Finite supersymmetric grand unified theory reexamined

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We analyze the finite supersymmetric SU(5) grand unified model taking into account the threshold corrections at the grand unified theory and supersymmetry breaking scales and the problem of electroweak symmetry breaking which is particularly important in the case of large tan β. We find that there are still parameter regions where the low-energy experimental values are consistently reproduced and the Higgs potential parameters actually satisfy both constraints for large tan β and radiative electroweak symmetry breaking, provided that a new free parameter is introduced in the boundary condition of the Higgs soft-mass parameter, while it preserves the finiteness requirements.

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I. INTRODUCTION

The appearance of infinity in quantum field theory has long been one of its annoying problems. Most particle physicists believe that the ultimate theory, if it exists, should not contain any infinities and needs no renormalization procedure. In the 1980s, it was pointed out that the requirement of the nonexistence of quadratic divergences leads to a kind of symmetry, i.e., supersymmetry (SUSY) [1]. It is interesting to investigate what symmetry will appear from the requirement of vanishing of even logarithmic divergences (vanishing β functions) in supersymmetric theories. Among these supersymmetric theories, $N=4$ and some $N=2$ theories have zero β functions, which is called finiteness, in all orders of perturbation theory [2] and it is believed that there are so-called duality symmetries in those theories [3]. In this way, imposing that there should be no infinity has corresponded to very important symmetries, until now. If that is the case, what happens in $N=1$ supersymmetric theories from the requirement of finiteness? In the perturbative region, there is a classification of models in which gauge and Yukawa couplings satisfy the conditions of finiteness in one-loop order [4]. Moreover the all-order finiteness conditions for these couplings have also been found [5]. The zero β functions are also strongly related to the nonperturbative dynamics such as the electromagnetic duality transformation proposed by Seiberg [6]. Along this line, dualities in finite $N=1$ gauge theories have been searched for and one interesting model was found to be a candidate of having *S*-dual symmetry [7]. However, what symmetry actually corresponds to the finiteness conditions has not been answered yet.

On the other hand, from the viewpoint of low-energy phenomenology, the $N=1$ supersymmetry, which would come from the requirement of vanishing quadratic divergences, has provided us with many interesting phenomena. Therefore it will surely be important and become a first step toward the understanding of the meanings of finiteness to analyze phenomenological results as a consequence of vanishing logarithmic divergences. The finiteness conditions for gauge and Yukawa couplings prohibit us from applying them to U(1) gauge theories and so to the minimal supersymmetric standard model (MSSM). Therefore we apply these conditions to grand unified theories (GUT's) and derive the boundary conditions at the GUT scale for couplings in low-energy theories, which is supposed to be the MSSM in this paper. In particular in case of SU(5) GUT models, several articles have obtained interesting results for the fermion masses, the superparticle masses, and so on [8–10].

In this paper we analyze the finite SU(5) GUT model taking care of the following two points which have not been considered so far. (i) The threshold corrections at GUT and SUSY breaking scales. Since we use the two-loop order β functions for the MSSM couplings we generally need to include the one-loop order threshold corrections [11]. Especially, it is important in large tan β cases to include a SUSY threshold correction to the bottom quark mass which can be $\lesssim 50\%$ of the uncorrected value [12]. (ii) We must check whether the Higgs potential can really generate the radiative electroweak symmetry breaking [13]. Since the finiteness conditions severely restrict the parameter space in the model it is not clear whether we can obtain, in particular, the desired values of tan β. To carry out this, we will actually see that it is necessary to take account of a new parameter in a boundary condition for the Higgs mixing mass while preserving the finiteness conditions in the SU(5) model.

This paper is organized as follows. We briefly review the finiteness conditions in $N=1$ supersymmetric gauge theories and its application to an SU(5) GUT model in Sec. II. In Sec. III we consider the matching conditions of this finite SU(5) model to the MSSM and then calculate low-energy predictions numerically. The GUT and SUSY threshold corrections which are characteristic to the model are discussed in Sec. IV. Section V is devoted to the summary and some comments. The appendixes contain the tree-level form of mass formulas and the SUSY threshold corrections in the MSSM.

II. FINITE SU(5) MODEL

We first review the finiteness conditions in the perturbation theory and explain our notations in this paper. First we describe the all-order finiteness conditions for gauge couplings and the couplings in the superpotential sector. We

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consider an anomaly free $N=1$ supersymmetric gauge theory based on a simple gauge group G with a gauge coupling g, and with the superpotential

$$W=\frac{1}{2}m_{ij}\Phi^i\Phi^j+\frac{1}{6}Y_{ijk}\Phi^i\Phi^j\Phi^k. \tag{2.1}$$

The one-loop β functions for these couplings are given by

$$\beta_g=\frac{-g^3}{16\pi^2}\left[3C_2(G)-\sum T(R_i)\right], \tag{2.2}$$

$$\beta_{ij}=m_{ik}\gamma_j^k+m_{jk}\gamma_i^k, \tag{2.3}$$

$$\beta_{ijk}=Y_{ijl}\gamma_k^l+Y_{jkl}\gamma_i^l+Y_{kil}\gamma_j^l, \tag{2.4}$$

$$\gamma_j^i=\frac{1}{32\pi^2}[Y_{ikl}^*Y_{jkl}-4g^2C_2(R_i)\delta_j^i], \tag{2.5}$$

where γ_j^i are a one-loop anomalous dimensions of field Φ^i and

$$\mathrm{tr}(T^aT^b)=T(R)\delta^{ab},\quad \sum_a T^aT^a=C_2(R)\underline{1},$$
$$\sum_{c,d} f_{acd}f_{bcd}=C_2(G)\delta_{ab}, \tag{2.6}$$

f_{abc} are the structure constants of the gauge group G. The requirement of finiteness implies that all the above β functions should be zero. It can be easily seen that under the condition $\beta_g=0$, the vanishing of β_{ij} and β_{ijk} is equivalent to the vanishing anomalous dimensions γ_j^i. Then the necessary and sufficient conditions for the one-loop order finiteness are

$$3C_2(G)-\sum T(R_i)=0, \tag{2.7}$$

$$Y_{ikl}^*Y_{jkl}-4g^2C_2(R_i)\delta_j^i=0. \tag{2.8}$$

That is, the field content of theories satisfies the first condition (2.7) and in addition the second condition (2.8) possesses the (g expansion) solutions of the form

$$Y_{ijk}=\rho_{ijk}g, \tag{2.9}$$

where ρ_{ijk} are constants. Because of the first condition, the field contents which can satisfy these conditions are limited and have been completely listed in Ref. [4]. Interestingly enough, these relations are also necessary and sufficient for two-loop order finiteness [14]. In addition to the above conditions it is necessary to impose one more condition so that the theory can have no divergence in all orders of perturbation theory. This condition says that the solutions of vanishing one-loop anomalous dimensions are not parametrized and cannot be multiple zeroes when considered as the solutions of vanishing one-loop Yukawa β functions, $\beta_{ijk}=0$ [5]. This ensures the existence of the all-order g-expansion solutions for vanishing anomalous dimensions when the one-loop level solutions (2.9) exist. The conditions for all-order finiteness can be expressed in term of the one-loop order quantities (β functions). Therefore one can easily apply these conditions to definite models.

Next we discuss the soft SUSY breaking sector. In general, this part of potential takes the following form (for a simple gauge group G):

$$V_{\mathrm{soft}}=\frac{1}{2}(\mu^2)_j^i\phi_i^*\phi^j+\frac{1}{2}B_{ij}\phi^i\phi^j+\frac{1}{6}h_{ijk}\phi^i\phi^j\phi^k$$
$$+\frac{1}{2}M\lambda\lambda+\mathrm{H.c.} \tag{2.10}$$

Here and hereafter the fields ϕ^i in V_{soft} denote the scalar components of corresponding superfields and λ stands for gaugino. In the leading order, the finiteness conditions (vanishing logarithmic divergences) for these parameters in V_{soft} are given by [15]

$$h_{ijk}=-MY_{ijk}, \tag{2.11}$$

$$(\mu^2)_j^i=\frac{1}{3}MM^*\delta_j^i, \tag{2.12}$$

together with Eqs. (2.7) and (2.8). It is interesting to note that these universal forms of the finiteness conditions are the same as those derived from superstring or four-dimensional $N=1$ supergravity models. This may indicate that the finiteness is originated from some higher-scale symmetries as stated before. The above relations also ensure the two-loop level finiteness for soft SUSY breaking parameters such as the dimensionless couplings [16]. Though the renormalization-group invariant relations which are valid to all orders have been found [17], we will apply these two-loop level conditions as a good approximation and calculate the low-energy predictions numerically. It is also noted that the requirement of finiteness has nothing to do with the parameters B_{ij}. However, other phenomenological requirements may determine the boundary conditions for these parameters (see, Sec. III).

As a concrete example of phenomenological applications of the finiteness conditions, we consider supersymmetric SU(5) GUT models. According to the classification tables in Ref. [4], there exists only one field content which can fulfill the following requirements. (i) It contains chiral three families [three $(\mathbf{10},\overline{\mathbf{5}})$ sets]. (ii) The other fields are vectorlike ones. (iii) It also contains fields in the adjoint representation in order to break the GUT gauge group. This model contains $(\mathbf{5},\overline{\mathbf{5}},\mathbf{10},\mathbf{24})$ with the multiplicities (4,7,3,1). Then the general superpotential for this content becomes

TABLE I. The charge assignments of the $Z_7 \times Z_3 \times Z_2$ (matter parity). The $\overline{H}_a$'s have opposite charges to those of H_a.

	10_1	10_2	10_3	$\overline{5_1}$	$\overline{5_2}$	$\overline{5_3}$	H_1	H_2	H_3	H_4	Σ
Z_7	1	2	4	4	1	2	5	3	6	0	0
Z_3	1	2	0	0	0	0	1	2	0	0	0
Z_2	1	1	1	1	1	1	0	0	0	0	0

$$W=\frac{1}{2}f_{ija}10_i10_jH_a+\overline{f}_{ija}10_i\overline{5}_j\overline{H}_a+\frac{1}{2}q_{ijk}10_i\overline{5}_j\overline{5}_k + f_{ab}\overline{H}_a\Sigma H_b+\frac{1}{2}q'_{iab}10_i\overline{H}_a\overline{H}_b+p\Sigma^3+f_{ia}\overline{5}_i\Sigma H_a+W_m \tag{2.13}$$

$$(i,j=1,2,3,\quad a,b=1,2,3,4),$$

where H_a, $\overline{H}_a$, and Σ denote the Higgs fields of the representations **5**, $\overline{\mathbf{5}}$, and **24**, respectively, and i,j stand for generation indices. The last term W_m contains the mass terms of the Higgs fields H_a, $\overline{H}_a$, and Σ. We now do not need to explicitly express these mass parameters because the finiteness of these couplings are automatically accomplished by the finiteness of other parameters. It is sufficient to impose the discrete symmetries in order to get isolate and nondegenerate solutions and in addition to suppress the rapid nucleon decay [10] (see Table I). With these symmetries, the superpotential is restricted to the form

$$W=\frac{1}{2}f_{iii}10_i10_iH_i+\overline{f}_{iii}10_i\overline{5}_i\overline{H}_i+f_{aa}\overline{H}_a\Sigma H_a+p\Sigma^3+W_m \tag{2.14}$$

and then we can find the following unique solution which guarantees the all-order finiteness for Yukawa couplings [8–10];

$$f_{111}=f_{222}=f_{333}=\sqrt{\frac{8}{5}}g,\quad \overline{f}_{111}=\overline{f}_{222}=\overline{f}_{333}=\sqrt{\frac{6}{5}}g,$$
$$f_{11}=f_{22}=f_{33}=0,\quad f_{44}=g,\quad p=\sqrt{\frac{15}{7}}g, \tag{2.15}$$

where g is the SU(5) gauge coupling.

Then we consider the finiteness conditions for the soft SUSY breaking parameters in this SU(5) model. The general form of the potential becomes under the above symmetries:

$$V_{\text{soft}}=(m_{\overline{5}}^2)_{ij}\overline{5}_i^{\dagger}\overline{5}_j+(m_{10}^2)_{ij}10_i^{\dagger}10_j+(m_H^2)_{ab}H_a^{\dagger}H_b + (m_{\overline{H}}^2)_{ab}\overline{H}_a^{\dagger}\overline{H}_b+m_\Sigma^2\Sigma^{\dagger}\Sigma+\Bigg(\frac{1}{2}h_{iii}10_i10_iH_i + \overline{h}_{iii}10_i\overline{5}_i\overline{H}_i+h_{aa}\overline{H}_a\Sigma H_a+h_p\Sigma^3+B_{ab}\overline{H}_aH_b + B_\Sigma\Sigma\Sigma+\frac{1}{2}M\lambda\lambda+\text{H.c.}\Bigg). \tag{2.16}$$

Taking into account the form of the superpotential (2.14), we can get the relations among the soft breaking parameters satisfying the two-loop order finiteness conditions:

$$h_{iii}=-Mf_{iii},\quad \overline{h}_{iii}=-M\overline{f}_{iii},\quad h_{44}=-Mf_{44},\quad h_p=-Mp,$$
$$(m_H^2)_{ab}=(m_{\overline{H}}^2)_{ab}=\frac{1}{3}M^2\delta_{ab},\quad m_\Sigma^2=\frac{1}{3}M^2,$$
$$(m_{10}^2)_{ij}=(m_{\overline{5}}^2)_{ij}=\frac{1}{3}M^2\delta_{ij}, \tag{2.17}$$

and all the other elements are zero. The relations (2.15) and (2.17) altogether provide us with the finite SU(5) model above GUT scale (at least, two-loop order finite for all couplings). Note that from the requirements of finiteness there are no constraints for the B parameters in the potential (2.16) as well as the supersymmetric Higgs mass parameters in W_m. They are to be determined by the requirements of some low-energy assumptions, which is one of the tasks in the next section.

III. MATCHING TO THE MSSM AND LOW-ENERGY PREDICTIONS

In this section, we analyze the low-energy predictions of the finite SU(5) model as explained in the previous section. This model, which is supposed to be broken spontaneously to the MSSM, casts the boundary conditions for couplings in the MSSM from the finiteness conditions at the GUT scale M_G. Leaving the threshold corrections to the gauge and Yukawa couplings in the next section, we first consider the tree-level matching of parameters between the MSSM and the finite SU(5) model. With these matching conditions, the parameter space of the MSSM couplings is highly restricted and then it is a nontrivial problem whether the experimentally required values of couplings, in particular, the radiative electroweak symmetry breaking are surely realized.

The matching conditions for gauge couplings are trivial,

$$g_1(M_G)=g_2(M_G)=g_3(M_G)=g, \tag{3.1}$$

where g_1, g_2, and g_3 are the gauge couplings of the MSSM, $U(1)_Y$, $SU(2)_W$, and $SU(3)_C$, respectively (in a GUT normalization).

Next, we consider the Yukawa couplings. At first sight, one may think that a pair of light Higgs doublet does not couple to any of matter fields because the doublet-triplet splitting mechanism seems to act only on H_4 with the solutions (2.15). However, as mentioned at the end of Sec. II, the Higgs mass parameters in the superpotential W_m are not constrained from finiteness requirements. Therefore we have a freedom for tunings of mass parameters of $H_a\overline{H}_b$ to cause a Higgs mixing at M_G scale as seen in the following. In this paper we consider nonzero Yukawa couplings only for the third generation matter fields but the Yukawa couplings for the first and second generations could be obtained in the same way [18].

After the SU(5) gauge symmetry breaking, we simply suppose the supersymmetric $H\bar{H}$ mass terms take the following form:

$$W'_m = f_{44}\bar{H}_4\langle\Sigma\rangle H_4 + M_1\bar{H}_1H_1 + M_2\bar{H}_2H_2 + \sum_{a,b=3,4} M_{ab}\bar{H}_aH_b, \tag{3.2}$$

$$M_1, M_2, M_{ab} \sim M_G.$$

In this case, the Higgs fields H_1 and H_2 just decouple at GUT scale. Substituting the vacuum expectation value of the adjoint Higgs field, $\langle\Sigma\rangle = \omega\ \mathrm{diag}(2,2,2,-3,-3)$, into W_m, the mass terms of H_3 and H_4 become

$$\bar{H}_a^{(3)}M_{ab}^{(3)}H_b^{(3)} \equiv \bar{H}_a^{(3)}\left[M_{ab} + \begin{pmatrix} 0 & \\ & 2 \end{pmatrix}\omega f_{44}\right]H_b^{(3)}, \tag{3.3}$$

$$\bar{H}_a^{(2)}M_{ab}^{(2)}H_b^{(2)} \equiv \bar{H}_a^{(2)}\left[M_{ab} + \begin{pmatrix} 0 & \\ & -3 \end{pmatrix}\omega f_{44}\right]H_b^{(2)}, \tag{3.4}$$

$$H_a = \begin{pmatrix} H_a^{(3)} \\ H_a^{(2)} \end{pmatrix}, \quad \bar{H}_a = \begin{pmatrix} \bar{H}_a^{(3)} \\ \bar{H}_a^{(2)} \end{pmatrix}, \tag{3.5}$$

where the indices ''(2)'' and ''(3)'' stand for doublet and triplet components of each quantity. Assuming the mass parameters M_{ab} are real, we diagonalize $M^{(2)}$ by a rotation of Higgs fields

$$\begin{pmatrix} H'_3 \\ H'_4 \end{pmatrix} = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix}\begin{pmatrix} H_3 \\ H_4 \end{pmatrix},$$

$$\begin{pmatrix} \bar{H}'_3 \\ \bar{H}'_4 \end{pmatrix} = \begin{pmatrix} \cos\bar{\theta} & \sin\bar{\theta} \\ -\sin\bar{\theta} & \cos\bar{\theta} \end{pmatrix}\begin{pmatrix} \bar{H}_3 \\ \bar{H}_4 \end{pmatrix}, \tag{3.6}$$

$$M^{(2)\prime} = \begin{pmatrix} \cos\bar{\theta} & \sin\bar{\theta} \\ -\sin\bar{\theta} & \cos\bar{\theta} \end{pmatrix} M^{(2)} \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix} \equiv \begin{pmatrix} \mu & 0 \\ 0 & \mu' \end{pmatrix}, \tag{3.7}$$

$$\mu \ll M_G, \quad \mu' \sim M_G \tag{3.8}$$

which leads to the doublet-triplet splitting and leaves a pair of light Higgs doublet to low-energy region. The triplet Higgs mass terms are

$$M^{(3)\prime} = \begin{pmatrix} \mu & 0 \\ 0 & \mu' \end{pmatrix} + 5\omega f_{44}\begin{pmatrix} \sin\theta\sin\bar{\theta} & \cos\theta\sin\bar{\theta} \\ \sin\theta\cos\bar{\theta} & \cos\theta\cos\bar{\theta} \end{pmatrix}. \tag{3.9}$$

After all, the third generation Yukawa couplings at GUT scale become

$$f'_{333} = f_{333}\cos\theta, \quad \bar{f}'_{333} = \bar{f}_{333}\cos\bar{\theta} \tag{3.10}$$

with the values of f_{333} and $\bar{f}_{333}$ from the finiteness conditions. From this, we can consider two phenomenologically separate cases

Case (1): $\cos\theta \sim \cos\bar{\theta} \sim 1$ (large $\tan\beta$),

Case (2): $\cos\theta \sim 1$, $\cos\bar{\theta} \sim 0$ (small $\tan\beta$).

In the following, we adopt a typical value for each case; (1) $\cos\theta = \cos\bar{\theta} = 0.9856$ ($\sin\theta = \sin\bar{\theta} = 0.169$) and (2) $\cos\theta = 0.954$, $\cos\bar{\theta} = 0.03$ ($\sin\theta = 0.3$, $\sin\bar{\theta} = 0.9995$). We can realize these desired values of μ, μ' and the rotation angles by choosing the mass parameters[1] M_{ab}. In the end, the superpotential in the MSSM takes the form

$$W = y_tHQ_3\bar{t} + y_b\bar{H}Q_3\bar{b} + y_\tau\bar{H}L_3\bar{\tau} + \rho\bar{H}H, \tag{3.11}$$

with the matching conditions at M_G given by

$$y_t(M_G) = f'_{333} = f_{333}\cos\theta,$$

$$y_b(M_G) = y_\tau(M_G) = \bar{f}'_{333} = \bar{f}_{333}\cos\bar{\theta}, \tag{3.12}$$

$$\rho(M_G) = \mu. \tag{3.13}$$

The values of f_{333} and $\bar{f}_{333}$ are given by the finiteness conditions in the SU(5) GUT and other parameters can be determined by phenomenological requirements.

Finally, we consider the matching conditions in the soft SUSY breaking sector. In the MSSM, a general form for this sector is

$$\begin{aligned} V_{\text{soft}} &= m_1^2\bar{H}^\dagger\bar{H} + m_2^2H^\dagger H + (m_3^2\bar{H}H + \text{H.c.}) \\ &+ \sum_{i=1,2,3}(m_{\tilde{Q}_i}^2|Q_i|^2 + m_{\tilde{L}_i}^2|L_i|^2 + m_{\tilde{u}_i}^2|\bar{u}_i|^2 + m_{\tilde{d}_i}^2|\bar{d}_i|^2 \\ &+ m_{\tilde{e}_i}^2|\bar{e}_i|^2) + (h_tHQ_3\bar{t} + h_b\bar{H}Q_3\bar{b} + h_\tau\bar{H}L_3\bar{\tau} + \text{H.c.}) \\ &+ \frac{1}{2}(M_{\lambda_1}\lambda_1\lambda_1 + M_{\lambda_2}\lambda_2\lambda_2 + M_{\lambda_3}\lambda_3\lambda_3 + \text{H.c.}). \end{aligned} \tag{3.14}$$

The first two lines are the soft mass terms for squarks, sleptons, and Higgs scalars. The third line denotes the scalar

[1]For example, we can take the following mass parameters: Case (1) $M_{33} \sim 0.6$, $M_{34} = M_{43} \sim 3.4$, $M_{44} \sim 20$ and Case (2) $M_{33} \sim 3.0$, $M_{34} \sim 9.5$, $M_{43} \sim 0.1$, $M_{44} \sim 0.3$ ($\times 10^{16}$ GeV) for $\mu \sim 100$ GeV and $\mu' \sim 10^{16}$ GeV.

trilinear couplings which corresponds to the Yukawa couplings in the superpotential (3.11) and the last line represents the gaugino masses for the standard gauge group. Taking into account the above Higgs rotations, the matching conditions for these parameters (except for m_3) become as follows:

$$m^2_{\tilde{Q}_i}(M_G)=m^2_{\tilde{u}_i}(M_G)=m^2_{\tilde{e}_i}(M_G)=m^2_{10\,i}, \quad (3.15)$$

$$m^2_{\tilde{L}_i}(M_G)=m^2_{\tilde{d}_i}(M_G)=m^2_{5\,i}, \quad (3.16)$$

$$m^2_1(M_G)=(m^2_{\overline{H}'})_{33}, \quad m^2_2(M_G)=(m^2_{H'})_{33}, \quad (3.17)$$

$$h_t(M_G)=h_{333}\cos\theta, \quad h_b(M_G)=h_\tau(M_G)=\overline{h}_{333}\cos\overline{\theta}, \quad (3.18)$$

$$M_{\lambda_1}(M_G)=M_{\lambda_2}(M_G)=M_{\lambda_3}(M_G)=M. \quad (3.19)$$

[For the definitions of soft SUSY breaking parameters in the SU(5) model, see Eq. (2.16).] Together with the finiteness conditions (2.15) and (2.17), we finally obtain the boundary conditions for the MSSM couplings at GUT scale:

$$g_1=g_2=g_3=g, \quad (3.20)$$

$$y_t=\sqrt{\frac{8}{5}}g\cos\theta, \quad y_b=y_\tau=\sqrt{\frac{6}{5}}g\cos\overline{\theta}, \quad (3.21)$$

$$h_t=-\sqrt{\frac{8}{5}}Mg\cos\theta, \quad h_b=h_\tau=-\sqrt{\frac{6}{5}}Mg\cos\overline{\theta}, \quad (3.22)$$

$$m^2_{\tilde{Q}_i}=m^2_{\tilde{L}_i}=m^2_{\tilde{u}_i}=m^2_{\tilde{d}_i}=m^2_{\tilde{e}_i}=m^2_1=m^2_2=\frac{1}{3}M^2 \quad (i=1,2,3), \quad (3.23)$$

$$M_{\lambda_1}=M_{\lambda_2}=M_{\lambda_3}=M, \quad (3.24)$$

$$\rho=\mu. \quad (3.25)$$

Furthermore we can determine a boundary condition for m_3 when we consider the doublet-triplet splitting in the soft SUSY breaking sector as well as in the supersymmetric sector W'_m. After the SU(5) breaking, the soft mass terms of the scalar components $H^{(2)}_a$ can be found from the finiteness conditions (2.17),

$$V_{\rm soft}=\overline{H}^{(2)}_a\left[B_{ab}-M\begin{pmatrix}0 & \\ & -3\end{pmatrix}\omega f_{44}\right]H^{(2)}_b. \quad (3.26)$$

It is easily seen from the doublet-triplet splitting in W'_m that to complete the doublet-triplet splitting we should take

$$B_{ab}=-MM_{ab}. \quad (3.27)$$

Then we can see that a pair of Higgs (scalar) doublet actually survives down to the low energy:

$$(3.26)=-M\overline{H}^{(2)}_a\left[M_{ab}+\begin{pmatrix}0 & \\ & -3\end{pmatrix}\omega f_{44}\right]H^{(2)}_b \quad (3.28)$$

$$=-M\overline{H}^{(2)\prime}\begin{pmatrix}\mu & 0\\ 0 & \mu'\end{pmatrix}H^{(2)\prime}. \quad (3.29)$$

Now, exactly speaking, there are uncertainties in the condition for B_{ab} (3.27) and/or the finiteness conditions for h_{44} in Eq. (2.17), whose order is less than the SUSY breaking scale $M_{\rm SUSY}$. This uncertainty does not disturb the electroweak symmetry breaking and/or the finiteness criterion in SUSY gauge theories. Then we can introduce a free parameter δm^2_3 in the matching condition for soft mixing mass parameters m^2_3;

$$m^2_3(M_G)=-M\mu+\delta m^2_3 \quad (|\delta m^2_3|\lesssim M^2_{\rm SUSY}). \quad (3.30)$$

This new parameter actually plays an important role in the following numerical analyses. That is, we find that it is necessary to cause the radiative electroweak symmetry breaking.

With the boundary conditions (3.20)–(3.25) and (3.30) for the MSSM couplings, we analyze the low-energy predictions of the finite SU(5) GUT in each case (1) and (2). Although these conditions are universal and very restrictive, we will find the parameter region where these predictions are all consistent with the present low-energy experimental values. When the threshold corrections are neglected, the analysis procedure is as follows.

A. Case (1): Large tan β case

In this case we have five free parameters, g, M, μ, δm^2_3, and a scale M_G. All other couplings are determined by the finiteness conditions explained above. In addition, since we do not deal with the threshold corrections in this section, we can treat $M_{\rm SUSY}$ and $\tan\beta$ as free parameters in the procedure. First, we input g, M_G, $M_{\rm SUSY}$, and $\tan\beta$, and run the gauge and Yukawa couplings down to M_Z scale by using two-loop β functions in the MSSM [19] and the standard model. Then we tune these four input parameters so that we can reproduce the low-energy values which are consistent with the experimental data [20],

$$\alpha_1(M_Z)=0.01689\pm0.00005, \quad (3.31)$$

$$\alpha_2(M_Z)=0.03322\pm0.00025, \quad (3.32)$$

$$\alpha_3(M_Z)=0.12\pm0.01, \quad (3.33)$$

$$m_b(M_Z)=3.1\pm0.4 \text{ GeV}, \quad (3.34)$$

$$m_\tau(M_Z)=1.75\pm0.01 \text{ GeV}. \quad (3.35)$$

$$(M_Z=91.187 \text{ GeV}).$$

Since the dimensionful parameters are not contained in these β functions and we now neglect the threshold corrections, the values of M, μ, and δm_3^2 give no effects to this tuning. Next, we calculate the dimensionful parameters with an input value of gaugino mass M in addition to the parameter set obtained above. We tune the value of M so that $M_{\rm SUSY}$ in the above parameter set can be equal to an average of squark masses defined by

$$M_{\rm SB}^2 \equiv \frac{1}{4}[2m_{\tilde{Q}_3}^2(M)+m_{\tilde{u}_3}^2(M)+m_{\tilde{d}_3}^2(M)]. \quad (3.36)$$

Similarly, this adjustment of M is independent of the input values of μ and δm_3^2 since none of the β functions of other couplings in the MSSM contain ρ and m_3^2. In the last step, we tune μ and δm_3^2 so that the low-energy Higgs potential can actually realize the value of $\tan\beta$ in the above parameter set and can fulfill the constraints for radiative electroweak symmetry breaking,

$$\tan^2\beta = \frac{m_1^2+\rho^2+\frac{1}{2}M_Z^2}{m_2^2+\rho^2+\frac{1}{2}M_Z^2}, \quad (3.37)$$

$$\sin 2\beta = \frac{-2m_3^2}{m_1^2+m_2^2+2\rho^2}. \quad (3.38)$$

At this stage, we should incorporate the one-loop radiative corrections to the minimization of the Higgs potential in order to improve a very sensitive dependence of the vacuum expectation values on the renormalization point Q [21]. The resultant one-loop corrected Higgs potential becomes

$$V = V^{(0)} + V^{(1)}, \quad (3.39)$$

$$V^{(0)} = (m_1^2+\rho^2)\overline{H}^\dagger\overline{H} + (m_2^2+\rho^2)H^\dagger H + (m_3^2\overline{H}H + \text{H.c.}) + (D\text{-term contributions}), \quad (3.40)$$

$$V^{(1)} = \frac{1}{64\pi^2}\,\mathrm{STr}\left[\mathcal{M}^4\left(\ln\frac{\mathcal{M}^2}{Q^2}-\frac{3}{2}\right)\right], \quad (3.41)$$

where $\mathcal{M}^2$ is a field-dependent mass-squared matrix and $\mathrm{STr}\mathcal{A} = \Sigma_j(-1)^{2j}(2j+1)\,\mathrm{Tr}\mathcal{A}_j$ is a weighted supertrace. The explicit expression for $\mathcal{M}^2$ can be found in Ref. [22]. This exact one-loop correction takes, however, a very complicated form. So we here adopt the handy calculating method [23] which incorporates the corrections only to quadratic terms from $V^{(1)}$. The rapid Q dependence of the potential mainly comes from that of the running mass parameters and this method can make it mild. This method practically gives a good approximation to the full one-loop potential and it is numerically found that the values of vacuum expectation values evaluated by this method are almost equal to those obtained from the full one-loop potential [23].

In this way, we can determine the input values of free parameters g, M_G, M, μ, and δm_3^2, and calculate the low-energy predictions (the gauge couplings, the masses of fermions, superparticles, Higgs bosons, etc.). We show typical two types of example in Tables II and III. One is with the parameter set for the highest $M_{\rm SB}$ (Table II) and the other is for the lowest one (Table III). In the tables, $m_{\tilde{x}}$ denote the superparticle masses and $m_{H^\pm}$, m_A, $m_{H,h}$ are the Higgs scalar masses which correspond to the charged Higgs bosons, the neutral CP-odd Higgs and the neutral CP-even Higgs bosons, respectively. Their explicit tree-level forms are given in Appendix A.

TABLE II. The low-energy predictions in case (1) (the high $M_{\rm SB}$ case).

Case (1): Example 1			
M_G	1.246×10^{16} (GeV)		
$\alpha_{\rm GUT}$	0.0388		
M	612.315 (GeV)		
μ	1467.828 (GeV)		
δm_3^2	$-(636.086)^2$ (GeV^2)		
$M_{\rm SB}$	1000.0 (GeV)	$\tan\beta$	54.0
$\alpha_1(M_Z)$	0.016850	$m_t(M_Z)$	179.24 (GeV)
$\alpha_2(M_Z)$	0.033333	$m_b(M_Z)$	3.21 (GeV)
$\alpha_3(M_Z)$	0.112	$m_\tau(M_Z)$	1.745 (GeV)
$m_{\tilde{t}_+}$	1064.3 (GeV)	$m_{\tilde{u}_+}$	1243.4 (GeV)
$m_{\tilde{t}_-}$	892.4 (GeV)	$m_{\tilde{u}_-}$	1198.3 (GeV)
$m_{\tilde{b}_+}$	1051.2 (GeV)	$m_{\tilde{d}_+}$	1245.8 (GeV)
$m_{\tilde{b}_-}$	915.1 (GeV)	$m_{\tilde{d}_-}$	1193.4 (GeV)
$m_{\tilde{\tau}_+}$	516.2 (GeV)	$m_{\tilde{e}_+}$	536.2 (GeV)
$m_{\tilde{\tau}_-}$	191.2 (GeV)	$m_{\tilde{e}_-}$	420.5 (GeV)
$m_{\tilde{\nu}_\tau}$	478.5 (GeV)	$m_{\tilde{\nu}_e}$	530.6 (GeV)
$m_{\tilde{\chi}_1^+}$	837.5 (GeV)	$m_{\tilde{\chi}_2^+}$	481.5 (GeV)
$m_{\tilde{\chi}_1^0}$	275.4 (GeV)	$m_{\tilde{\chi}_2^0}$	500.5 (GeV)
$m_{\tilde{\chi}_3^0}$	794.9 (GeV)	$m_{\tilde{\chi}_4^0}$	814.9 (GeV)
$m_{H^\pm}$	331.8 (GeV)	m_A	322.6 (GeV)
m_H	322.7 (GeV)	m_h	89.0 (GeV)
M_{λ_3}	1326.6 (GeV)		
h_t	-853.6 (GeV)	h_b	-834.0 (GeV)
h_τ	-107.1 (GeV)		

The sparticle mass predictions are enough within the experimental bounds [20]. Note that unlike the usual MSSM predictions, the lightest superparticle is not a neutral component but the tau slepton. This is because of the highly restrictive finiteness conditions. That is, with these conditions, we must use the large value of μ and relatively small M to realize the radiative electroweak symmetry breaking and also use large values of bottom and tau Yukawa couplings at M_G. This may be avoided by considering an R-parity violating interaction $Q\overline{d}L$ which is needed for one of the possible interpretations of the high-Q^2 anomaly observed at the DESY ep collider HERA [24]. In any case, this property is characteristic to the finite SU(5) model with large $\tan\beta$ and would be tested in the future experiments.

B. Case (2): Small $\tan\beta$ case

In the same way as the case (1), we have five free parameters for the radiative symmetry breaking in this case. Unlike the large $\tan\beta$ case, there is no problem of the fine-tuning of

TABLE III. The low-energy predictions in case (1) (the low M_{SB} case).

Case (1): Example 2			
M_G	1.116×10^{16} (GeV)		
α_{GUT}	0.0392		
M	473.385 (GeV)		
μ	1171.954 (GeV)		
δm_3^2	$-(510.579)^2$ (GeV^2)		
M_{SB}	790.0 (GeV)	$\tan\beta$	54.0
$\alpha_1(M_Z)$	0.016930	$m_t(M_Z)$	178.9 (GeV)
$\alpha_2(M_Z)$	0.033462	$m_b(M_Z)$	3.20 (GeV)
$\alpha_3(M_Z)$	0.113	$m_\tau(M_Z)$	1.745 (GeV)
$m_{\tilde{t}_+}$	858.0 (GeV)	$m_{\tilde{u}_+}$	978.3 (GeV)
$m_{\tilde{t}_-}$	694.5 (GeV)	$m_{\tilde{u}_-}$	944.2 (GeV)
$m_{\tilde{b}_+}$	841.5 (GeV)	$m_{\tilde{d}_+}$	981.3 (GeV)
$m_{\tilde{b}_-}$	708.2 (GeV)	$m_{\tilde{d}_-}$	940.9 (GeV)
$m_{\tilde{\tau}_+}$	415.0 (GeV)	$m_{\tilde{e}_+}$	416.5 (GeV)
$m_{\tilde{\tau}_-}$	111.5 (GeV)	$m_{\tilde{e}_-}$	326.4 (GeV)
$m_{\tilde{\nu}_\tau}$	369.4 (GeV)	$m_{\tilde{\nu}_e}$	409.2 (GeV)
$m_{\tilde{\chi}_1^+}$	666.4 (GeV)	$m_{\tilde{\chi}_2^+}$	367.9 (GeV)
$m_{\tilde{\chi}_1^0}$	215.0 (GeV)	$m_{\tilde{\chi}_2^0}$	391.1 (GeV)
$m_{\tilde{\chi}_3^0}$	613.3 (GeV)	$m_{\tilde{\chi}_4^0}$	638.6 (GeV)
$m_{H^\pm}$	261.8 (GeV)	m_A	250.1 (GeV)
m_H	250.2 (GeV)	m_h	88.9 (GeV)
M_{λ_3}	1042.7 (GeV)		
h_t	-677.2 (GeV)	h_b	-662.5 (GeV)
h_τ	-78.5 (GeV)		

TABLE IV. The low-energy prediction in case (2).

Case (2)			
M_G	1.206×10^{16} (GeV)		
α_{GUT}	0.0392		
M	332.38 (GeV)		
μ	642.65 (GeV)		
δm_3^2	$-(255.15)^2$ (GeV^2)		
M_{SB}	620.0 (GeV)	$\tan\beta$	3.1
$\alpha_1(M_Z)$	0.016852	$m_t(M_Z)$	177.5 (GeV)
$\alpha_2(M_Z)$	0.033119	$m_b(M_Z)$	3.42 (GeV)
$\alpha_3(M_Z)$	0.110	$m_\tau(M_Z)$	1.751 (GeV)
$m_{\tilde{t}_+}$	685.5 (GeV)	$m_{\tilde{u}_+}$	690.8 (GeV)
$m_{\tilde{t}_-}$	453.8 (GeV)	$m_{\tilde{u}_-}$	667.4 (GeV)
$m_{\tilde{b}_+}$	665.8 (GeV)	$m_{\tilde{d}_+}$	694.4 (GeV)
$m_{\tilde{b}_-}$	619.2 (GeV)	$m_{\tilde{d}_-}$	665.8 (GeV)
$m_{\tilde{\tau}_+}$	293.6 (GeV)	$m_{\tilde{e}_+}$	292.8 (GeV)
$m_{\tilde{\tau}_-}$	229.0 (GeV)	$m_{\tilde{e}_-}$	231.0 (GeV)
$m_{\tilde{\nu}_\tau}$	284.4 (GeV)	$m_{\tilde{\nu}_e}$	284.4 (GeV)
$m_{\tilde{\chi}_1^+}$	559.0 (GeV)	$m_{\tilde{\chi}_2^+}$	246.3 (GeV)
$m_{\tilde{\chi}_1^0}$	188.7 (GeV)	$m_{\tilde{\chi}_2^0}$	296.9 (GeV)
$m_{\tilde{\chi}_3^0}$	411.5 (GeV)	$m_{\tilde{\chi}_4^0}$	493.9 (GeV)
$m_{H^\pm}$	654.1 (GeV)	m_A	649.5 (GeV)
m_H	651.6 (GeV)	m_h	72.0 (GeV)
M_{λ_3}	734.5 (GeV)		
h_t	-523.1 (GeV)	h_b	-63.2 (GeV)
h_τ	-17.6 (GeV)		

Higgs mass parameters (and a large threshold correction to bottom quark mass discussed in the next section) in small $\tan\beta$ cases. On the other hand, when we adopt the handy calculating method to estimate the one-loop corrections to the Higgs potential, we should take care of another respect as mentioned in Ref. [23]. This is that in the small $\tan\beta$ case, the contributions to quartic terms from $V^{(1)}$ near the flat direction ($\tan\beta\sim1$) are no longer small compared to that from the tree-level potential $V^{(0)}$. Therefore the method which includes only mass corrections may be no more good approximation. The contribution to the quartic terms from $V^{(0)}$ is

$$\sim\frac{1}{8}(g_1^2+g_2^2)|v|^4\cos^2 2\beta \tag{3.42}$$

and the typical contribution from $V^{(1)}$ is

$$\sim\frac{1}{32\pi^2}g_2^2|v|^4. \tag{3.43}$$

Therefore requiring that, for instance, Eq. (3.43) is within 10% of Eq. (3.42), we need

$$\tan\beta\gtrsim1.35. \tag{3.44}$$

In all the other respects, we can follow the analysis procedure in the case (1) and show a representative result in Table IV.

We can see from this that for example, this model predicts that the lightest superparticle is the lightest neutralino component because the μ parameter can be taken smaller than in the large $\tan\beta$ case and therefore the result bears a close resemblance to the typical one in the usual MSSM [25].

IV. THRESHOLD CORRECTIONS

In this section we consider the threshold corrections at GUT and SUSY breaking scales. Two types of corrections are important. One is the GUT corrections to the gauge and Yukawa couplings [26,27] which may be characteristic to the finite SU(5) model. The GUT corrections to the other parameters (gaugino masses etc.) are so small that the following analyses are not affected too much [29]. The other important corrections are the SUSY scale ones [27,28], especially a correction to the bottom quark mass m_b which is important in the large $\tan\beta$ cases. These corrections to the dimensionless couplings are important in a sense that the parameters are now precisely measured by experiments and so to include these corrections may further restrict the allowed parameter regions. Since the MSSM couplings are rather restricted at GUT scale by the finiteness conditions and the low-energy physics (the experimental data and the constraints from the Higgs potential), there is only a little room for varying the threshold corrections except for their signs. Therefore we

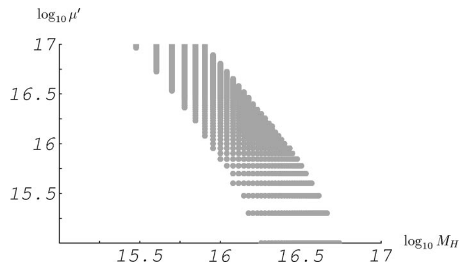

FIG. 1. The allowed region for M_H and μ' in the large $\tan\beta$ case.

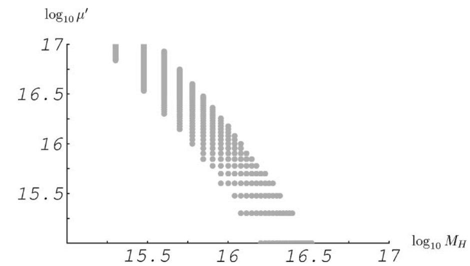

FIG. 2. The allowed region for M_H and μ' in the small $\tan\beta$ case.

should check whether the low-energy experimental values of couplings are consistently reproduced by tuning the GUT threshold corrections. In this paper we neglect the one-loop corrections from the electroweak gauge boson and the top quark [30] except for an important correction to the mass of the lightest CP-even Higgs boson [31].

First, we discuss the one-loop GUT threshold corrections. These corrections to the standard gauge couplings α_i ($i=1,2,3$) are found to be

$$\frac{2\pi}{\alpha_i(\Lambda)}=\frac{2\pi}{\alpha}-\Delta_i^G(\Lambda), \tag{4.1}$$

$$\Delta_1^G(\Lambda)=\frac{5}{2}\ln\left(\frac{M_V}{\Lambda}\right)-\frac{1}{4}\sum_{i=1,2}\ln\left(\frac{M_i}{\Lambda}\right)-\frac{1}{10}\sum_{i=3,4}\ln\left(\frac{M_i}{\Lambda}\right)-\frac{3}{20}\ln\left(\frac{\mu'}{\Lambda}\right), \tag{4.2}$$

$$\Delta_2^G(\Lambda)=\frac{3}{2}\ln\left(\frac{M_V}{\Lambda}\right)-\frac{1}{2}\ln\left(\frac{M_\Sigma}{\Lambda}\right)-\frac{1}{4}\sum_{i=1,2}\ln\left(\frac{M_i}{\Lambda}\right)-\frac{1}{4}\ln\left(\frac{\mu'}{\Lambda}\right), \tag{4.3}$$

$$\Delta_3^G(\Lambda)=\ln\left(\frac{M_V}{\Lambda}\right)-\frac{4}{3}\ln\left(\frac{M_\Sigma}{\Lambda}\right)-\frac{1}{4}\sum_{i=1,\cdots,4}\ln\left(\frac{M_i}{\Lambda}\right), \tag{4.4}$$

where M_V is a superheavy SU(5) gauge boson mass [which is just equal to the mass of the $(3,2,\pm\frac{5}{6})$ component of adjoint Higgs field Σ], M_Σ is the mass of color octet and SU(2) triplet component of Σ, and M_i and μ' are the masses of color triplet parts of the fundamental Higgs $H_a(\overline{H}_a)$. These mass parameters are defined by the conditions for the SU(5) symmetry breaking and the finiteness as follows:

$$M_V=5\sqrt{2}g\omega=\frac{10}{3}\sqrt{\frac{14}{15}}M_{24}, \tag{4.5}$$

$$M_\Sigma=5M_{24}, \tag{4.6}$$

$$M_3=5\omega f_{44}\sin\theta\sin\overline{\theta}=\frac{10}{3}\sqrt{\frac{7}{15}}\sin\theta\sin\overline{\theta}M_{24}, \tag{4.7}$$

$$M_4=\mu'+5\omega f_{44}\cos\theta\cos\overline{\theta}=\mu'+\frac{10}{3}\sqrt{\frac{7}{15}}\cos\theta\cos\overline{\theta}M_{24}. \tag{4.8}$$

The parameters M_1, M_2, μ', and ω are defined in Sec. III [see Eqs. (3.2) and (3.7)] and M_{24} is a supersymmetric mass of the adjoint Higgs Σ field:

$$W_m=W_m'+M_{24}\Sigma^2. \tag{4.9}$$

When we set $M_G\equiv M_V=\Lambda$ there are three free parameters M_1, M_2, and μ' in the above correction formulas. Furthermore since the H_1 and H_2 sector have same structures, we can set $M_1=M_2\equiv M_H$ without loss of generality. We show in Figs. 1 and 2 the allowed region for M_H and μ' which reproduces the desired low-energy values of the gauge couplings $\alpha_{1,2,3}(M_Z)$ for the case 1 (large $\tan\beta$) and case 2 (small $\tan\beta$). When evaluating the allowed regions, we have also included the SUSY threshold corrections to the gauge couplings given in Appendix B. These SUSY threshold corrections are of the order of 1–2%. Therefore it is found that they can be cancelled by $\Delta_{1,2,3}^G$ and the allowed regions obtained in the previous section are still valid. However, the regions in Figs. 1 and 2 may be rather narrowed if the nucleon decay constraints ($M_H, \mu'\gtrsim 10^{16}$ GeV) are taken into account.

Next we consider the GUT corrections to Yukawa couplings. The one-loop corrected values y_i^- ($i=t,b,\tau$) are given by

$$y_t^-(\Lambda)=y_t^+[1+\Delta_t^G(\Lambda)], \tag{4.10}$$

$$y_b^-(\Lambda)=y_b^+[1+\Delta_b^G(\Lambda)], \tag{4.11}$$

$$y_\tau^-(\Lambda)=y_\tau^+[1+\Delta_\tau^G(\Lambda)], \tag{4.12}$$

$$
\begin{aligned}
16\pi^2\Delta_t^G(\Lambda) = {} & -\frac{g^2}{2}[5F(M_V^2,0)+3F(M_3^2,M_V^2)]+\frac{3}{2}(y_t^{+2}\cos^2\theta+y_b^{+2}\cos^2\overline{\theta})F(M_3^2,0)+\frac{3}{2}(y_t^{+2}\sin^2\theta+y_b^{+2}\sin^2\overline{\theta})F(M_4^2,0)\\
& +\frac{1}{2}f_{44}^2\sin^2\theta\sin^2\overline{\theta}\left(3F(M_3^2,M_V^2)+\frac{3}{2}F(M_\Sigma^2,0)+\frac{3}{10}F[(0.2M_\Sigma)^2,0]\right)+\frac{1}{2}f_{44}^2(\cos^2\theta\sin^2\overline{\theta}+\sin^2\theta\cos^2\overline{\theta})\\
& \times\left(3F(M_4^2,M_V^2)+\frac{3}{2}F(M_\Sigma^2,\mu'^2)+\frac{3}{10}F[(0.2M_\Sigma)^2,\mu'^2]\right), \qquad (4.13)
\end{aligned}
$$

$$
\begin{aligned}
16\pi^2\Delta_b^G(\Lambda) = {} & -\frac{g^2}{2}[5F(M_V^2,0)+3F(M_3^2,M_V^2)]+\left(y_t^{+2}\cos^2\theta+\frac{3}{2}y_b^{+2}\cos^2\overline{\theta}\right)F(M_3^2,0)+\left(y_t^{+2}\sin^2\theta\right.\\
& \left.+\frac{3}{2}y_b^{+2}\sin^2\overline{\theta}\right)F(M_4^2,0)+\frac{1}{2}f_{44}^2\sin^2\theta\sin^2\overline{\theta}\left(3F(M_3^2,M_V^2)+\frac{3}{2}F(M_\Sigma^2,0)+\frac{3}{10}F[(0.2M_\Sigma)^2,0]\right)\\
& +\frac{1}{2}f_{44}^2(\cos^2\theta\sin^2\overline{\theta}+\sin^2\theta\cos^2\overline{\theta})\left(3F(M_4^2,M_V^2)+\frac{3}{2}F(M_\Sigma^2,\mu'^2)+\frac{3}{10}F[(0.2M_\Sigma)^2,\mu'^2]\right), \qquad (4.14)
\end{aligned}
$$

$$
\begin{aligned}
16\pi^2\Delta_\tau^G(\Lambda) = {} & -\frac{g^2}{2}[9F(M_V^2,0)+3F(M_3^2,M_V^2)]+\frac{3}{2}(y_t^{+2}\cos^2\theta+y_b^{+2}\cos^2\overline{\theta})F(M_3^2,0)+\frac{3}{2}(y_t^{+2}\sin^2\theta+y_b^{+2}\sin^2\overline{\theta})F(M_4^2,0)\\
& +\frac{1}{2}f_{44}^2\sin^2\theta\sin^2\overline{\theta}\left(3F(M_3^2,M_V^2)+\frac{3}{2}F(M_\Sigma^2,0)+\frac{3}{10}F[(0.2M_\Sigma)^2,0]\right)+\frac{1}{2}f_{44}^2(\cos^2\theta\sin^2\overline{\theta}+\sin^2\theta\cos^2\overline{\theta})\\
& \times\left(3F(M_4^2,M_V^2)+\frac{3}{2}F(M_\Sigma^2,\mu'^2)+\frac{3}{10}F[(0.2M_\Sigma)^2,\mu'^2]\right), \qquad (4.15)
\end{aligned}
$$

where a superscript + denotes that the coupling is a GUT scale parameter. From the finiteness conditions (2.15) discussed in Sec. II, we have

$$y_t^+=\sqrt{\frac{8}{5}}g,\quad y_b^+=\sqrt{\frac{6}{5}}g,\quad f_{44}=g. \qquad (4.16)$$

The threshold function $F(a,b)$ is defined as follows;

$$F(m_a^2,m_b^2)=\frac{1}{m_a^2-m_b^2}\left[m_a^2\ln\left(\frac{m_a^2}{\Lambda^2}\right)-m_b^2\ln\left(\frac{m_b^2}{\Lambda^2}\right)\right]-1. \qquad (4.17)$$

The typical values of these corrections to the low-energy fermion masses and the bottom-tau ratio $R_{b/\tau}$ are shown in Figs. 3 and 4 (for the large $\tan\beta$ and small $\tan\beta$ cases).

We first investigate the large $\tan\beta$ case. In this case, the absolute value of a SUSY threshold correction to the bottom quark mass $m_b(M_Z)$ could be very large [12,28] (about 25%). This is due to the contributions from the gluino-squark and chargino-squark loop diagrams and large values of α_3 and y_t, especially in the models with universal soft SUSY breaking terms as in the present model. The sign of this correction, however, depends on that of the supersymmetric Higgs mass parameters which can be easily changed. This change gives only a slight effect to the other low-energy parameters and then to the SUSY threshold corrections. The SUSY correction to the tau lepton mass $m_\tau(M_Z)$ is also important and becomes about 5% which always has the opposite sign to that of m_b in the large $\tan\beta$ case. Taking into account the above facts, it is found from Figs. 3 and 5 that we cannot predict the proper value of $m_b(M_Z)$ even though $m_b(M_Z)$ has an experimental uncertainty of about 15%.

On the other hand in the small $\tan\beta$ case, the SUSY threshold corrections to the Yukawa couplings $\Delta^S_{t,\,b,\,\tau}$ are all about a few percent. Therefore we may tune $\Delta_i^G(M_H,\mu')$ so

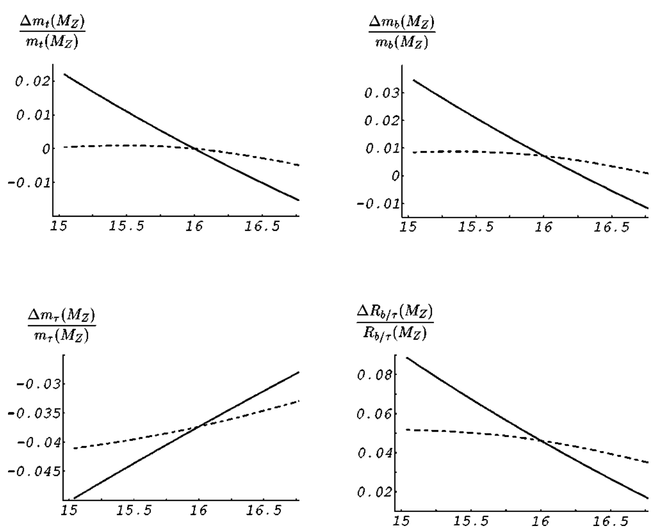

FIG. 3. The GUT threshold corrections to the low-energy fermion masses in the large $\tan\beta$ case. The solid and dashed lines indicate the M_H and μ' dependences, respectively.

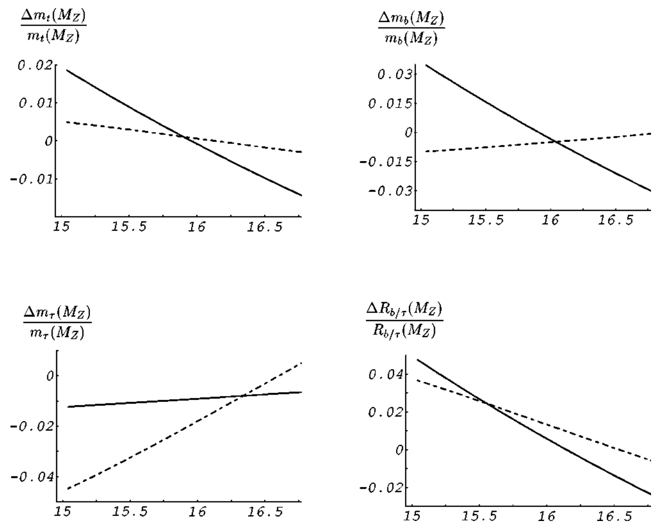

FIG. 4. The GUT threshold corrections to the low-energy fermion masses in the small $\tan\beta$ case. The solid and dashed lines indicate the M_H and μ' dependences, respectively.

that the experimental values may be properly reproduced. However, there are some other difficulties in this case. That is, the ratio $R_{b/\tau}$ becomes larger because the bottom Yukawa coupling y_b is small and then it does not come into play in the renormalization-group evolutions. To make matters worse, the correction Δ_b^S always makes a positive contribution which is independent of the sign of Higgs mixing mass parameter unlike in the large $\tan\beta$ case. Then the experimental bound $R_{b/\tau}(M_Z)\lesssim 2.0$ highly constrains the parameter spaces. For example, in Fig. 2, only the left and above narrow region is now allowed. A typical result in the allowed parameter space is shown in Table V in which the one-loop corrections to mass of the lightest Higgs boson m_h are also included. In this way, there is still room to reproduce correct low-energy values but it seems that only very narrow parameter regions are left due to the corrections to m_b and m_τ and other experimental constraints.

V. SUMMARY AND COMMENTS

We have investigated the SU(5) model with the finiteness conditions and its low-energy predictions. In this model, all the β-functions of gauge, Yukawa, and other dimensionful

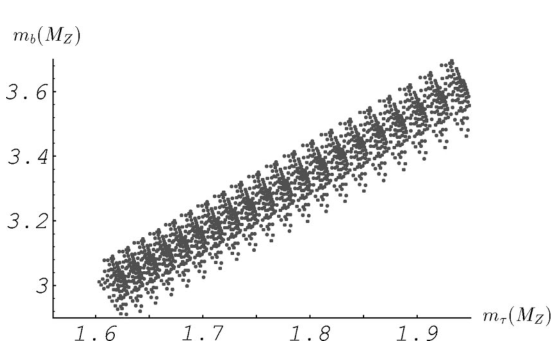

FIG. 5. The typical one-loop corrected values of $m_b(M_Z)$ and $m_\tau(M_Z)$ for the allowed parameter space in the large $\tan\beta$ case.

TABLE V. The low-energy predictions in case (2) including the GUT and SUSY threshold corrections (for $M_H=0.4\times10^{16}$ GeV and $\mu'=2.5\times10^{16}$ GeV).

Case (2) with threshold corrections			
M_G	1.206×10^{16} (GeV)		
$\alpha_{\rm GUT}$	0.0392		
M	332.38 (GeV)		
μ	642.65 (GeV)		
δm_3^2	$-(255.15)^2$ (GeV^2)		
$M_{\rm SB}$	620.0 (GeV)	$\tan\beta$	3.1
$\alpha_1(M_Z)$	0.016888	$m_t(M_Z)$	180.5 (GeV)
$\alpha_2(M_Z)$	0.033014	$m_b(M_Z)$	3.49 (GeV)
$\alpha_3(M_Z)$	0.115	$m_\tau(M_Z)$	1.747 (GeV)
$m_{\tilde t_+}$	694.7 (GeV)	$m_{\tilde u_+}$	700.2 (GeV)
$m_{\tilde t_-}$	461.5 (GeV)	$m_{\tilde u_-}$	677.2 (GeV)
$m_{\tilde b_+}$	675.6 (GeV)	$m_{\tilde d_+}$	703.7 (GeV)
$m_{\tilde b_-}$	627.8 (GeV)	$m_{\tilde d_-}$	675.6 (GeV)
$m_{\tilde \tau_+}$	293.7 (GeV)	$m_{\tilde e_+}$	292.9 (GeV)
$m_{\tilde \tau_-}$	229.0 (GeV)	$m_{\tilde e_-}$	231.1 (GeV)
$m_{\tilde \nu_\tau}$	284.5 (GeV)	$m_{\tilde \nu_e}$	284.5 (GeV)
$m_{\tilde\chi_1^+}$	560.8 (GeV)	$m_{\tilde\chi_2^+}$	245.9 (GeV)
$m_{\tilde\chi_1^0}$	188.2 (GeV)	$m_{\tilde\chi_2^0}$	296.4 (GeV)
$m_{\tilde\chi_3^0}$	411.8 (GeV)	$m_{\tilde\chi_4^0}$	495.1 (GeV)
$m_{H^\pm}$	650.7 (GeV)	m_A	646.1 (GeV)
m_H	648.2 (GeV)	m_h	94.8 (GeV)
M_{λ_3}	743.7 (GeV)		
h_t	-537.3 (GeV)	h_b	-64.9 (GeV)
h_τ	-17.6 (GeV)		

couplings are vanishing in, at least, two-loop orders of perturbation theory. Especially we have analyzed the low-energy phenomena taking into account the Higgs potential problems. That is, we have checked whether the Higgs potential actually satisfy both constraints of large $\tan\beta$ and the radiative electroweak symmetry breaking including the one-loop corrections to the Higgs mass parameters from heavy ($\sim M_{\rm SUSY}$) sector. As a result, it is found that without disturbing the finiteness conditions we need to introduce a new free parameter to satisfy these constraints. We have also estimated the GUT and SUSY threshold corrections to the dimensionless couplings. Include these corrections we are left with very narrow available parameter spaces in the model. Especially, the large $\tan\beta$ case is completely excluded unlike usual analyses of the MSSM. In this paper, we have discussed a particular form of the Higgs mass matrix at GUT scale and it is an interesting problem to analyze more general forms of matrix in order to investigate the proton decay constraints, the light fermion masses, the *CP* violation, and so on.

We now comment on an alternative way to construct realistic and restricted (GUT) models. It is the coupling constant reduction method [32] based on renormalization-group invariant relations among couplings which are solutions of the so-called reduction equations [33]. Though with these relations the models are not necessarily finite, one can reduce the number of free parameters and increase the predictive power as well as in the models with finiteness conditions. Moreover, an application of this method to the soft SUSY breaking sector in the ordinary minimal SU(5) model leads non-universal boundary conditions for the soft mass parameters at GUT scale. This may improve both problems of the large SUSY threshold correction to m_b and of the tau slepton as the lightest superparticle in large $\tan\beta$ cases unlike the finite SU(5) model. In any case, the success or failure of the models and the determinations of allowed parameter regions entirely depend on the near future experiments.

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APPENDIX A: TREE-LEVEL MASS FORMULAS IN THE MSSM

In this appendix we express the tree-level formulas for the mass eigenvalues for the MSSM particles [25]. The parameters used in the formulas are defined in Sec. III. In the following, since we neglect the small Yukawa couplings for the first and second generations their masses are identical with each other.

Sfermion masses for the third generation:

$$M^2_{\tilde{t}_\pm}=\frac{1}{2}\left\{m^2_{\tilde{Q}_3}+m^2_{\tilde{t}}+2m_t^2+\frac{1}{2}M_Z^2\cos 2\beta\pm\sqrt{\left[m^2_{\tilde{Q}_3}-m^2_{\tilde{t}}+\left(\frac{1}{2}-\frac{4}{3}\sin^2\theta_W\right)M_Z^2\cos 2\beta\right]^2+4|m_t|^2|A_t-\rho\cot\beta|^2}\right\}, \quad \text{(A1)}$$

$$M^2_{\tilde{b}_\pm}=\frac{1}{2}\left\{m^2_{\tilde{Q}_3}+m^2_{\tilde{b}}+2m_b^2-\frac{1}{2}M_Z^2\cos 2\beta\pm\sqrt{\left[m^2_{\tilde{Q}_3}-m^2_{\tilde{b}}-\left(\frac{1}{2}-\frac{2}{3}\sin^2\theta_W\right)M_Z^2\cos 2\beta\right]^2+4|m_b|^2|A_b-\rho\tan\beta|^2}\right\}, \quad \text{(A2)}$$

$$M^2_{\tilde{\tau}_\pm}=\frac{1}{2}\left\{m^2_{\tilde{L}_3}+m^2_{\tilde{\tau}}+2m_\tau^2-\frac{1}{2}M_Z^2\cos 2\beta\pm\sqrt{\left[m^2_{\tilde{L}_3}-m^2_{\tilde{\tau}}-\left(\frac{1}{2}-2\sin^2\theta_W\right)M_Z^2\cos 2\beta\right]^2+4|m_\tau|^2|A_\tau-\rho\tan\beta|^2}\right\}. \quad \text{(A3)}$$

Sfermion masses for the first and second generations:

$$M^2_{\tilde{u}_+}=m^2_{\tilde{Q}_{1,2}}+\left(\frac{1}{2}-\frac{2}{3}\sin^2\theta_W\right)M_Z^2\cos 2\beta, \quad \text{(A4)}$$

$$M^2_{\tilde{u}_-}=m^2_{\tilde{u}_{1,2}}+\frac{2}{3}\sin^2\theta_W M_Z^2\cos 2\beta, \quad \text{(A5)}$$

$$M^2_{\tilde{d}_+}=m^2_{\tilde{Q}_{1,2}}-\left(\frac{1}{2}-\frac{1}{3}\sin^2\theta_W\right)M_Z^2\cos 2\beta, \quad \text{(A6)}$$

$$M^2_{\tilde{d}_-}=m^2_{\tilde{d}_{1,2}}-\frac{1}{3}\sin^2\theta_W M_Z^2\cos 2\beta, \quad \text{(A7)}$$

$$M^2_{\tilde{e}_+}=m^2_{\tilde{L}_{1,2}}-\left(\frac{1}{2}-\sin^2\theta_W\right)M_Z^2\cos 2\beta, \quad \text{(A8)}$$

$$M^2_{\tilde{e}_-}=m^2_{\tilde{e}_{1,2}}-\sin^2\theta_W M_Z^2\cos 2\beta. \quad \text{(A9)}$$

Chargino masses:

$$m^2_{\tilde{\chi}_{1,2}}=\frac{1}{2}\Big[(M^2_{\lambda_2}+\rho^2+2M_W^2) \pm\sqrt{(M^2_{\lambda_2}+\rho^2+2M_W^2)^2-4(M_{\lambda_2}\rho-M_W^2\sin 2\beta)^2}\Big]. \quad \text{(A10)}$$

Neutralino masses: The neutralino mass term is

$$\mathcal{L}_{\rm nm}=-\frac{1}{2}(\tilde{B}_L\ \ \tilde{W}_L^3\ \ \tilde{H}^0_{1L}\ \ \tilde{H}^0_{2L})\times M_n\begin{pmatrix}\tilde{B}_L\\ \tilde{W}_L^3\\ \tilde{H}^0_{1L}\\ \tilde{H}^0_{2L}\end{pmatrix}+\text{H.c.}, \quad \text{(A11)}$$

where $\tilde{B}_L,\tilde{W}_L^3,\tilde{H}^0_{iL}$ are the $U(1)_Y$ gaugino, the third component of the $SU(2)_W$ gaugino, and the neutral components of Higgsinos, respectively. The matrix M_n is

$$M_n=\begin{pmatrix} M_{\lambda_1} & 0 & -M_Z\sin\theta_W\cos\beta & M_Z\sin\theta_W\sin\beta \\ 0 & M_{\lambda_2} & M_Z\cos\theta_W\cos\beta & -M_Z\cos\theta_W\sin\beta \\ -M_Z\sin\theta_W\cos\beta & M_Z\cos\theta_W\cos\beta & 0 & -\rho \\ M_Z\sin\theta_W\sin\beta & -M_Z\cos\theta_W\sin\beta & -\rho & 0 \end{pmatrix}. \tag{A12}$$

When the SUSY breaking scale $M_{\rm SUSY}$ is rather larger than the electroweak breaking scale M_Z, the analytic expressions for the eigenvalues of M_n are given by

$$m_{\tilde\chi_1^0}=M_{\lambda_1}+\frac{M_Z^2\sin^2\theta_W}{M_{\lambda_1}^2-\rho^2}(M_{\lambda_1}+\rho\sin2\beta), \tag{A13}$$

$$m_{\tilde\chi_2^0}=M_{\lambda_2}+\frac{M_Z^2\cos^2\theta_W}{M_{\lambda 2}^2-\rho^2}(M_{\lambda_2}+\rho\sin2\beta), \tag{A14}$$

$$m_{\tilde\chi_3^0}=\rho+\frac{M_Z^2(1+\sin2\beta)}{2(\rho-M_{\lambda_1})(\rho-M_{\lambda_2})}(\rho-M_{\lambda_1}\cos^2\theta_W-M_{\lambda_2}\sin^2\theta_W), \tag{A15}$$

$$m_{\tilde\chi_4^0}=\rho+\frac{M_Z^2(1-\sin2\beta)}{2(\rho+M_{\lambda_1})(\rho+M_{\lambda_2})}(\rho+M_{\lambda_1}\cos^2\theta_W+M_{\lambda_2}\sin^2\theta_W). \tag{A16}$$

Higgs scalar masses:

$$m_{H^\pm}^2=m_1^2+m_2^2+2\rho^2+M_W^2, \tag{A17}$$

$$m_A^2=m_1^2+m_2^2+2\rho^2, \tag{A18}$$

$$m_H^2=\frac{1}{2}\left[m_A^2+M_Z^2+\sqrt{(m_A^2+M_Z^2)^2-4m_A^2M_Z^2\cos^22\beta}\right], \tag{A19}$$

$$m_h^2=\frac{1}{2}\left[m_A^2+M_Z^2-\sqrt{(m_A^2+M_Z^2)^2-4m_A^2M_Z^2\cos^22\beta}\right]. \tag{A20}$$

APPENDIX B: THE SUSY THRESHOLD CORRECTIONS TO THE STANDARD GAUGE COUPLINGS

In this appendix we present the explicit forms of the SUSY threshold corrections $\Delta_i^S(i=1,2,3)$ to the MSSM gauge couplings [27,28] in the same approximation as Appendix A. For the explicit forms of the Yukawa coupling thresholds, see Refs. [27,28]. The corrected values $\alpha_i^-(\Lambda)$ are given by

$$\frac{2\pi}{\alpha_i^-(\Lambda)}=\frac{2\pi}{\alpha_i(\Lambda)}-\Delta_i^S(\Lambda)-\Delta_i^{DR}, \tag{B1}$$

$$\begin{aligned}\Delta_1^S(\Lambda)={}&\frac{1}{15}\ln\left(\frac{M_{\tilde t_+}}{\Lambda}\right)+\frac{4}{15}\ln\left(\frac{M_{\tilde t_-}}{\Lambda}\right)-\frac{1}{5}\sin^2\theta_{\tilde t}\ln\left(\frac{M_{\tilde t_+}}{M_{\tilde t_-}}\right)-\frac{1}{30}\ln\left(\frac{M_{\tilde b_+}}{\Lambda}\right)+\frac{1}{15}\ln\left(\frac{M_{\tilde b_-}}{\Lambda}\right)\\ &-\frac{1}{10}\sin^2\theta_{\tilde b}\ln\left(\frac{M_{\tilde b_+}}{M_{\tilde b_-}}\right)+\frac{1}{10}\ln\left(\frac{M_{\tilde\tau_+}}{\Lambda}\right)+\frac{1}{5}\ln\left(\frac{M_{\tilde\tau_-}}{\Lambda}\right)-\frac{1}{10}\sin^2\theta_{\tilde\tau}\ln\left(\frac{M_{\tilde\tau_+}}{M_{\tilde\tau_-}}\right)\\ &+\sum_{i=1,2}\left\{\frac{1}{15}\ln\left(\frac{M_{\tilde u_{Li}}}{\Lambda}\right)+\frac{4}{15}\ln\left(\frac{M_{\tilde u_{Ri}}}{\Lambda}\right)-\frac{1}{30}\ln\left(\frac{M_{\tilde d_{Li}}}{\Lambda}\right)+\frac{1}{15}\ln\left(\frac{M_{\tilde d_{Ri}}}{\Lambda}\right)\right.\\ &\left.+\frac{1}{10}\ln\left(\frac{M_{\tilde e_{Li}}}{\Lambda}\right)+\frac{1}{5}\ln\left(\frac{M_{\tilde e_{Ri}}}{\Lambda}\right)\right\}+\frac{2}{5}\ln\left(\frac{m_{\tilde\chi_2}}{\Lambda}\right)-\frac{1}{5}(\sin^2\theta_L+\sin^2\theta_R)\ln\left(\frac{m_{\tilde\chi_2}}{m_{\tilde\chi_1}}\right),\end{aligned} \tag{B2}$$

$$\Delta_2^S(\Lambda)=\frac{1}{3}\ln\left(\frac{M_{\tilde{t}_+}}{\Lambda}\right)-\frac{1}{3}\sin^2\theta_{\tilde{t}}\ln\left(\frac{M_{\tilde{t}_+}}{M_{\tilde{t}_-}}\right)+\frac{1}{3}\sum_{i=1,2}\ln\left(\frac{M_{\tilde{u}_{Li}}}{\Lambda}\right)+\frac{1}{6}\ln\left(\frac{M_{\tilde{b}_+}}{\Lambda}\right)-\frac{1}{6}\sin^2\theta_{\tilde{b}}\ln\left(\frac{M_{\tilde{b}_+}}{M_{\tilde{b}_-}}\right)+\frac{1}{6}\sum_{i=1,2}\ln\left(\frac{M_{\tilde{d}_{Li}}}{\Lambda}\right)+\frac{1}{6}\ln\left(\frac{M_{\tilde{\tau}_+}}{\Lambda}\right)-\frac{1}{6}\sin^2\theta_{\tilde{\tau}}\ln\left(\frac{M_{\tilde{\tau}_+}}{M_{\tilde{\tau}_-}}\right)+\frac{1}{6}\sum_{i=1,2}\ln\left(\frac{M_{\tilde{e}_{Li}}}{\Lambda}\right)+\frac{4}{3}\ln\left(\frac{m_{\tilde{\chi}_1}}{\Lambda}\right)+\frac{2}{3}\ln\left(\frac{m_{\tilde{\chi}_2}}{\Lambda}\right)+\frac{1}{3}(\sin^2\theta_L+\sin^2\theta_R)\ln\left(\frac{m_{\tilde{\chi}_2}}{m_{\tilde{\chi}_1}}\right), \tag{B3}$$

$$\Delta_3^S(\Lambda)=2\ln\left(\frac{M_{\lambda_3}}{\Lambda}\right)+\frac{1}{6}\sum_{i=\pm}\left\{\ln\left(\frac{M_{\tilde{t}_i}}{\Lambda}\right)+\ln\left(\frac{M_{\tilde{b}_i}}{\Lambda}\right)\right\}+\frac{1}{6}\sum_{i=1,2}\left\{\ln\left(\frac{M_{\tilde{u}_{Li}}}{\Lambda}\right)+\ln\left(\frac{M_{\tilde{u}_{Ri}}}{\Lambda}\right)+\ln\left(\frac{M_{\tilde{d}_{Li}}}{\Lambda}\right)+\ln\left(\frac{M_{\tilde{d}_{Ri}}}{\Lambda}\right)\right\}, \tag{B4}$$

$$\Delta_i^{DR}=-\frac{C_2(G_i)}{12\pi}=\begin{cases}0 & (i=1),\\ -\dfrac{1}{6\pi} & (i=2),\\ -\dfrac{1}{4\pi} & (i=3).\end{cases} \tag{B5}$$

All the mass parameters in the above formulas are defined in Sec. III and Appendix A. The squark and chargino mixing angles $\theta_{\tilde{i}}$ and $\theta_{L,R}$ are given by

$$\tan 2\theta_{\tilde{t}}=\frac{2|m_t(A_t-\mu\cot\beta)|}{m_{\tilde{Q}_3}^2-m_{\tilde{t}}^2+\left(\frac{1}{2}-\frac{4}{3}\sin^2\theta_W\right)M_Z^2\cos 2\beta}, \tag{B6}$$

$$\tan 2\theta_{\tilde{b}}=\frac{2|m_b(A_b-\mu\tan\beta)|}{m_{\tilde{Q}_3}^2-m_{\tilde{b}}^2-\left(\frac{1}{2}-\frac{2}{3}\sin^2\theta_W\right)M_Z^2\cos 2\beta}, \tag{B7}$$

$$\tan 2\theta_{\tilde{\tau}}=\frac{2|m_\tau(A_\tau-\mu\tan\beta)|}{m_{\tilde{L}_3}^2-m_{\tilde{\tau}}^2-\left(\frac{1}{2}-2\sin^2\theta_W\right)M_Z^2\cos 2\beta}, \tag{B8}$$

$$\tan 2\theta_L=\frac{2\sqrt{2}M_W(M_{\lambda_2}\cos\beta+\mu\sin\beta)}{M_{\lambda_2}^2-\mu^2-2M_W^2\cos 2\beta}, \tag{B9}$$

$$\tan 2\theta_R=\frac{2\sqrt{2}M_W(M_{\lambda_2}\sin\beta+\mu\cos\beta)}{M_{\lambda_2}^2-\mu^2+2M_W^2\cos 2\beta}. \tag{B10}$$

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