Renormalizability and the model independent observables for the Abelian Z' search

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The observables that are useful for the model independent search for signals of the Abelian Z' in the processes $e^+e^- \to \overline{f}f$ are introduced. They are based on the renormalization group relations between the Z' couplings to the standard model particles developed recently and extend the variables suggested by Osland, Pankov, and Paver. The bounds on the values of the observables at the center-of-mass energy $\sqrt{s}=500$ GeV are derived.

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I. INTRODUCTION

The Abelian Z' boson with a mass much larger than the W-boson mass ($m_{Z'} \gg m_W$) is predicted by a number of extensions of the standard model (SM) of elementary particles [1]. At the current energies $\sim m_W$ the Z' is decoupled. It can be described by a model based on the effective gauge group $SU(2)_L \times U(1)_Y \times \tilde{U}(1)$ which is assumed to be a low energy remnant of some unknown underlying theory [the grand unified theory (GUT), for example]. However, the Z' would be light enough to give the first signal in future experiments.

Because of the lower mass limit from the Fermilab Tevatron, $m_{Z'} > \mathcal{O}(500)$ GeV, only ''indirect'' Z' manifestations caused by virtual heavy states can be searched for at the energies of present day accelerators. In general, one is not able to estimate the magnitude of the Z' signal because of the unknown couplings and the mass. However, numerous strategies to evidence manifestations of the Z' in experiments at high energy e^+e^- and hadronic colliders have been developed. The analysis can be done with or without assumptions on the specific underlying theory containing the Z'. Hence, there are model dependent and model independent variables allowing the detection of the Z'.

One of the model independent approaches useful in searching for the Z' signal in the leptonic processes $e^+e^- \to l^+l^-$ has been proposed in Ref. [2]. The basic idea was to replace the standard observables, the total cross section σ_T and the forward-backward asymmetry A_{FB}, by a new set of variables defined as the differences of the cross sections integrated over suitable ranges of the polar angle θ:

$$\sigma_{\pm} \equiv \pm \int_{\mp z'}^{1} \frac{d\sigma}{d\cos\theta} d\cos\theta \mp \int_{-1}^{\mp z'} \frac{d\sigma}{d\cos\theta} d\cos\theta. \quad (1)$$

Because of the SM values of the leptonic charges and the kinematic properties of the fermionic currents, they have chosen the value $z' = 2^{2/3}-1 = 0.5874$ to make the leading order deviations from the SM predictions $\Delta\sigma_{\pm}$ dependent on the combinations $v^e_{Z'} v^l_{Z'} \pm a^e_{Z'} a^l_{Z'}$, where $v^f_{Z'}$ and $a^f_{Z'}$ parametrize the vector and the axial-vector coupling of the Z' to the fermion f. Therefore, assuming the lepton universality one obtains the sign definite observable $\Delta\sigma_+$.

Usually, the parameters describing at low energies the Z' coupling to the SM fermions (like $v^f_{Z'}$ and $a^f_{Z'}$) are assumed to be arbitrary numbers which must be fixed in experiments. However, this is not the case if the renormalizability of the underlying theory is taken into account. In Refs. [3,4] it has been shown that, if one uses the principles of the renormalization group (RG) and the decoupling theorem, the correlations between the parameters describing interactions of light particles with heavy virtual states of new physics beyond the SM can be derived. It is most important that the relations obtained, being the consequence of the renormalizability formulated in the framework of scattering in the external field, are independent of the specific underlying (GUT) model.

In Ref. [4] the method was applied to find signals of the heavy Abelian Z' in the four-fermion scattering processes. It was found that the renormalizability of the theory results in the following constraints on the Z' couplings to the SM fermions:

$$v^{f'}_{Z'} - a^{f'}_{Z'} = v^f_{Z'} - a^f_{Z'}, \quad a^f_{Z'} = I^f_3 Y_\phi, \quad (2)$$

where f' denotes the isopartner of f ($l' = \nu_l$, $\nu'_l = l$, $q'_d = q_u$, $q'_u = q_d$, where $l = e,\mu,\tau$ stands for leptons), I^f_3 is the third component of the weak isospin, and Y_ϕ is the hypercharge parametrizing the Z' coupling to the SM scalar doublet. Since the parameters of various fermionic processes appear to be correlated, one could expect that it is possible to introduce specific observables sensitive to the Z' manifestations. In the present paper we propose such observables which generalize the variables $\sigma_{\pm}$ (1).

The content is as follows. In Sec. II the structure of neutral currents induced by the Z' boson as well as Z-Z' mixing is briefly discussed. Employing the relations (2) between the Z' parameters the optimal observables for searching for signals of the heavy Abelian Z' are constructed in Sec. III. In Sec. IV the experimental bounds on the observables are predicted. The obtained results are summarized in Sec. V.

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II. NEUTRAL CURRENTS AND VECTOR-BOSON MIXING

Considering the interactions of the Z' with the SM particles, one has to conclude that at low energies, $E \ll m_{Z'}$, the renormalizable interactions are to be dominant. Terms of nonrenormalizable type [for example, $\sim(\partial_\mu Z'_\nu - \partial_\nu Z'_\mu)\bar{\psi}\sigma_{\mu\nu}\psi$], being generated at the GUT (or some intermediate $\Lambda^{GUT} > \Lambda' > m_{Z'}$) mass scale, are suppressed by the factors $1/\Lambda^{GUT}$, $1/\Lambda'$, and can be neglected. Thus, the interaction of the Z' boson with the fermionic currents can be specified by the effective Lagrangian

$$\mathcal{L}_{NC} = eA_\mu J^\mu_A + g_Z Z_\mu J^\mu_Z + g_{Z'} Z'_\mu J^\mu_{Z'}, \tag{3}$$

where A, Z, and Z' are the photon, the Z boson, and Z' boson, respectively, $e=\sqrt{4\pi\alpha}$, $g_Z = e/\sin\theta_W \cos\theta_W$, and $g_{Z'}$ stands for the $\tilde{\mathrm{U}}(1)$ coupling constant. θ_W denotes the SM value of the Weinberg angle [$\tan\theta_W = g'/g$, where the charges g, g' correspond to the gauge groups $\mathrm{SU}(2)_L$, $\mathrm{U}(1)_Y$, respectively]. The neutral currents can be parametrized as

$$J^\mu_V = \sum_f \bar{f}\gamma^\mu (v^f_V + a^f_V \gamma^5) f, \tag{4}$$

with $V \equiv A, Z, Z'$. The vector and the axial-vector couplings of the vector bosons to the fermion f are

$$v^f_A = Q_f, \quad a^f_A = 0, \tag{5}$$

$$v^f_Z = \left(\frac{I^f_3}{2} - Q_f \sin^2\theta_W\right)\cos\theta_0 + \frac{g_{Z'}}{g_Z} Y^v_f \sin\theta_0,$$

$$a^f_Z = -\frac{I^f_3}{2}\cos\theta_0 + \frac{g_{Z'}}{g_Z} Y^a_f \sin\theta_0, \tag{6}$$

$$v^f_{Z'} = Y^v_f \cos\theta_0 - \frac{g_Z}{g_{Z'}}\left(\frac{I^f_3}{2} - Q_f \sin^2\theta_W\right)\sin\theta_0,$$

$$a^f_{Z'} = Y^a_f \cos\theta_0 + \frac{g_Z}{g_{Z'}}\frac{I^f_3}{2}\sin\theta_0, \tag{7}$$

where Q_f is the fermion charge in the positron charge units. The constants Y^v_f and Y^a_f parametrize the vector and the axial-vector coupling of the fermion f to the $\tilde{\mathrm{U}}(1)$ symmetry eigenstate, whereas θ_0 is the mixing angle relating the mass eigenstates Z_μ, Z'_μ to the massive neutral components of the $\mathrm{SU}(2)_L \times \mathrm{U}(1)_Y$ and the $\tilde{\mathrm{U}}(1)$ gauge fields, respectively. Its value can be determined from the relation [5]

$$\tan^2\theta_0 = \frac{m^2_W/\cos^2\theta_W - m^2_Z}{m^2_{Z'} - m^2_W/\cos^2\theta_W}. \tag{8}$$

Because of the mixing between the Z and Z' bosons, the mass m_Z differs from the SM value $m_W/\cos\theta_W$ by a small quantity of order $m^2_W/m^2_{Z'}$ [4],

$$m^2_Z = \frac{m^2_W}{\cos^2\theta_W}\left(1 - \frac{4g^2_{Z'}Y^2_\phi}{g^2}\frac{m^2_W}{m^2_{Z'} - m^2_W/\cos^2\theta_W}\right), \tag{9}$$

and, as a consequence, the parameter $\rho \equiv m^2_W/m^2_Z \cos^2\theta_W > 1$. Therefore, the mixing angle θ_0 is also small $\theta_0 \simeq \tan\theta_0 \simeq \sin\theta_0 \sim m^2_W/m^2_{Z'}$.

The difference $m^2_Z - m^2_W/\cos^2\theta_W$ is negative and completely determined by the Z' coupling to the scalar doublet. Thus, constraints on the Z' interaction with the scalar field can be obtained by experimental detecting this observable:

$$\frac{g^2_{Z'}Y^2_\phi}{m^2_{Z'}} = \left(1 - \frac{m^2_Z \cos^2\theta_W}{m^2_W}\right)\frac{g^2}{4m^2_W} + O\left(\frac{m^4_W}{m^4_{Z'}}\right). \tag{10}$$

As has been proved in Ref. [4], the following relations hold for the constants Y^v_f and Y^a_f:

$$Y^L_{f'} = Y^L_f, \quad Y^a_f = I^f_3 Y_\phi, \tag{11}$$

where $Y^L_f \equiv Y^v_f - Y^a_f$, $Y^R_f \equiv Y^v_f + Y^a_f$, and Y_ϕ is the hypercharge parametrizing the coupling of the SM scalar doublet to the vector boson associated with the $\tilde{\mathrm{U}}(1)$ symmetry. As is noted in Sec. I, the notation f' stands for the isopartner of f.

In fact, relations (11) mean that the Z' couplings to the SM axial-vector currents have the universal absolute value, if the single light scalar doublet exists. Among the four values Y^v_f, Y^a_f, $Y^v_{f'}$, $Y^a_{f'}$ parametrizing the interaction of the Z' boson with the SU(2) fermionic isodoublet only one is independent. The rest can be expressed through it and the hypercharge Y_ϕ of the Z' coupling to the SM scalar doublet. If the hypercharges are treated as unknown parameters, these relations are to be taken into account in order to preserve the gauge symmetry [4]. Relations (11) also show that the fermion and the scalar sectors of the new physics are strongly correlated. As a result, the couplings of the Z' boson to the SM axial-vector currents are completely determined by its interaction with the scalar fields. Therefore, one is able to predict the Z' coupling to the SM axial-vector currents by measuring the ρ parameter. When the Z' does not interact with the scalar doublet, the Z-boson mass is to be identical to its SM value. In this case the Z' couplings to the axial currents are produced by loops and to be suppressed by the additional small factor $g^2/16\pi^2$.

III. OBSERVABLES

In the present section we consider the electron-positron annihilation into fermion pairs, $e^+e^- \to V^* \to \bar{f}f$, for energies $\sqrt{s} \sim 500$ GeV. In this case all the fermions except for the t quark can be treated as massless particles $m_f \sim 0$. The Z'-boson existence causes the deviations ($\sim m^{-2}_{Z'}$) of the cross section from its SM value:

$$\Delta\frac{d\sigma}{d\Omega} = \frac{d\sigma}{d\Omega} - \frac{d\sigma_{SM}}{d\Omega} = \frac{\mathrm{Re}[T^*_{SM}\Delta T]}{32\pi s} + O\left(\frac{s^2}{m^4_{Z'}}\right), \tag{12}$$

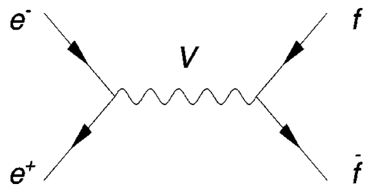

FIG. 1. The amplitude T_V of the process $e^+e^- \to V^* \to \overline{f}f$ at the Born level.

with

$$T_{SM} = T_A + T_Z|_{\theta_0=0}, \quad \Delta T = T_{Z'} + \left.\frac{dT_Z}{d\theta_0}\right|_{\theta_0=0} \theta_0, \quad (13)$$

where T_V denotes the Born amplitude of the process $e^+e^- \to V^* \to \overline{f}f$ with the virtual V-boson ($V=A,Z,Z'$) state in the s channel (the corresponding diagram is shown in Fig. 1).

The quantity $\Delta d\sigma/d\Omega$ can be calculated in the form

$$\Delta \frac{d\sigma}{d\Omega} = \frac{\alpha I_3^f N_f}{4\pi} \sum_{\lambda,\xi} \frac{g_{Z'}^2 \zeta_{\lambda\xi}^{ef}}{m_{Z'}^2} \{|Q_f| + \chi(s)(\mathrm{sgn}\,\lambda - \varepsilon) \times [\mathrm{sgn}\,\xi + |Q_f|(1-\varepsilon) - 1]\}(z + \mathrm{sgn}\,\lambda\,\xi)^2, \quad (14)$$

where $N_f=3$ for quarks and $N_f=1$ for leptons, $\lambda,\xi = L,R$ denotes the fermion state helicities, $\chi(s) = [16 \sin^2\theta_W \cos^2\theta_W (1 - m_Z^2/s)]^{-1}$, $z \equiv \cos\theta$ (where θ is the angle between the incoming electron and the outgoing fermion), $\varepsilon \equiv 1 - 4\sin^2\theta_W \sim 0.08$, and

$$\zeta_{\lambda\xi}^{ef} \equiv Y_e^\lambda Y_f^\xi - \frac{m_W^2/\cos^2\theta_W}{s - m_Z^2}[Y_\phi Y_f^\xi (2\sin^2\theta_W - \delta_{\lambda,L}) + 2 I_3^f Y_\phi Y_e^\lambda (-2|Q_f|\sin^2\theta_W + \delta_{\xi,L})], \quad (15)$$

with $\delta_{\lambda,\xi}=1$ when $\lambda=\xi$ and $\delta_{\lambda,\xi}=0$ otherwise.

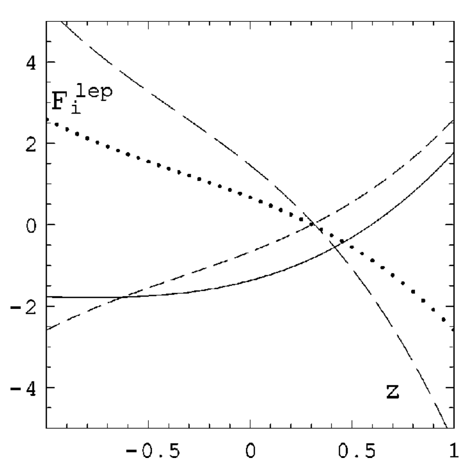

FIG. 2. The leptonic functions F_0^l (the solid curve), F_1^l (the long-dashed curve), F_2^l (the dashed curve), and F_3^l (the dotted curve) at $\sqrt{s} = 500$ GeV.

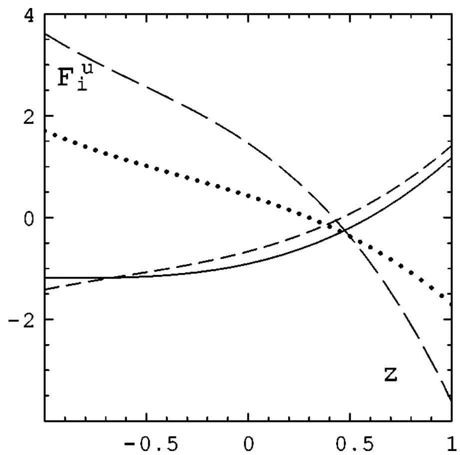

FIG. 3. The quark functions ($q_u = u,c$) $F_0^{q_u}$ (the solid curve), $F_1^{q_u}$ (the long-dashed curve), $F_2^{q_u}$ (the dashed curve), and $F_3^{q_u}$ (the dotted curve) at $\sqrt{s} = 500$ GeV.

To discuss the consequences due to Eq. (11) we introduce the function $\sigma(z)$ defined as the difference of cross sections integrated in a suitable range of $\cos\theta$:

$$\sigma(z) \equiv \int_z^1 \frac{d\sigma}{dz} dz - \int_{-1}^z \frac{d\sigma}{dz} dz. \quad (16)$$

The value of z differs from that in Ref. [2] and will be chosen later. Actually, this observable is the generalized σ_+ of Ref. [2] $[\sigma_+ = \sigma(-z')]$. The two conventionally used observables, the total cross section σ_T and the forward-backward asymmetry A_{FB}, can be obtained by the special choice of z $[\sigma_T = \sigma(-1),\ A_{FB} = \sigma(0)/\sigma_T]$. One can express $\sigma(z)$ in terms of σ_T and A_{FB}:

$$\sigma(z) = \sigma_T \left(A_{FB}(1 - z^2) - \frac{1}{4} z(3 + z^2) \right). \quad (17)$$

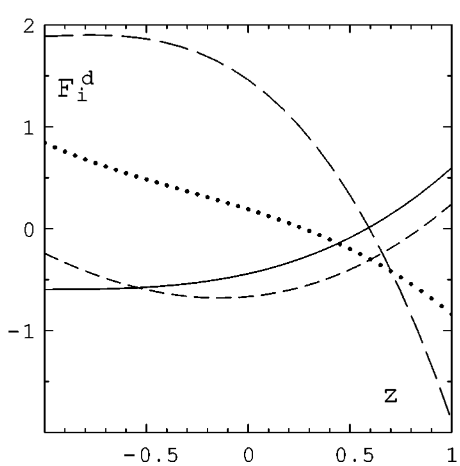

FIG. 4. The quark functions ($q_d = d,s,b$) $F_0^{q_d}$ (the solid curve), $F_1^{q_d}$ (the long-dashed curve), $F_2^{q_d}$ (the dashed curve), and $F_3^{q_d}$ (the dotted curve) at $\sqrt{s} = 500$ GeV.

Then let us introduce the quantity $\Delta\sigma(z)\equiv\sigma(z)-\sigma_{SM}(z)$, which owing to the relations (11) can be written in the form

$$\Delta\sigma(z)=\frac{\alpha N_f}{8}\frac{g_{Z'}^2}{m_{Z'}^2}[F_0^f(z,s)Y_\phi^2+2F_1^f(z,s)I_3^fY_f^LY_e^L+2F_2^f(z,s)I_3^fY_f^LY_\phi+F_3^f(z,s)Y_e^LY_\phi]. \qquad (18)$$

The factor functions $F_i^f(z,s)$ depend on the fermion type through the $|Q_f|$, only. In Figs. 2-4 they are shown as the functions of z for $\sqrt{s}=500$ GeV. The leading contributions to $F_i^f(z,s)$,

$$F_0^f(z,s)=-\frac{4}{3}|Q_f|\left(1-z-z^2-\frac{z^3}{3}\right)+O\left(\varepsilon,\frac{m_Z^2}{s}\right),$$

$$F_1^f(z,s)=\frac{4}{3}[1-z^2-|Q_f|(3z+z^3)]+O\left(\varepsilon,\frac{m_Z^2}{s}\right),$$

$$F_2^f(z,s)=-\frac{2}{3}(1-z^2)+\frac{2}{9}(3z+z^3)\times(4|Q_f|-1)+O\left(\varepsilon,\frac{m_Z^2}{s}\right),$$

$$F_3^f(z,s)=\frac{2}{3}|Q_f|(1-3z-z^2-z^3)+O\left(\varepsilon,\frac{m_Z^2}{s}\right), \qquad (19)$$

are given by the Z' exchange diagram [the first term of Eq. (15)], since the contribution of the Z exchange diagram to ΔT [the second term of Eq. (15)] is suppressed by the factor m_Z^2/s.

From Eqs. (19) one can see that the leading contributions to the leptonic factors F_1^l, F_2^l, F_3^l are found to be proportional to the same polynomial in z. This is the characteristic feature of the leptonic functions F_i^l originating due to the kinematic properties of fermionic currents and the specific values of the SM leptonic charges. Therefore, it is possible to choose the value of $z=z^*$ which switches off three leptonic factors F_1^l, F_2^l, F_3^l simultaneously. Moreover, the quark function F_3^q in the lower order is proportional to the leptonic one and therefore is switched off, too. As is seen from Figs. 2–4, the appropriate value of z^* is about ~ 0.3. By choosing this value of z^* one can simplify Eq. (18). It is also follows from Eq. (18) that neglecting the factors F_1^l, F_2^l, F_3^l one obtains the sign definite quantity $\Delta\sigma_l(z^*)$.

Comparing the observable $\Delta\sigma_l(z^*)$ with $\Delta\sigma_+=\Delta\sigma_l(-0.5874)$ or $\Delta\sigma_-=-\Delta\sigma_l(0.5874)$ one has to conclude that the sign of the variables $\Delta\sigma_\pm$ is completely undetermined in the case of arbitrary leptonic couplings Y_l^L. Therefore, in order to predict the sign of the observables $\Delta\sigma_\pm$ one must assume an additional restriction such as the lepton universality.

Let the value of z in Eq. (16) be determined from the relation

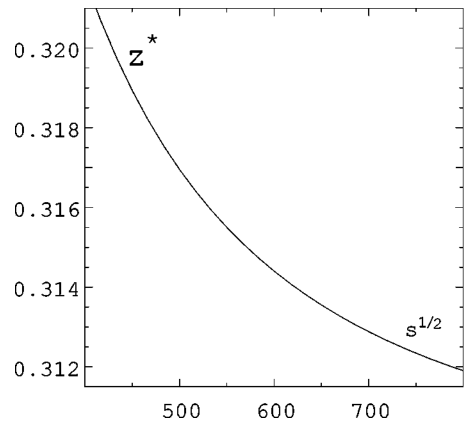

FIG. 5. z^* as the function of $\sqrt{s}$ (GeV).

$$F_1^l(z^*,s)=0. \qquad (20)$$

The solution $z^*(s)$ is shown in Fig. 5. As is seen, z^* decreases from 0.3170 at $\sqrt{s}=500$ GeV to 0.3129 at $\sqrt{s}=700$ GeV. Table I demonstrates the corresponding behavior of the functions $F_i^f(z^*,s)$. Since $F_i^f(z^*,s)$ depend on the center-of-mass energy through the small quantity m_Z^2/s, the order of the shifts is about 3%. Therefore, in what follows the value of $\sqrt{s}$ is taken to be 500 GeV.

Assuming $Y_e^L\sim Y_l^L\sim Y_\phi\sim Y_u^L\sim 1$, one can derive, for the lepton and quark observables,

$$\Delta\sigma_l(z^*)=-0.10\frac{\alpha g_{Z'}^2Y_\phi^2}{m_{Z'}^2}[1+O(0.04)],$$

$$\Delta\sigma_{q_u}(z^*)=1.98\Delta\sigma_l(z^*)+0.32\frac{\alpha g_{Z'}^2Y_\phi}{2m_{Z'}^2}\times[(Y_e^L/Y_\phi-0.6)Y_{q_u}^L+O(0.07)],$$

TABLE I. Energy dependence of $F_i^f(z^*)$.

$\sqrt{s}$ (GeV)	500	600	700
z^*	0.3170	0.3144	0.3129
$F_0^l(z^*)$	-0.8012	-0.7889	-0.7815
$F_2^l(z^*)$	0.0346	0.0341	0.0338
$F_3^l(z^*)$	-0.0346	-0.0341	-0.0338
$F_0^{q_u}(z^*)$	-0.5277	-0.5216	-0.5179
$F_1^{q_u}(z^*)$	0.4250	0.4215	0.4194
$F_2^{q_u}(z^*)$	-0.2532	-0.2499	-0.2479
$F_3^{q_u}(z^*)$	-0.0331	-0.0296	-0.0276
$F_0^{q_d}(z^*)$	-0.2513	-0.2522	-0.2527
$F_1^{q_d}(z^*)$	0.8500	0.8430	0.8388
$F_2^{q_d}(z^*)$	-0.5410	-0.5339	-0.5297
$F_3^{q_d}(z^*)$	-0.0362	-0.0282	-0.0235

$$\Delta\sigma_{q_d}(z^*)=0.94\Delta\sigma_l(z^*)-0.32\frac{\alpha g_{Z'}^2 Y_\phi}{m_{Z'}^2}\times[(Y_e^L/Y_\phi-0.6)Y_{q_u}^L+O(0.08)]. \quad (21)$$

Hence it is seen that the observable $\Delta\sigma_l(z^*)$ is negative. Using Eq. (10) it can be written in terms of the parameter ρ:

$$\Delta\sigma_l(z^*)\simeq 0.10\frac{\alpha g^2(1-\rho)}{4m_W^2\rho}<0. \quad (22)$$

One also can introduce sign definite observables for quarks of the same generation:

$$\Delta\sigma_q(z^*)=\Delta\sigma_{q_u}+0.5\Delta\sigma_{q_d}. \quad (23)$$

As follows from Eq. (21),

$$\Delta\sigma_q(z^*)\simeq 2.45\Delta\sigma_l(z^*)<0. \quad (24)$$

Hence, one can conclude that the values of $\Delta\sigma_{q_u}(z^*)$ and $\Delta\sigma_{q_d}(z^*)$ in the $\Delta\sigma_{q_u}(z^*)$–$\Delta\sigma_{q_d}(z^*)$ plane have to be at the line crossing axes at the points $\Delta\sigma_{q_u}(z^*)=2.45\Delta\sigma_l(z^*)$ and $\Delta\sigma_{q_d}(z^*)=4.9\Delta\sigma_l(z^*)$, respectively. It also follows from Eq. (22) that the observable $\Delta\sigma_q(z^*)$ is negative.

Thus, the dependences (2) between the Z' couplings to SM fermions allows one to construct three negative-valued observables, $\Delta\sigma_l(z^*)$, $1-\rho$, and $\Delta\sigma_q(z^*)$, which are correlated by Eqs. (22)–(24). These observables are the most general model independent ones which can be introduced without any assumptions about the specific form of the Z' interaction with fermions (such as the lepton or quark universality).

IV. EXPERIMENTAL CONSTRAINTS ON THE OBSERVABLES

The present day experimental data constrain the magnitude of the four-fermion contact interactions, allowing one to derive bounds on the Z' coupling to the axial-vector currents and, consequently, on the observables introduced in the previous section. Our analysis is based on the data presented in Refs. [6] where a study of the experimental bounds on the lepton-quark four-fermion contact couplings has been performed. In general, the contributions of new physics beyond the SM to the processes considered therein (the atomic parity violation, the electron-nuclea, the muon-nuclea, and the ν_μ-nuclea scattering) are described by 20 parameters, namely,

$$\eta_{\lambda\xi}^{lq}\equiv-\frac{g_{Z'}^2 Y_l^\lambda Y_q^\xi}{m_{Z'}^2}, \quad \eta_{L\xi}^{\nu_\mu q}\equiv-\frac{g_{Z'}^2 Y_{\nu_\mu}^L Y_q^\xi}{m_{Z'}^2}, \quad (25)$$

where $l=e,\mu$, $q=u,d$, and $\lambda,\xi=L,R$. In order to reduce the number of independent $\eta_{\lambda\xi}^{lq}$ one usually assumes $SU(2)_L$ invariance and lepton universality. As a result, six variables (for example, η_{LL}^{lu}, η_{LR}^{lu}, η_{LR}^{ld}, η_{RL}^{lu}, η_{RR}^{lu}, η_{RR}^{ld}) can be chosen as the basis.

However, the number of independent $\eta_{\lambda\xi}^{lq}$ can be decreased by employing the correlations (2) instead of the assumption of lepton universality. In this case it is convenient to introduce the couplings η_{AA}^{lq}, η_{LA}^{lq}, η_{AL}^{lq} parametrizing the four-fermion interactions between the left-handed and the axial-vector currents. These couplings are the linear combinations of the variables (25):

$$\eta_{AA}^{lq}\equiv\eta_{RR}^{lq}-\eta_{RL}^{lq}-\eta_{LR}^{lq}+\eta_{LL}^{lq},$$
$$\eta_{LA}^{lq}\equiv\eta_{LR}^{lq}-\eta_{LL}^{lq},$$
$$\eta_{AL}^{lq}\equiv\eta_{RL}^{lq}-\eta_{LL}^{lq}. \quad (26)$$

As follows from Eq. (2), one has six independent parameters

$$\eta_{AA}^{eu}=\frac{g_{Z'}^2 Y_\phi^2}{4m_{Z'}^2}, \quad \eta_{LA}^{lu}=-\frac{g_{Z'}^2 Y_l^L Y_\phi}{2m_{Z'}^2},$$
$$\eta_{AL}^{eu}=\frac{g_{Z'}^2 Y_u^L Y_\phi}{2m_{Z'}^2}, \quad \eta_{LL}^{lu}=-\frac{g_{Z'}^2 Y_l^L Y_u^L}{m_{Z'}^2}, \quad (27)$$

which can be used as the basis.

The experiment constrains the specific linear combinations of the variables $\eta_{\lambda\xi}^{lq}$ (see Ref. [6]). Introducing the normalized couplings

$$\Delta C_{1q}^l=-\frac{1}{2\sqrt{2}G_F}(\eta_{RR}^{lq}-\eta_{LR}^{lq}+\eta_{RL}^{lq}-\eta_{LL}^{lq}),$$
$$\Delta C_{2q}^l=-\frac{1}{2\sqrt{2}G_F}(\eta_{RR}^{lq}+\eta_{LR}^{lq}-\eta_{RL}^{lq}-\eta_{LL}^{lq}),$$
$$\Delta C_{3q}^l=-\frac{1}{2\sqrt{2}G_F}(\eta_{RR}^{lq}-\eta_{LR}^{lq}-\eta_{RL}^{lq}+\eta_{LL}^{lq}),$$
$$\Delta q_L=-\frac{1}{2\sqrt{2}G_F}\eta_{LL}^{\nu_\mu q},$$
$$\Delta q_R=-\frac{1}{2\sqrt{2}G_F}\eta_{LR}^{\nu_\mu q}, \quad (28)$$

where G_F is the Fermi constant, one can write down the experimental bounds as follows:

$$2\Delta C_{1u}^e-\Delta C_{1d}^e=0.217\pm0.26,$$
$$2\Delta C_{2u}^e-\Delta C_{2d}^e=-0.765\pm1.23, \quad (29)$$
$$2\Delta C_{3u}^\mu-\Delta C_{3d}^\mu=-1.51\pm4.9,$$

$$2\Delta C^{\mu}_{2u}-\Delta C^{\mu}_{2d}=1.74\pm 6.3, \tag{30}$$

$$\Delta C^{e}_{1u}+\Delta C^{e}_{1d}=0.0152\pm 0.033, \tag{31}$$

$$-2.73\Delta C^{e}_{1u}+0.65\Delta C^{e}_{1d}-2.19\Delta C^{e}_{2u}+2.03\Delta C^{e}_{2d} = -0.065\pm 0.19, \tag{32}$$

$$376\Delta C^{e}_{1u}+422\Delta C^{e}_{1d}=0.96\pm 0.92,$$

$$572\Delta C^{e}_{1u}+658\Delta C^{e}_{1d}=1.58\pm 4.2, \tag{33}$$

$$\Delta u_L=-0.0032\pm 0.0169,$$

$$\Delta u_R=-0.0084\pm 0.0251,$$

$$\Delta d_L=0.002\pm 0.0136,$$

$$\Delta d_R=-0.0109\pm 0.0631. \tag{34}$$

Equations (29)–(34) represent the results of SLAC e-D, CERN μ-C, Bates e-C, Mainz e-Be, the atomic parity violation, and the ν_μ-nucleon scattering experiments, respectively [6]. They determine the allowed region in the space of η parameters.

By employing Eqs. (26), (29)–(34) it is easy to obtain the bounds on the quantities (27):

$$0<\eta^{eu}_{AA}<0.114 \ \ \mathrm{TeV}^{-2},$$

$$-0.018 \ \ \mathrm{TeV}^{-2}<\eta^{eu}_{AL}<0.006 \ \ \mathrm{TeV}^{-2},$$

$$-0.437 \ \ \mathrm{TeV}^{-2}<\eta^{\mu u}_{LA}<0.661 \ \ \mathrm{TeV}^{-2},$$

$$-0.667 \ \ \mathrm{TeV}^{-2}<\eta^{eu}_{LA}<0.238 \ \ \mathrm{TeV}^{-2},$$

$$-0.423 \ \ \mathrm{TeV}^{-2}<\eta^{\mu u}_{LL}<0.358 \ \ \mathrm{TeV}^{-2}. \tag{35}$$

The first of the relations (35) gives the possibility of determining the allowed magnitude of the observables $\Delta\sigma_l(z^*)$ and $\Delta\sigma_q(z^*)$

$$-0.13 \ \ \mathrm{pb}<\Delta\sigma_l(z^*)<0,$$

$$-0.32 \ \ \mathrm{pb}<\Delta\sigma_q(z^*)<0. \tag{36}$$

Thus, if the low energy physics is described by the minimal SM, the signals of the Abelian Z' should respect the above relations.

It is instructive to compare the first of Eqs. (36) based on the analysis of lepton-quark interactions with direct constraints derived from experiments on the $e^+e^-\to l^+l^-$ scattering. Introducing the normalized Z' coupling to the axial leptonic current,

$$|A_l|=\sqrt{\frac{g^2_{Z'}Y^{a2}_l m^2_Z}{4\pi m^2_{Z'}}}, \tag{37}$$

one can write down the bounds obtained from the processes $e^+e^-\to l^+l^-$ as follows: $|A_l|<0.025$ (see Fig. 2.7 of Ref. [1]). However, relations (36) lead to the constraint $|A_l|<0.0087$. Thus, the bounds (36) are about one order stronger than ones derived from the analysis of pure leptonic interactions.

V. DISCUSSION

In the lack of reliable information on the model describing physics beyond the SM and predicting Z' boson it is of great importance to find model independent variables to search for this particle. In this regard, it could be useful to employ the method of Ref. [4] which gives the possibility of reducing the amount of unknown numbers parametrizing effects of new heavy virtual particles.

In Ref. [2] observables $\sigma_\pm$ alternative to the familiar σ_T and A_{FB} and perspective in searching for the Abelian Z' signal in the leptonic processes $e^+e^-\to l^+l^-$ were proposed. It was pointed out that the sign of the σ_+ deviation from the SM value can be uniquely predicted when lepton universality is assumed. The sign remains undetermined in the case of arbitrary interactions of Z' with leptons.

The observables introduced in the present paper are the extension of that in Ref. [2] with the choice, specified above, of the boundary angle. The observable $\Delta\sigma_l(z^*)$ is the negative defined quantity even in the case when lepton universality is not assumed. Moreover, the observable $\Delta\sigma_q(z^*)$ introduced for quarks of the same generation is proportional to the leptonic one $\Delta\sigma_l(z^*)$ being negative, too.

As was mentioned in Sec. III, the magnitude of the leptonic observable $\Delta\sigma_l(z^*)$ is expressed in terms of the ρ parameter [Eq. (22)]. Thus, the Z' contributions to the quantities $1-\rho=1-m^2_W/m^2_Z\cos^2\theta_W$, $\Delta\sigma_l(z^*)$ and $\Delta\sigma_q(z^*)$ have to be negative, allowing one to detect the Z' signal.

Now let us discuss the most important properties of relations (2) and (11). The key point in deriving the RG relations is that the basis theory, being assumed to describe processes at low energies, must be fixed. As the underlying theory describing interactions at high energies $E\gg m_W$ is concerned, it is unknown and therefore not specified in our approach. In Ref. [4] the minimal SM was chosen as the basis model. However, a number of theories, based on the underlying E_6 group and predicting the Abelian Z', are reduced at low energies to the two-Higgs-doublet model (THDM). In this regard, let us note that RG relations (2) and (11) respect the $\tilde{\mathrm{U}}(1)$ gauge invariance of the Yukawa terms of the SM Lagrangian. That is why one could expect that if the THDM is chosen as the basis theory, the same RG relations have to be obtained for each of the scalar doublets. But, of course, the relations have to be quite different if we consider the non-Abelian Z' boson (see for details [4]).

It is interesting to check whether RG relations (2) and (11) hold for theories which extend the SM at low energies. For definiteness, let us consider the E_6 gauge theory [1,7] with specific values of the Z' couplings, generated in different scenarios of symmetry breaking. In Table II (see Ref. [1]) we show the Z' couplings to the SM fermions predicted by the E_6 theory (notice that the sign of the axial couplings in Ref. [1] is opposite to the sign of $a^f_{Z'}$). At first glance, one

TABLE II. The Z' couplings to the SM fermions in the E_6 and LR models.

f	E_6: $a^f_{Z'}$	$v^f_{Z'}$	LR: $a^f_{Z'}$	$v^f_{Z'}$
ν	$-3\frac{\cos\beta}{\sqrt{40}}-\frac{\sin\beta}{\sqrt{24}}$	$3\frac{\cos\beta}{\sqrt{40}}+\frac{\sin\beta}{\sqrt{24}}$	$-\frac{1}{2\alpha}$	$\frac{1}{2\alpha}$
e	$-\frac{\cos\beta}{\sqrt{10}}-\frac{\sin\beta}{\sqrt{6}}$	$2\frac{\cos\beta}{\sqrt{10}}$	$-\frac{\alpha}{2}$	$\frac{1}{\alpha}-\frac{\alpha}{2}$
u	$\frac{\cos\beta}{\sqrt{10}}-\frac{\sin\beta}{\sqrt{6}}$	0	$\frac{\alpha}{2}$	$-\frac{1}{3\alpha}+\frac{\alpha}{2}$
d	$-\frac{\cos\beta}{\sqrt{10}}-\frac{\sin\beta}{\sqrt{6}}$	$-2\frac{\cos\beta}{\sqrt{10}}$	$-\frac{\alpha}{2}$	$-\frac{1}{3\alpha}-\frac{\alpha}{2}$

would find a discrepancy between the couplings in the Table II and the ones following from RG relations (2) or (11). This requires to be discussed in more detail.

There are different symmetry breaking schemes in the E_6 group. One of them is

$$E_6 \to SO(10)\times U(1)_\psi,$$
$$SO(10)\to SU(3)_c\times SU(2)_L\times SU(2)_R\times U(1)_{B-L}. \tag{38}$$

It leads to the so-called left-right (LR) model. In this case the values of the Z' couplings to the charged leptons and quarks satisfy relations (2) and (11). The Z' interactions with neutrinos will be discussed later. Another scheme,

$$E_6\to SO(10)\times U(1)_\psi \to SU(5)\times U(1)_\chi \times U(1)_\psi, \tag{39}$$

predicts the Abelian Z' which is the linear combination of the neutral vector bosons ψ and χ:

$$Z' = \chi\cos\beta + \psi\sin\beta. \tag{40}$$

The mixing of the states χ and ψ, naturally, influences the Z' couplings. In our problem, two possibilities to choose β are of interest. First is if the mass of the field ψ is much heavier than that of the χ field. As a consequence, the field ψ is decoupled and the mixing angle β is small ($\beta \ll 1$). In this case RG relations (2) and (11) hold in lower order in β for the Z' couplings to quarks and charged leptons. Second is if the masses are of the same order. This case cannot be treated straightforwardly on the basis of relations (2) and (11) since the mixed states of Z' have to be included into consideration explicitly. Our approach is also applicable in this case. However, it requires additional investigations which we will present in a separate publication.

As is usually supposed in theories based on the E_6 group, the Yukawa terms responsible for the generation of the Dirac masses of neutrinos must be set to zero [7]. So there are no RG relations for the Z' interactions with the neutrino axial-vector currents. In this case the couplings $a^\nu_{Z'}$ given in Table II are not restricted by the RG relations. Moreover, the numerical estimates (36) are completely independent of these parameters.

In general, one can consider the problem of searching for the Abelian Z' when the basis low-energy theory is not the minimal SM. In this case, the existence of new light fields (for instance, light scalar particles) must be allowed for. These additional light fields will enter the β and γ functions calculated in the external field substituting the virtual Z' state and in this way modify the RG relations [4].

As a corollary of our analysis we note that the introduced observables allow one to identify (or to discard) the Abelian Z' effects. They can usefully complement the conventional analysis of Z' couplings based on the observables σ_T and A_{FB}.

If one wishes to find variables giving the possibility of searching for Z' (or other heavy particles) and independent of neither high-energy nor low-energy theory, then in our approach it is necessary to derive a series of RG relations for different versions of the low-energy theory and the corresponding observables. Overlap of the variables will give such desired model independent constraints.

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