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Measuring β in $B \to D^{(*)+} D^{(*)-} K_s$ decays

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We consider the possibility of measuring both sin(2β) and cos(2β) in the KM unitarity triangle using the process $B^0 \to D^{*+} D^{*-} K_s$. This decay mode has a higher branching fraction $[O(1\%)]$ than the mode $B^0 \to D^{*+} D^{*-}$. We use the factorization assumption and heavy hadron chiral perturbation theory to estimate the branching fraction and polarization. The time dependent rate for $B^0(t) \to D^{*+} D^{*-} K_s$ can be used to measure sin(2β) and cos(2β). Furthermore, examination of the $D^{*+} K_s$ mass spectrum may be the best way to experimentally find the broad 1^+ p-wave D_s meson.

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I. INTRODUCTION

The decay $B^0 \to J/\psi K_s$ is expected to provide a clean measurement of the angle sin(2β) in the unitarity triangle [1]. However, other modes can also provide relevant information on the angle β. An example of such a mode is the decay $B^0 \to D^{(*)} \overline{D}^{(*)}$. In this mode $B^0 \to D^{*+} D^{*-}$, the vector-vector final state in general is an admixture of CP odd and even eigenstates, because s, p, and d partial waves with different CP parities can contribute. Since the CP asymmetry has opposite sign for the two CP states, they tend to cancel or dilute the overall asymmetry. The amount of dilution of the CP asymmetry is represented by the dilution factor, D, which depends on the CP composition of the final state. The presence of two CP components in the final state of $B^0 \to D^{*+} D^{*-}$ makes the dilution factor, $D<1$, for this decay. This is unlike the case for a mode such as $B^0 \to D^+ D^-$ where the final state is a CP eigenstate and $D=1$ as there is no dilution of the CP asymmetry. An angular analysis can extract the contribution of the different CP eigenstates, leading to a measurement of D and hence of sin(2β) [2,3]. However, in the factorization approximation and using heavy quark effective theory (HQET) it can be shown that the final state in $B^0 \to D^{*+} D^{*-}$ is dominated by a single CP eigenstate [4]. To the extent that this is valid, the angle sin(2β) can be determined without the need for an angular analysis. The decay $B^0 \to D^{*+} D^{*-}$ may be preferred to $B^0 \to D^+ D^-$ because contamination from penguin contributions and final state interactions (FSI's) is expected to be smaller in the former decay [3].

In this work we consider the possibility of extracting β from the decay $B^0 \to D^{(*)} \overline{D}^{(*)} K_s$. These modes are enhanced relative to $B^0 \to D^{(*)} \overline{D}^{(*)}$ by the factor $|V_{cs}/V_{cd}|^2 \sim 20$. As in the case of $B^0 \to J/\psi K_s$ decay, the penguin contamination is expected to be small in these decays. Moreover, these decays can be used to probe both $\sin 2\beta$ and $\cos 2\beta$ which can resolve the $\beta \to \pi/2 - \beta$ ambiguity [5].

The possibility that a large portion of the $b \to c\overline{c}s$ decays materialize as $B \to D\overline{D}K$ was first suggested by Buchalla *et al.* [6]. Using wrong sign D-lepton correlations, experimental evidence for this possibility was found by CLEO, who observed $\mathcal{B}(B \to DX) = (7.9 \pm 2.2)\%$ [7]. Later, CLEO [8], ALEPH [9] and DELPHI [10] reported full reconstruction of exclusive $D\overline{D}K$ final states with branching fractions that are consistent with the result from D-lepton correlations.

CLEO obtained $\mathcal{B}(\overline{B^0} \to D^{*+} \overline{D}^{*0} K^-) = (1.30^{+0.61}_{-0.47} \pm 0.27)\%$ and $\mathcal{B}(B^- \to D^{*0} \overline{D}^{*0} K^-) = (1.45^{+0.78}_{-0.58} \pm 0.36)\%$. These values should be approximately equal to the branching fraction for $\mathcal{B}(B^0 \to D^{*+} D^{*-} K^0)$. We use the latter value for the purpose of a sensitivity estimate. Taking into account $\mathcal{B}(K^0 \to K_s) = 0.5$, $\mathcal{B}(K_s \to \pi^+ \pi^-) = 0.667$, and assuming that the K_s reconstruction efficiency is ~ 0.5, we can estimate the ratio of the tagged $B^0 \to D^{*+} D^{*-} K_s$ events to the tagged $D^{*+} D^{*-}$ events. Assuming $\mathcal{B}(B^0 \to D^{*+} D^{*-}) = 6 \times 10^{-4}$, which is the central value of the recent CLEO measurement [11], we find that the ratio of the number of events is ~ 4.0. Therefore, this mode could be more sensitive to the CP violation angle sin(2β) than $B^0 \to D^{*+} D^{*-}$. However, if the final state contains a resonance, then B^0 and $\overline{B}^0$ can be distinguished and there is additional dilution of the CP asymmetry. For the decay $B \to f$ and $\overline{B} \to \overline{f}$ the dilution factor, D, measures the ratio of the overlap of the amplitudes for $B \to f$ and $\overline{B} \to \overline{f}$ to the average of the decay rate for $B \to f$ and $\overline{B} \to \overline{f}$. Clearly $D=1$ when the amplitudes for $B \to f$ and $\overline{B} \to \overline{f}$ decays are equal. When the final state in the decay $B \to D^{*+} D^{*-} K_s$ contains a resonance the amplitudes for B and $\overline{B}$ decays are different because the resonance in the B and $\overline{B}$ final states occurs at different kinematical points. This causes additional mismatch of the B and $\overline{B}$ amplitudes which results in the further dilution of the CP asymmetry. A similar conclusion is obtained in the comparison of $B^0 \to D^+ D^- K_s$ to $B^0 \to D^+ D^-$. The above conclusions are detector dependent; a somewhat pessimistic estimate of the K_s reconstruction efficiency is used here while the detection efficiency for the $D^{*+}D^{*-}$ final state is assumed to be similar for both cases. Better determination of the CP sensitivities will require more precise measurements of the branching fractions for the $D^*\bar{D}^*K$ decay modes and will also depend on details of the experimental apparatus and reconstruction programs.

The amplitude for the decay $B^0 \to D^*\bar{D}^*K_s$ can have a resonant contribution and a nonresonant contribution. For the resonant contribution the D^*K_s in the final state comes dominantly from an excited $D_s(1^+)$ state. In the approximation of treating $D^*\bar{D}^*K_s$ as D^*D_s (excited), there are four possible excited p-wave D_s states which might contribute. These are the two states with the light degrees of freedom in a $j^P=3/2^+$ state and the two states with light degrees of freedom in a $j^P=1/2^+$ state. Since the states with $j^P=3/2^+$ decay via d wave to D^*K_s, they are suppressed. Of the states with light degrees of freedom in $j^P=1/2^+$ states, only the 1^+ state contributes. The 0^+ state is forbidden to decay to the final state D^*K_s.

To estimate the above contribution and to calculate the nonresonant amplitude, we use heavy hadron chiral perturbation theory (HHChPT) [12]. The momentum p_k of K_s can have a maximum value of about 1 GeV for $B^0 \to D^{*+}\bar{D}^{*-}K_s$. This is of the same order as Λ_χ which sets the scale below which we expect HHChPT to be valid. It follows that in the present case it is reasonable to apply HHChPT to calculate the three body decays.

In the lowest order in the HHChPT expansion, contributions to the decay amplitude come from the contact interaction terms and the pole diagrams which give rise to the nonresonant and resonant contributions, respectively. The pole diagrams get contributions from the various multiplets involving D_s type resonances as mentioned above. In the framework of HHChPT, the ground state heavy meson has the light degrees of freedom in a spin-parity state $j^P=\frac{1}{2}^-$, corresponding to the usual pseudoscalar-vector meson doublet with $J^P=(0^-,1^-)$. The first excited state involves a p-wave excitation, in which the light degrees of freedom have $j^P=\frac{1}{2}^+$ or $\frac{3}{2}^+$. In the latter case we have a heavy doublet with $J^P=(1^+,2^+)$. These states can probably be identified with $D_{s1}(2536)$ and $D_{sJ}(2573)$ [13]. Heavy quark symmetry rules out any pseudoscalar coupling of this doublet to the ground state at lowest order in the chiral expansion [14]; hence the effects of these states will be suppressed and we will ignore them in our analysis. In fact there is an experimental upper limit on inclusive $B \to D_{s1}(2536)X < 0.95\%$ at 90% C.L. [15]. Since the total $D^*\bar{D}^*K$ rate is about 8%, this confirms that the narrow p-wave states do not account for a significant fraction of the total $D^*\bar{D}^*K$ rate.

The other excited doublet has $J^P=(0^+,1^+)$. These states are expected to decay rapidly through s-wave pion emission and have large widths [16]. Observation of the 1^+ state in the D system was recently reported by CLEO [17]. Only the 1^+ can contribute in this case. For later reference, we denote this state by $D_{s1}^{*'}$. However, quark model estimates suggest [18] that these states should have masses near $m+\delta m$ with $\delta m = 500$ MeV, where m is the mass of the lowest multiplet.

We will assume that the leading order terms in HHChPT give the dominant contribution to the decay amplitude and so we will neglect all sub-leading effects suppressed by $1/\Lambda_\chi$ and $1/m$, where m is the heavy quark mass. We show that from the time-dependent analysis of $B^0(t) \to D^{*+}D^{*-}K_s$ one can extract $\sin(2\beta)$ and $\cos(2\beta)$. Measurement of both $\sin(2\beta)$ and $\cos(2\beta)$ can resolve the $\beta \to \pi/2-\beta$ ambiguity [5,19,20]. The measurement of $\sin(2\beta)$ can be made from the time dependent partial rate asymmetry while a fit to the time dependent rate for $\Gamma[B^0(t) \to D^{*+}D^{*-}K_s] + \Gamma[\bar{B}^0(t) \to D^{*+}D^{*-}K_s]$ may be used for the extraction of $\cos(2\beta)$. Note that the $\cos(2\beta)$ term measures the overlap of the imaginary part of the amplitudes for $B \to D^{*+}D^{*-}K_s$ and $\bar{B} \to D^{*+}D^{*-}K_s$ decays and is nonzero only if there is a resonance contribution.

As in the case for $B \to D^{*+}D^{*-}$ the asymmetry in $B \to D^{*+}D^{*-}K_s$ is also diluted. For the nonresonant contribution to $B \to D^{*+}D^{*-}K_s$ the final state is an admixture of CP states with different CP parities. This leads to the dilution of the asymmetry and this is the same dilution of the asymmetry as in the case for $B \to D^{*+}D^{*-}$. As already mentioned above, when the resonant contribution is included there is further dilution of the asymmetry from the additional mismatch of the amplitudes for B and $\bar{B}$ decays. One can reduce the additional dilution of the CP asymmetry by imposing cuts to remove the resonance. A narrow resonance is preferable as it can be more effectively removed from the signal region than a broad resonance. In this work we examine several cuts that can be used to remove the resonance and lessen the dilution of the CP asymmetry. When we include the resonance contribution we find that a broader resonance leads to a larger value of D and is a more useful probe of $\cos(2\beta)$ because of the larger overlap of the amplitudes for $B \to D^{*+}D^{*-}K_s$ and $\bar{B} \to D^{*+}D^{*-}K_s$ decays.

We also point out that from the differential decay distribution of the time-independent process $B^0 \to D^{*+}D^{*-}K_s$ one can discover the 1^+ resonance $D_{s1}^{*'}$. We show that the differential decay distribution for small values of E_k, the kaon energy, shows a clear resonant structure which comes from the pole contribution to the amplitude with the excited $J^P=1^+$ intermediate state. Therefore, examination of the D^*K_s mass spectrum may be the best experimental way to find the broad 1^+ p-wave D_s meson and a fit to the decay distribution will measure its mass and the coupling.

A similar analysis can be performed for $B^0 \to D^+D^-K_s$ [5,21]. However, the predictions of HHChPT for this mode may be less reliable because of the larger energy of the K_s. The effects of penguin contributions, though small, may also be more important in $B^0 \to D^+D^-K_s$ than in $B^0 \to D^*D^*K_s$ as in the two body case [3].

In the next section we describe the extraction of $\sin 2\beta$ and $\cos 2\beta$ from the time dependent rate for $B(t) \to D^{*+}D^{*-}K_s$. In the next section we present the amplitude for $B \to D^*\bar{D}^*K_s$ in the factorization approximation and using HHChPT. In the last section we discuss and present our results.

II. EXTRACTION OF $\sin 2\beta$ AND $\cos 2\beta$

In this section we discuss the extraction of $\sin 2\beta$ and $\cos 2\beta$ from the time dependent rate for $B(t)\to D^{*+}D^{*-}K_s$. We define the following amplitudes:

$$a^{\lambda_1,\lambda_2}\equiv A(B^0(p)\to D^{+*}_{\lambda_1}(p_+)D^{-*}_{\lambda_2}(p_-)K_s(p_k)),$$

$$\overline{a}^{\lambda_1,\lambda_2}\equiv A(\overline{B}^0(p)\to D^{+*}_{\lambda_1}(p_+)D^{-*}_{\lambda_2}(p_-)K_s(p_k)), \quad (1)$$

where B^0 and $\overline{B}^0$ represent unmixed neutral B and λ_1 and λ_2 are the polarization indices of the D^{*+} and D^{*-} respectively.

The time-dependent amplitudes for an oscillating state $B^0(t)$ which has been tagged as a B^0 meson at time $t=0$ is given by

$$A^{\lambda_1,\lambda_2}(t)=a^{\lambda_1,\lambda_2}\cos\left(\frac{\Delta m\, t}{2}\right)+ie^{-2i\beta}\overline{a}^{\lambda_1,\lambda_2}\sin\left(\frac{\Delta m\, t}{2}\right), \quad (2)$$

and the time-dependent amplitude squared summed over polarizations and integrated over the phase space angles is

$$|A(s^+,s^-;t)|^2=\frac{1}{2}[G_0(s^+,s^-)+G_c(s^+,s^-)\cos\Delta mt - G_s(s^+,s^-)\sin\Delta m\, t] \quad (3)$$

with

$$G_0(s^+,s^-)=|a(s^+,s^-)|^2+|\overline{a}(s^+,s^-)|^2, \quad (4)$$

$$G_c(s^+,s^-)=|a(s^+,s^-)|^2-|\overline{a}(s^+,s^-)|^2, \quad (5)$$

$$G_s(s^+,s^-)=2\Im[e^{-2i\beta}\overline{a}(s^+,s^-)a^*(s^+,s^-)] = -2\sin(2\beta)\,\Re(\overline{a}a^*) + 2\cos(2\beta)\,\Im(\overline{a}a^*). \quad (6)$$

The variables s^+ and s^- are the Dalitz plot variables:

$$s^+=(p_++p_k)^2, \quad s^-=(p_-+p_k)^2.$$

The transformation defining the CP-conjugate channel $\overline{B}^0(t)\to D^{*-}D^{*+}K_s$ is $s^+\leftrightarrow s^-$, $a\leftrightarrow\overline{a}$ and $\beta\to-\beta$. Then

$$|\overline{A}(s^-,s^+;t)|^2=\frac{1}{2}[G_0(s^-,s^+)-G_c(s^-,s^+)\cos\Delta mt + G_s(s^-,s^+)\sin\Delta mt]. \quad (7)$$

Note that for simplicity the $e^{-\Gamma t}$ and constant phase space factors have been omitted in the above equations.

It is convenient in our case to replace the variables s^+ and s^- by the variables y and E_k where E_k is the K_s energy in the rest frame of the B and $y=\cos\theta$ with θ being the angle between the momentum of K_s and D^{*+} in a frame where the two D^* are moving back to back along the z axis. This frame is boosted with respect to the rest frame of the B with $\vec{\beta}=-(\vec{p}_k/m_B)[1/(1-E_k/m_B)]$. Note that $s^+\leftrightarrow s^-$ corresponds to $y\leftrightarrow -y$. The variable y can be expressed in terms of variables in the rest frame of B. For instance

$$E_+=\frac{E'_BE'_+-p'_Bp'_+y}{m_B}$$

where E_+ and E'_+ are the energy of the D^{*+} in the rest frame of the B and in the boosted frame while E'_B is the energy of the B in the boosted frame. The magnitudes of the momentum of the B and the D^{*+} in the boosted frame are given by p'_B and p'_+ respectively.

In the approximation of neglecting the penguin contributions to the amplitude there is no direct CP violation. This leads to the relation

$$a^{\lambda_1,\lambda_2}(\vec{p}_{k1},E_k)=\overline{a}^{-\lambda_1,-\lambda_2}(-\vec{p}_{k1},E_k) \quad (8)$$

where $\vec{p}_{k1}$ is the momentum of the of the K_s in the boosted frame. The above relations then leads to

$$G_0(-y,E_k)=G_0(y,E_k) \quad (9)$$

$$G_c(-y,E_k)=-G_c(y,E_k) \quad (10)$$

$$G_{s1}(-y,E_k)=G_{s1}(y,E_k) \quad (11)$$

$$G_{s2}(-y,E_k)=-G_{s2}(y,E_k) \quad (12)$$

where we have defined

$$G_{s1}(y,E_k)=\Re(\overline{a}a^*) \quad (13)$$

$$G_{s2}(-y,E_k)=\Im(\overline{a}a^*). \quad (14)$$

Carrying out the integration over the phase space variables y and E_k one gets the following expressions for the time-dependent total rates for $B^0(t)\to D^{*+}D^{*-}K_s$ and the CP conjugate process:

$$\Gamma(t)=\frac{1}{2}[I_0+2\sin(2\beta)\sin(\Delta mt)I_{s1}] \quad (15)$$

$$\overline{\Gamma}(t)=\frac{1}{2}[I_0-2\sin(2\beta)\sin(\Delta mt)I_{s1}] \quad (16)$$

where I_0 and I_{s1} are the integrated $G_0(y,E_k)$ and $G_{s1}(y,E_k)$ functions. One can then extract $\sin(2\beta)$ from the rate asymmetry

$$\frac{\Gamma(t)-\overline{\Gamma}(t)}{\Gamma(t)+\overline{\Gamma}(t)}=D\sin(2\beta)\sin(\Delta mt) \quad (17)$$

where

$$D=\frac{2I_{s1}}{I_0} \quad (18)$$

is the dilution factor.

The cos(2β) term can be probed by integrating over half the range of the variable y which can be taken for instance to be $y \geqslant 0$. In this case we have

$$\Gamma(t) = \frac{1}{2}[J_0 + J_c \cos(\Delta m t) + 2\sin(2\beta)\sin(\Delta m t) J_{s1} - 2\cos(2\beta)\sin(\Delta m t) J_{s2}] \quad (19)$$

$$\overline{\Gamma}(t) = \frac{1}{2}[J_0 + J_c \cos(\Delta m t) - 2\sin(2\beta)\sin(\Delta m t) J_{s1} - 2\cos(2\beta)\sin(\Delta m t) J_{s2}] \quad (20)$$

where J_0, J_c, J_{s1} and J_{s2}, are the integrated $G_0(y,E_k)$, $G_c(y,E_k)$, $G_{s1}(y,E_k)$ and $G_{s2}(y,E_k)$ functions integrated over the range $y \geqslant 0$. One can measure cos(2β) by fitting to the time distribution of $\Gamma(t) + \overline{\Gamma}(t)$. Measurement of cos(2$\beta$) can resolve the $\beta \to \pi/2 - \beta$ ambiguity.

III. AMPLITUDE AND DECAY DISTRIBUTION

In this section we present the amplitude and decay distribution for the decay $B \to D^{*+} D^{*-} K_s$. Details of the calculation of the amplitudes using the factorization assumption and HHChPT are given in Appendix A.

The nonresonant amplitude for the three body decay $\overline{B^0}(v,m) \to D^{*+}(\epsilon_1, v_+, m_1) D^{*-}(\epsilon_2, v_-, m_2) K_s(p_k)$, after setting $m_2 = m_1$, is given by

$$\overline{a}_{non\text{-}res} = K\sqrt{m}\sqrt{m_1} m_1 \xi(v\cdot v_+) \frac{f_{D^*}}{f_K}[i\varepsilon^{\mu\nu\alpha\beta}\epsilon^*_{1\mu}\epsilon^*_{2\nu} v_\alpha v_{+\beta} + \epsilon^*_1\cdot v\,\epsilon^*_2\cdot v_+ - \epsilon^*_1\cdot\epsilon^*_2(v\cdot v_+ + 1)] \quad (21)$$

where $K = (G_F/\sqrt{2}) V_c (c_1/N_c + c_2)$. Note that the amplitude above is the same as the amplitude for $B^0 \to D^{*-} D^{*+}$ [4] except for a constant multiplicative factor $\sim 1/f_K$.

To a good approximation one can use $\vec{v} \sim 0$ where $\vec{v}$ is the velocity of the $\overline{B}^0$ in the boosted frame where the two D^* are moving back to back. The K_s, in this limit, is emitted in an *s*-wave configuration as the amplitude is independent of the angles that specify the K_s momentum in the boosted frame. Then, as in the $\overline{B}^0 \to D^{*+} D^{*-}$ case there are three helicity states allowed, $(+,+)$, $(-,-)$ and (0,0), with the corresponding helicity amplitudes H_{++}, H_{--} and H_{00}. The helicity states are not CP eigenstates but one can go to the partial wave basis or the transverse basis where the states are CP eigenstates. The transverse basis amplitudes are related to the helicity amplitudes as

$$A_\parallel = \frac{H_{++} + H_{--}}{\sqrt{2}}$$

$$A_\perp = \frac{H_{++} - H_{--}}{\sqrt{2}}$$

$$A_0 = H_{00}. \quad (22)$$

The three partial waves that are allowed in this case, *s*, *p* and *d*, are then given by

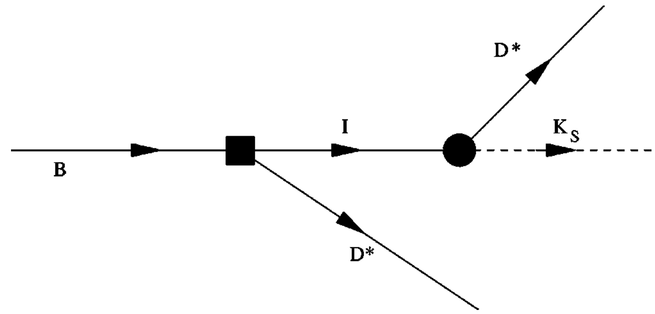

FIG. 1. The pole contribution to the process $B \to D^* D^* K_s$. The intermediate state I can be $D^{*\prime}_{s1}$ or D^*_s. The solid square represents the weak vertex while the solid circle represents the strong vertex.

$$s = \frac{\sqrt{2}A_\parallel - A_0}{\sqrt{3}}$$

$$p = A_\perp$$

$$d = \frac{\sqrt{2}A_0 + A_\parallel}{\sqrt{3}}. \quad (23)$$

The CP of the final state is given by $\eta(-)^L$ where η is the intrinsic parity of the final states and L is the relative angular momentum between D^{*+} and D^{*-}. In the approximation $\vec{v} \sim 0$ one can write the nonresonant amplitude for $B^0(v,m) \to D^{*+}(\epsilon_1, v_+, m_1) D^{*-}(\epsilon_2, v_-, m_1) K_s(p_k)$:

$$a_{non-res} = K\sqrt{m}\sqrt{m_1} m_1 \xi(v\cdot v_-) \frac{f_{D^*}}{f_K} \times [-i\varepsilon^{\mu\nu\alpha\beta}\epsilon^*_{2\mu}\epsilon^*_{1\nu} v_\alpha v_{-\beta} + \epsilon^*_2\cdot v\,\epsilon^*_1\cdot v_- - \epsilon^*_1\cdot\epsilon^*_2(v\cdot v_- + 1)]. \quad (24)$$

There can also be pole contributions of the type shown in Fig. 1.

These give the decay sequences

$$\overline{B^0} \to D^{*+} D^{*\prime -}_{s1} \to D^{*+} D^{*-} K^0$$

and

$$\overline{B^0} \to D^{*+} D^{*-}_s \to D^{*+} D^{*-} K^0.$$

The propagator for the vector resonance is given by

$$S_{\mu\nu} = \frac{i(V_\mu V_\nu - g_{\mu\nu})}{2V\cdot k} \quad (25)$$

where the momentum of the propagating particle $P = m_I V + k$ where m_I is the mass of the intermediate particle in Fig. 1.

The contributions from the pole diagrams are given by $\overline{a}_{1res}$ and $\overline{a}_{2res}$, where $\overline{a}_{1res}$ is, with $m_I = m^{*\prime}$,

$$\overline{a}_{1res}=K\sqrt{m}\sqrt{m_1}\sqrt{m_1}\sqrt{m^{*\prime}}\xi(v\cdot v_+)\times\frac{f_{D_{s1}^{*\prime}}}{f_K}\frac{hp_k\cdot v_-}{\left(p_k\cdot v_-+m_1-m^{*\prime}+\frac{i\Gamma_{D_{s1}^{*\prime}}}{2}\right)}\times[-i\varepsilon^{\mu\nu\alpha\beta}\epsilon_{1\mu}^*\epsilon_{2\nu}^*v_\alpha v_{+\beta}-\epsilon_1^*\cdot v\epsilon_2^*\cdot v_+ +\epsilon_1^*\cdot\epsilon_2^*(v\cdot v_++1)]. \quad (26)$$

Note that the above amplitude can be rewritten as

$$\overline{a}_{1res}=-\overline{a}_{non-res}\frac{f_{D_{s1}^{*\prime}}}{f_{D^*}}\sqrt{\frac{m^{*\prime}}{m_1}}\times\frac{hp_k\cdot v_-}{\left(p_k\cdot v_-+m_1-m^{*\prime}+\frac{i\Gamma_{D_{s1}^{*\prime}}}{2}\right)}. \quad (27)$$

$\overline{a}_{2res}$ is given by, with $m_I=m^*$ where m^* is the 1^- D_s^* mass,

$$\overline{a}_{2res}=K\sqrt{m}\sqrt{m_1}\sqrt{m_1}\sqrt{m^*}\xi(v\cdot v_+)\frac{f_{D_s^*}}{f_K}\frac{g}{\left(p_k\cdot v_-+(m_1-m^*)+\frac{i\Gamma_{D_s^*}}{2}\right)}X$$

$$X=-i\varepsilon^{\mu\nu\alpha\beta}\epsilon_{2\mu}^*p_{k\nu}v_{+\alpha}v_{-\beta}\epsilon_1^*\cdot v+i\varepsilon^{\mu\nu\alpha\beta}\epsilon_{1\mu}^*\epsilon_{2\nu}^*p_{k\alpha}v_{-\beta}(v\cdot v_++1)+(\epsilon_1^*\cdot v_-\epsilon_2^*\cdot vp_k\cdot v_+ -\epsilon_1^*\cdot v_-\epsilon_2^*\cdot v_+p_k\cdot v)+(\epsilon_1^*\cdot p_k\epsilon_2^*\cdot v_+v\cdot v_--\epsilon_1^*\cdot p_k\epsilon_2^*\cdot vv_+\cdot v_-) +(\epsilon_1^*\cdot\epsilon_2^*p_k\cdot vv_+\cdot v_--\epsilon_1^*\cdot\epsilon_2^*p_k\cdot v_+v\cdot v_-). \quad (28)$$

The amplitude $\overline{a}_{2res}$ gives a tiny contribution to the total amplitude and can be neglected. In fact, this amplitude vanishes in the small velocity limit where the D^* are almost at rest [22]. We note that the process with the 0^+ intermediate state

$$\overline{B}^0\to D^{*+}D_{s0}^-\to D^{*+}D^{*-}K^0$$

is not allowed due to parity conservation while the amplitude with the 0^- intermediate state

$$\overline{B}^0\to D^{*+}D_s^-\to D^{*+}D^{*-}K^0$$

is expected to be small compared to $\overline{a}_{1res}$. The propagator term in the above amplitude goes as approximately $1/[E_K+(m_{D^*}-m_{D_s})]$ which does not have a pole as in $\overline{a}_{1res}$. Moreover, the amplitude is further suppressed with respect to $\overline{a}_{1res}$ by a factor $\sim p_k/E_k$ or $|\vec{v}|/v_0$, where $\vec{v}$ and v_0 are the three-velocity and the time component of the velocity four-vector of the D^*, from the $D_s^+D^{*+}K^0$ vertex.

The total amplitude for $\overline{B}^0(v,m)\to D^{*+}(\epsilon_1,v_+,m_1)D^{*-}(\epsilon_2,v_-,m_1)K_s(p_k)$ can be written as

$$\overline{a}=\overline{a}_{non\text{-}res}[1-P_1] \quad (29)$$

and the total amplitude for $B^0(v,m)\to D^{*+}(\epsilon_1,v_+,m_1)D^{*-}(\epsilon_2,v_-,m_1)K_s(p_k)$ can be written as

$$a=a_{non\text{-}res}[1-P_2] \quad (30)$$

with

$$P_1=\frac{f_{D_{s1}^{*\prime}}}{f_{D^*}}\sqrt{\frac{m^{*\prime}}{m_1}}\frac{hp_k\cdot v_-}{\left(p_k\cdot v_-+m_1-m^{*\prime}+\frac{i\Gamma_{D_{s1}^{*\prime}}}{2}\right)} \quad (31)$$

$$P_2=\frac{f_{D_{s1}^{*\prime}}}{f_{D^*}}\sqrt{\frac{m^{*\prime}}{m_1}}\frac{hp_k\cdot v_+}{\left(p_k\cdot v_++m_1-m^{*\prime}+\frac{i\Gamma_{D_{s1}^{*\prime}}}{2}\right)}. \quad (32)$$

Note that P_1 and P_2 can be expressed in terms of E_k and y and $P_1(y,E_k)=P_2(-y,E_k)$. The relation between quantities in the boosted frame and the rest frame of the B and the calculation of the squared amplitude are given in Appendix B.

The double differential decay distribution for the time independent process

$$\overline{B}^0(v,m)\to D^{*+}(\epsilon_1,v_+,m_1)D^{*-}(\epsilon_2,v_-,m_1)K_s(p_k)$$

can be written as

$$\frac{1}{\Gamma}\frac{d\Gamma}{dydE_k}=\frac{f(y,E_k)}{\int f(y,E_k)\frac{p_k'p_+'}{m}dydE_k} \quad (33)$$

where p_k' and p_+' are the magnitudes of the three-momentum of the K_s and D^{*+} in the boosted frame and the expression for $f(y,E_k)$ can be found in Appendix B. The differential distribution depends only on $f_{D_{s1}^{*\prime}}/f_{D^*}$, the mass $m^{*\prime}$ and the coupling h of the $D_{s1}^{*\prime}$ state. It is expected that $f_{D_{s1}^{*\prime}} \approx f_{D_s^*}$ and in the $SU(3)$ limit $f_{D_s^*}=f_{D^*}$. So in the $SU(3)$ limit a two parameter fit to the differential decay distribution can determine the mass and the coupling of the $D_{s1}^{*\prime}$ state.

The widths of the positive parity excited states are expected to be saturated by single kaon transitions [14]. In our calculation we require the width of the $D_{s1}^{*\prime}$ state. Assuming

$$\Gamma_{D_{s1}^{*+\prime}} \approx \Gamma(D_{s1}^{*+\prime} \to D^{*+}K^0) + \Gamma(D_{s1}^{*+\prime} \to D^{*0}K^+) \tag{34}$$

one can write

$$\Gamma_{D_{s1}^{*\prime}} = \frac{h^2}{\pi f_K^2} \frac{m_1}{m^{*\prime}} (m^{*\prime} - m_1)^2 p \tag{35}$$

where p is the magnitude of the three-momentum of the decay products in the rest frame of $D_{s1}^{*\prime}$ and m_1 and $m^{*\prime}$ are the masses of the D_s^* and $D_{s1}^{*\prime}$ state.

It is clear that if $a=\bar{a}$, then the dilution factor $D=1$. However, that is not the case here. For the nonresonant contribution, in the approximation of small velocity of the B, the final state is an admixture of CP states with different CP parities. This leads to $D<1$. This is the same dilution of the asymmetry as in the case for $B \to D^{*+}D^{*-}$. When the resonant contribution is included the amplitudes a and $\bar{a}$ have an asymmetric dependence on the variable y. This reflects the fact that in the process $\bar{B}^0 \to D^{*+}D_{s1}^{*\prime -} \to D^{*+}D^{*-}K^0$ the kaon emerges most of the time closer to D^{*-} than the D^{*+}. The situation is reversed for B^0 decays. Consequently there is additional mismatch between the amplitudes a and $\bar{a}$ which leads to further dilution of the asymmetry. One can reduce the dilution of the asymmetry, i.e., increase D, by imposing cuts so as to reduce the resonant contribution. We consider several cases where cuts may be employed to decrease the dilution of the asymmetry. From Eqs. (31),(32) it is clear that resonance occurs when the following condition is met:

$$p_k \cdot v_+ = m^{*\prime} - m_1 \tag{36}$$

$$p_k \cdot v_- = m^{*\prime} - m_1. \tag{37}$$

If, in the allowed region of E_k, we can find a value E_{k0} such that for values of $E_k \geqslant E_{k0}$ the above conditions are not satisfied for $-1 \leqslant y \leqslant 1$, then we can remove the resonance by using the cut $E_k \geqslant E_{k0}$. The value of $E_{k0} \sim 0.76$ GeV in our case. We will call this case cut 1 for future reference.

Another possible cut is to include the whole range of E_k but in the region $E_k \leqslant E_{k0}$ we remove the resonance by cutting on the variable y. We can use the region $-0.5 \leqslant y \leqslant 0.5$ since for most values of E_k the resonance condition is satisfied in the range $-1 \leqslant y \leqslant -0.5$ and $0.5 \leqslant y \leqslant 1$. We will call this case cut 2 for future reference. In any event, the cuts can be optimized after the resonance has been seen experimentally. However, as we try to increase the value of D by cutting on the resonance, we reduce the usable part of the branching fraction.

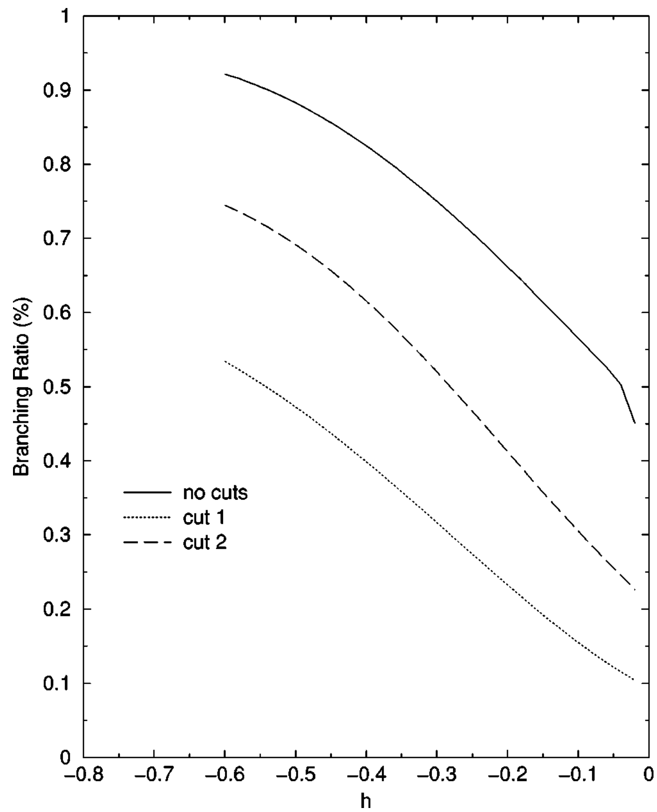

FIG. 2. The branching fraction for $\bar{B}^0 \to D^{*+}D^{*-}K_s$ as a function of the h with and without cuts.

IV. RESULTS AND DISCUSSIONS

As inputs to the calculation, we use $f_{D_s^*} \approx f_{D_{s1}^{*\prime}} = 200$ MeV and take the mass of the $D_{s1}^{*\prime}$ state to be 2.6 GeV. For the Isgur-Wise function we use the form

$$\xi(\omega) = \left(\frac{2}{1+\omega}\right)^2.$$

QCD sum rules have been used to compute the strong coupling constants g and h [23]. We will use $g=0.3$ as obtained in Ref. [23] but keep h as a free parameter because this coupling plays a more important role in the decay widths.

Figure 2 shows the branching fraction for $\bar{B}^0 \to D^{*+}D^{*-}K_s$ as a function of the coupling h. A QCD sum rule calculations gives $h \sim -0.5$ [23]. We use the same sign of h as obtained in QCD sum rule calculation but vary h from -0.6 to -0.1. For this range of h the branching fraction can vary in the range 0.45–0.93 % when we employ no cuts. For $h=-0.4$ which corresponds to a $D_{s1}^{*\prime}$ state with a width of about 150 MeV the branching fraction is 0.83%. In our calculation this corresponds to a branching ratio

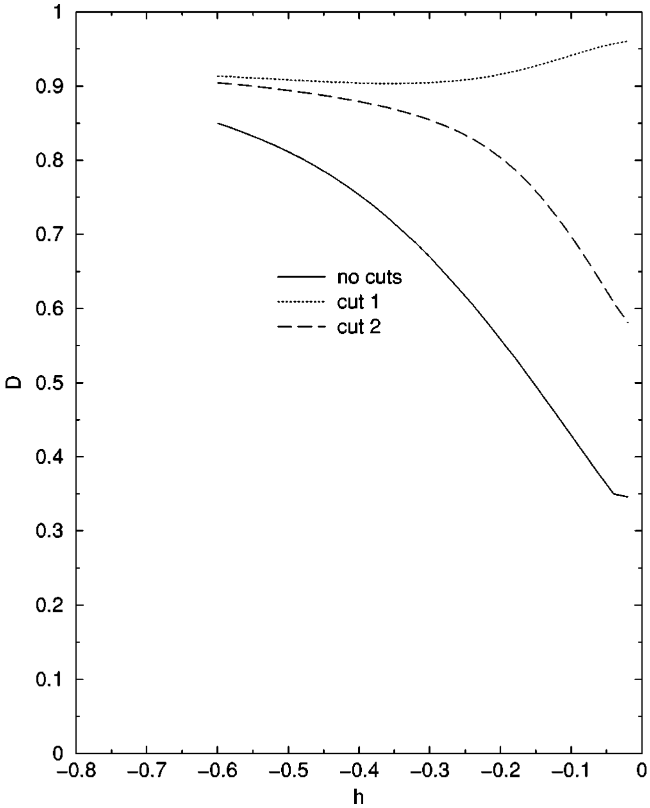

FIG. 3. The dilution factor D as a function of the h with and without cuts.

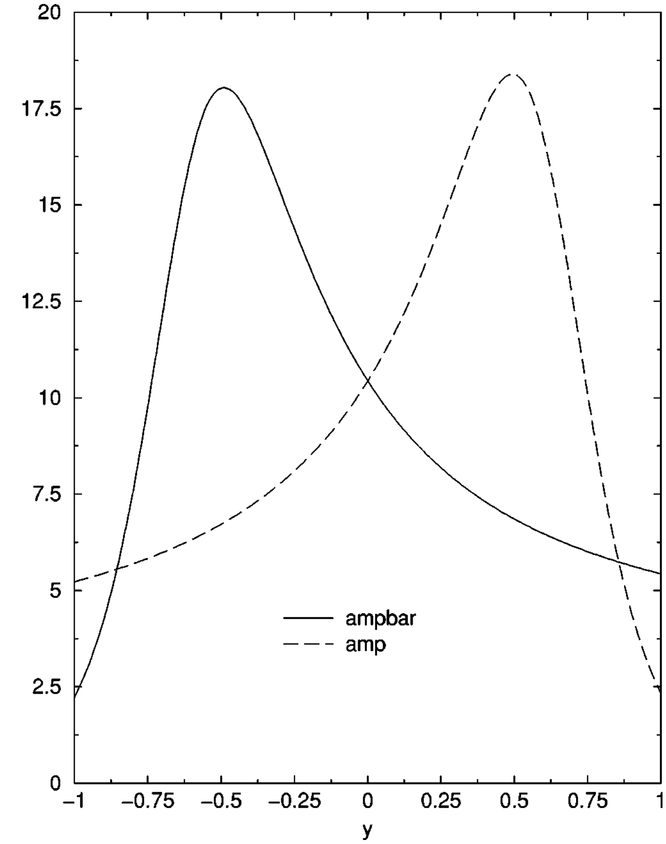

FIG. 4. The squared amplitude for $B^0\rightarrow D^{*+}D^{*-}K_s\overline{B}^0\rightarrow D^{*+}D^{*-}K_s$ as a function of the variable y for $h=-0.4$ which corresponds to a $D_{s1}^{*\prime}$ state with a width of about 150 MeV.

$$\begin{aligned}\mathcal{B}(B^0\rightarrow D^{*-}D^{*+}K^0)&\approx\mathcal{B}(B^0\rightarrow D^{*-}D^{*0}K^+)\\&\approx\mathcal{B}(B^+\rightarrow\overline{D}^{*0}D^{*0}K^+)\\&\approx\mathcal{B}(B^+\rightarrow\overline{D}^{*0}D^{*+}K^0)\\&\approx 0.9-1.86\%.\end{aligned}$$

This is consistent with the CLEO measurements mentioned above. In the figure we also show the branching fraction with the cuts which are designed to reduce the dilution of the *CP* asymmetry.

Figure 3 shows a plot of the dilution factor D versus the coupling h. In the absence of any cuts we find that larger values of $|h|$ give a larger value of D and hence less dilution in the asymmetry because for a broad $D_{s1}^{*\prime}$ state there is more overlap between the amplitudes for $B^0\rightarrow D^{*+}D^{*-}K_s$ and $\overline{B}^0\rightarrow D^{*+}D^{*-}K_s$. For $h=-0.4$ the dilution factor is about 0.75 with no cuts. For the case of cut 1, where we use the cut $E_k>E_{k0}$ to effectively remove the resonance, the dilution factor increases with smaller $|h|$. This is because for smaller $|h|$ and $E_k>E_{k0}$ the resonant amplitude is small and the total amplitude is dominated by the nonresonant amplitude which gives a larger value for D. For the case of cut 2, as in the case with no cuts, the dilution factor D decreases with smaller $|h|$. This is because we are using the entire region of E_k and not removing the resonance by the cut $E_k>E_{k0}$ as in the case of cut 1. Consequently a broader resonance and hence a larger value of $|h|$ give a larger value of D and vice versa.

Figure 4 shows the squared amplitude for $B^0\rightarrow D^{*+}D^{*-}K_s$ and $\overline{B}^0\rightarrow D^{*+}D^{*-}K_s$ as a function of the variable y for $h=-0.4$. As mentioned above the nature of the two curves reflects the fact that in the process $\overline{B}^0\rightarrow D^{*+}D_{s1}^{*\prime -}\rightarrow D^{*+}D^{*-}K^0$ the kaon emerges most of the time closer to D^{*-} than the D^{*+} while the situation is reversed for B^0 decays.

Figure 5 shows the plot of the functions G_0, G_c, G_{s1} and G_{s2} as a function of y for $E_k=0.6$ GeV and for $h=-0.4$. From the figure we see that the functions G_0 and G_{s1} are symmetric in y while G_c and G_{s2} are antisymmetric in y. This follows from the absence of direct *CP* violation as shown in Eqs. (19)–(22).

In Fig. 6 we show the decay distribution $d\Gamma/dE_k$ versus the kaon energy E_k. For small values of E_k the decay distribution shows a clear resonant structure which comes from the pole contribution to a_{1res} with the excited $J^P=1^+$ intermediate state. Therefore, examination of the D^*K_s mass spectrum may be the best experimental way to find the broad 1^+ p-wave D_s meson and as mentioned in the previous section a fit to the decay distribution will measure its mass and the coupling.

In Fig. 7 we show the functions G_0, G_c, G_{s1} and G_{s2}

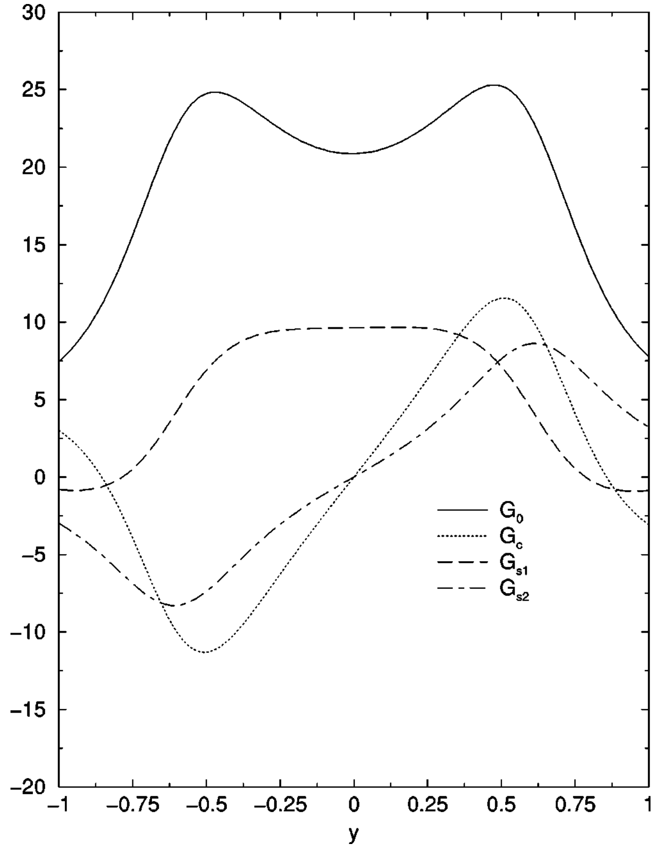

FIG. 5. The functions G_0, G_c, G_{s1} and G_{s2} as a function of y for $E_k = 0.6$ GeV and $h = -0.4$.

integrated over the $y \geqslant 0$ as a function of h. J_0 and J_c refer to the integrated G_0 and G_c functions while J_{s1} and J_{s2} refer to the integrated G_{s1} and G_{s2} functions. As already mentioned, restricting the integration range to $y \geqslant 0$ allows a probe of the $\cos(2\beta)$ term in the time dependent rate for $B^0(t) \to D^{(*+)}D^{(*-)}K_s$ decays. It is clear from the figure that a broader resonance is more favorable to probe G_{s2} which is the coefficient of the $\cos(2\beta)$ term.

In summary, we have studied the possibility of extracting $\sin(2\beta)$ and $\cos(2\beta)$ from time dependent $B^0 \to D^{(*)}\bar{D}^{(*)}K_s$ decays. These decays are expected to have less penguin contamination and much larger branching fractions than the two body modes $B^0 \to D^{(*)}\bar{D}^{(*)}$. Using HHChPT we have calculated the branching fractions and the various coefficient functions that appear in the time dependent rate for $B^0 \to D^{(*+)}D^{(*-)}K_s$. We also showed that a examination of the D^*K_s mass spectrum may be the best experimental way to find the broad 1^+ p-wave D_s meson and measure its mass and coupling.

ACKNOWLEDGMENTS

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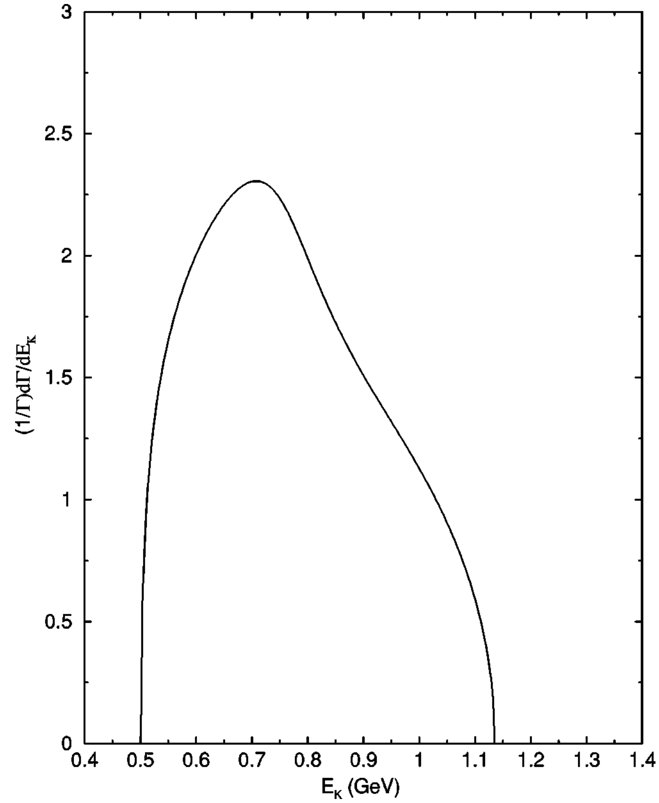

FIG. 6. The decay distribution $d\Gamma/dE_k$ versus the kaon energy E_k.

APPENDIX A

In the standard model (SM) the amplitudes for $B \to D^{(*)}\bar{D}^{(*)}K_s$ are generated by the following effective Hamiltonian [24,25]:

$$H^q_{eff} = \frac{G_F}{\sqrt{2}} \Bigg[V_{fb}V^*_{fq}(c_1 O^q_{1f} + c_2 O^q_{2f}) - \sum_{i=3}^{10} (V_{ub}V^*_{uq}c^u_i + V_{cb}V^*_{cq}c^c_i + V_{tb}V^*_{tq}c^t_i)O^q_i \Bigg] + \text{H.c.}, \tag{A1}$$

where the superscripts u, c, t indicate the internal quark, f can be a u or c quark and q can be either a d or a s quark depending on whether the decay is a $\Delta S = 0$ or $\Delta S = -1$ process. The operators O^q_i are defined as

$$O^q_{1f} = \bar{q}_\alpha \gamma_\mu L f_\beta \bar{f}_\beta \gamma^\mu L b_\alpha, \quad O^q_{2f} = \bar{q}\gamma_\mu L f \bar{f} \gamma^\mu L b, \tag{A2}$$

$$O^q_{3,5} = \bar{q}\gamma_\mu L b \bar{q}' \gamma_\mu L(R) q',$$

$$O^q_{4,6} = \bar{q}_\alpha \gamma_\mu L b_\beta \bar{q}'_\beta \gamma_\mu L(R) q'_\alpha,$$

$$O^q_{7,9} = \frac{3}{2}\bar{q}\gamma_\mu L b e_{q'} \bar{q}' \gamma^\mu R(L) q',$$

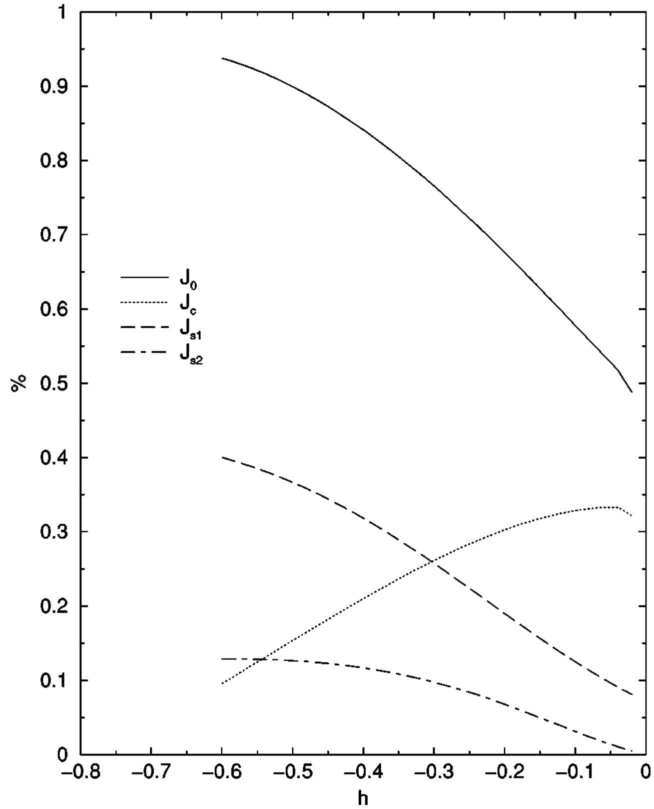

FIG. 7. The functions G_0, G_c, G_{s1} and G_{s2} integrated over the $y\geqslant 0$ as a function of h. J_0 and J_c refer to the integrated G_0 and G_c functions while J_{s1} and J_{s2} refer to the integrated G_{s1} and G_{s2} functions. The values of the integral can be obtained by multiplying by Γ_B where Γ_B is the width of the B.

$$O^q_{8,10}=\frac{3}{2}\overline{q}_\alpha\gamma_\mu L b_\beta e_{q'}\overline{q'_\beta}\gamma_\mu R(L)q'_\alpha,$$

where $R(L)=1\pm\gamma_5$, and q' is summed over all flavors except t. $O_{1f,2f}$ are the current-current operators that represent tree level processes. O_{3-6} are the strong gluon induced penguin operators, and the operators O_{7-10} are due to γ and Z exchange (electroweak penguins) and ''box'' diagrams at the loop level. The Wilson coefficients c_i^f are defined at the scale $\mu\approx m_b$ and have been evaluated to next-to-leading order in QCD. The c_i^t are the regularization scheme independent values obtained in Ref. [26]. We give the nonzero c_i^f below for $m_t=176$ GeV, $\alpha_s(m_Z)=0.117$, and $\mu=m_b=5$ GeV,

$$\begin{gathered}c_1=-0.307,\quad c_2=1.147,\quad c_3^t=0.017,\\ c_4^t=-0.037,\quad c_5^t=0.010,\quad c_6^t=-0.045,\\ c_7^t=-1.24\times10^{-5},\quad c_8^t=3.77\times10^{-4},\\ c_9^t=-0.010,\quad c_{10}^t=2.06\times10^{-3},\\ c_{3,5}^{u,c}=-c_{4,6}^{u,c}/N_c=P_s^{u,c}/N_c,\quad c_{7,9}^{u,c}=P_e^{u,c},\quad c_{8,10}^{u,c}=0\end{gathered}\tag{A3}$$

where N_c is the number of colors. The leading contributions to $P^i_{s,e}$ are given by: $P^i_s=(\alpha_s/8\pi)c_2[\frac{10}{9}+G(m_i,\mu,q^2)]$ and $P^i_e=(\alpha_{em}/9\pi)(N_c c_1+c_2)[\frac{10}{9}+G(m_i,\mu,q^2)]$. The function $G(m,\mu,q^2)$ is given by

$$G(m,\mu,q^2)=4\int_0^1 x(1-x)\ln\frac{m^2-x(1-x)q^2}{\mu^2}dx.\tag{A4}$$

All the above coefficients are obtained up to one loop order in electroweak interactions. The momentum q is the momentum carried by the virtual gluon in the penguin diagram. When $q^2>4m^2$, $G(m,\mu,q^2)$ becomes imaginary. In our calculation, we use $m_u=5$ MeV, $m_d=7$ MeV, $m_s=200$ MeV, $m_c=1.35$ GeV [27,13].

In the factorization assumption the amplitude for $B\to D^{(*)}\overline{D}^{(*)}K_s$ can now be written as

$$M=M_1+M_2+M_3+M_4\tag{A5}$$

where

$$\begin{aligned}M_1&=\frac{G_F}{\sqrt{2}}X_1\langle\overline{D}^{(*)}K_s|\overline{s}\gamma^\mu(1-\gamma^5)\,c|0\rangle\\&\quad\times\langle D^{(*)}|\overline{c}\,\gamma_\mu(1-\gamma^5)\,b|B\rangle\\ M_2&=\frac{G_F}{\sqrt{2}}X_2\langle\overline{D}^{(*)}D^{(*)}|\overline{c}\gamma^\mu(1-\gamma^5)\,c|0\rangle\\&\quad\times\langle K_s|\overline{s}\,\gamma_\mu(1-\gamma^5)\,b|B\rangle\\ M_3&=\frac{G_F}{\sqrt{2}}X_3\langle\overline{D}^{(*)}D^{(*)}|\overline{c}\gamma^\mu(1+\gamma^5)\,c|0\rangle\\&\quad\times\langle K_s|\overline{s}\,\gamma_\mu(1-\gamma^5)\,b|B\rangle\\ M_4&=\frac{G_F}{\sqrt{2}}X_4\langle\overline{D}^{(*)}K_s|\overline{s}(1+\gamma^5)\,c|0\rangle\\&\quad\times\langle D^{(*)}|\overline{c}\,(1-\gamma^5)\,b|B\rangle\end{aligned}\tag{A6}$$

where

$$\begin{aligned}X_1&=V_c\left(\frac{c_1}{N_c}+c_2\right)+\frac{B_3}{N_c}+B_4+\frac{B_9}{N_c}+B_{10}\\ X_2&=V_c\left(c_1+\frac{c_2}{N_c}\right)+B_3+\frac{1}{N_c}B_4+B_9+\frac{1}{N_c}B_{10}\\ X_3&=B_5+\frac{1}{N_c}B_6+B_7+\frac{1}{N_c}B_8\\ X_4&=-2\left(\frac{1}{N_c}B_5+B_6+\frac{1}{N_c}B_7+B_8\right).\end{aligned}\tag{A7}$$

We have defined

$$B_i = -\sum_{q=u,c,t} c_i^q V_q \tag{A8}$$

with

$$V_q = V_{qs}^* V_{qb}. \tag{A9}$$

In the above equations N_c represents the number of colors. It is usually the practice in the study of two body nonleptonic decays to include nonfactorizable effects by the replacement $N_c \to N_{eff}$. Since it is not obvious that N_{eff} for two body nonleptonic decays is the same for nonleptonic three body decays, we will use $N_c = 3$ in our calculation.

As already mentioned, we expect the contribution from penguin diagrams to be small and so as a first approximation we will neglect M_3 and M_4. Furthermore, from the values of the Wilson coefficients $c_{1,2}$ given above in the previous section it is clear that the amplitude M_2 is suppressed with respect to M_1 with the Wilson coefficients associated with M_2 being about 7% of the Wilson coefficients associated with M_1. We also note that the currents $\langle \overline{D}^{(*)} K_s | \overline{s} \gamma^\mu (1 - \gamma^5) c | 0 \rangle$ and $\langle K_S | \overline{s} \gamma_\mu (1 - \gamma^5) b | B \rangle$, which appear in M_1 and M_2 respectively, receive contributions from both the contact terms and the pole terms. For the former current the pole terms are proportional to $1/(E_K - \delta m)$ while for the latter the pole term goes as $1/(E_K + \delta m)$. This also leads to a further suppression of M_2 relative to M_1. We therefore neglect M_2 and only retain M_1 in our calculation. We will also neglect CP violation in the K^0-$\overline{K^0}$ system and so (with an appropriate choice of phase convention) we can write

$$K_s = \frac{K^0 - \overline{K^0}}{\sqrt{2}}. \tag{A10}$$

To calculate the various matrix elements in M_1 above we use heavy hadron chiral perturbation theory (HHChPT). In HHChPT, the ground state ($j^P = \frac{1}{2}^-$) heavy mesons are described by the 4×4 Dirac matrix

$$H_a = \frac{(1 + \not{v})}{2} [P_{a\mu}^* \gamma^\mu - P_a \gamma_5] \tag{A11}$$

where v is the heavy meson velocity, and $P_a^{*\mu}$ and P_a are annihilation operators of the 1^- and 0^- $Q\overline{q}_a$ mesons ($a = 1,2,3$ for u,d and s): for charm, they are D^* and D respectively. The field $\overline{H}_a$ is defined by

$$\overline{H}_a = \gamma^0 H^\dagger \gamma^0.$$

Similarly, the positive parity 1^+ and 0^+ states ($j^P = \frac{1}{2}^+$) are described by

$$S_a = \frac{(1 + \not{v})}{2} [D_{1\mu}^{*'} \gamma^\mu \gamma_5 - D_0]. \tag{A12}$$

In the above equations v generically represents the heavy meson four-velocity and $D^{*\mu}$ and D are annihilation operators normalized as follows:

$$\langle 0 | D | c\overline{q}(0^-) \rangle = \sqrt{M_H}$$

$$\langle 0 | D^{*\mu} | c\overline{q}(1^-) \rangle = \epsilon^\mu \sqrt{M_H}. \tag{A13}$$

Similar equations hold for the positive parity states $D_{1\mu}^{*'}$ and D_0 . The vector states in the multiplet satisfy the transversality conditions

$$v^\mu D_\mu^* = v^\mu D_{1\mu}^{*'} = 0.$$

For the octet of the pseudo Goldstone bosons, one uses the exponential form

$$\xi = \exp\left(\frac{iM}{f_\pi}\right) \tag{A14}$$

where

$$M = \begin{pmatrix} \sqrt{\frac{1}{2}}\pi^0 + \sqrt{\frac{1}{6}}\eta & \pi^+ & K^+ \\ \pi^- & -\sqrt{\frac{1}{2}}\pi^0 + \sqrt{\frac{1}{6}}\eta & K^0 \\ K^- & \overline{K^0} & -\sqrt{\frac{2}{3}}\eta \end{pmatrix} \tag{A15}$$

and $f_\pi = 132$ MeV.

The Lagrangian describing the fields H, S and ξ and their interactions, under the hypothesis of chiral and spin-flavor symmetry and at the lowest order in light mesons derivatives, is [14]

$$\begin{aligned}\mathcal{L} = {} & \frac{f_\pi^2}{8} \mathrm{Tr}[\partial^\mu \Sigma \partial_\mu \Sigma^\dagger + i H_b v^\mu D_{\mu ba} \overline{H}_a] \\ & + \mathrm{Tr}[S_b (i v^\mu D_{\mu ba} - \delta_{ba} \Delta) \overline{S}_a] + ig\, \mathrm{Tr}[H_b \gamma_\mu \gamma_5 \mathcal{A}_{ba}^\mu \overline{H}_a] \\ & + ig' \mathrm{Tr}[S_b \gamma_\mu \gamma_5 \mathcal{A}_{ba}^\mu \overline{S}_a] + ih \mathrm{Tr}[S_b \gamma_\mu \gamma_5 \mathcal{A}_{ba}^\mu \overline{H}_a] + \mathrm{H.c.}\end{aligned} \tag{A16}$$

where "Tr" means the trace, and

$$D_{\mu ba} = \delta_{ba} \partial_\mu + \mathcal{V}_{\mu ba} = \delta_{ba} \partial_\mu + \frac{1}{2} (\xi^\dagger \partial_\mu \xi + \xi \partial_\mu \xi^\dagger)_{ba} \tag{A17}$$

$$\mathcal{A}_{\mu ba} = \frac{1}{2} (\xi^\dagger \partial_\mu \xi - \xi \partial_\mu \xi^\dagger)_{ba} \tag{A18}$$

$\Sigma = \xi^2$ and Δ is the mass splitting of the S_a states from the ground state H_a .

The currents involving the heavy b and c quarks,

$$J_V^\mu = \langle D^*(\epsilon_1, p_1) | \overline{c} \gamma^\mu (1 - \gamma_5) b | B(p) \rangle, \tag{A19}$$

can be expressed in general in terms of form factors [28]

$$J_V^\mu = \frac{-2iV(q^2)}{m+m_1}\varepsilon^{\mu\nu\alpha\beta}\epsilon^*_{1\nu}p_\alpha p_{1\beta} - (m+m_1)A_1(q^2)\epsilon_1^{*\mu} + \frac{A_2(q^2)}{m+m_1}\epsilon_1^*\cdot q(p+p_1)^\mu + 2m_1A_3(q^2)\frac{\epsilon_1^*\cdot q}{q^2}q^\mu - 2m_1A_0(q^2)\frac{\epsilon_1^*\cdot q}{q^2}q^\mu \tag{A20}$$

with

$$A_3(q^2) = \frac{m+m_1}{2m_1}A_1(q^2) - \frac{m-m_1}{2m_1}A_2(q^2)$$

$$A_3(0) = A_0(0) \tag{A21}$$

where $q=p-p_1$ is the momentum transfer and m and m_1 are the masses of B and D^*. In the heavy quark limit the various form factors are related to a universal Isgur-Wise function $\xi(v\cdot v_1)$ where v and v_1 are the four-velocities of the B and D^* mesons. One can write

$$J_V^\mu = \sqrt{m}\sqrt{m_1}\xi(v\cdot v_1)[-i\varepsilon^{\mu\nu\alpha\beta}\epsilon^*_{1\nu}v_\alpha v_{1\beta} + v_1^\mu\epsilon_1^*\cdot v - \epsilon_1^{*\mu}(v\cdot v_1+1)]. \tag{A22}$$

The weak current $L_a^\mu = \overline{q}^a\gamma^\mu(1-\gamma_5)Q$ can be written in the effective theory as

$$L_a^\mu = \frac{if_H\sqrt{m_H}}{2}\mathrm{Tr}[\gamma^\mu(1-\gamma_5)H_b\xi^+_{ba}] \tag{A23}$$

where f_Q is the heavy meson decay constant. One can therefore write

$$\langle\overline{D}^*(\epsilon_2,v_2)\overline{K}^0|\overline{s}\gamma_\mu(1-\gamma_5)c|0\rangle = i\frac{m_2f_{D^*}\epsilon^*_{2\mu}}{f_K}. \tag{A24}$$

APPENDIX B

The total amplitude for $\overline{B}^0(v,m)\to D^{*+}(\epsilon_1,v_+,m_1)D^{*-}(\epsilon_2,v_-,m_1)K_s(p_k)$ can be written as

$$\overline{a} = \overline{a}_{non\text{-}res}[1-P_1] \tag{B1}$$

and the total amplitude for $B^0(v,m)\to D^{*+}(\epsilon_1,v_+,m_1)D^{*-}(\epsilon_2,v_-,m_1)K_s(p_k)$ can be written as

$$a = a_{non\text{-}res}[1-P_2] \tag{B2}$$

with

$$P_1 = \frac{f_{D_{s1}^{*\prime}}}{f_{D^*}}\sqrt{\frac{m^{*\prime}}{m_1}}\frac{hp_k\cdot v_-}{\left(p_k\cdot v_- + m_1 - m^{*\prime} + \frac{i\Gamma_{D_{s1}^{*\prime}}}{2}\right)} \tag{B3}$$

$$P_2 = \frac{f_{D_{s1}^{*\prime}}}{f_{D^*}}\sqrt{\frac{m^{*\prime}}{m_1}}\frac{hp_k\cdot v_+}{\left(p_k\cdot v_+ + m_1 - m^{*\prime} + \frac{i\Gamma_{D_{s1}^{*\prime}}}{2}\right)}. \tag{B4}$$

In the boosted frame we can write

$$p_k\cdot v_- = \frac{E_k'E_-' + p_k'p_-'y}{m_1} \tag{B5}$$

$$p_k\cdot v_+ = \frac{E_k'E_+' - p_k'p_+'y}{m_1} \tag{B6}$$

where E_k' and p_k' are the energy and the magnitude of the momentum of the kaon in the boosted frame, $E_\pm'$ and $p_\pm'$ are the energies and the magnitude of the momenta of the $D^{*\pm}$ in the boosted frame and m_1 is the D^* mass. In the boosted frame we have the following relations:

$$E_k' = \gamma(E_k - \vec{\beta}\cdot\vec{p}_k) \tag{B7}$$

$$= \frac{1}{\sqrt{1-\dfrac{E_k^2-m_k^2}{m^2\left(1-\dfrac{E_k}{m}\right)^2}}}\left[E_k + \frac{E_k^2-m_k^2}{m\left(1-\dfrac{E_k}{m}\right)}\right] \tag{B8}$$

$$p_k' = p_B' = \sqrt{E_k'^2 - m_k^2} \tag{B9}$$

$$p_+' = p_-' = \sqrt{E_+'^2 - m_1^2} \tag{B10}$$

$$E_+' = E_-' = \frac{E_B' - E_k'}{2} \tag{B11}$$

where E_k and p_k are the energy and magnitude of the momentum of the K_s in the B rest frame, E_B' and p_B' are the energy and magnitude of the momentum of the B in the boosted frame and m, m_1 and m_k are the B, D^* and K_s masses.

Note from the above relations that P_1 and P_2 can be expressed in terms of E_k and y and $P_1(y,E_k) = P_2(-y,E_k)$.

Squaring the amplitudes and summing over polarizations one can write

$$|\overline{a}|^2 = |\overline{a}_{non\text{-}res}|^2|1-P_1|^2 \tag{B12}$$

$$|a|^2 = |a_{non\text{-}res}|^2|1-P_2|^2 \tag{B13}$$

$$a^*\overline{a} = a^*_{non\text{-}res}\overline{a}_{non\text{-}res}(1-P_2)^*(1-P_1) \tag{B14}$$

where

$$|\overline{a}_{non\text{-}res}|^2 = \kappa^2[-x^2 + 2(2x_1x_2+x_2)x + 2x_1^2 - x_2^2 + 4x_1 + 2] \tag{B15}$$

$$|a_{non\text{-}res}|^2=\kappa^2[-x^2+2(2x_1x_2+x_1)x+2x_2^2-x_1^2+4x_2+2] \tag{B16}$$

$$a^*{}_{non\text{-}res}\overline{a}_{non\text{-}res}=\kappa^2[x^2+(x_1+x_2-2)x+2x_1+5x_1x_2+2x_2+2+O(p_k^2/m^2)] \tag{B17}$$

where

$$\kappa=\frac{G_F}{\sqrt{2}}V_c\left(\frac{c_1}{N_c}+c_2\right)\sqrt{m}\sqrt{m_1}m_1$$

$$x_1=v\cdot v_+=\frac{E'_BE'_+-p'_Bp'_+y}{mm_1}$$

$$x_2=v\cdot v_-=\frac{E'_BE'_-+p'_Bp'_-y}{mm_1}$$

$$x=v_+\cdot v_-=\frac{E'_+E'_-+p'_+p'_-}{m_1^2}.$$

The double differential decay distribution for the time independent process

$$\overline{B}^0(v,m)\rightarrow D^{*+}(\epsilon_1,v_+,m_1)D^{*-}(\epsilon_2,v_-,m_1)K_s(p_k)$$

can be written as

$$\frac{1}{\Gamma}\frac{d\Gamma}{dydE_k}=\frac{f(y,E_k)}{\int f(y,E_k)\frac{p'_kp'_+}{m}dydE_k} \tag{B18}$$

$$f(y,E_k)=[-x^2+2(2x_1x_2+x_2)x+2x_1^2-x_2^2+4x_1+2]|1-P_1|^2 \tag{B19}$$

where p'_k and p'_+ are the magnitudes of the three-momentum of the K_s and D^{*+} in the boosted frame.

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