Measuring the finite width and unitarity corrections to the $\phi\omega$ mixing amplitude

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It is shown that the phase of $\phi\omega$ interference in the reaction $e^+e^-\to\pi^+\pi^-\pi^0$ at energies close to the $\phi(1020)$ peak can be calculated in a way that is practically independent of the model of $\phi\omega$ mixing. The magnitude of the presently measured interference phase, still of poor accuracy, is in agreement with the predictions based on extending the $\omega(782)$ resonance tail from the peak position to the ϕ mass upon assuming the $\omega\to\rho\pi\to3\pi$ model. The calculated ω width at the ϕ mass is about 200 MeV.

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I. INTRODUCTION

Recent measurements of the $e^+e^-\to\pi^+\pi^-\pi^0$ reaction cross section at energies in the vicinity of the $\phi(1020)$ resonance reached by the CMD-2 team in Novosibirsk have revealed the $\phi\omega$ interference phase $\chi_{\phi\omega}=162°\pm17°$ [1], provided the phases of the complex propagators of ϕ and ω mesons are properly included:

$$\sigma_{3\pi}\propto\left|\frac{1}{m_\omega^2-s-i\sqrt{s}\Gamma_\omega(s)}+\frac{A\exp(i\chi_{\phi\omega})}{m_\phi^2-s-i\sqrt{s}\Gamma_\phi(s)}+A_{\rm bg}\right|^2, \tag{1.1}$$

A being a real positive number, and $A_{\rm bg}$ denoting the contribution of the nonresonant background. Hereafter s is the total center-of-mass energy squared. The accuracy of the measurements is expected to be drastically improved by the Novosibirsk SND and CMD-2 teams at the VEPP-2M facility, not to mention the DAΦNE machine, with its huge number of expected ϕ mesons. The measured phase is still consistent (within 1σ) with the canonical value of 180° predicted in approaches based on the flavor SU(3) and the simplest quark model with real coupling constants [2]. The canonical phase explains correctly the location of the $\phi\omega$ interference minimum in the energy behavior of the $e^+e^-\to\pi^+\pi^-\pi^0$ reaction cross section above the ϕ mass, as observed in experiment [1,3]. However, the deviation of the central value of the measured $\chi_{\phi\omega}$ from 180° points, possibly, to some dynamical source. The aim of the present work is to reveal the latter. To this end we will demonstrate that $\chi_{\phi\omega}$ can be calculated in a way that is practically independent of the specific model of $\phi\omega$ mixing. As will become clear, this is due to the compensation between the $\rho\pi$ state contribution to the $\phi\omega$ mixing amplitude and the direct transition. The deviation of $\chi_{\phi\omega}$ from 180° will be shown to be explained mainly by the finite width effects. The precise measurement of this phase could offer the firm ground for the extension of the ω excitation curve to the energies up to the ϕ mass.

Below, in Sec. II, the basic models of the decay $\phi\to\rho\pi$ are outlined. Section III is devoted to the discussion of the unitarity corrections to the coupling constants and the $\phi\omega$ mixing amplitude. The $\phi\omega$ interference phase $\chi_{\phi\omega}$ is calculated in Sec. IV. Section V contains conclusions drawn from the work.

II. BASIC SOURCES OF THE $\phi\to\rho\pi$ DECAY

All the necessary theoretical background for analyzing the $\phi\omega$ interference pattern in the cross section of the reaction $e^+e^-\to\pi^+\pi^-\pi^0$ was developed earlier [4–6], so one may find the details in these papers. The problem of to what extent the $\omega(782)$ and $\phi(1020)$ mesons are ideally mixed states,

$$\omega^{(0)}=(u\bar{u}+d\bar{d})/\sqrt{2},$$

$$\phi^{(0)}=s\bar{s}, \tag{2.1}$$

is as old as these mesons themselves [7]. The fact is that the decay $\phi\to\rho\pi\to\pi^+\pi^-\pi^0$ which violates the Okubo-Zweig-Iizuka (OZI) rule [7–9] is usually considered as evidence in favor of an admixture of the nonstrange quarks in the wave function of ϕ meson:

$$\phi(1020)=s\bar{s}+\varepsilon_{\phi\omega}(s)(u\bar{u}+d\bar{d})/\sqrt{2}, \tag{2.2}$$

where the $\phi\omega$ mixing amplitude is described by the complex mixing parameter $\varepsilon_{\phi\omega}(s)$ dependent on energy, $|\varepsilon_{\phi\omega}(s)|\ll1$. It can be expressed through the nondiagonal polarization operator $\Pi_{\phi\omega}$ according to the relation

$$\varepsilon_{\phi\omega}(s)=-\frac{{\rm Re}\Pi_{\phi\omega}(s)+i{\rm Im}\Pi_{\phi\omega}(s)}{\Delta M^2_{\phi\omega}(s)}, \tag{2.3}$$

where

$$\Delta M^2_{\phi\omega}(s)=\Delta m^{(0)2}_{\phi\omega}-i\sqrt{s}[\Gamma^{(0)}_\phi(s)-\Gamma^{(0)}_\omega(s)], \tag{2.4}$$

and $\Delta m^{(0)2}_{\phi\omega}=m^{(0)2}_\phi-m^{(0)2}_\omega$. Hereafter $m^{(0)}_V$, $\Gamma^{(0)}_V(s)$ are, respectively, the mass and width of the ideally mixed states in Eq. (2.1), and all quantities with the superscript (0) refer to these states. Below we will call this mechanism the model of strong $\phi\omega$ mixing. In QCD, the real part of the mixing operator ${\rm Re}\,\Pi_{\phi\omega}(s)$ arises qualitatively either via the perturbative three-gluon intermediate state shown in Fig. 1(a) [10,11] or the nonperturbative effects [12] diagrammatically shown in Fig. 1(b). Quantitatively, the contribution of Fig. 1(a) is

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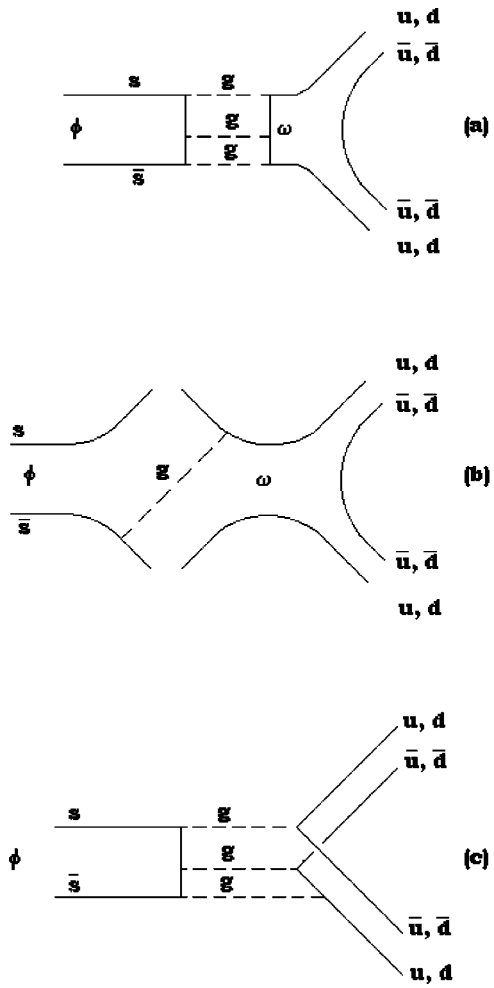

FIG. 1. Models of the decay $\phi(1020)\to\rho(770)\pi$. (a) The $\phi\omega$ mixing caused by the three-gluon mechanism. (b) The $\phi\omega$ mixing due to the nonperturbative QCD effects. (c) The three-gluon mechanism of the direct transition $\phi\to\rho\pi$. Gluon is denoted by g.

small and of the wrong sign [10,11] while the calculations of $\varepsilon_{\phi\omega}(m_\phi^2)$ according to Fig. 1(b) [12] can be considered as order-by-magnitude estimates at best. The one photon contribution to $\mathrm{Re}\,\Pi_{\phi\omega}(s)$ is by two orders of magnitude smaller than the value necessary to explain the 3π branching ratio of the ϕ. The non-one-photon contribution to $\mathrm{Re}\,\Pi_{\phi\omega}(s)$ is assumed to be independent on energy. As it was pointed out in Ref. [4], this assumption does not contradict the data.

An alternative to the conventional $\phi\omega$ mixing is the direct decay, $\mathrm{Re}\,g^{(0)}_{\phi\rho\pi}\neq 0$, $\mathrm{Re}\,\Pi_{\phi\omega}(s)\equiv 0$ diagrammatically shown in Fig. 1(c). It is essentially the famous Appelquist-Politzer mechanism [13] of the OZI rule violation in the decays of heavy quarkonia into the light hadrons, extrapolated to the ϕ mass region. As is shown in [6], the direct decay can be considered as a viable contribution to the $\phi\to\rho\pi$ amplitude [14]. An order-of-magnitude estimate of $\mathrm{Re}\,g^{(0)}_{\phi\rho\pi}$ [6] is in agreement with the value extracted from the $\phi\to 3\pi$ branching ratio. This model will be called the model of weak $\phi\omega$ mixing. Intermediate variants are possible, of course.

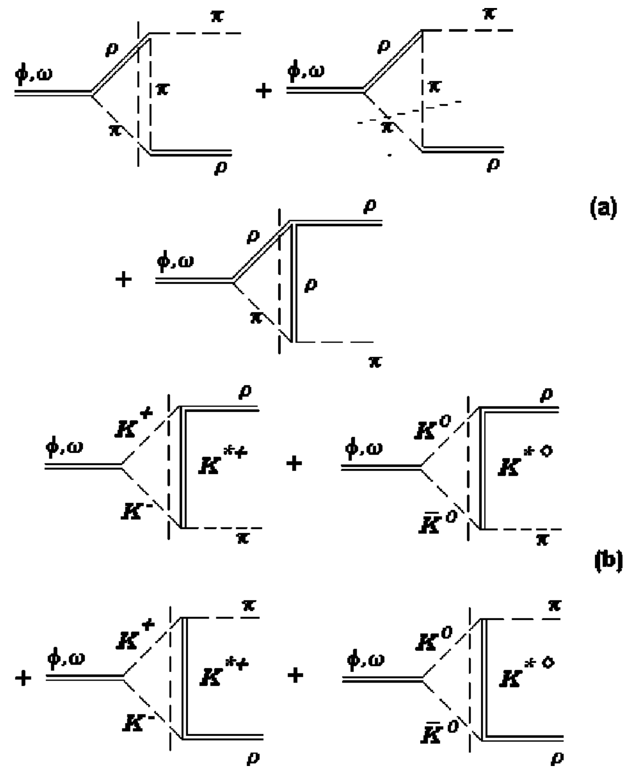

FIG. 2. The contributions to $2\,\mathrm{Im}\,g^{(0)}_{\phi(\omega)\rho\pi}$ from the $\rho\pi$ intermediate state (a) and the $K\bar{K}$ intermediate state (b).

III. UNITARITY CORRECTIONS TO COUPLINGS AND $\phi\omega$ MIXING AMPLITUDE

Contrary to $\mathrm{Re}\,g^{(0)}_{\phi\rho\pi}$ and $\mathrm{Re}\,\Pi_{\phi\omega}(s)$, which are in fact unknown, their imaginary counterparts can be evaluated reliably via the unitarity relation. The dominant contributions to $2\,\mathrm{Im}\,g^{(0)}_{\phi(\omega)\rho\pi}$ come from the diagrams shown in Fig. 2. The sum of the first two diagrams, upon extending the results of works [15,16] to include the form factor of the π exchange, $\exp(-\lambda_\pi|t-m_\pi^2|)$, is

$$\Phi_{\rho\pi}(s,m^2)=-\frac{g^2_{\rho\pi\pi}}{8\pi\sqrt{s}q_f^3}\int_{2m_\pi}^{\sqrt{s}-m_\pi}d\mu\,\frac{2\mu^2\Gamma(\rho\to\pi\pi,\mu)}{\pi|D_\rho(\mu^2)|^2}\times\Bigg\{(q_iq_f)^2\mathrm{vp}\int_{-1}^{+1}dx\,\frac{1-x^2}{a+x}\times[\exp2(-\lambda_\pi)q_iq_f|a+x|-1]+\Phi_0(s,m^2,\mu^2)\Bigg\},\quad(3.1)$$

where vp means the principal value and m and μ are, respectively, the invariant masses of the final and intermediate ρ meson whose propagator is $D_\rho(\mu^2)=m_\rho^2-\mu^2-i\mu\Gamma(\rho\to\pi\pi,\mu)$, and

$$\Phi_0(s,m^2,\mu^2)=(q_iq_f)^2\left(2a+\ln\left|\frac{a+1}{a-1}\right|\right)+(q_{\pi\pi}q_f)^2\left(2b+\ln\left|\frac{b+1}{b-1}\right|\right).$$

The notations in the above expressions are

$$a=(\mu^2/2-E_iE_f)/q_iq_f,$$
$$b=m(E_i+E_f-E_\rho)/2q_{\pi\pi}q_f, \tag{3.2}$$

where

$$q_i=q(\sqrt{s},m_\pi,\mu),\ E_i=E(\sqrt{s},m_\pi,\mu),$$
$$q_f=q(\sqrt{s},m_\pi,m),\ E_f=E(\sqrt{s},m_\pi,m),$$
$$q_{\pi\pi}=q(m,m_\pi,m_\pi),\ E_\rho=E(\sqrt{s},\mu,m_\pi), \tag{3.3}$$

and

$$E(M,m_1,m_2)=(M^2+m_1^2-m_2^2)/2M,$$
$$q(M,m_1,m_2)=\{[M^2-(m_1+m_2)^2]\times[M^2-(m_1-m_2)^2]\}^{1/2}/2M \tag{3.4}$$

are the expressions for energy and momentum, respectively. The decay kinematics of the first two diagrams in Fig. 2(a) result in a very slow variation of their contribution with the change of λ_π. This is because the $\pi\pi$ cutting contributes considerably and it does not depend on λ_π (see the details in [15,16]). Numerically, one obtains $\Phi_{\rho\pi}(m_\phi^2,m_\rho^2)=0.44$, 0.45, 0.47, 0.49 at $\lambda_\pi=0$, 1, 2, 4 GeV^{-2}, respectively. The slight increase with λ_π is due to the fact that the first two diagrams in Fig. 2(a) are opposite in sign at $\sqrt{s}<1.1$ GeV. The third diagram in Fig. 2(a), at $\sqrt{s}=m_\phi$, amounts to -3.4×10^{-2}, provided the slope of the ρ exchange is $\lambda_\rho=2\ \mathrm{GeV}^{-2}$. The latter value is chosen from the demand that the phase of the $\pi\pi$ scattering at this energy range is given by the phase of the ρ propagator with an accuracy of about 10%. Hence, its contribution can be neglected in comparison with $\Phi_{\rho\pi}$. The contribution of the diagrams in Fig. 2(b) come from the $K\bar{K}$ intermediate states with the K^* exchange. In the case of ϕ meson it can be written as

$$g_{\phi\rho\pi}^{(K\bar{K})}(s,m^2)=g_{\phi K\bar{K}}^{(0)}\Phi_{K\bar{K}}(s,m^2), \tag{3.5}$$

where

$$\Phi_{K\bar{K}}(s,m^2)=g_{K^{*+}K^+\pi^0}g_{K^{*+}K^+\rho^0}\times\frac{q_{K\bar{K}}^2}{8\pi\sqrt{s}q_{\rho\pi}}\int_{-1}^{+1}dx\,\frac{1-x^2}{a_{K\bar{K}}+x}\times\exp[2\lambda_{K^*}q_{K\bar{K}}q_{\rho\pi}(a_{K\bar{K}}+x)]. \tag{3.6}$$

Here $a_{K\bar{K}}=(m_K^2-m_{K^*}^2+m^2)/2q_{K\bar{K}}q_{\rho\pi}$, $q_{K\bar{K}}=q(\sqrt{s},m_K,m_K)$, and $q_{\rho\pi}=q(\sqrt{s},m,m_\pi)$. The $K\bar{K}$ intermediate state contribution to $g_{\omega\rho\pi}$ is written in a similar way, with the SU(3) relation

$$g_{\omega K\bar{K}}^{(0)}=-g_{\phi K\bar{K}}^{(0)}/\sqrt{2} \tag{3.7}$$

being taken into account. Note also that SU(3) predicts $g_{K^{*+}K^+\rho^0}=g_{\omega\rho\pi}^{(0)}/2$ and fixes the relative signs of bare coupling constants in the VPP and VVP vertices. Numerically, the effect of $\Phi_{K\bar{K}}\neq0$ is negligible for ω meson because $|g_{\omega\rho\pi}^{(K\bar{K})}(m_\phi^2,m_\rho^2)/g_{\omega\rho\pi}|\simeq3\times10^{-3}$. In the case of ϕ meson, at first sight this effect being expressed as the phase of the coupling constant $g_{\phi\rho\pi}$ is proportional to $g_{\phi\rho\pi}^{(K\bar{K})}(m_\phi^2,m_\rho^2)/g_{\phi\rho\pi}$ and seems to be enhanced by the factor of $g_{\omega\rho\pi}/g_{\phi\rho\pi}\simeq17$. Yet even in this case the contribution of the $K\bar{K}$ intermediate state is smaller, at $\sqrt{s}=1020$ (1050) MeV, than 6% (18%) of the magnitude of the $\phi\rho\pi$ effective coupling constant. These estimates are obtained at $\lambda_{K^*}=0\ \mathrm{GeV}^{-2}$ and $m=m_\rho$. A more realistic $\lambda_{K^*}=1\ \mathrm{GeV}^{-2}$, together with the fact that it is the averaging of $\Phi_{K\bar{K}}(s,m^2)$ over $\pi\pi$ mass spectrum that enters into the expression for the $\phi\omega$ interference phase [see Eq. (4.3) below], both result in dividing the above estimates by the factor of two. In the meantime, the dominant effect of $\Phi_{\rho\pi}\neq0$ is relatively large; one should take into account the entire chain of rescatterings in the diagrams of Fig. 2(a). This can be made in a manner resembling the solution of the Dyson-like equation for the vertex function. Taking the above remarks into account, the coupling constants of ϕ and ω with $\rho\pi$ can be written as

$$g_{\omega\rho\pi}^{(0)}(s,m^2)\simeq\mathrm{Re}\,g_{\omega\rho\pi}^{(0)}/[1-i\Phi_{\rho\pi}(s,m^2)],$$
$$g_{\phi\rho\pi}^{(0)}(s,m^2)\simeq[\mathrm{Re}\,g_{\phi\rho\pi}^{(0)}+ig_{\phi\rho\pi}^{(K\bar{K})}(s,m^2)]/[1-i\Phi_{\rho\pi}(s,m^2)]. \tag{3.8}$$

Of course, $\mathrm{Re}\,g_{\phi(\omega)\rho\pi}^{(0)}$ should be determined from the partial width of the decay $\phi(\omega)\to\pi^+\pi^-\pi^0$ on the $\phi(\omega)$ mass shell. As is evident from Eq. (3.8), the most essential contribution to the imaginary parts of coupling constants coming from the $\rho\pi$ intermediate state cancels from their ratio. However, a nonzero $\Phi_{\rho\pi}$ enters the expression for the 3π decay width of ω and ϕ mesons [15,16],

$$\Gamma_{\omega(\phi)3\pi}^{(0)}(s)=[\mathrm{Re}\,g_{\omega(\phi)\rho\pi}^{(0)}]^2W(s)/4\pi, \tag{3.9}$$

where the dynamical phase space factor for the decay $\omega\to\rho^0\pi^0+\rho^+\pi^-+\rho^-\pi^+\to\pi^+\pi^-\pi^0$ is

$$W(s)=\frac{1}{2\pi}\int_{2m_\pi}^{\sqrt{s}-m_\pi}dm\,m^2\Gamma_{\rho\pi\pi}(m^2)q_{\rho\pi}^3(m)\times\int_{-1}^{1}dx(1-x^2)\left|\frac{1}{D_\rho(m^2)Z(m^2)}+\frac{1}{D_\rho(m_+^2)Z(m_+^2)}+\frac{1}{D_\rho(m_-^2)Z(m_-^2)}\right|^2. \tag{3.10}$$

In the above equation, the invariant squared masses of the charged ρ mesons are

$$m_{\pm}^2=(s+3m_\pi^2-m^2)/2\pm 2xq_{\rho\pi}q_{\pi\pi}\sqrt{s}/m, \quad (3.11)$$

with $q_{\rho\pi}=q(\sqrt{s},m,m_\pi)$, $q_{\pi\pi}=q(m,m_\pi,m_\pi)$ evaluated via Eq. (3.4), and $Z(m^2)=1-i\Phi_{\rho\pi}(s,m^2)$. The effect of $\Phi_{K\bar{K}}\neq 0$ on the $\phi\to 3\pi$ partial width is negligible.

The dominant contributions to $\mathrm{Im}\,\Pi_{\phi\omega}(s)$ come from the real $K\bar{K}$ and $\rho\pi$ intermediate states,

$$\mathrm{Im}\,\Pi_{\phi\omega}(s)=\sqrt{s}\left[\frac{\mathrm{Re}\,g^{(0)}_{\phi\rho\pi}}{\mathrm{Re}\,g^{(0)}_{\omega\rho\pi}}\Gamma^{(0)}_{\omega 3\pi}(s)-\frac{\Gamma^{(0)}_{\phi K\bar{K}}(s)}{\sqrt{2}}\right], \quad (3.12)$$

where

$$\Gamma^{(0)}_{\phi K\bar{K}}(s)=g^{(0)2}_{\phi K\bar{K}}\{[q(\sqrt{s},m_{K^+},m_{K^-})]^3+[q(\sqrt{s},m_{K_L},m_{K_S})]^3\}/6\pi s \quad (3.13)$$

is the $K\bar{K}$ partial width of the ϕ that includes different thresholds for the charged and neutral kaons. To gain an impression of the role of these contributions to $\mathrm{Im}\,\Pi_{\phi\omega}(s)$, we evaluate them at $\sqrt{s}=m_\phi$. The $\pi^+\pi^-\pi^0$ intermediate state contribution is, at most, $\simeq 0.015$ GeV^2 in the model of weak $\phi\omega$ mixing and vanishes in the model of strong $\phi\omega$ mixing. The contribution of the $K\bar{K}$ intermediate state amounts to $\simeq 3\times 10^{-3}$ GeV^2. Note that the difference between the considered models of the mixing in their predictions for this intermediate state is far below the accuracy (see below) of the SU(3) relation (3.7) necessary to obtain the numbers given above. Here we set this accuracy to be, conservatively, 20%. The radiative $\pi^0\gamma$ and $\eta\gamma$ intermediate states do not exceed, respectively, 4% and 2% of the $K\bar{K}$ intermediate state. These figures are far below the accuracy of SU(3) symmetry necessary to relate the couplings of the ϕ and ω to $K\bar{K}$. Hence, the radiative intermediate states can be neglected [17].

Note, for the sake of completeness, that although the effects of $\Phi_{\rho\pi}\neq 0$ are important for the $\omega\rho$ interference pattern in the $\pi^+\pi^-$ mass spectrum [15,16], in the case of the calculation of the branching ratio of the decay to 3π they can be modeled, at given s, by inclusion of the form factor of the type

$$c_{\rho\pi}(s)=[1+(R_{\rho\pi}m_\omega)^2]/(1+R^2_{\rho\pi}s), \quad (3.14)$$

so that the $\omega\to\rho\pi$ vertex should now include the substitution

$$\mathrm{Re}\,g^{(0)}_{\omega\rho\pi}\to\mathrm{Re}\,\tilde{g}^{(0)}_{\omega\rho\pi}(s)=c_{\rho\pi}(s)\mathrm{Re}\,g^{(0)}_{\omega\rho\pi}. \quad (3.15)$$

The effect of this substitution on the $e^+e^-\to 3\pi$ cross section behavior was discussed in Ref. [15].

IV. EVALUATING THE $\phi\omega$ INTERFERENCE PHASE

The expression for the cross section of the reaction $e^+e^-\to\pi^+\pi^-\pi^0$ that incorporates the above features of the decay $\phi\to\pi^+\pi^-\pi^0$ can be written, near $\sqrt{s}=m_\phi$, as [4,5]

$$\sigma_{3\pi}(s)=\frac{4\pi\alpha^2W(s)}{s^{3/2}}\left|\frac{g_{\gamma\omega}(s)g_{\omega\rho\pi}(s)}{m_\omega^2-s-i\sqrt{s}\Gamma_\omega(s)}+\frac{g_{\gamma\phi}(s)g_{\phi\rho\pi}(s)}{m_\phi^2-s-i\sqrt{s}\Gamma_\phi(s)}\right|^2, \quad (4.1)$$

where the equations

$$\begin{aligned}
g_{\gamma\omega}(s)&=g^{(0)}_{\gamma\omega}-\varepsilon_{\phi\omega}(s)g^{(0)}_{\gamma\phi},\\
g_{\gamma\phi}(s)&=g^{(0)}_{\gamma\phi}+\varepsilon_{\phi\omega}(s)g^{(0)}_{\gamma\omega},\\
g_{\omega\rho\pi}(s)&=\mathrm{Re}\,\tilde{g}^{(0)}_{\omega\rho\pi}(s)-\varepsilon_{\phi\omega}(s)\mathrm{Re}\,g^{(0)}_{\phi\rho\pi}\\
&\simeq\mathrm{Re}\,\tilde{g}^{(0)}_{\omega\rho\pi}(s),\\
g_{\phi\rho\pi}(s)&\simeq\mathrm{Re}\,g^{(0)}_{\phi\rho\pi}+\varepsilon_{\phi\omega}(s)\mathrm{Re}\,\tilde{g}^{(0)}_{\omega\rho\pi}(s)\\
&\quad+i\langle g^{(K\bar{K})}_{\phi\rho\pi}(s)\rangle,
\end{aligned} \quad (4.2)$$

relate the coupling constants of physical states whose total widths are $\Gamma_{\phi,\omega}(s)$, with those ideally mixed. We omit here the contribution of heavier ω', ω'' resonances for the reason explained in the end of the section. In principle, they can be incorporated in a way presented in Ref. [19]. In the above formula, $\langle g^{(K\bar{K})}_{\phi\rho\pi}(s)\rangle=g_{\phi K\bar{K}}\langle\Phi_{K\bar{K}}(s)\rangle$, and

$$\langle\Phi_{K\bar{K}}(s)\rangle=\int_{2m_\pi}^{\sqrt{s}-m_\pi}dm\frac{2m^2\Gamma_\rho(m)}{\pi|D_\rho(m^2)|^2}\Phi_{K\bar{K}}(s,m^2) \quad (4.3)$$

is the averaging over the $\pi\pi$ mass spectrum, which corresponds to some approximate way of taking into account the dependence of $\Phi_{K\bar{K}}$ on the invariant mass. Numerically, it reduces, at $\sqrt{s}\simeq m_\phi$, to the diminishing of $\Phi_{K\bar{K}}$ by 33% from its value at the ρ mass. Note that $g^{(0)}_{\gamma V}=m_V^{(0)2}/f_V^{(0)}$ ($V=\omega,\phi$) is the $\gamma\to V$ transition amplitude, and $f_V^{(0)}$ enters the leptonic width of an unmixed state $V^{(0)}$ as

$$\Gamma(V^{(0)}\to e^+e^-,m_V^{(0)2})=\frac{4\pi\alpha^2m_V^{(0)}}{3f_V^{(0)2}}, \quad (4.4)$$

with $\alpha=1/137$ being the fine structure constant. If all coupling constants and the $\phi\omega$ mixing parameter in Eq. (4.1) were real, the phase of the $\phi\omega$ interference would be given by the sign of the ratio

$$R_0(s)=\frac{g_{\gamma\phi}(s)\mathrm{Re}\,g_{\phi\rho\pi}}{g_{\gamma\omega}(s)\mathrm{Re}\,\tilde{g}_{\omega\rho\pi}(s)}. \quad (4.5)$$

In the meantime, the location of the $\phi\omega$ interference minimum in the energy behavior of the $e^+e^-\to\pi^+\pi^-\pi^0$ reaction cross section,

$$s_{\rm min}^{1/2}=\left[\frac{m_\phi^2+R_0(m_\phi^2)m_\omega^2}{1+R_0(m_\phi^2)}\right]^{1/2}, \quad (4.6)$$

is experimentally determined to be at $s_{\rm min}^{1/2}=1.05$ GeV [1,3]. This corresponds to $R_0=-0.13$, hence the canonical phase 180°. However, the above discussion shows that considerable imaginary parts to both the coupling constants and mixing parameter arise via unitarity, due to the real intermediate states. As can be observed by comparing Eqs. (1.1) and (4.1) [see also Eq. (4.2)], a sizable additional phase $\Delta\chi_{\phi\omega}$ comes from the phase of the combination of the coupling constants from Eq. (4.2),

$$r(s)=\frac{{\rm Re}\,g^{(0)}_{\phi\rho\pi}}{{\rm Re}\,\tilde g^{(0)}_{\omega\rho\pi}(s)}+\varepsilon_{\phi\omega}(s)+i\frac{\langle g^{K\bar K}_{\phi\rho\pi}(s)\rangle}{{\rm Re}\,\tilde g^{(0)}_{\omega\rho\pi}(s)}. \quad (4.7)$$

The first two terms in the above equation, taken separately, are drastically different in magnitude in the models of strong and weak $\phi\omega$ mixing. This is because ${\rm Re}\,g^{(0)}_{\phi\rho\pi}$ [${\rm Re}\,\Pi_{\phi\omega}(s)$] vanishes in the former [latter] model. However, this dramatic difference cancels almost completely from the sum in Eq. (4.7) that determines the measured quantity. Indeed, one obtains, upon using Eqs. (2.3) and (4.2), that

$$\begin{aligned}r(s)=&\frac{{\rm Re}\,g^{(0)}_{\phi\rho\pi}}{{\rm Re}\,\tilde g^{(0)}_{\omega\rho\pi}(s)}+i\frac{\langle g^{(K\bar K)}_{\phi\rho\pi}(s)\rangle}{{\rm Re}\,\tilde g^{(0)}_{\omega\rho\pi}(s)}-\frac{1}{\Delta m^{(0)2}_{\phi\omega}-i\sqrt{s}[\Gamma^{(0)}_\phi(s)-\Gamma^{(0)}_\omega(s)]}\\&\times\left\{{\rm Re}\,\Pi_{\phi\omega}(s)+i\sqrt{s}\left[\frac{{\rm Re}\,g^{(0)}_{\phi\rho\pi}}{{\rm Re}\,\tilde g^{(0)}_{\omega\rho\pi}(s)}\Gamma^{(0)}_{\omega3\pi}(s)-\frac{\Gamma^{(0)}_{\phi K\bar K}(s)}{\sqrt{2}}\right]\right\}\\=&\frac{\Delta m^{(0)2}_{\phi\omega}}{\Delta M^2_{\phi\omega}(s)}\left\{\frac{{\rm Re}\,g^{(0)}_{\phi\rho\pi}}{{\rm Re}\,\tilde g^{(0)}_{\omega\rho\pi}(s)}-\frac{{\rm Re}\,\Pi_{\phi\omega}(s)}{\Delta m^{(0)2}_{\phi\omega}}+i\frac{\sqrt{s}\Gamma^{(0)}_{\phi K\bar K}(s)}{\sqrt{2}\Delta m^{(0)2}_{\phi\omega}}-i\sqrt{\frac{s}{\Delta m^{(0)2}_{\phi\omega}}}\frac{{\rm Re}\,g^{(0)}_{\phi\rho\pi}}{{\rm Re}\,\tilde g^{(0)}_{\omega\rho\pi}(s)}[\Gamma^{(0)}_\phi(s)-\Gamma^{(0)}_\omega(s)+\Gamma^{(0)}_{\omega3\pi}(s)]\right\}\\&+i\frac{\langle g^{(K\bar K)}_{\phi\rho\pi}(s)\rangle}{{\rm Re}\,\tilde g^{(0)}_{\omega\rho\pi}(s)}, \quad (4.8)\end{aligned}$$

and $\Delta M^2_{\phi\omega}(s)$ is given by Eq. (2.4). Since the dominant 3π decay mode of the ω is cancelled from the expression in the square parentheses of the last line of the above equation, and the combination of remaining $K\bar K$ and radiative decay widths appear to be multiplied by the factor ${\rm Re}\,g^{(0)}_{\phi\rho\pi}/{\rm Re}\,\tilde g^{(0)}_{\omega\rho\pi}(s)$, which is either small, $\sim 1/17$, as it takes place in the model of weak $\phi\omega$ mixing, or even vanishing, as it does in the model of strong $\phi\omega$ mixing, the last term in curly brackets can be safely neglected. As a result, the following simplified expression for valid r with a good accuracy can be written as

$$r(s)\simeq\frac{\Delta m^2_{\phi\omega}}{\Delta M^2_{\phi\omega}(s)}\left[\frac{{\rm Re}\,g^{(0)}_{\phi\rho\pi}}{{\rm Re}\,\tilde g^{(0)}_{\omega\rho\pi}(s)}-\frac{{\rm Re}\,\Pi_{\phi\omega}(s)}{\Delta m^2_{\phi\omega}}+i\frac{\sqrt{s}\Gamma_{\phi KK}(s)}{\sqrt{2}\Delta m^2_{\phi\omega}}\right]+i\frac{\langle g^{(K\bar K)}_{\phi\rho\pi}(s)\rangle}{{\rm Re}\,\tilde g^{(0)}_{\omega\rho\pi}(s)}. \quad (4.9)$$

With the accuracy of about 5%, the masses and widths of ideally mixed states are replaced hereafter with those of the physical states. Note that the combination

$$g(s)={\rm Re}\,g^{(0)}_{\phi\rho\pi}/{\rm Re}\,\tilde g^{(0)}_{\omega\rho\pi}(s)-{\rm Re}\,\Pi_{\phi\omega}(s)/\Delta m^2_{\phi\omega} \quad (4.10)$$

standing in the right hand side of Eq. (4.9) determines the branching ratio of the ϕ decay into 3π. Hence, its magnitude coincides in both models of $\phi\omega$ mixing mentioned earlier. One can obtain from the 3π branching ratios of the ω and ϕ at their respective mass shells that

$$|g(m_\phi^2)|=c^{-1}_{\rho\pi}(m_\phi^2)\left[\frac{B_{\phi3\pi}\Gamma_\phi/W(m_\phi^2)}{B_{\omega3\pi}\Gamma_\omega/W(m_\omega^2)}\right]^{1/2}\simeq0.06. \quad (4.11)$$

When obtaining this number, the dynamical phase space factors $W(m_\omega^2)=4.5\times10^{-4}$ GeV^3 and $W(m_\phi^2)=1.3\times10^{-2}$ GeV^3, evaluated from Eq. (3.10) under the assumption of no rescattering correction [$Z(m^2)=1$, etc.], are used and we set $R_{\rho\pi}=0$ GeV^{-1} here. Keeping $\varepsilon_{\phi\omega}(s)\neq0$ in the transition amplitude $g_{\gamma\phi}(s)$ gives the phase shift $\delta\chi_{\phi\omega}=1.4°$, which is below the accuracy of calculation. Hence, the calculation of $\chi_{\phi\omega}$ is practically model independent.

First, let us give rough estimates of the phase deviation at the ϕ mass. They are obtained upon neglecting the unitarity corrections to the coupling constants of ω and ϕ mesons. Then one can obtain the above deviation as

$$\Delta\chi_{\phi\omega}\simeq\tan^{-1}\left[\frac{m_\phi\Gamma_{\phi KK}(m_\phi^2)}{\sqrt{2}g(m_\phi^2)\Delta m^2_{\phi\omega}}\right]-\tan^{-1}\frac{m_\phi[\Gamma_\omega(m_\phi^2)-\Gamma_\phi(m_\phi^2)]}{\Delta m^2_{\phi\omega}}. \quad (4.12)$$

The first term in Eq. (4.12) gives $6°\pm 1°$ to $\Delta\chi_{\phi\omega}$ and the uncertainty is solely due to the 20% uncertainty of the SU(3) predictions for the vector meson couplings to $K\bar{K}$. We obtain these values upon inserting the Particle Data Group entries [18] for masses, total widths, and branching ratios, together with the numerical value of the combination (4.11). The sign of the latter (positive) is fixed in accord with the position of the $\phi\omega$ interference minimum in the $e^+e^-\to\pi^+\pi^-\pi^0$ reaction cross section located on the right from the ϕ peak [3]. The contribution of the second term is opposite in sign to the first one and is strongly dependent on the ω width at the ϕ mass, $\Gamma_\omega(m_\phi)$. Varying $R_{\rho\pi}$ in Eq. (3.14) from 0 to 1 GeV^{-1}, which corresponds to the variation of the ω width from 200 to 120 MeV, gives the second contribution varying from $-26°$ to $-13°$. Larger values of $R_{\rho\pi}$ would destroy the description of the data on the cross section of the reaction $e^+e^-\to\pi^+\pi^-\pi^0$ at the energies above the $\phi(1020)$ mass. In fact, our previous fits [19] gave $R_{\rho\pi}=0.8^{+0.6}_{-0.3}\ \mathrm{GeV}^{-1}$.

The results of more accurate numerical evaluations are as follows. The uncertainties of the calculations come from the poor knowledge of the slopes of the form factors that enter the unitarity relations. If one includes the $\rho\pi$ rescattering effects, $\Phi_{\rho\pi}\neq 0$, in the consideration, the variation of λ_π in the range from 0 to 4 GeV^{-2} results in a small, 0.5° variation of the phase $\chi_{\phi\omega}$. The variation of λ_{K*} in the same range results in the phase variation at about $2°$. If one includes the 20% uncertainty of the flavor SU(3) predictions in $\mathrm{Im}\,\Pi_{\phi\omega}(s)$, the total uncertainty amounts to $\pm 3°$. This figure is far below the current accuracy of the data, $\Delta\chi_{\phi\omega}=\pm 17°$, and is comparable with the accuracy expected in the future. The calculated phase depends on the form factor (3.14) that restricts the growth of the ω width with an energy increase. Taking into account the above uncertainty, we find $\chi_{\phi\omega}=165°\pm 3°$ at $R_{\rho\pi}=0\ \mathrm{GeV}^{-1}$ and $\chi_{\phi\omega}=172°\pm 3°$ at $R_{\rho\pi}=1\ \mathrm{GeV}^{-1}$. The present accuracy of the $\chi_{\phi\omega}$ measurement still admits very large bounds for $R_{\rho\pi}$, but the future goal of the $\pm 10°$ accuracy of the phase determination will permit one to put the restriction $R_{\rho\pi}\lesssim 2\ \mathrm{GeV}^{-1}$ with the perspective to give the reliable value of this parameter upon further improvement of the accuracy. Second, if one does not take into account the $\rho\pi$ rescattering effect in the 3π decay width then, including the uncertainties pointed out above, one obtains $\chi_{\phi\omega}=162°\pm 4°$ at $R_{\rho\pi}=0\ \mathrm{GeV}^{-1}$, and $\chi_{\phi\omega}=170°\pm 4°$ at $R_{\rho\pi}=1\ \mathrm{GeV}^{-1}$. Unfortunately, the difference between the predictions of the strong and weak $\phi\omega$ mixing models for $\chi_{\phi\omega}$ at the ϕ mass 0.6° is too small to be measured. However, the two mixing models can be distinguished by their predictions for the $e^+e^-\to\pi^+\pi^-\pi^0$ reaction cross section at energies near the $\phi\omega$ interference minimum [5]. This is due to the influence of the $K\bar{K}$ intermediate state on imaginary parts of the coupling constants and the mixing parameter which is strongly energy dependent. At the ϕ mass, its contribution is within the uncertainties of the calculation, but it grows upon the energy increase, so that at energies near the interference minimum, an additional phase due to this intermediate state could be observed [5]. Of course, the study of the energy behavior of $\chi_{\phi\omega}$ illustrated by the curve in Fig. 3 would be of interest.

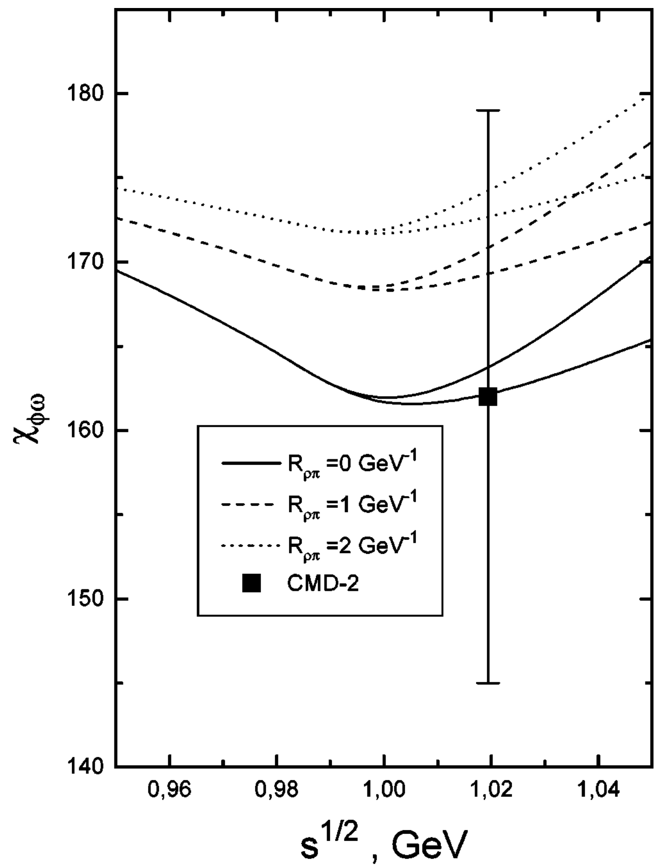

FIG. 3. Energy behavior of the $\phi\omega$ interference phase in the case of no rescattering correction to the 3π decay width, calculated at $\lambda_{K*}=1\ \mathrm{GeV}^{-2}$. The splitting of each curve at $\sqrt{s}\geqslant 2m_K=0.992$ GeV illustrates the opening of the $K\bar{K}$ channel in the $\phi\rho\pi$ coupling (see text for explanation). The lower curve in each pair corresponds to the latter being taken into account.

As far as the contribution of heavier ω', ω'' resonances is concerned, we neglect it here. At the present time, this is justifiable. Indeed, the data [1] give $\sigma_{\mathrm{bg}}=0.32\pm 0.22$ nb for the cross section corresponding to the amplitude A_{bg} in Eq. (1.1) and the $\omega(782)$ tail contribution at the ϕ mass is $\simeq 3$ nb. On the other hand, there are estimates [19] of the ω', ω'' resonance parameters which imply the contribution to the 3π cross section $\sigma_{3\pi}(\omega'+\omega'')\simeq 0.3$ nb at the ϕ mass compatible with the background σ_{bg} from [1]. The $\omega(782)$ tail at the same energy is estimated to be $\simeq 3$ nb. Because the data on which the work [19] is based are rather contradictory, it would be misleading now to include the contribution of heavier resonances, whose parameters are extracted from these imperfect data. Of course, the upcoming improvement of the ω', ω'' resonance parameters will by no means invalidate the present calculation of the interference phase because their contributions can be properly taken into account in a manner similar to Eq. (1.1).

V. CONCLUSION

Upon isolating possible contributions to the $\phi\omega$ interference phase $\chi_{\phi\omega}$ in the reaction $e^+e^-\to\pi^+\pi^-\pi^0$, we point to the imaginary part of the $\phi\omega$ mixing parameter arising mainly due to the $\rho\pi$ state as responsible for the deviation of $\chi_{\phi\omega}$ from 180° observed in the experiment [1]. The uncovered source of the deviation of $\chi_{\phi\omega}$ from the naively expected phase 180° is far from being trivial. The fact is that the tails of resonances are often treated as some substitution to unknown background. The value of information about the $\phi\omega$ interference phase obtained in [1], still to be supported by further precise measurements, is that it give the evidence in favor of applicability of usual field theoretical methods to such complicated objects as hadronic resonances. The confirmation of the observed [1] deviation of the phase would mean that the tail of the ω is essential at the ϕ mass, which is as distant from the ω as 28 widths of the latter. It can hardly be represented by the normally used nonresonant background. Further evidence in favor of this view could be provided by the measurements of the energy dependence of the $\phi\omega$ interference phase as illustrated in Fig. 3. Except for the behavior of $\chi_{\phi\omega}$, the accurate measurements of the $\pi^+\pi^-\pi^0$ cross section in between the ω and ϕ peaks are necessary. They could help both in an unambiguous answer to the question of the magnitude of $R_{\rho\pi}$ [Eq. (3.14)], because the cross section evaluated at $R_{\rho\pi}=1$ GeV^{-1} is lower than that evaluated at $R_{\rho\pi}=0$ GeV^{-1} by 20% (28%) at $\sqrt{s}=900$ MeV (950 MeV), and in elucidating the role of heavier $\omega',\omega''\cdots$ resonances at $\sqrt{s}\lesssim m_\phi$.

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