_{\text{inv}},
\end{aligned} \tag{20}$$

where $\beta(g)$ is the β function [17]

$$\beta(g) = -\frac{g^3}{16\pi^2}\cdot\frac{3N_c-N_f+N_f\gamma_m(g)}{1-N_cg^2/8\pi^2+\mathcal{O}(g^4)} \tag{21}$$

and $\gamma_m(g)$ is the anomalous dimension of mass. The renormalization-group invariant quantities Λ_{N_c,N_f} and $[m_i]_{\text{inv}}$ are defined as

$$\Lambda_{N_c,N_f} = \mu\exp\left(-\int_g^{g(\mu)}\frac{dg'}{\beta(g')}\right), \tag{22}$$

$$[m_i]_{\text{inv}} = m_i(\mu)\exp\left(-\int_g^{g(\mu)}dg'\frac{\gamma_m(g')}{\beta(g')}\right), \tag{23}$$

where g satisfies

$$g^{2N_c}\exp\left(\frac{8\pi^2}{g^2}(1+\mathcal{O}(g^2))\right)=1. \tag{24}$$

Therefore, in the case of $N_c=2$ and $N_f=3$ and if all masses are degenerate we have

$$\left\langle\frac{g_0^2}{32\pi^2}\lambda^{a\dot{\alpha}}\lambda^{a}_{\dot{\alpha}}\right\rangle = \pm(C_2(\Lambda_{2,3})^3[m]^3_{\text{inv}})^{1/2}, \quad C_2=\frac{16}{5}. \tag{25}$$

The mass parameter in the effective theory, m, can be identified with $[m]_{\text{inv}}$ since the mass term in the effective theory is introduced through the replacement of the renormalization-group invariant operator $m(\mu)(\epsilon_{\alpha\beta}Q^{\alpha}_iQ^{\beta}_j)_\mu/[m]_{\text{inv}}$ by the effective field $\tilde{V}_{ij}$ in the superpotential. Therefore, by solving Eqs. (18) and (25) we obtain the Yukawa coupling

$$g_Y=\left(\frac{1}{C_2}\left(\frac{\Lambda}{\Lambda_{2,3}}\right)^3\right)^{1/4}, \tag{26}$$

which is the function of the ratio $\Lambda/\Lambda_{2,3}$. These two scales are not always equal since the scale Λ is introduced without any concrete relation to the fundamental theory. The Yukawa

[3]It is known that this instanton calculation gives incorrect numerical coefficients [16]. However, it does not affect the result of this paper, since the difference is a factor of the order of unity in the case of the SU(2) gauge group.

coupling can be determined if Λ is described by $\Lambda_{2,3}$.[4] We need another independent quantity which can be calculated both in the effective theory and the fundamental theory. The mass of the gluino-gluino bound state can be the quantity.

III. GLUINO-GLUINO BOUND STATE IN THE EFFECTIVE THEORY

We introduce the chiral superfield

$$S \sim -\frac{g_0^2}{32\pi^2} W^{a\dot{\alpha}} W^a_{\dot{\alpha}}, \tag{27}$$

whose scalar component is the gluino-gluino bound state to the low energy effective theory using the method of integrating in [11], and calculate its mass. Following the conjecture of Ref. [11], we consider the effective superpotential after integrating in as follows:

$$\tilde{W}'_{eff} = \mathcal{G}(\tilde{V},\tilde{S}) - \frac{1}{2} m \,\mathrm{tr}(J\tilde{V}) + \ln \Lambda_S^3 \cdot \tilde{S}, \tag{28}$$

where $\tilde{S}$ is Seiberg's effective field with dimension three and is directly related to the operator $-(g_0^2/32\pi^2) W^{a\dot{\alpha}} W^a_{\dot{\alpha}}$ in the fundamental theory. The conjecture is that in the effective superpotential the scale Λ_S is included only as a coefficient of the field $\tilde{S}$ with the form of $\ln \Lambda_S^{3N_c - N_f}$. The function $\mathcal{G}(\tilde{V},\tilde{S})$ satisfies

$$\frac{\partial \mathcal{G}}{\partial \tilde{S}} = -\ln \Lambda_S^3 \tag{29}$$

due to the supersymmetric vacuum condition $\partial \tilde{W}'_{eff}/\partial \tilde{S} = 0$. On the other hand, since $\tilde{W}_{eff}$ is equivalent to $\tilde{W}'_{eff}$ as the effective superpotential, the relation

$$\frac{\partial \tilde{W}_{eff}}{\partial \ln \Lambda_S^3} = \frac{\partial \tilde{W}'_{eff}}{\partial \ln \Lambda_S^3} = \tilde{S} \tag{30}$$

should be satisfied. This relation gives

$$\ln \Lambda_S^3 = \ln \frac{\mathrm{Pf}\tilde{V}}{\tilde{S}} \tag{31}$$

and we can integrate Eq. (29) and obtain

$$\mathcal{G}(\tilde{V},\tilde{S}) = \tilde{S}\left(\ln \frac{\tilde{S}}{\mathrm{Pf}\tilde{V}} - 1 \right) + \mathcal{F}(\tilde{V}), \tag{32}$$

where $\mathcal{F}(\tilde{V})$ is a function of $\tilde{V}$. Therefore, we have

$$\tilde{W}'_{eff} = \tilde{S}\left(\ln \frac{\Lambda^3 \tilde{S}}{g_Y^4 \mathrm{Pf}\tilde{V}} - 1 \right) - \frac{1}{2} m \,\mathrm{tr}(J\tilde{V}) + \mathcal{F}(\tilde{V}), \tag{33}$$

where the relation $\Lambda_S^3 = \Lambda^3 a^6 = \Lambda^3/g_Y^4$ was used. This effective superpotential correctly gives the gluino pair condensate of Eq. (18).[5]

To obtain the mass of the gluino-gluino bound state, the canonically normalized effective field S has to be defined. We assume the Kähler potential

$$\tilde{K}'_{eff} = \frac{a}{\Lambda_S^2} \frac{1}{2} \mathrm{tr}(\tilde{V}^\dagger \tilde{V}) + b(\tilde{S}^\dagger \tilde{S})^{1/3} \tag{34}$$

following Ref. [18], where b is a positive constant. If the effective field $\tilde{S}$ is scaled appropriately to the dimensionless one, $\hat{S}$, together with the scalings of $\tilde{V}$ to $\hat{V}$ and so on, all of the couplings and coefficients in the effective Lagrangian should become of the order of unity with the overall factor F^2. Since the first term of $\tilde{W}'_{eff}$ is proportional to $\tilde{S}$, the scaling has to be

$$\hat{S} = \frac{\Lambda}{F^2} \tilde{S}. \tag{35}$$

The effective Lagrangian becomes

$$\mathcal{L}_{eff} = F^2 \left\{ -\int d^4\hat{\theta}\, \hat{K}'_{eff} + \left(\int d^2\hat{\theta}\, \hat{W}'_{eff} + \mathrm{H.c.} \right) \right\} \tag{36}$$

with

$$\hat{K}'_{eff} = \frac{1}{2} \mathrm{tr}(\hat{V}^\dagger \hat{V}) + b \left(\frac{\Lambda^2}{F} \right)^{2/3} (\hat{S}^\dagger \hat{S})^{1/3}, \tag{37}$$

$$\hat{W}'_{eff} = \hat{S}\left(\ln \frac{\hat{S}}{\mathrm{Pf}\hat{V}} - 1 \right) - \frac{1}{2} \hat{m} \,\mathrm{tr}(J\hat{V}) + \hat{\mathcal{F}}(\hat{V}). \tag{38}$$

The requirement that the coefficient of $(\hat{S}^\dagger \hat{S})^{1/3}$ in $\hat{K}'_{eff}$ is unity gives $b = g_Y^{-2/3}$.

Next we expand $(\tilde{S}^\dagger \tilde{S})^{1/3}$ in $\tilde{K}'_{eff}$ around the vacuum expectation value of $\langle \tilde{S} \rangle$ and define the canonical normalization. Namely, we set

$$\tilde{S} = \langle \tilde{S} \rangle + \tilde{S}^q \tag{39}$$

and obtain

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[4] If we use the relation $\Lambda^3 = \Lambda_S^3/a^6 = \Lambda_s^3 g_Y^4$, Eq. (26) gives just a relation between Λ_S and $\Lambda_{2,3}$. The difference between Λ_S and Λ is important.

[5] It must be noticed that this effective superpotential does not include any other heavy bound states which are as heavy as the gluino-gluino bound state. This fact is an uncontrolled approximation in the result.

$$\tilde{K}'_{eff} = \frac{a}{\Lambda_S^2}\frac{1}{2}\mathrm{tr}(\tilde{V}^\dagger \tilde{V}) + \frac{b}{3}\frac{\tilde{S}^{q\dagger}\tilde{S}^q}{(\langle\tilde{S}^\dagger\rangle\langle\tilde{S}\rangle)^{2/3}} + (\langle\tilde{S}^\dagger\rangle\langle\tilde{S}\rangle)^{1/3}\cdot\mathcal{O}((\langle\tilde{S}^\dagger\rangle\langle\tilde{S}\rangle)^{-2}). \tag{40}$$

Then the canonically normalized field is defined as

$$S = \sqrt{\frac{b}{3}\frac{1}{(\langle\tilde{S}^\dagger\rangle\langle\tilde{S}\rangle)^{2/3}}}\tilde{S} = \frac{g_Y}{\sqrt{3}m\Lambda}\tilde{S}. \tag{41}$$

Therefore, the mass of the gluino-gluino bound state is obtained as

$$m_S^2 = \left|\left(\frac{\sqrt{3}m\Lambda}{g_Y}\right)^2\left\langle\frac{\partial^2\tilde{W}'_{eff}}{\partial\tilde{S}^2}\right\rangle\right|^2 = \left(\frac{\sqrt{3}m\Lambda}{g_Y}\right)^4\frac{1}{|\langle\tilde{S}\rangle|^2} = 9m\Lambda. \tag{42}$$

In the limit of $m\to\infty$ the theory becomes supersymmetric SU(2) Yang-Mills theory with scale $\Lambda_{\mathrm{SYM}} = \sqrt{m\Lambda_S}$, and the mass of the gluino-gluino bound state is expected to be of the order of Λ_{SYM}. Therefore, the result of Eq. (42) is correct for large $m > \Lambda_S$ assuming no mass dependence of g_Y. However, it cannot be a correct formula for small $m \ll \Lambda_S$, since m_S is expected to remain finite in the $m\to 0$ limit with finite g_Y. This means that the assumption of Eq. (34) is not justified for small $m \ll \Lambda_S$.

IV. GLUINO-GLUINO BOUND STATE IN THE FUNDAMENTAL THEORY

We calculate the mass of the gluino-gluino bound state using Shifman-Vainshtein-Zakharov (SVZ) sum rule (QCD sun rule) [12] in the fundamental theory.[6] The bound state couples to both the scalar and auxiliary components of the operator

$$\mathcal{O}_S(y,\theta) = -\frac{g_0^2}{32\pi^2}W^{a\dot{\alpha}}(y,\theta)W^a_{\dot{\alpha}}(y,\theta) = \frac{g_0^2}{32\pi^2}\lambda^{a\dot{\alpha}}(x)\lambda^a_{\dot{\alpha}}(x) + \cdots, \tag{43}$$

where $y = x + i\bar{\theta}\sigma\theta$. Then we consider the quantity

$$\Pi(Q^2) = i\int d^4x\, e^{iqx}\left\langle T\int d^2\theta\,\mathcal{O}_S(y,\theta)\mathcal{O}_S(0,0)\right\rangle, \tag{44}$$

where $Q^2 = -q^2$. This quantity can be described in the spectral function representation as

$$\Pi(Q^2) = \int_0^\infty ds\frac{\rho(s)}{s+Q^2-i\epsilon} \tag{45}$$

with

$$\rho(s=k^2)\epsilon(k_0) = (2\pi)^3\sum_n \delta^4(p_n-k)\langle 0| \times \int d^2\theta\,\mathcal{O}_S(y,\theta)\Big|_{x=0}|n\rangle\langle n|\mathcal{O}_S(0,0)|0\rangle, \tag{46}$$

where the summation is taken over all the states. On the other hand, $\Pi(q^2)$ can be directly calculated in the limit of $Q^2\to\infty$ by the operator product expansion (OPE). Namely,

$$\begin{aligned}
&\lim_{Q^2\to\infty} i\int d^4x\, e^{iqx}T\left\{\int d^2\theta\,\mathcal{O}_S(y,\theta),\mathcal{O}_S(0,0)\right\}\\
&= 2\left(\frac{g^2}{32\pi^2}\right)^2\lim_{Q^2\to\infty} i\int d^4x\, e^{iqx}\Bigg[T\left\{\frac{1}{4}(v^{a\mu\nu}v^a_{\mu\nu} + iv^{a\mu\nu}\tilde{v}^a_{\mu\nu})_{(x)},\ (\lambda^b\lambda^b)_{(0)}\right\}\\
&\quad + T\{((\lambda^\dagger i\sigma^\mu\overleftrightarrow{D}_\mu)^a\lambda^a)_{(x)},\ (\lambda^b\lambda^b)_{(0)}\} + T\left\{\left(-\frac{g^2}{2}(A_Q^\dagger T^a A_Q)(A_Q^\dagger T^a A_Q)\right)_{(x)},\ (\lambda^b\lambda^b)_{(0)}\right\}\Bigg]\\
&= A(Q^2)\frac{g^2}{32\pi^2}(\lambda^a\lambda^a)_{(0)} + B(Q^2)\frac{1}{2}m(J^{ij}\epsilon_{\alpha\beta}A^\alpha_{Q\,i}A^\beta_{Q\,j})_{(0)} + C(Q^2)\frac{g^2}{32\pi^2}(\lambda^a\lambda^a A_Q^\dagger A_Q)_{(0)}\\
&\quad + D(Q^2)(\epsilon_{abc}\lambda^a\bar{\sigma}^{\mu\nu}\lambda^b v^c_{\mu\nu})_{(0)} + E(Q^2)(\epsilon_{abc}\lambda^a\bar{\sigma}^{\mu\nu}\lambda^b\tilde{v}^c_{\mu\nu})_{(0)} + \mathcal{O}(1/Q^4),
\end{aligned} \tag{47}$$

where $v^a_{\mu\nu}$ is the gluon field strength and $\tilde{v}^a_{\mu\nu}$ is its dual. All quantities are the renormalized quantities. The Wilsonian coefficients $A(Q^2)$, $B(Q^2)$, $C(Q^2)$, $D(Q^2)$, and $E(Q^2)$ can be determined by the perturbation theory. Note that the gluino number plus the squark number [anomalous $\mathrm{U}(1)_R$ symmetry] is conserved in the perturbation theory.

[6] The mass has already been calculated using a similar technique in Ref. [19].

By estimating the vacuum expectation values of the T products of both sides, multiplied by two $\lambda^{\dagger}$'s or two $A_Q^{\dagger}$'s in the first order of the perturbation theory, we obtain

$$A(Q^2)=\frac{\alpha(\mu)}{2\pi}\left(1+\frac{3}{2\pi}\alpha(\mu)\ln\left(\frac{Q^2}{\mu^2}\right)\right), \tag{48}$$

$$B(Q^2)=0, \tag{49}$$

where $\alpha(\mu)=g(\mu)^2/4\pi$. We consider only the lowest dimensional operators in OPE as an approximation. In the following, we take the renormalization point as $\mu=\sqrt{Q^2}$, by which the higher-order logarithmic correction is suppressed. Then, we have the sum rule

$$\int_0^{\infty} ds\,\frac{\rho(s)}{s+Q^2-i\epsilon}=-\frac{\alpha(\sqrt{Q^2})}{2\pi}\langle\mathcal{O}_S(0,0)\rangle \tag{50}$$

for large Q^2. Following Ref. [12], we consider the Borel transform of this sum rule. Namely,

$$\int_0^{\infty} ds\, e^{-s/M^2}\rho(s)=-M^2\frac{\alpha(\sqrt{M^2})}{2\pi}\langle\mathcal{O}_S(0,0)\rangle, \tag{51}$$

where M^2 is a parameter of dimension two which corresponds to Q^2. This is the SVZ sum rule in our case. If there is a value of M^2 so that $\alpha(\sqrt{M^2})$ in the right hand side is kept small enough and so that the integral in the left hand side is dominated by the lowest-lying state, we can reliably extract the information of the lowest-lying state. In the following we first assume that this is the case and estimate the goodness of the approximation later.

By differentiating the sum rule of Eq. (51), we obtain

$$\int_0^{\infty} ds\, e^{-s/M^2}s\rho(s)=-M^4\frac{\alpha(\sqrt{M^2})}{2\pi}\langle\mathcal{O}_S(0,0)\rangle, \tag{52}$$

where we neglect the $\mathcal{O}(\alpha(\sqrt{M^2})^2)$ term in the right hand side. The ratio of the two sum rules of Eqs. (51) and (52) gives

$$\int_0^{\infty} ds\, e^{-s/M^2}s\rho(s)\bigg/\int_0^{\infty} ds\, e^{-s/M^2}\rho(s)=M^2. \tag{53}$$

If the lowest-lying state dominates the integrals in the left hand side, we can set them as

$$\rho(s=k^2)\simeq\delta(k^2-m_S^2)\langle 0|\times\int d^2\theta\,\mathcal{O}_S(y,\theta)\bigg|_{x=0}|k\rangle_S\,{}_S\langle k|\mathcal{O}_S(0,0)|0\rangle, \tag{54}$$

and obtain $M^2=m_S^2$, where $|k\rangle_S$ is the one-particle state of S with momentum k. Then the sum rule of Eq. (51) becomes

$$\int_0^{\infty} ds\, e^{-s/m_S^2}\rho(s)=-m_S^2\frac{\alpha(\sqrt{m_S^2})}{2\pi}\langle\mathcal{O}_S(0,0)\rangle. \tag{55}$$

The vacuum expectation value $\langle\mathcal{O}_S(0,0)\rangle$ and the matrix elements in the spectral function of Eq. (54) can be estimated in the effective theory. It is clear that

$$\langle\mathcal{O}_S(0,0)\rangle=\langle\tilde{S}\rangle=\pm\frac{\sqrt{m^3\Lambda^3}}{g_Y^2} \tag{56}$$

and

$$\begin{aligned}{}_S\langle k|\mathcal{O}_S(0,0)|0\rangle&={}_S\langle k|A_{\tilde{S}}(0)|0\rangle\\&=\frac{\sqrt{3}m\Lambda}{g_Y}\,{}_S\langle k|A_S(0)|0\rangle\\&=\frac{\sqrt{3}m\Lambda}{g_Y},\end{aligned} \tag{57}$$

where $A_{\tilde{S}}$ and A_S are the scalar components of the effective fields $\tilde{S}$ and S, respectively. Moreover,

$$\begin{aligned}\langle 0|\int d^2\theta\,\mathcal{O}_S(y,\theta)\bigg|_{x=0}|k\rangle_S&=\langle 0|F_{\tilde{S}}(0)|k\rangle_S\\&=\frac{\sqrt{3}m\Lambda}{g_Y}\langle 0|F_S(0)|k\rangle_S,\end{aligned} \tag{58}$$

where $F_{\tilde{S}}$ and F_S are the auxiliary components of the effective fields $\tilde{S}$ and S, respectively. The auxiliary field F_S can be calculated using the effective superpotential of Eq. (33) to be

$$F_S=-\frac{\sqrt{3}m\Lambda}{g_Y}\frac{\partial\tilde{W}'^{\dagger}_{eff}}{\partial\tilde{S}^{\dagger}}\bigg|_{\text{scalar}}=-\frac{\sqrt{3}m\Lambda}{g_Y}\ln\frac{\sqrt{3}m\Lambda A_S^{\dagger}}{g_Y^2\mathrm{Pf}A_V^{\dagger}}, \tag{59}$$

where A_V is the scalar component of the effective field V. We expand this expression by $A_S^{\dagger}$ around its vacuum expectation value:

$$F_S=-\frac{\sqrt{3}m\Lambda}{g_Y}\frac{A_S^{q\dagger}}{\langle A_S^{\dagger}\rangle}+\mathcal{O}(1/\langle A_S^{\dagger}\rangle^2)-\frac{\sqrt{3}m\Lambda}{g_Y}\ln\frac{\sqrt{3}m\Lambda\langle A_S^{\dagger}\rangle}{g_Y^2\mathrm{Pf}A_V^{\dagger}}. \tag{60}$$

The first term describes the coupling with the one-particle state. Then we obtain

$$\langle 0|\int d^2\theta\,\mathcal{O}_S(y,\theta)\bigg|_{x=0}|k\rangle_S=-\left(\frac{\sqrt{3}m\Lambda}{g_Y}\right)^3\frac{1}{\langle A_{\tilde{S}}^{\dagger}\rangle}. \tag{61}$$

Therefore, the spectral function can be written as

$$\rho(s)\simeq-\left(\frac{\sqrt{3}m\Lambda}{g_Y}\right)^4\frac{1}{\langle\tilde{S}\rangle}\delta(s-m_S^2), \tag{62}$$

where we use $\langle A_{\tilde{S}}^{\dagger}\rangle=\langle\tilde{S}\rangle$.

This result and the sum rule of Eq. (55) give

$$m_S^2\alpha(\sqrt{m_S^2})=2\pi\left(\frac{\sqrt{3}m\Lambda}{g_Y}\right)^4\frac{1}{\langle\tilde{S}\rangle^2}=2\pi\cdot 9m\Lambda. \quad (63)$$

Using Eq. (42) we have

$$\alpha(\sqrt{m_S^2})=2\pi. \quad (64)$$

This is the condition which has to be satisfied by the mass of the gluino-gluino bound state. The expansion parameter on the gauge coupling in the OPE is

$$\frac{g(\sqrt{m_S^2})^2}{(4\pi)^2}=\frac{\alpha(\sqrt{m_S^2})}{4\pi}=\frac{1}{2}. \quad (65)$$

This is not much smaller than unity, and we are using the perturbation theory near the limit where it is breaking down. However, the approximation is enough for the order estimate since the higher-order logarithmic correction is suppressed by the appropriate selection of the renormalization point.

Now we use the formula of Eq. (42). Since it is reliable only for $m>\Lambda_S$, we should not use the running coupling for the case of $N_c=2$ and $N_f=3$, but for the case of $N_c=2$ and $N_f=0$. Furthermore, we have to use the running coupling which follows the β function [17]

$$\beta(\alpha)=-\frac{\alpha^2}{2\pi}\cdot\frac{3N_c}{1-N_c\alpha/2\pi+\mathcal{O}(\alpha^2)}, \quad N_c=2, \quad (66)$$

since the scale of dynamics which is nonperturbatively defined by the instanton calculation [see Eq. (22)] has to be introduced. The solution of the renormalization group equation is

$$\frac{1}{\alpha(\mu)}+\frac{1}{\pi}\ln\alpha(\mu)=\frac{3}{\pi}\ln\frac{\mu}{\Lambda_{2,0}}, \quad (67)$$

where the $\mathcal{O}(\alpha^2)$ term in the denominator of the β function is neglected as a small contribution. We can impose the one-loop matching relation, $\Lambda_{2,0}=\sqrt{m\Lambda_{2,3}}$.[7]

Now we can determine the value of the ratio $\Lambda/\Lambda_{2,3}$ using Eqs. (67), (64), and (42) as follows:

$$\frac{\Lambda}{\Lambda_{2,3}}=\frac{1}{9}(2\pi)^{2/3}e^{1/3}\simeq 0.5. \quad (68)$$

The scale Λ is the same order of $\Lambda_{2,3}$ as expected. Now it is possible to estimate the magnitude of the higher-order operator correction in OPE. The expansion parameter should be

$$\frac{(\Lambda_{2,0})^2}{M^2}=\frac{(\Lambda_{2,0})^2}{m_S^2}=\frac{1}{9}\frac{\Lambda_{2,3}}{\Lambda}\simeq 0.2. \quad (69)$$

This is small and independent from the mass m. Then the present approximation is good for the order estimate.

[7]The one-loop matching relation is satisfied in the results of the explicit instanton calculation.

Finally, we can determine the value of the Yukawa coupling g_Y using Eqs. (68) and (26);

$$g_Y=\left(\frac{5}{16}\frac{(2\pi)^2e}{9^3}\right)^{1/4}\simeq 0.5. \quad (70)$$

Namely, the resultant value of the dynamically generated Yukawa coupling (which is independent from the mass m) is of the order of unity for large $m>\Lambda_S$, which is different from the result of NDA, $4\pi\sim 10$, for small $m<\Lambda_S$.

Here we have to stress that the obtained value of the Yukawa coupling is for the theory with $m>\Lambda_S$, though it is independent from m. We may consider the simple $m\to 0$ limit, but there are several problems. For example, the mass of the gluino-gluino bound state vanishes in this limit [see Eq. (42)], which seems to contradict 't Hooft anomaly matching conditions, although the coupling in the spectral function also vanishes in this limit [see Eq. (62)] and the bound state disappears from the spectrum. To take the massless limit, we have to consider the bound state which couples to the operator $\mathrm{Pf}(\epsilon_{\alpha\beta}A_Q^\alpha A_Q^\beta)$ in Eq. (34), for example. Since the bound state has the same quantum number of S, there must be mixing between them, and we can expect that there is no massless bound state in the limit of $m\to 0$, except for V.

V. CONCLUSION

The value of the Yukawa coupling among the low energy effective fields (composite fields) was calculated in the $N=1$ supersymmetric SU(2) gauge theory with massive three flavors. First, the value of the squark pair condensate (or gluino pair condensate) and the mass of the gluino-gluino bound state were calculated in the effective theory considering the uniqueness of the scale of dynamics in the theory. These quantities are described by the parameters in the effective theory: Λ, m and g_Y. Next, these quantities were evaluated directly in the fundamental theory using the technique of the instanton calculation and SVZ sum rule. The results are described by the parameters in the fundamental theory, $\Lambda_{2,3}$ and m. Then, we obtained the expression of the parameters in the effective theory by those of the fundamental theory;

$$\frac{\Lambda}{\Lambda_{2,3}}=\frac{1}{9}(2\pi)^{2/3}e^{1/3}\simeq 0.5, \quad (71)$$

$$g_Y=\left(\frac{5}{16}\left(\frac{\Lambda}{\Lambda_{2,3}}\right)^3\right)^{1/4}\simeq 0.5. \quad (72)$$

These results are for large mass $m>\Lambda_S$, although they are independent from the mass. Unfortunately, the value cannot be directly compared with the result from NDA, $g_Y\simeq 4\pi$, for small mass.

We made some approximations in using the SVZ sum rule. The higher order in the perturbative gauge coupling in Wilson coefficients and the higher-order operator were neglected in the OPE. The approximations are good for the order estimate, since the expansion parameters are not so large; $\alpha(\sqrt{m_S^2})/4\pi = 0.5$ and $\Lambda_{2,0}^2/m_S^2 \simeq 0.2$. Note that the appropriate selection of the renormalization point suppresses the higher-order logarithmic correction in Wilson coefficients.

The method which is developed in this paper can be applied to determine the effective coupling constants in the low energy effective theories of the other supersymmetric gauge theories.

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APPENDIX: NOTATION

The metric we use is $g = \mathrm{diag}(1,-1,-1,-1)$, and the σ matrices for the two-component spinor are $(\sigma_\mu)_{\alpha\dot\beta} = (1,\tau^i)$ and $(\bar\sigma_\mu)_{\dot\alpha\beta} = (1,-\tau^i)$, where τ^i are the Pauli matrices. The convention on the contraction of the index of the two-component spinor is

$$\theta\theta = \theta^{\dot\alpha}\theta_{\dot\alpha}, \quad \overline{\theta\theta} = \bar\theta^{\alpha}\bar\theta_{\alpha}, \tag{A1}$$

with $\theta^{\dot\alpha} = \epsilon^{\dot\alpha\dot\beta}\theta_{\dot\beta}$ and $\bar\theta^{\alpha} = \epsilon^{\alpha\beta}\bar\theta_{\beta}$, where $\epsilon^{\dot\alpha\dot\beta} = \epsilon_{\dot\alpha\dot\beta}$ and $\epsilon^{\alpha\beta} = \epsilon_{\alpha\beta}$. The integration over the spinors is defined as

$$\int d^2\theta\, \theta^2 = 1, \quad \int d^2\bar\theta\, \bar\theta^2 = 1. \tag{A2}$$

In the following we give the correspondence between the standard notation by Wess and Bagger [20] and our own.

(i) On the metric and spinors:

$$\eta^{mn}|_{W-B} = -g^{\mu\nu}. \tag{A3}$$

$$\epsilon^{\alpha\beta}|_{W-B} = \epsilon^{\alpha\beta}, \quad \epsilon_{\alpha\beta}|_{W-B} = -\epsilon_{\alpha\beta}. \tag{A4}$$

$$(\sigma^m)_{\alpha\dot\beta}|_{W-B} = -(\sigma^\mu)_{\alpha\dot\beta}, \quad (\bar\sigma^m)^{\dot\alpha\beta}|_{W-B} = -(\bar\sigma^\mu)^{\dot\alpha\beta}. \tag{A5}$$

$$\theta^\alpha|_{W-B} = \bar\theta^\alpha, \quad \bar\theta^{\dot\alpha}|_{W-B} = \theta^{\dot\alpha}. \tag{A6}$$

$$\theta\theta|_{W-B} = \overline{\theta\theta} = \bar\theta^\alpha\bar\theta_\alpha, \quad \overline{\theta\theta}|_{W-B} = -\theta\theta = -\theta^{\dot\alpha}\theta_{\dot\alpha}. \tag{A7}$$

$$d^2\theta|_{W-B} = d^2\bar\theta, \quad d^2\bar\theta|_{W-B} = -d^2\theta. \tag{A8}$$

(ii) On the chiral superfields:

$$W_\alpha(y,\theta)|_{W-B} = \bar W_\alpha(y^\dagger,\bar\theta), \quad \bar W_{\dot\alpha}(y^\dagger,\bar\theta)|_{W-B} = W_{\dot\alpha}(y,\theta). \tag{A9}$$

$$\Phi(y,\theta)|_{W-B} = \Phi^\dagger(y^\dagger,\bar\theta), \quad \Phi^\dagger(y^\dagger,\bar\theta)|_{W-B} = \Phi(y,\theta). \tag{A10}$$

$$y^m|_{W-B} \equiv x^m + i\theta\sigma^m\bar\theta|_{W-B} = y^{\dagger\mu} \equiv x^\mu - i\bar\theta\sigma^\mu\theta. \tag{A11}$$

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