## **Searching for**  $\nu_{\mu} \rightarrow \nu_{\tau}$  oscillations with extragalactic neutrinos

Sharada Iyer

*Department of Physics, University of Arizona, Tucson, Arizona 85721*

Mary Hall Reno

*Department of Physics and Astronomy, University of Iowa, Iowa City, Iowa 52242*

Ina Sarcevic

*Department of Physics, University of Arizona, Tucson, Arizona 85721* (Received 17 September 1999; published 7 February 2000)

We propose a novel approach for studying  $\nu_{\mu} \rightarrow \nu_{\tau}$  oscillations with extragalactic neutrinos. Active galactic nuclei and gamma ray bursts are believed to be sources of ultrahigh energy muon neutrinos. With distances of 100 Mpc or more, they provide an unusually long baseline for possible detection of  $v_{\mu} \rightarrow v_{\tau}$  with mixing parameters  $\Delta m^2$  down to  $10^{-17}$  eV<sup>2</sup>, many orders of magnitude below the current accelerator experiments. By solving the coupled transport equations, we show that high-energy  $v_r$ 's, as they propagate through the Earth, cascade down in energy, producing the enhancement of the incoming  $\nu_{\tau}$  flux in the low energy region, in contrast with the high-energy  $\nu_{\mu}$ 's, which get absorbed. For an AGN quasar model we find the  $\nu_{\tau}$  flux to be a factor of 2 to 2.5 larger than the incoming flux in the energy range between  $10^2$  GeV and  $10^4$  GeV, while for a GRB fireball model, the enhancement is 10–27 % in the same energy range and for zero nadir angle. This enhancement decreases with larger nadir angle, thus providing a novel way to search for  $\nu_{\tau}$  appearance by measuring the angular dependence of the muons. To illustrate how the cascade effect and the  $v<sub>r</sub>$  final flux depend on the steepness of the incoming  $\nu_{\tau}$ , we show the energy and angular distributions for several generic cases of the incoming tau neutrino flux,  $F_v^0 \sim E^{-n}$  for  $n=1$ , 2 and 3.6. We show that for the incoming flux that is not too steep, the signal for the appearance of high-energy  $\nu_{\tau}$  is the enhanced production of lower energy  $\mu$ and their distinctive angular dependence, due to the contribution from the  $\tau$  decay into  $\mu$  just below the detector.

PACS number(s): 96.40.Tv, 14.60.Pq, 98.54.Cm, 98.70.Rz

Recent Super-Kamiokande (SuperK) measurements of the low atmospheric  $v_{\mu}/v_e$  ratio and the strong zenith angle dependence of the  $\nu_{\mu}$  events [1] suggest oscillations of  $\nu_{\mu}$  into  $\nu_{\tau}$  with the parameters  $\sin^2 2\theta > 0.7$  and  $1.5 \times 10^{-3} < \Delta m^2$  $\leq 1.5 \times 10^{-2}$  eV<sup>2</sup> [1]. This is in agreement with previously reported results on the atmospheric anomaly by Kamiokande  $[2]$  and MACRO  $[3]$  and is consistent with limits from other experiments, e.g., CHOOZ [4]. Confirmation of  $\nu_\mu \rightarrow \nu_\tau$  oscillations and determination of neutrino mixing angles would be a crucial indication of the nature of physics beyond the standard model. The firmest confirmation of this hypothesis would be via detection of  $\tau$  leptons produced by charged current interactions of  $v<sub>\tau</sub>$ 's resulting from oscillations of  $v_{\mu}$ 's, which is extremely difficult with current neutrino experiments.

In this paper, we propose a study of  $v_{\mu} \rightarrow v_{\tau}$  oscillations with extragalactic neutrinos. Large volume neutrino detectors and the prospect of astrophysical neutrino sources put  $v_{\tau}$ detection in the realm of possibility  $[5]$ . The large distances involved for astrophysical sources, on the order of one to thousands of Megaparsecs, make the next generation of neutrino experiments potentially sensitive to neutrino mass differences as low as  $\Delta m^2 \sim 10^{-17}$  eV<sup>2</sup> [6]. Over such long baselines, half of the neutrinos arriving at the Earth would be  $\nu_{\tau}$ 's in oscillation scenarios, the other half being  $\nu_{\mu}$ 's. By observing both  $v_{\mu}$  and  $v_{\tau}$  from extragalactic sources such as gamma ray bursts  $(GRBs)$   $[7]$  and active galactic nuclei  $(AGN)$  [8], neutrino oscillation hypothesis would be confirmed and models of these sources would be tested.

The effect of attenuation of the neutrino flux due to interactions of neutrinos in the Earth is qualitatively different for  $v_{\mu}$  and  $v_{\tau}$ . Muon neutrinos are absorbed by charged current interactions, while tau neutrinos are regenerated by tau decays. The Earth never becomes opaque to  $v_{\tau}$ , though the effect of  $\nu_{\tau} \rightarrow \tau \rightarrow \nu_{\tau}$  interaction and decay processes is to degrade the energy of the incident  $v_{\tau}$ . The identical spectra of  $v_n$  and  $v_\tau$  incident on the Earth emerge after passage through the Earth with distinctly different spectra. The preferential penetration of  $v<sub>\tau</sub>$  through the Earth is of great importance for high energy neutrino telescopes such as AMANDA, NESTOR and ANTARES.

We consider  $\nu_{\mu}$  and  $\nu_{\tau}$  propagation through the Earth using a similar procedure to the one outlined for  $v_n$  in Ref. [9]. We show that the energy spectrum of the  $v<sub>\tau</sub>$  becomes enhanced at low energy, providing a distinctive signature for its detection. The degree of enhancement depends on the initial neutrino flux. We consider initial fluxes  $F_v^0 \sim E^{-n}$  for  $n=1,2,3.6$ , a GRB flux [7] and an AGN flux [8]. We solve the coupled transport equations for lepton and neutrino fluxes as indicated below.

Let  $F_{\nu_{\tau}}(E, X)$  and  $F_{\tau}(E, X)$  be the differential energy spectrum of tau neutrinos and tau respectively at a column depth *X* in the medium defined by

$$
X = \int_0^L \rho(L') dL',
$$

where  $\rho(L)$  is the density of the medium at a distance *L* from the boundary measured along the neutrino beam path. Then, one can derive the following cascade equation for neutrinos as

$$
\frac{\partial F_{\nu_{\tau}}(E,X)}{\partial X} = -\frac{F_{\nu_{\tau}}(E,X)}{\lambda_{\nu_{\tau}}(E)}
$$
  
+ 
$$
\int_{E}^{\infty} dE_{y} \left[ \frac{F_{\nu_{\tau}}(E_{y},X)}{\lambda_{\nu_{\tau}}(E_{y})} \right] \frac{dn}{dE}
$$
  

$$
\times (\nu_{\tau}N \to \nu_{\tau}X;E_{y},E)
$$
  
+ 
$$
\int_{E}^{\infty} dE_{y} \left[ \frac{F_{\tau}(E_{y},X)}{\rho_{\tau}^{dec}(E_{y})} \right] \frac{dn}{dE}
$$
  

$$
\times (\tau \to \nu_{\tau}X;E_{y},E) + \int_{E}^{\infty} dE_{y} \left[ \frac{F_{\tau}(E_{y},X)}{\lambda_{\tau}(E_{y})} \right] \frac{dn}{dE}
$$
  

$$
\times (\tau N \to \nu_{\tau}X;E_{y},E) \tag{1}
$$

and for taus as

$$
\frac{\partial F_{\tau}(E,X)}{\partial X} = -\frac{F_{\tau}(E,X)}{\lambda_{\tau}(E)} - \frac{F_{\tau}(E,X)}{\rho_{\tau}^{dec}(E,X,\theta)} + \int_{E}^{\infty} dE_{y} \left[ \frac{F_{\nu_{\tau}}(E_{y},X)}{\lambda_{\nu_{\tau}}(E_{y})} \right] \frac{dn}{dE} (\nu_{\tau} N \to \tau X; E_{y}, E).
$$
\n(2)

The first term in Eq.  $(1)$  is a loss due to the neutrino interactions, the second is the regeneration term due to the neutral current, the third term is a contribution due to the tau decay and the last term is the contribution due to tau interactions.

In Eq.  $(2)$ , the first term is a loss due to tau interactions, the second term is a loss due to the tau decay, while the last term is a contribution from neutrino charged current interactions. Tau decays are more important than tau interactions at the energies considered here, below  $10^6$  GeV. Tau charged current interaction length and the photonuclear interaction length both become comparable to the tau decay length at  $E > 10^8$  GeV. The tau energy loss, in principle, affects the shape of the tau neutrino energy spectrum by enhancing the lower energy part [10]. However, for the fluxes that we consider, which are quite steep at energies above  $10^8$  GeV, this effect is negligible. Thus, we neglect the tau interaction terms in Eqs.  $(1)$  and  $(2)$  in what follows.

Here  $\lambda(E)$  is the interaction length and  $\rho_{\tau}^{dec}(E, X, \theta)$  is the decay length for tau. They are defined as

$$
\frac{1}{\lambda_{\nu}(E)} = \sum_{T} N_{T} \sigma_{\nu T}^{tot}(E),
$$
  

$$
\rho_{\tau}^{dec}(E, X, \theta) = \gamma c \zeta_{\tau} \varrho(X, \theta)
$$

where  $N_T$  is the number of scatterers T in 1 g of the medium,  $\sigma_{\nu T}^{tot}(E)$  is the total cross section for the  $\nu T$  interactions and the sum is over all scatterer types  $(T=N,e, \ldots), \zeta_{\tau}$  is the mean lifetime of tau and  $\rho$  is the density of matter in the earth. Scatterings of neutrinos and taus with nucleons (N) are most important, so we approximate

$$
\frac{1}{\lambda_{\nu}(E)} \simeq N_0 \sigma_{\nu N}^{tot}(E),
$$

where  $N_0$  is Avogadro's number. We use the CTEQ5 parton distribution functions to evaluate neutrino cross sections [11]. We have previously calculated charged and neutral current energy distributions, *dn*/*dE*, and the total cross section,  $\sigma_{vT}^{tot}(E)$  taking into account recent improvements in our knowledge of the small-x behavior of the structure functions  $|12|$ .

To simplify the solution to the equation for the tau flux, we approximate the *X* and  $\theta$  dependent density of the earth by the average of the density along the column depth of angle  $\theta$ :

$$
\varrho(X,\theta) \simeq \varrho^{avg}(\theta).
$$

Following Ref.  $[9]$ , let us define the effective absorption length  $\Lambda_{\nu}(E,X)$  by

$$
F_{\nu}(E,X) = F_{\nu}^{0}(E) \exp\bigg[-\frac{X}{\Lambda_{\nu}(E,X)}\bigg].
$$
 (3)

It is convenient to define

$$
\Lambda_{\nu}(E,X) = \frac{\lambda_{\nu}(E)}{1 - Z_{\nu}(E,X)}\tag{4}
$$

where  $Z_{\nu}(E, X)$  is a positive function (we will call it *Z* factor in analogy with the hadronic cascade theory) which contains the complete information about neutrino interaction and regeneration in matter.

Assuming that there is no significant contribution to the neutrino flux from decaying particles (as would be the case for muon neutrinos) using the above equation we can find an implicit equation for  $Z$  from the transport equation [9],

$$
Z_{\nu}(E,X) = \int_0^1 \eta_{\nu}(y,E) \Phi_{\nu}^{nc}(y,E) \left[ \frac{1 - e^{-XD_{\nu}(E,E_y,X)}}{XD_{\nu}(E,E_y,X)} \right] dy,
$$
\n(5)

where

$$
D_{\nu}(E, E_{y}, X) = \frac{1}{\Lambda_{\nu}(E_{y}, X)} - \frac{1}{\Lambda_{\nu}(E, X)}
$$

$$
\eta_{\nu}(y, E) = \frac{F_{\nu}^{0}(E_{y})}{F_{\nu}^{0}(E)(1 - y)}
$$

$$
\frac{d\sigma_{\nu N \to \nu X}(y, E_{y})}{dy} = \Phi_{\nu}^{nc}(y, E) \sigma_{\nu N}^{tot}(E)
$$

and  $d\sigma_{\nu N\rightarrow\nu X}(y,E)/dy$  is the differential cross section for the inclusive reaction  $\nu N \rightarrow \nu X$  (with  $E<sub>v</sub>$  the incoming neutrino energy and *y* the fraction of energy lost) and  $E_y$  $\equiv$ *E*/(1-*y*).

Naumov and Perrone [9] have shown that by iteratively evaluating Eq. (5), starting with  $Z^{(0)}=0$ , the solution for muon neutrinos quickly converges for a wide range of starting fluxes.

By a similar procedure, the coupled differential equations for tau neutrinos including tau production and decay can be iteratively solved. The tau flux generated by charged current interactions including the loss term due to its decay is

$$
\frac{F_{\tau}(E,X)}{F_{\nu}^{0}(E)} = \exp\left[-\frac{X}{\rho_{\tau}^{dec}(E,\theta)}\right] \int_{0}^{X} \int_{0}^{1} \frac{\Phi_{\nu}^{cc}(y,E)}{\lambda_{\nu}(E)} \eta_{\nu}(y,E)
$$

$$
\times \exp\left[-\frac{X'}{\Lambda_{\nu}(E_{y},X')}\right] \exp\left[\frac{X'}{\rho_{\tau}^{dec}(E,\theta)}\right] dX'dy.
$$
(6)

The *Z* factor for the tau neutrino flux is then

$$
Z = Z_{\nu} + Z_{\tau} \tag{7}
$$

where  $Z_{\nu}$  is given by Eq. (5) and

$$
Z_{\tau} = \left[\frac{1}{X}\right] \int_0^X \int_0^1 \frac{\lambda_{\nu}(E)}{\rho_{\tau}^{dec}(E,\theta)} \Phi_{\nu}^{dec}(y,E) \eta_{\nu}(y,E)
$$

$$
\times \exp\left[-\frac{X'}{\Lambda_{\nu}(E_y,X')}\right] \frac{F_{\tau}(E_y,X')}{F_{\nu}^0(E_y)} dX'dy. \tag{8}
$$

We include decay modes in  $\Phi_{\nu}^{dec}(y, E)$  as in Ref. [13] and a constant energy distribution for the remaining branching fraction not included there. In Eqs.  $(5)$  and  $(8)$ , the *Z* factors implicit in  $\Lambda_{\nu}$  are  $Z = Z_{\nu} + Z_{\tau}$ .

In the iterative solution of the equation for *Z*, one has the option of picking the initial value  $Z^{(0)}$ . We have chosen the *X* and *E* dependent solution to the cascade equation for the  $\nu_{\mu}$  flux, namely the solution to Eq. (5).

To demonstrate the importance of regeneration of tau neutrinos from tau decays, we evaluate the tau neutrino flux for several input neutrino spectra and compare to the attenuated  $\nu_{\mu}$  flux. For the incoming neutrino spectrum we use [9]

$$
F_{\nu}^{0}(E) = K \left(\frac{E_{0}}{E}\right)^{n} \phi \left(\frac{E}{E_{\text{cut}}}\right),\tag{9}
$$

where *K*, *n*,  $E_0$  and  $E_{\text{cut}}$  are parameters and  $\phi(t)$  is a function equal to 0 at  $t \ge 1$  and 1 at  $t \le 1$ . We use  $\phi(t) = 1/[1]$  $+tan(\pi t/2)$  ( $t<1$ ) and  $E_{\text{cut}}=3\times10^{10}$  GeV and  $E_0=1$ PeV. For  $n=1$ , we introduce a smooth cutoff by multiplying Eq. (9) by a factor  $(1 + E/E_0)^{-2}$  and by setting  $E_0 = 100$ PeV. To evaluate the depth as a function of nadir angle, the density profile of the earth described in Ref.  $[12]$  is used.

In Fig. 1 we show the nadir angle dependence of the ratios of the fluxes calculated via Eqs.  $(1)$ – $(8)$  to the input flux  $F_{\nu}^{0}(E)$ . All fluxes are evaluated as a function of nadir angle,



FIG. 1. The nadir angle dependence of the ratio of fluxes for energies 10<sup>4</sup> GeV, 10<sup>5</sup> GeV and 10<sup>6</sup> GeV for the  $v_\mu$  and  $v_\tau$  assuming  $F_v^0(E) \sim E^{-n}$  with  $n = 1, 2$  and 3.6.

at the *X* value for the surface of the earth and for energies  $10^4$  GeV,  $10^5$  GeV and  $10^6$  GeV for  $\nu_\mu$  and  $\nu_\tau$  assuming  $F_p^0(E)$  given by Eq. (9). For an incoming flux,  $n=1$ , we find that the  $\nu_{\tau}$  flux is enhanced (relative to the incoming  $\nu_{\tau}$  flux) for all nadir angles for  $E_{\nu_{\tau}} = 10^4$  GeV and  $10^5$  GeV, and for  $\theta$ >50° for  $E_{\nu_{\tau}}$ =10<sup>6</sup> GeV. The peak of the enhancement gets shifted toward the higher nadir angles as the energy increases. This is due to the fact that high energy  $v<sub>\tau</sub>$  can remain high energy if the column depth is small, i.e. for large nadir angles. In case of the steeper incoming flux,  $n=2$ , we find that  $\nu_{\tau}$ 's are less attenuated than the  $\nu_{\mu}$ 's, and the expected enhancement at low energy is not evident due to the steepness of the flux. For even steeper flux,  $n=3.6$ , the difference between  $\nu_{\tau}$  and  $\nu_{\mu}$  flux is very small.

In Fig. 2 we show the energy dependence of the ratio of fluxes for nadir angles  $\theta = 0$ ,  $\theta = 30^{\circ}$  and  $\theta = 60^{\circ}$  for  $\nu_{\mu}$  and  $\nu_{\tau}$  with  $F_{\nu}^{0}(E)$  given by Eq. (9). For small nadir angles,  $\theta$  $=0$  and 30° and  $F_v^0(E) \sim 1/E$  we find that enhancement of tau neutrinos is in the energy range of  $10^2$  GeV and  $10^5$ GeV, while for  $\theta$ =60°, the enhancement extends up to 10<sup>6</sup>



FIG. 2. The energy dependence of the ratio of fluxes for nadir angles  $\theta = 0$ ,  $\theta = 30^{\circ}$  and  $\theta = 60^{\circ}$  for  $\nu_{\mu}$  and  $\nu_{\tau}$  assuming  $F_{\nu}^{0}(E)$  $\sim E^{-n}$  with  $n=1$ , 2 and 3.6.



FIG. 3. The energy dependence of the ratio of fluxes for a Stecker-Salamon AGN model [8].

GeV. In contrast the  $\nu_{\mu}$  flux is attenuated for all the nadir angles. When the incoming flux is steeper,  $n=2$ , the  $\nu_{\tau}$  flux appears to be attenuated at high energies, although less than the  $v_{\mu}$  flux. For  $n=3.6$ , the energy dependence of these two fluxes is very similar, they are both reduced at high energies, and the effect is stronger for smaller nadir angle, since in this case the column depth is larger and there are more charged current interactions possible.

In case of the AGN quasar model [8], we find that the  $v_{\tau}$ flux is a factor of 2 to 2.5 times larger than the input flux, for nadir angle,  $\theta = 0$ . This is shown in Fig. 3. For larger angles, the effect is smaller. Detection of AGN neutrinos would be optimal for small nadir angles and for  $v<sub>\tau</sub>$  with energy of 10<sup>2</sup> GeV to  $10^4$  GeV.

We also present results for the  $v<sub>\tau</sub>$  flux for the case of GRB fireball model  $[7]$ . We find that due to the steepness of the input flux for  $E_{\nu_{\tau}} > 100 \text{ TeV}$ , the  $\nu_{\tau}$  flux is enhanced only by about 10–27%, depending on the energy and nadir angle. We show the energy spectrum of the ratio of  $\nu_{\tau}$  flux to the input flux  $F^0_{\nu}$  in Fig. 4.

We expect that the next generation of neutrino telescopes will be able to detect the high energy neutrinos from AGN and GRBs. We have previously shown that most of the extragalactic neutrino fluxes exceed the atmospheric neutrino background for neutrino energy greater than  $\sim$  10 TeV which may enable the detection of the extragalactic neutrinos  $[12]$ . Detection of PeV  $\nu_{\tau}$  might be possible via "double-bang" events, namely a big hadronic shower from the incoming  $\nu_{\tau}$ charged-current interaction, a  $\tau$  track and after about 100 m, another big particle cascade from the  $\tau$  decay [5].



FIG. 4. The energy dependence of the ratio of the fluxes for a Waxman-Bahcall GRB model [7].

For TeV energies, searches for  $v<sub>\tau</sub>$  appearance would look for enhanced muon rates. The enhanced rates of muons come from muonic decays of the  $\tau$  produced by  $\nu_{\tau}$  charged current interactions in or near the instrumented detector volume. The muon rates would be enhanced at low energy  $(\sim10-100$ TeV) and for small nadir angles. In the case of an AGN quasar model, we find that the  $v<sub>\tau</sub>$  flux enhancement is very distinct, while for a GRB fireball model, the effect is only 10–27%. This is due to the fact that GRB input  $\nu_{\tau}$  flux is much steeper, thus there are not many high-energy  $v<sub>r</sub>$ 's that would contribute to the enhancement at low energy. We have also shown that the low energy pile up is significant only for incoming fluxes that are less steep than  $1/E<sup>2</sup>$  in the high energy region. For an incoming flux which is proportional to  $1/E$ , we find the angular distribution of  $\nu_\tau$ 's to be significantly different than for the  $v_n$ 's.

We have proposed a novel way of detecting appearance of extragalactic high-energy  $v<sub>\tau</sub>$  by measuring the angular and energy distribution of muons with energy above 10 TeV. This would give an experimental signature of  $v_n \rightarrow v_\tau$  oscillations with  $\Delta m^2$  as low as  $10^{-17}$  eV<sup>2</sup>.

We are grateful to F. Halzen and D. Seckel for stimulating discussions. The work of S.I. and I.S. has been supported in part by the DOE under Contract DE-FG02-95ER40906. The work of M.H.R. has been supported in part by National Science Foundation Grant No. PHY-9802403. M.H.R and I.S. thank Aspen Center for Physics for its hospitality while part of this work was completed. Upon completion of this work, we received a conference paper by Bottai and Becattini who had obtained similar results using a Monte Carlo calculation for a  $1/E$  flux  $|14|$ .

- [1] Super-Kamiokande Collaboration, Y. Fukuda et al., Phys. Rev. Lett. 82, 2430 (1999); hep-ex/9908049; Phys. Lett. B **433**, 9 (1998); **436**, 33 (1998); Phys. Rev. Lett. **81**, 1562  $(1998).$
- [2] Kamiokande Collaboration, S. Hatakeyama *et al.*, Phys. Rev. Lett. **81**, 2016 (1998).
- [3] MACRO Collaboration, M. Ambrosio *et al.*, Phys. Lett. B 434, 451 (1998).
- @4# CHOOZ Collaboration, M. Apollonio *et al.*, Phys. Lett. B **420**, 397 (1998).
- [5] J. G. Learned and S. Pakvasa, Astropart. Phys. 3, 267 (1995).
- [6] F. Halzen and D. Saltzberg, Phys. Rev. Lett. **81**, 4305 (1998); T. J. Weiler, W. A. Simmons, S. Pakvasa, and J. G. Learned, hep-ph/9411432.
- @7# E. Waxman and J. N. Bahcall, Phys. Rev. Lett. **78**, 2292  $(1997).$
- [8] F. W. Stecker and M. Salomon, Space Sci. Rev. **75**, 341  $(1995).$
- @9# V. A. Naumov and L. Perrone, Astropart. Phys. **10**, 239  $(1999).$
- [10] M. H. Reno, I. Sarcevic, and D. Seckel (in preparation).
- [11] CTEQ Collaboration, H. L. Lai *et al.*, hep-ph/9903282.
- [12] R. Gandhi, C. Quigg, M. H. Reno, and I. Sarcevic, Phys. Rev. D 58, 093009 (1998).
- [13] L. Pasquali and M. H. Reno, Phys. Rev. D **59**, 093003 (1998).
- [14] S. Bottai and F. Becattini, in Proceedings of the Cosmic Ray Conference (HE 4.2.15), Salt Lake City, Utah, 1999.