SO(10) GUT and quark-lepton mass matrices

K. Matsuda and T. Fukuyama
Department of Physics, Ritsumeikan University, Kusatsu, Shiga, 525-8577 Japan

H. Nishiura
Department of General Education, Junior College of Osaka Institute of Technology, Asahi-ku, Osaka, 535-8585 Japan

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The phenomenological model that all quark and lepton mass matrices have the same zero texture, namely their (1,1), (1,3), and (3,1) components are zeros, is discussed in the context of SO(10) grand unified theories (GUTs). The mass matrices of type I for quarks are consistent with the experimental data in the quark sector. For the lepton sector, consistent fitting to the data of neutrino oscillation experiments forces us to use the mass matrix for the charged leptons which is slightly deviated from type I. Given quark masses and charged lepton masses, the model includes 19 free parameters, whereas the SO(10) GUTs give 16 constrained equations. Changing the remaining three parameters freely, we can fit all the entries of the CKM quark mixing matrix and the MNS lepton mixing matrix, and three neutrino masses consistently with the present experimental data.

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I. INTRODUCTION

The downward and upward discrepancy in the atmospheric neutrino deficit in SuperKamiokande [1] together with other neutrino oscillation experiments such as solar neutrino [2], reactor [3], and accelerator [4] experiments drives us to the definite conclusion that neutrinos have mass. These experiments enable us to get a glimpse of high energy physics beyond the standard model. In these situations our strategy is as follows. First, we search for the most suitable phenomenological quark and lepton mass matrices which satisfies miscellaneous experiments in the hadron and electroweak interactions. Next, in order to search for its uniqueness and for its physical implications, such mass matrices are incorporated into grand unified theories (GUTs). Of course phenomenological mass matrices and GUTs are closely correlated and the real model building is performed going back and forth between these two approaches. Indeed, we consider the seesaw mechanism in the neutrino mass matrix [5], which supports minimally SO(10) GUTs. Conversely SO(10) GUTs prefer the mass matrices reflecting some similarity in the quark and lepton sectors. In the seminal work of phenomenological quark mass matrix models [6], Fritzsch proposed a symmetric or Hermitian matrix later called a six texture zero model which has vanishing (1,1), (1,3), (3,1), and (2,2) components in both the mass matrices, M_u for up-type quarks (u,c,t) and M_d for down-type quarks (d,s,b). Here n texture zero means that two types of quark mass matrices have totally n zeros in the upper half of Hermitian mass matrices, in this case (1,1), (1,3), and (2,2) in each mass matrix. However, this model failed to predict a large top quark mass. Symmetric or Hermitian six and five texture zero models were systematically discussed by Ramond *et al.* [7]. They found that the Hermitian M_u and M_d compatible with experiments can have at most five texture zero. Before the work of Ramond *et al.* nonsymmetric or non-Hermitian six texture zero quark mass matrices model [nearest-neighbor interaction (NNI) model] was proposed by Branco, Lavoura, and Mota [8], and Takasugi showed that, by rebasing and rephasing of weak bases, always one of M_u and M_d can have the symmetric Fritzsch form and the other can have the NNI form [9]. Demanding to deal with the quark and lepton mass matrices on the same footing, we have proposed a four texture zero model [10], in which all the quark and lepton mass matrices, M_u, M_d, M_e, and M_ν are Hermitian and have the same textures. Here M_ν and M_e are mass matrices of neutrinos (ν_e,ν_μ,ν_τ) and charged leptons (e,μ,τ), respectively. Namely their (1,1), (1,3), and (3,1) components are zeros and the others are nonzero valued. This model was also discussed by Du and Xing [11], by Fritzsch and Xing [12], by Kang and Kang [13], by Kang, Kang, Kim, and Kim [14], and by Chkareuli and Froggatt [15], mainly in the quark sector. This model is compatible with the large top quark mass, the small quark mixing angles, and the large ν_μ-ν_τ neutrino mixing angles via the seesaw mechanism. In this article, we discuss the above four texture zero model embedding in the SO(10) GUTs. The SO(10) GUTs impose some further constraints on the mass matrices. Using those constraints we predict all the entries of the lepton mixing matrix and neutrino masses, which are consistent with the experimental data, in terms of three free parameters left in the model.

This article is organized as follows. In Sec. II we review four texture zero model. In Sec. III we present a mass matrix model motivated by SO(10) GUTs . This model is combined with the four texture zero *Ansätze* in Sec. IV. Section V is devoted to summary.

II. FOUR TEXTURE ZERO QUARK-LEPTON MASS MATRICES

Phenomenological quark mass matrices have been discussed from various points of view [16]. In this section we review our quark and lepton mass matrix model [10]. The mass term in the Lagrangian is given by

$$L_M=-\overline{q^u_{R,i}}M_{uij}q^u_{L,j}-\overline{q^d_{R,i}}M_{dij}q^d_{L,j}-\overline{l_{R,i}}M_{eij}l_{L,j}-\overline{\nu'_{R,i}}M_{Dij}\nu_{L,j}-\frac{1}{2}\overline{(\nu_{L,i})^c}M_{Lij}\nu_{L,j}-\frac{1}{2}\overline{(\nu'_{R,i})^c}M_{Rij}\nu'_{R,j}+\text{H.c.},\tag{1}$$

with

$$q^u_{L,R}=\begin{pmatrix} u \\ c \\ t \end{pmatrix}_{L,R}, \quad q^d_{L,R}=\begin{pmatrix} d \\ s \\ b \end{pmatrix}_{L,R}, \quad l_{L,R}=\begin{pmatrix} e \\ \mu \\ \tau \end{pmatrix}_{L,R},$$

$$\nu_L=\begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix}_L, \quad \nu'_R=\begin{pmatrix} \nu'_e \\ \nu'_\mu \\ \nu'_\tau \end{pmatrix}_R, \tag{2}$$

where M_u, M_d, M_e, M_D, M_L, and M_R are the mass matrices for up quarks, down quarks, charged leptons, Dirac neutrinos, left-handed Majorana neutrinos, and right-handed Majorana neutrinos, respectively. The mass matrix of light Majorana neutrinos M_ν is given by

$$M_\nu=M_L-M_D^T M_R^{-1} M_D, \tag{3}$$

which is constructed via the seesaw mechanism [5] from the block-diagonalization of neutrino mass matrix,

$$\begin{pmatrix} M_L & M_D^T \\ M_D & M_R \end{pmatrix}. \tag{4}$$

We set an *Ansätz* that the mass matrices M_u, M_d, M_e, and M_ν are Hermitian and have the same textures. Our model is different from the Fritzsch model in the sense that (2,2) components are not zeros and that our model deals with the quark and lepton mass matrices on the same footing. The mass matrices M_D, M_L, and M_R are, furthermore, assumed to have the same zero texture as M_ν. This ansatz restricts the texture forms [10] and we choose our following texture because it is most closely related with the NNI form [8]:

$$\text{NNI:}\begin{pmatrix} 0 & * & 0 \\ * & 0 & * \\ 0 & * & * \end{pmatrix}, \quad \textbf{Our Texture}:\begin{pmatrix} 0 & * & 0 \\ * & * & * \\ 0 & * & * \end{pmatrix}. \tag{5}$$

The nonvanishing (2,2) component distinguishes our form from NNI's. Thus the quark and lepton mass matrices are described as follows:

$$M_u=\begin{pmatrix} 0 & A_u & 0 \\ A_u & B_u & C_u \\ 0 & C_u & D_u \end{pmatrix},$$

$$M_d=P_d\begin{pmatrix} 0 & A_d & 0 \\ A_d & B_d & C_d \\ 0 & C_d & D_d \end{pmatrix}P_d^\dagger =\begin{pmatrix} 0 & A_d e^{i\alpha_{12}} & 0 \\ A_d e^{-i\alpha_{12}} & B_d & C_d e^{i\alpha_{23}} \\ 0 & C_d e^{-i\alpha_{23}} & D_d \end{pmatrix}, \tag{6}$$

$$M_e=P_e\begin{pmatrix} 0 & A_e & 0 \\ A_e & B_e & C_e \\ 0 & C_e & D_e \end{pmatrix}P_e^\dagger =\begin{pmatrix} 0 & A_e e^{i\beta_{12}} & 0 \\ A_e e^{-i\beta_{12}} & B_e & C_e e^{i\beta_{23}} \\ 0 & C_e e^{-i\beta_{23}} & D_e \end{pmatrix},$$

$$M_\nu=\begin{pmatrix} 0 & A_\nu & 0 \\ A_\nu & B_\nu & C_\nu \\ 0 & C_\nu & D_\nu \end{pmatrix},$$

where $P_d\equiv\text{diag}(e^{i\alpha_1},e^{i\alpha_2},e^{i\alpha_3})$, $\alpha_{ij}\equiv\alpha_i-\alpha_j$, and $P_e\equiv\text{diag}(e^{i\beta_1},e^{i\beta_2},e^{i\beta_3})$, $\beta_{ij}\equiv\beta_i-\beta_j$.

Let us discuss the relations between the following texture's components of mass matrix M:

$$M=\begin{pmatrix} 0 & A & 0 \\ A & B & C \\ 0 & C & D \end{pmatrix} \tag{7}$$

and its eigenmass m_i. They satisfy

$$B+D=m_1+m_2+m_3,$$

$$BD-C^2-A^2=m_1m_2+m_2m_3+m_3m_1,$$

$$DA^2=-m_1m_2m_3. \tag{8}$$

Therefore, the mass matrix is classified into two types by choosing B and D as follows:

$$[\text{type I}] \quad B=m_2, \quad D=m_3+m_1,$$

$$[\text{type II}] \quad B=m_1, \quad D=m_3+m_2. \tag{9}$$

In the previous paper [10] we showed that type I is compatible with the experimental data both for the quark and lepton mass matrices. So, we concentrate on the type I case.

In the type I case ($B=m_2,\quad D=m_3+m_1$), the other A and C take the following value from Eq. (8):

$$A=\sqrt{\frac{(-m_1)m_2m_3}{m_3+m_1}},\quad C=\sqrt{\frac{(-m_1)m_3(m_3-m_2+m_1)}{m_3+m_1}}. \tag{10}$$

Transforming m_1 into $-m_1$ by rephasing, the mass matrix M becomes

$$M=\begin{pmatrix} 0 & \sqrt{\frac{m_1m_2m_3}{m_3-m_1}} & 0 \\ \sqrt{\frac{m_1m_2m_3}{m_3-m_1}} & m_2 & \sqrt{\frac{m_1m_3(m_3-m_2-m_1)}{m_3-m_1}} \\ 0 & \sqrt{\frac{m_1m_3(m_3-m_2-m_1)}{m_3-m_1}} & m_3-m_1 \end{pmatrix}$$
$$\simeq\begin{pmatrix} 0 & \sqrt{m_1m_2} & 0 \\ \sqrt{m_1m_2} & m_2 & \sqrt{m_1m_3} \\ 0 & \sqrt{m_1m_3} & m_3-m_1 \end{pmatrix} \quad (\text{for } m_3 \gg m_2 \gg m_1). \tag{11}$$

The orthogonal matrix O which diagonalizes M in Eq. (11) as

$$O^T\begin{pmatrix} 0 & \sqrt{m_1m_2} & 0 \\ \sqrt{m_1m_2} & m_2 & \sqrt{m_1m_3} \\ 0 & \sqrt{m_1m_3} & m_3-m_1 \end{pmatrix}O=\begin{pmatrix} -m_1 & 0 & 0 \\ 0 & m_2 & 0 \\ 0 & 0 & m_3 \end{pmatrix} \tag{12}$$

is given by

$$O=\begin{pmatrix} \sqrt{\frac{m_2m_3^2}{(m_2+m_1)(m_3^2-m_1^2)}} & \sqrt{\frac{m_1m_3(m_3-m_2-m_1)}{(m_2+m_1)(m_3-m_2)(m_3-m_1)}} & \sqrt{\frac{m_1^2m_2}{(m_3-m_2)(m_3^2-m_1^2)}} \\ -\sqrt{\frac{m_1m_3}{(m_2+m_1)(m_3+m_1)}} & \sqrt{\frac{m_2(m_3-m_2-m_1)}{(m_2+m_1)(m_3-m_2)}} & \sqrt{\frac{m_1m_3}{(m_3-m_2)(m_3+m_1)}} \\ \sqrt{\frac{m_1^2(m_3-m_2-m_1)}{(m_2+m_1)(m_3^2-m_2^2)}} & -\sqrt{\frac{m_1m_2m_3}{(m_3-m_2)(m_2+m_1)(m_3-m_1)}} & \sqrt{\frac{(m_3)^2(m_3-m_2-m_1)}{(m_3^2-m_2^2)(m_3-m_2)}} \end{pmatrix}$$
$$\simeq\begin{pmatrix} 1 & \sqrt{\frac{m_1}{m_2}} & \sqrt{\frac{m_1m_2^2}{m_3^3}} \\ -\sqrt{\frac{m_1}{m_2}} & 1 & \sqrt{\frac{m_1}{m_3}} \\ \sqrt{\frac{m_1^2}{m_2m_3}} & -\sqrt{\frac{m_1}{m_3}} & 1 \end{pmatrix} \quad (\text{for } m_3 \gg m_2 \gg m_1). \tag{13}$$

The mass matrices for quarks and charged leptons, M_d, M_u, and M_e, are considered to be of this type I and are given by

$$M_d \simeq P_d \begin{pmatrix} 0 & \sqrt{m_d m_s} & 0 \\ \sqrt{m_d m_s} & m_s & \sqrt{m_d m_b} \\ 0 & \sqrt{m_d m_b} & m_b - m_d \end{pmatrix} P_d^\dagger,$$

$$M_u \simeq \begin{pmatrix} 0 & \sqrt{m_u m_c} & 0 \\ \sqrt{m_u m_c} & m_c & \sqrt{m_u m_t} \\ 0 & \sqrt{m_u m_t} & m_t - m_u \end{pmatrix},$$

$$M_e \simeq P_e \begin{pmatrix} 0 & \sqrt{m_e m_\mu} & 0 \\ \sqrt{m_e m_\mu} & m_\mu & \sqrt{m_e m_\tau} \\ 0 & \sqrt{m_e m_\tau} & m_\tau - m_e \end{pmatrix} P_e^\dagger. \quad (14)$$

Those M_d, M_u, and M_e are, respectively, diagonalized by matrices $P_d O_d$, O_u, and $P_e O_e$. Here the orthogonal matrices O_d, O_u, and O_e which diagonalize $P_d^\dagger M_d P_d$, M_u, and $P_e^\dagger M_e P_e$ are obtained from Eq. (13) by replacing m_1, m_2, m_3 by m_d, m_s, m_b, by m_u, m_c, m_t, and by m_e, m_μ, m_τ, respectively. In this case, the Cabibbo-Kobayashi-Maskawa (CKM) quark mixing matrix V can be written as

$$V = P_q^{-1} P_d^{-1} O_u^T P_d O_d P_q \simeq \begin{pmatrix} |V_{11}| & |V_{12}| & |V_{13}| e^{-i\phi} \\ -|V_{12}| & |V_{22}| & |V_{23}| \\ |V_{12}V_{23}| - |V_{13}| e^{i\phi} & -|V_{23}| & |V_{33}| \end{pmatrix}, \quad (15)$$

where the P_d^{-1} factor is included to put V in the form with diagonal elements real to a good approximation. Furthermore, the P_q^{-1} and $P_q = \mathrm{diag}(e^{i\phi_1}, e^{i\phi_2}, e^{i\phi_3})$ with $\phi_1 - \phi_2 = \arg(P_d^{-1} O_u^T P_d O_d)_{12}$ and $\phi_1 - \phi_3 = \arg(P_d^{-1} O_u^T P_d O_d)_{23}$ come from the choice of phase convention as Eq. (15). The explicit forms of the components of V are obtained [10] as

$$|V_{12}| \simeq \left| \sqrt{\frac{m_d}{m_s}} - \sqrt{\frac{m_u}{m_c}} e^{-i\alpha_{12}} \right|,$$

$$|V_{23}| \simeq \left| \sqrt{\frac{m_d}{m_b}} - \sqrt{\frac{m_u}{m_t}} e^{-i\alpha_{23}} \right|,$$

$$|V_{13}| \simeq \left| \sqrt{\frac{m_d^2 m_s}{m_b^3}} - \sqrt{\frac{m_u}{m_c}} \left(\sqrt{\frac{m_d}{m_b}} - \sqrt{\frac{m_u}{m_t}} e^{-i\alpha_{23}} \right) e^{-i\alpha_{12}} \right|,$$

$$\cos\phi \simeq \frac{|V_{12}|^2 + m_u/m_c - m_d/m_s}{2|V_{12}|\sqrt{m_u/m_c}}. \quad (16)$$

The lepton mixing matrix U (hereafter we call it the Maki-Nakagawa-Sakata (MNS) mixing matrix [17]) is given by

$$U = P_e^\dagger O_e^T P_e O_\nu = \begin{pmatrix} U_{11} & U_{12} & U_{13} \\ U_{21} & U_{22} & U_{23} \\ U_{31} & U_{32} & U_{33} \end{pmatrix}, \quad (17)$$

where the $P_e^\dagger$ factor is included to put U in the form with diagonal elements real to a good approximation. Here the O_ν is the orthogonal matrix which diagonalizes the light Majorana neutrino mass matrices M_ν given by Eq. (3).

III. MASS MATRICES IN THE CONTEXT OF SO(10) GUTS

Even if we succeeded in constructing the quark mass matrices M_u and M_d consistent with experiments, we have infinitely many mass matrices equivalent to the M_u and M_d which are defined as

$$M_u' = F^\dagger M_u G_u, \quad M_d' = F^\dagger M_d G_d, \quad (18)$$

with arbitrary unitary matrices F, G_u, and G_d in the standard $SU_L(2) \times U_Y(1)$ model, and with $G_u = G_d$ in the $SU_L(2) \times SU_R(2) \times U_Y(1)$ model. The fact that quark and lepton mass matrices have the same form strongly suggests that the quarks and leptons belong to the same multiplets. So in this section we try to incorporate our mass matrix in the context of SO(10) GUTs. We consider two SO(10) symmetry breaking patterns:

$$\text{(i)}\quad SO(10) \to SU(4) \times SU_L(2) \times SU_R(2) \to SU_c(3) \times SU_L(2) \times SU_R(2) \times U(1) \to G_s,$$

$$\text{(ii)}\quad SO(10) \to SU(5) \to G_s, \quad (19)$$

where $G_s = SU_c(3) \times SU_L(2) \times U(1)$.

A. The case of SO(10) breaking down to $SU(4) \times SU_L(2) \times SU_R(2)$

Here we consider the charge-conjugation-conserving (CCC) version [18–20] of the SO(10) model in which left-right discrete (not manifest) symmetry is imposed.

In the SO(10) model [21–23], the left- (right-) handed fermions $\psi_{L(R)i}$ in a given ith generation are assigned to a single irreducible **16**. Since $\mathbf{16} \times \mathbf{16} = \mathbf{10}_S + \mathbf{120}_A + \mathbf{126}_S$, the fermion masses are generated when the Higgs fields of **10**, and **120**, and **126** dimensional SO(10) representation (denoted by ϕ_{10}, ϕ_{120}, and ϕ_{126}, respectively) develop nonvanishing expectation values. Their decomposition under $SU(4) \times SU_L(2) \times SU_R(2)$ are given by

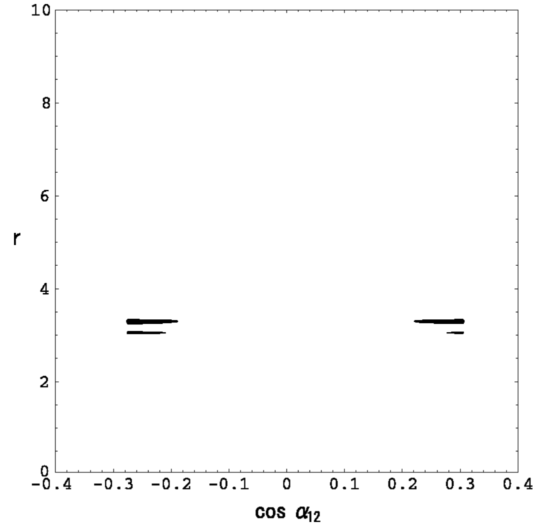

FIG. 1. The case where M_u, M_d, and M_e are all of purely type I is analyzed. The experimental constraint on $|U_{23}|$ $(0.28 \leqslant |U_{23}|^2 \leqslant 0.72)$ gives the allowed region (four thick lines) in the cos α_{12}-r plane. Here the r is treated as a free parameter.

$$\mathbf{10} = (\mathbf{6},\mathbf{1},\mathbf{1}) + (\mathbf{1},\mathbf{2},\mathbf{2}),$$

$$\mathbf{120} = (\mathbf{15},\mathbf{2},\mathbf{2}) + (\mathbf{6},\mathbf{3},\mathbf{1}) + (\mathbf{6},\mathbf{1},\mathbf{3}) + (\mathbf{1},\mathbf{2},\mathbf{2}) + (\mathbf{10},\mathbf{1},\mathbf{1}) + (\overline{\mathbf{10}},\mathbf{1},\mathbf{1}), \tag{20}$$

$$\mathbf{126} = (\mathbf{10},\mathbf{1},\mathbf{3}) + (\overline{\mathbf{10}},\mathbf{3},\mathbf{1}) + (\mathbf{15},\mathbf{2},\mathbf{2}) + (\mathbf{6},\mathbf{1},\mathbf{1}).$$

On the other hand, the fermion field of 16-dimensinal SO(10) representation is decomposed as

$$\mathbf{16} = (\mathbf{4},\mathbf{2},\mathbf{1}) + (\overline{\mathbf{4}},\mathbf{1},\mathbf{2}). \tag{21}$$

With respect to $SU(4) \times SU_L(2) \times SU_R(2)$, the left- and right-handed quarks and leptons of a given ith generation are assigned as

$$\begin{pmatrix} u_r & u_y & u_b & \nu_e \\ d_r & d_y & d_b & e \end{pmatrix}_{L(R)} \equiv F_{L(R)1}, \tag{22}$$

and $F_{L(R)2}$ and $F_{L(R)3}$ are likewise defined for the 2nd and 3rd generations. Note that their transformation properties are $F_{Li} = (\mathbf{4},\mathbf{2},\mathbf{1})$ and $F_{Ri} = (\mathbf{4},\mathbf{1},\mathbf{2})$ and that $(F_{Li} + \overline{F_{Ri}})$ yields the **16** of SO(10). Since $(\mathbf{4},\mathbf{2},\mathbf{1}) \times (\overline{\mathbf{4}},\mathbf{1},\mathbf{2}) = (\mathbf{15},\mathbf{2},\mathbf{2}) + (\mathbf{1},\mathbf{2},\mathbf{2})$, the Dirac masses for quarks and leptons are generated when neutral components in a $(\mathbf{1},\mathbf{2},\mathbf{2})$ multiplet in ϕ_{10}, $(\mathbf{1},\mathbf{2},\mathbf{2})$ and $(\mathbf{15},\mathbf{2},\mathbf{2})$ in ϕ_{120}, and $(\mathbf{15},\mathbf{2},\mathbf{2})$ in ϕ_{126} of $SU(4) \times SU_L(2) \times SU_R(2) \subset SO(10)$ develop nonvanishing expectation values. On the other hand, the $(\overline{\mathbf{10}},\mathbf{3},\mathbf{1})$ and $(\mathbf{10},\mathbf{1},\mathbf{3})$ in ϕ_{126} are responsible for the left- and the right-handed Majorana neutrino masses through the Higgs-lepton-lepton interactions $(\overline{\mathbf{10}},\mathbf{3},\mathbf{1})(\mathbf{4},\mathbf{2},\mathbf{1})(\mathbf{4},\mathbf{2},\mathbf{1})$ and $(\mathbf{10},\mathbf{1},\mathbf{3})(\overline{\mathbf{4}},\mathbf{1},\mathbf{2})(\overline{\mathbf{4}},\mathbf{1},\mathbf{2})$, respectively. Here the $(\overline{\mathbf{10}},\mathbf{3},\mathbf{1})$ is the $SU_L(2)$ Higgs triplet $[$denoted by $\phi(\overline{\mathbf{10}},\mathbf{3},\mathbf{1})]$ and the $(\mathbf{10},\mathbf{1},\mathbf{3})$ is the $SU_R(2)$ Higgs triplet $[\phi(\mathbf{10},\mathbf{1},\mathbf{3})]$.

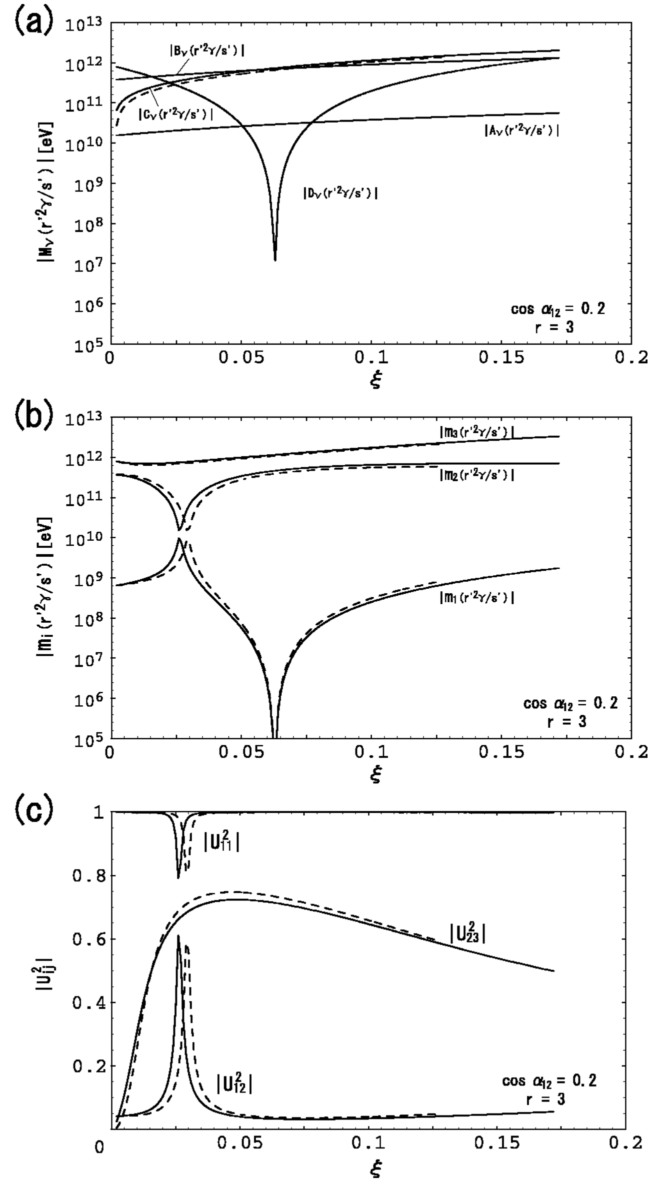

FIG. 2. The slight deviation from type I ($\xi \neq 0$) makes physical parameters change drastically . The dotted lines (solid lines) show the ξ dependence for cos $\alpha_{23} \geqslant 0$ (cos $\alpha_{23} \leqslant 0$) in each diagram. All lines terminate at the points from where $|\cos\alpha_{23}| \geqslant 1$ or $|\cos\beta_{23}| \geqslant 1$ as will be seen from Fig. 9 and Fig. 11. (a) The diagram of the elements in the neutrino mass matrix M_ν versus ξ. Except for $|C_\nu(r'^2\gamma/s')|$ the dotted lines are overlapped with the corresponding solid lines. (b) The diagram of the neutrino mass eigenvalues versus ξ. (c) The MNS mixing matrices versus ξ.

In the CCC version of the SO(10) model , the mass matrices M_u, M_d, M_e, M_D, M_L, and M_R, for up quarks, down quarks, charged leptons, Dirac neutrinos, left-handed Majorana neutrinos, and right-handed Majorana neutrinos, respectively, are given, in the lowest tree level, by

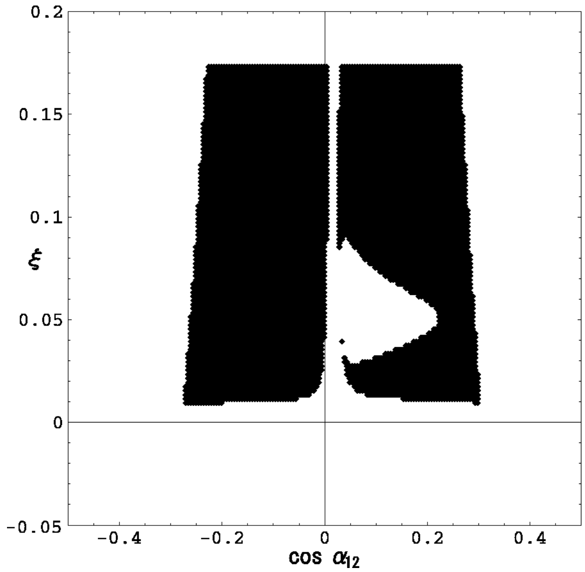

FIG. 3. The experimental constraint on $|U_{23}|$ $(0.28\leqslant|U_{23}|^2\leqslant 0.72)$ gives the allowed region (dotted area) in the $\cos\alpha_{12}$-ξ plane.

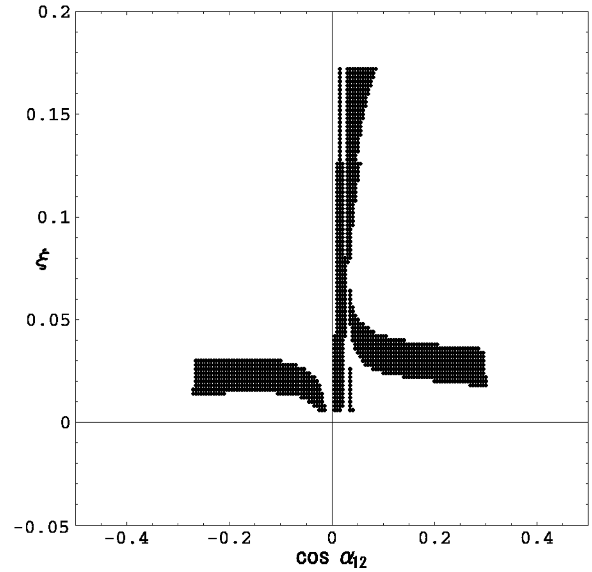

FIG. 4. The allowed region in the $\cos\alpha_{12}$-ξ plane from the experimental constraints $\Delta m_{12}^2/\Delta m_{23}^2\leqslant(1\times10^{-4})/(3.5\times10^{-3})=2.9\times10^{-2}$.

$$
\begin{aligned}
M_u &= S^{(10)}\langle\phi_+^1\rangle + A^{(120)}\left(\langle\phi_+^3\rangle + \frac{1}{3}\langle\phi_+^{3'}\rangle\right) + S^{(126)}\frac{1}{3}\langle\phi_+^5\rangle,\\
M_d &= S^{(10)}\langle\phi_-^1\rangle + A^{(120)}\left(-\langle\phi_-^3\rangle + \frac{1}{3}\langle\phi_-^{3'}\rangle\right) - S^{(126)}\frac{1}{3}\langle\phi_-^5\rangle,\\
rM_e &= S^{(10)}\langle\phi_-^1\rangle + A^{(120)}(-\langle\phi_-^3\rangle - \langle\phi_-^{3'}\rangle) + S^{(126)}\langle\phi_-^5\rangle, \qquad (23)\\
r'M_D &= S^{(10)}\langle\phi_+^1\rangle + A^{(120)}(\langle\phi_+^3\rangle - \langle\phi_+^{3'}\rangle) - S^{(126)}\langle\phi_+^5\rangle,\\
sM_L &= S^{(126)}\langle\phi(\overline{\mathbf{10}},\mathbf{3},\mathbf{1})\rangle,\\
s'M_R &= S^{(126)}\langle\phi(\mathbf{10},\mathbf{1},\mathbf{3})\rangle,
\end{aligned}
$$

where $\langle\phi_\pm^1\rangle$ are the vacuum expectation values of the Higgs fields of ϕ_{10}, $\langle\phi_\pm^3\rangle$ and $\langle\phi_\pm^{3'}\rangle$ of ϕ_{120}, and $\langle\phi_\pm^5\rangle$, $\langle\phi(\overline{\mathbf{10}},\mathbf{3},\mathbf{1})\rangle$ and $\langle\phi(\mathbf{10},\mathbf{1},\mathbf{3})\rangle$ of ϕ_{126}. See Ref. [20] for details and the notations. The matrices $S^{(10)}$ and $S^{(126)}$ are real symmetric matrices and $A^{(120)}$ is a pure imaginary matrix. These matrices are the coupling-constant matrices which appear in the Yukawa coupling of fermion fields with Higgs field, which is given by

$$
\begin{aligned}
2L_Y &= S_{ij}^{(10)}\overline{(\psi_{Li})^c}\phi_{10}\psi_{Lj} + A_{ij}^{(120)}\overline{(\psi_{Li})^c}\phi_{120}\psi_{Lj}\\
&\quad + S_{ij}^{(126)}\overline{(\psi_{Li})^c}\phi_{126}\psi_{Lj} + (L\leftrightarrow R) + \text{H.c.} \qquad (24)
\end{aligned}
$$

The $\psi_{L(R)i}$ are the 16 irreducible representations of the left- and right- handed fermion fields in a given ith generation. The property that $S^{(10)}$ and $S^{(126)}$ are symmetric and $A^{(120)}$ is

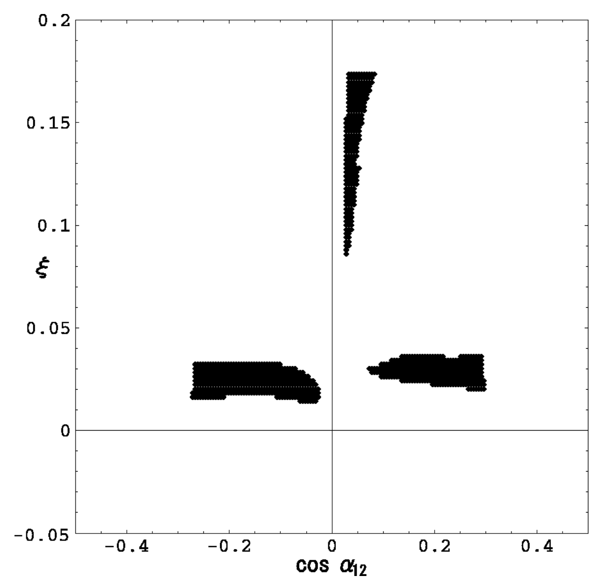

FIG. 5. The allowed region in the $\cos\alpha_{12}$-ξ plane from the experimental constraints $0.28\leqslant|U_{23}|^2\leqslant 0.72$ and $\Delta m_{12}^2/\Delta m_{23}^2\leqslant 2.9\times10^{-2}$.

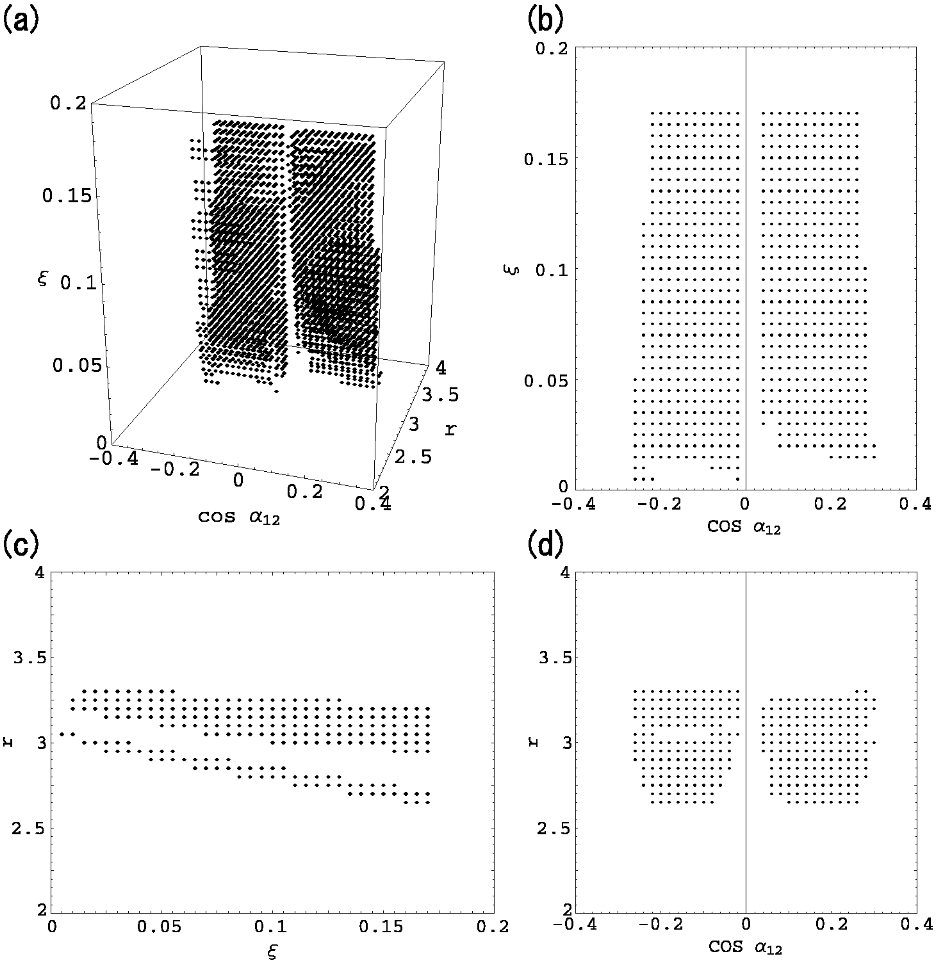

FIG. 6. The r is treated as a free parameter. Figure (a) shows the allowed region in the $\cos\alpha_{12}$-r-ξ space from the experimental constraints $0.28 \leqslant |U_{23}|^2 \leqslant 0.72$ and $\Delta m_{12}^2/\Delta m_{23}^2 \leqslant 2.9 \times 10^{-2}$. Figures (b), (c), and (d) show the projected allowed regions in the $\cos\alpha$-ξ, ξ-r, and $\cos\alpha$-r planes, respectively.

antisymmetric results from the decomposition $\mathbf{16}\times\mathbf{16}=\mathbf{10}_S+\mathbf{120}_A+\mathbf{126}_S$, whereas the property that $S^{(10)}$ and $S^{(126)}$ are real and $A^{(120)}$ is purely imaginary is a consequence of their being Hermitian, which in turn comes from the requirement of the invariance of L_Y under the discrete symmetry $\psi_L \leftrightarrow \psi_R^c$ [20]. In Eq. (23), the factors $r \simeq (2\sim 3)$, r', s, and s', all roughly of order unity, are the renormalization-group-equation factors [24,19] which arise from the differences in the renormalization of the lepton and quark masses due to the color quantum numbers of the quarks and so on. The overall factor comes from the loop correction of gauge boson in the renormalization group equation. Exactly we should consider the evolution equation of Yukawa coupling and in this case mass matrices get renormalized in a somewhat different form. Therefore, this form is an approximation. We will also discuss this point in the last section.

We now make the following assumptions.

(i) The contribution from **120** is assumed to be small compared with the contributions from **10** and **126**, and hence it is neglected in M_u and M_D. On the other hand, it is retained in M_d and M_e, for the main term $S^{(10)}\langle\phi_-^1\rangle$ is smaller by the factor $\alpha = \langle\phi_-^1\rangle/\langle\phi_+^1\rangle$, which is of order of (m_b/m_t) [see Eq. (27)]. This is an assumption for simplicity in order to incorporate Eq. (6).

(ii) All the vacuum expectation values of Higgs fields are assumed to be real so that all the fermion mass matrices are Hermitian.

With these assumptions, Eq. (24) becomes

$$\begin{aligned} M_u &= S+\epsilon S', \\ M_d &= \alpha S + S' + A' = \alpha M_u + A' - (\alpha\epsilon - 1)S', \\ rM_e &= \alpha S - 3S' + \delta A' = \alpha M_u + \delta A' - (\alpha\epsilon + 3)S', \\ r'M_D &= S - 3\epsilon S', \\ sM_L &= \beta S', \\ s'M_R &= \gamma S', \end{aligned} \tag{25}$$

where the matrices S, S', and A' and the real parameters α, β, γ, and δ are defined by

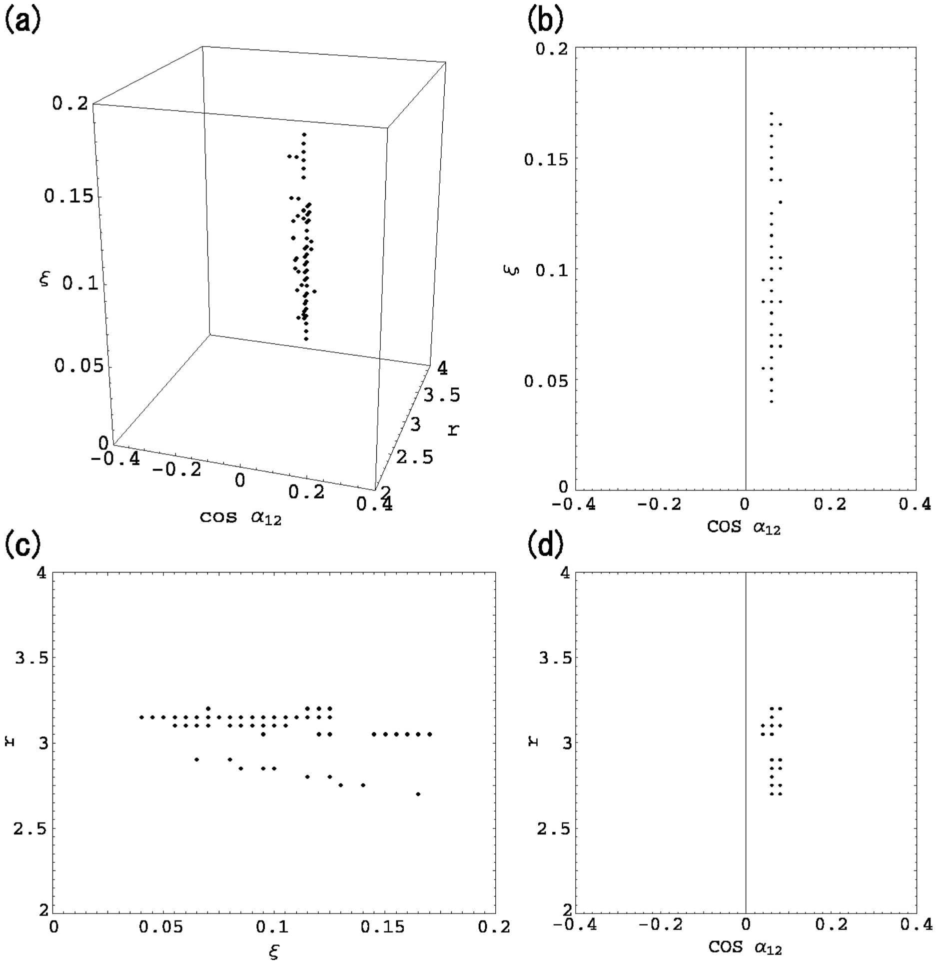

FIG. 7. The allowed region in the $\cos\alpha$-r-ξ space from the experimental constraints $0.28 \leqslant |U_{23}|^2 \leqslant 0.72$, the small mixing angle solution of the solar neutrino experiments $[\sin^2 2\theta_{12} = (2-10)\times 10^{-3}]$, and the up-to-date value of mass difference $\Delta m_{12}^2/\Delta m_{23}^2 = [(4-10)\times 10^{-6}]/[(1.5-6)\times 10^{-3}] = (0.67-6.7)\times 10^{-3}$.

$$S = S^{(10)}\langle \phi_+^1 \rangle, \quad S' = S^{(126)}\left(-\frac{1}{3}\langle \phi_-^5 \rangle \right),$$

$$A' = A^{(120)}\left(-\langle \phi_-^3 \rangle + \frac{1}{3}\langle \phi_-^{3'} \rangle \right),$$

$$\alpha = \langle \phi_-^1 \rangle / \langle \phi_+^1 \rangle, \quad \beta = \langle \phi(\overline{\mathbf{10}},\mathbf{3},\mathbf{1}) \rangle \Big/ \left(-\frac{1}{3}\langle \phi_-^5 \rangle \right), \tag{26}$$

$$\gamma = \langle \phi(\mathbf{10},\mathbf{1},\mathbf{3}) \rangle \Big/ \left(-\frac{1}{3}\langle \phi_-^5 \rangle \right),$$

$$\delta = (\langle \phi_-^3 \rangle + \langle \phi_-^{3'} \rangle) \Big/ \left(\langle \phi_-^3 \rangle - \frac{1}{3}\langle \phi_-^{3'} \rangle \right).$$

Note that solving diagonal elements of Eq. (25) for α, one finds

$$\alpha = \frac{3\,\mathrm{Tr}\,M_d + r\,\mathrm{Tr}\,M_e}{3\,\mathrm{Tr}\,M_u + r'\,\mathrm{Tr}\,M_D} \simeq \frac{m_b}{m_t}, \tag{27}$$

which is about 0.02. As mentioned already, this is why the $A^{(120)}$ and $S^{(126)}$ terms are kept in M_d and M_e. Equation (25) is our SO(10)-motivated model for fermion mass matrices.

B. The case of SO(10) breaking down to $SU(5)$

In this case, the fermion masses are also generated when the Higgs fields of **10**, and **120**, and **126** dimensional SO(10) representation (denoted by ϕ_{10}, ϕ_{120}, and ϕ_{126}, respectively) develop nonvanishing expectation values. Their decomposition under $SU(5)$ are given by

$$\mathbf{10} = \mathbf{5} + \overline{\mathbf{5}},$$

$$\mathbf{120} = \mathbf{5} + \overline{\mathbf{5}} + \mathbf{10} + \overline{\mathbf{10}} + \mathbf{45} + \overline{\mathbf{45}}, \tag{28}$$

$$\mathbf{126} = \mathbf{1} + \overline{\mathbf{5}} + \mathbf{10} + \overline{\mathbf{15}} + \mathbf{45} + \overline{\mathbf{50}}.$$

The Yukawa couplings in L_Y give the following fermion masses when the neutral components in a **5** and $\overline{\mathbf{5}}$ Higgs multiplet in ϕ_{10}, **5**, $\overline{\mathbf{5}}$, **45**, and $\overline{\mathbf{45}}$ in ϕ_{120}, and **1**, $\overline{\mathbf{5}}$, $\overline{\mathbf{15}}$, and **45** in ϕ_{126} of $SU(5) \subset SO(10)$ develop nonvanishing expectation values [25,23]:

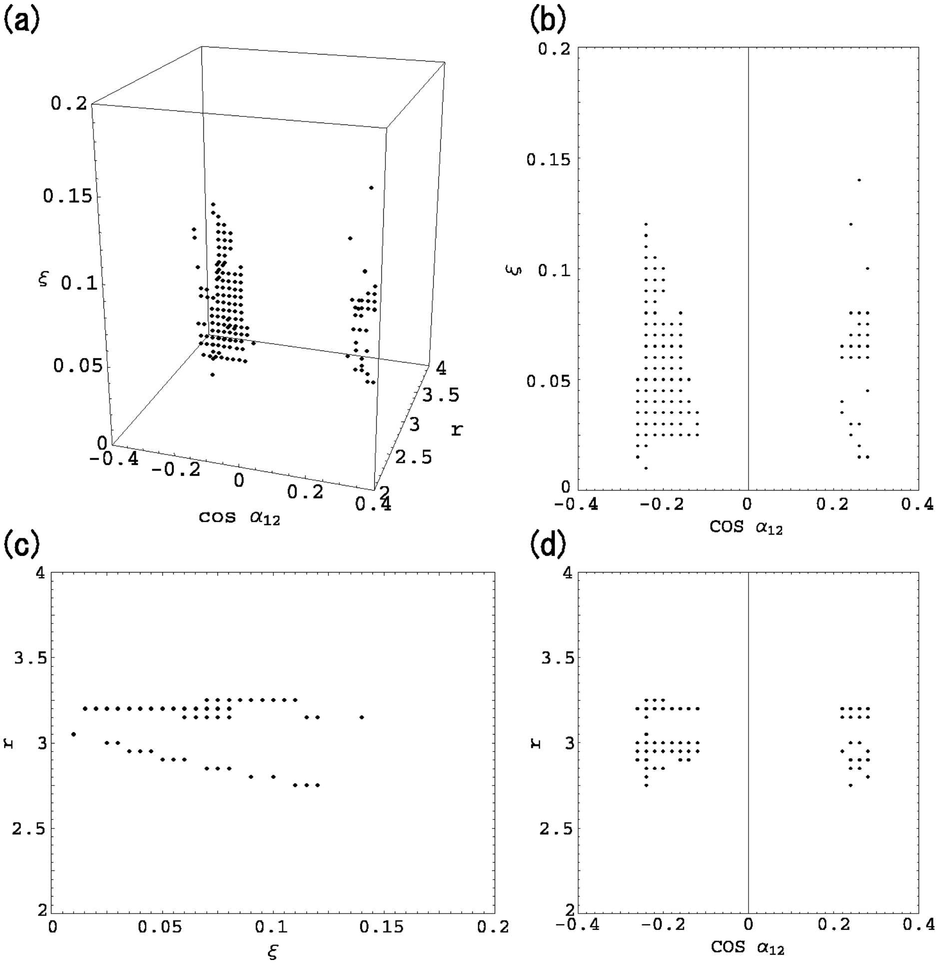

FIG. 8. The allowed region in the cos α_{12}-r-ξ space from the experimental constraints $0.28 \leqslant |U_{23}|^2 \leqslant 0.72$, the large mixing angle solution of the solar neutrino experiments $\sin^2 2\theta_{12} = (0.5\text{–}1)$, and the up-to-date value of mass difference $\Delta m_{12}^2/\Delta m_{23}^2 = [(8\text{–}30)\times 10^{-6}]/[(1.5\text{–}6)\times 10^{-3}] = (0.13\text{–}2.0)\times 10^{-3}$.

$$S_{ij}^{(10)}\overline{(\psi_{Li})^c}\phi_{10}\psi_{Lj} \rightarrow S_{ij}^{(10)}\{\langle\phi_{10}(\mathbf{5})\rangle(\overline{u_{R,i}}u_{L,j}+\overline{\nu'_{R,i}}\nu_{L,j}) + \langle\phi_{10}(\overline{\mathbf{5}})\rangle(\overline{d_{R,i}}d_{L,j}+\overline{e_{R,i}}e_{L,j})\},$$

$$A_{ij}^{(120)}\overline{(\psi_{Li})^c}\phi_{120}\psi_{Lj} \rightarrow A_{ij}^{(120)}\{\langle\phi_{120}(\overline{\mathbf{5}})\rangle(\overline{d_{R,i}}d_{L,j}+\overline{e_{R,i}}e_{L,j}) + \langle\phi_{120}(\overline{\mathbf{45}})\rangle(\overline{d_{R,i}}d_{L,j}-3\overline{e_{R,i}}e_{L,j}) + \langle\phi_{120}(\mathbf{5})\rangle\overline{\nu'_{R,i}}\nu_{L,j} + \langle\phi_{120}(\mathbf{45})\rangle\overline{u_{R,i}}u_{L,j}\}, \quad (29)$$

$$S_{ij}^{(126)}\overline{(\psi_{Li})^c}\phi_{126}\psi_{Lj} \rightarrow S_{ij}^{(126)}\{\langle\phi_{126}(\mathbf{5})\rangle(\overline{u_{R,i}}u_{L,j}-3\overline{\nu_{R,i}}\nu_{L,j}) + \langle\phi_{126}(\overline{\mathbf{45}})\rangle(\overline{d_{R,i}}d_{L,j}-3\overline{e_{R,i}}e_{L,j}) + \langle\phi_{126}(\mathbf{1})\rangle\overline{\nu'^{c}_{R,i}}\nu'_{R,j} + \langle\phi_{126}(\mathbf{15})\rangle\overline{\nu^{c}_{L,i}}\nu_{L,j}\},$$

Therefore, the mass matrices M_u, M_d, M_e, M_D, M_L, and M_R, for up quarks, down quarks, charged leptons, Dirac neutrinos, left-handed Majorana neutrinos, and right-handed Majorana neutrinos, respectively, are given by

$$M_u = S^{(10)}\langle\phi_{10}(\mathbf{5})\rangle + A^{(120)}\langle\phi_{120}(\mathbf{45})\rangle + S^{(126)}\langle\phi_{126}(\mathbf{5})\rangle,$$

$$M_d = S^{(10)}\langle\phi_{10}(\overline{\mathbf{5}})\rangle + A^{(120)}(\langle\phi_{120}(\overline{\mathbf{5}})\rangle + \langle\phi_{120}(\overline{\mathbf{45}})\rangle) + S^{(126)}\langle\phi_{126}(\overline{\mathbf{45}})\rangle,$$

$$rM_e = S^{(10)}\langle\phi_{10}(\overline{\mathbf{5}})\rangle + A^{(120)}(\langle\phi_{120}(\overline{\mathbf{5}})\rangle - 3\langle\phi_{120}(\overline{\mathbf{45}})\rangle) - 3S^{(126)}\langle\phi_{126}(\overline{\mathbf{45}})\rangle, \quad (30)$$

$$r'M_D = S^{(10)}\langle\phi_{10}(\mathbf{5})\rangle + A^{(120)}\langle\phi_{120}(\mathbf{5})\rangle - 3S^{(126)}\langle\phi_{126}(\mathbf{5})\rangle,$$

$$sM_L = S^{(126)}\langle\phi_{126}(\mathbf{15})\rangle,$$

$$s'M_R = S^{(126)}\langle\phi_{126}(\mathbf{1})\rangle,$$

These mass matrices reduce to the same form as Eq. (25) by assuming again that the contributions from the **120** Higgs representation in M_u and M_D are negligible and by defining the matrices S, S', and A' and the real parameters α, β, γ, and δ, instead of Eq. (26), as

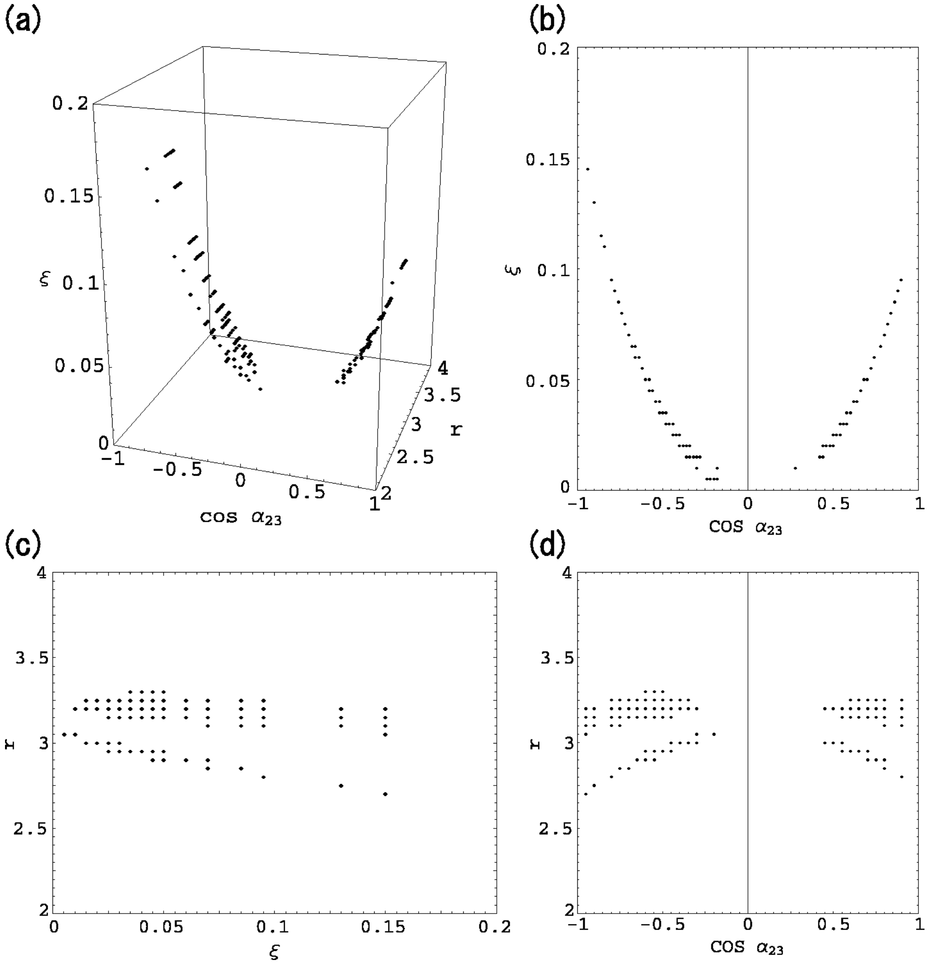

FIG. 9. The allowed region in the $\cos\alpha_{23}$-r-ξ space from the same constraints as in Fig. 6.

$$S=S^{(10)}\langle\phi_{10}(\mathbf{5})\rangle,\quad S'=S^{(126)}\langle\phi_{126}(\overline{\mathbf{45}})\rangle,$$

$$A'=A^{(120)}(\langle\phi_{120}(\overline{\mathbf{5}})\rangle+\langle\phi_{120}(\overline{\mathbf{45}})\rangle),$$

$$\alpha=\langle\phi_{10}(\overline{\mathbf{5}})\rangle/\langle\phi_{10}(\mathbf{5})\rangle,\quad \beta=\langle\phi_{126}(\mathbf{15})\rangle/\langle\phi_{126}(\overline{\mathbf{45}})\rangle, \tag{31}$$

$$\gamma=\langle\phi_{126}(\mathbf{1})\rangle/\langle\phi_{126}(\overline{\mathbf{45}})\rangle,$$

$$\delta=(\langle\phi_{120}(\overline{\mathbf{5}})\rangle-3\langle\phi_{120}(\overline{\mathbf{45}})\rangle)/(\langle\phi_{120}(\overline{\mathbf{5}})\rangle+\langle\phi_{120}(\overline{\mathbf{45}})\rangle),$$

Thus, Eq. (25) is our SO(10)-motivated model for fermion mass matrices for both the SO(10) breaking patterns (i) and (ii) in Eq. (19).

IV. FOUR TEXTURE ZERO MODEL IN SO(10)

The SO(10) model Eq. (25) is now combined with the four texture zero *Ansätze* for M_u, M_d, and M_e which are given by Eq. (6).

First it follows from Eq. (25) that S, S', and A' are represented in terms of the symmetric (antisymmetric) parts, M_{sym} ($M_{antisym}$), of M_u, M_d, and M_e:

$$(1-\alpha\epsilon)S=(M_u)_{sym}-\epsilon(M_d)_{sym},$$

$$S'=\frac{1}{4}\{(M_d)_{sym}-r(M_e)_{sym}\}, \tag{32}$$

$$A'=(M_d)_{antisym}.$$

We also find the constraints

$$(1-\alpha\epsilon)r(M_e)_{sym}=4\alpha(M_u)_{sym}-(3+\alpha\epsilon)(M_d)_{sym},$$

$$\delta(M_d)_{antisym}=r(M_e)_{antisym}. \tag{33}$$

Using the four texture zero *Ansätze* for M_u, M_d, and M_e given by Eq. (6), the respective elements of Eq. (33) become

$$(1-\alpha\epsilon)rA_e\cos\beta_{12}=4\alpha A_u-(3+\alpha\epsilon)A_d\cos\alpha_{12},$$

$$(1-\alpha\epsilon)rB_e=4\alpha B_u-(3+\alpha\epsilon)B_d,$$

$$(1-\alpha\epsilon)rC_e\cos\beta_{23}=4\alpha C_u-(3+\alpha\epsilon)C_d\cos\alpha_{23},$$

$$(1-\alpha\epsilon)rD_e=4\alpha D_u-(3+\alpha\epsilon)D_d, \tag{34}$$

$$\delta A_d\sin\alpha_{12}=rA_e\sin\beta_{12},$$

$$\delta C_d\sin\alpha_{23}=rC_e\sin\beta_{23}.$$

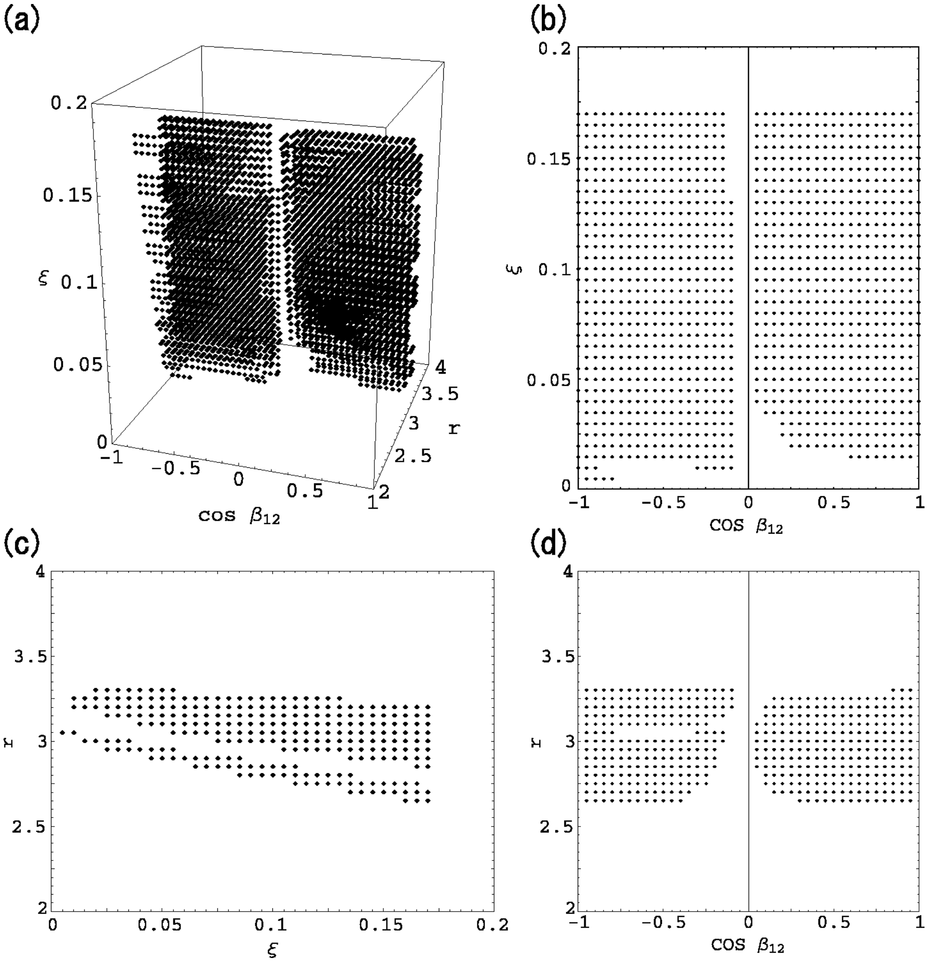

FIG. 10. The allowed region in the $\cos\beta_{12}$-r-ξ space from the same constraints as in Fig. 6.

In Eq. (34) there are six equations and eight unknown parameters, namely α, ϵ, δ, α_{12}, α_{23}, β_{12}, β_{23}, and r provided that $A_u, \quad B_u, \ldots, D_e$ are given. In the following, we treat $\cos\alpha_{12}$ and r as free parameters so that all the other parameters are functions of them. Here we treat r as a free parameter too, although we know $r \simeq (2\text{–}3)$. Let us present the following useful expressions which are derived from Eq. (34):

$$\cos\beta_{12} = \left(\frac{B_eD_d - D_eB_d}{B_uD_d - D_uB_d}\right)\left(\frac{A_u}{A_e}\right) - \left(\frac{B_eD_u - D_eB_u}{B_uD_d - D_uB_d}\right)\left(\frac{A_d}{A_e}\right)\cos\alpha_{12},$$

$$\cos\beta_{23} = \left(\frac{B_eD_d - D_eB_d}{B_uD_d - D_uB_d}\right)\left(\frac{C_u}{C_e}\right) - \left(\frac{B_eD_u - D_eB_u}{B_uD_d - D_uB_d}\right)\left(\frac{C_d}{C_e}\right)\cos\alpha_{23},$$

$$\frac{\sin\beta_{23}}{\sin\alpha_{23}} = \left(\frac{A_eC_d}{A_dC_e}\right)\frac{\sin\beta_{12}}{\sin\alpha_{12}},$$

$$\alpha = \frac{r\left(\dfrac{B_eD_d - D_eB_d}{B_uD_d - D_uB_d}\right)}{r\left(\dfrac{B_eD_u - D_eB_u}{B_uD_d - D_uB_d}\right) + 1}, \quad \epsilon = \frac{r\left(\dfrac{B_eD_u - D_eB_u}{B_uD_d - D_uB_d}\right) - 3}{r\left(\dfrac{B_eD_d - D_eB_d}{B_uD_d - D_uB_d}\right)},$$

$$\delta = r\left(\frac{A_e}{A_d}\right)\frac{\sin\beta_{12}}{\sin\alpha_{12}}. \tag{35}$$

Now we discuss the Maki-Nakagawa-Sakata (MNS) lepton mixing matrix and neutrino masses. The light Majorana neutrino mass matrix M_ν is given by Eq. (3), where the Dirac neutrino, left- and right-handed Majorana neutrino mass matrices M_D, M_L, and M_R are expressed in terms of the entries of the quarks and charged lepton mass matrices due to the SO(10) constraints and their expressions are given, from Eqs. (25) and (32), by

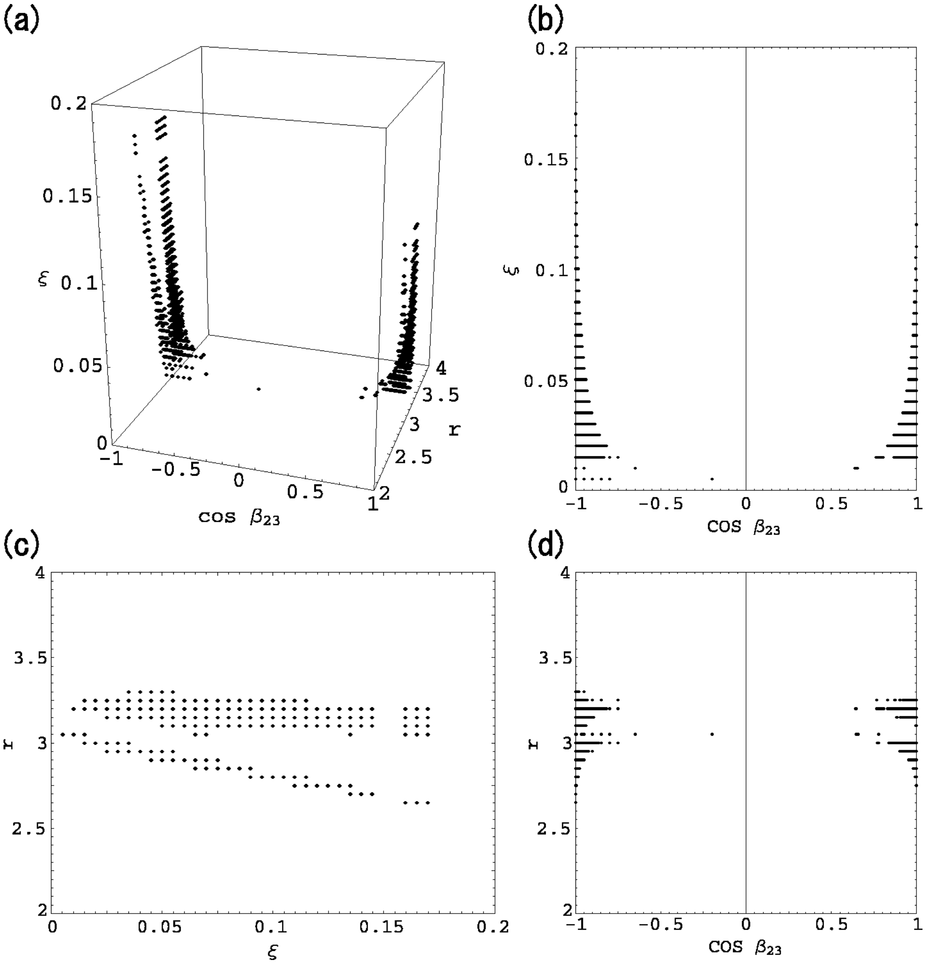

FIG. 11. The allowed region in the $\cos\beta_{23}$-r-ξ space from the same constraints as in Fig. 6.

$$r'M_D=S-3\epsilon S'=\begin{pmatrix}0 & A_D & 0\\ A_D & B_D & C_D\\ 0 & C_D & D_D\end{pmatrix},$$

$$sM_L=\beta S'=\beta\begin{pmatrix}0 & A_{S'} & 0\\ A_{S'} & B_{S'} & C_{S'}\\ 0 & C_{S'} & D_{S'}\end{pmatrix}, \tag{36}$$

$$s'M_R=\gamma S'=\gamma\begin{pmatrix}0 & A_{S'} & 0\\ A_{S'} & B_{S'} & C_{S'}\\ 0 & C_{S'} & D_{S'}\end{pmatrix},$$

where the elements A_D, B_D, C_D, C_D, $A_{S'}$, $B_{S'}$, $C_{S'}$, and $C_{S'}$ are obtained as

$$A_D=A_u-\epsilon(A_d\cos\alpha_{12}-rA_e\cos\beta_{12}),$$

$$B_D=B_u-\epsilon(B_d-rB_e),$$

$$C_D=C_u-\epsilon(C_d\cos\alpha_{23}-rC_e\cos\beta_{23}), \tag{37}$$

$$D_D=D_u-\epsilon(D_d-rD_e),$$

and

$$A_{S'}=\frac{1}{4}(A_d\cos\alpha_{12}-rA_e\cos\beta_{12}),$$

$$B_{S'}=\frac{1}{4}(B_d-rB_e),$$

$$C_{S'}=\frac{1}{4}(C_d\cos\alpha_{23}-rC_e\cos\beta_{23}), \tag{38}$$

$$D_{S'}=\frac{1}{4}(D_d-rD_e).$$

In the following analysis, we assume that the contribution of M_L to M_ν in Eq. (3) is much smaller than that of the second term so that we have $M_\nu=M_L-M_D^TM_R^{-1}M_D\simeq -M_D^TM_R^{-1}M_D$. Then, all the components A_ν, B_ν, C_ν, and

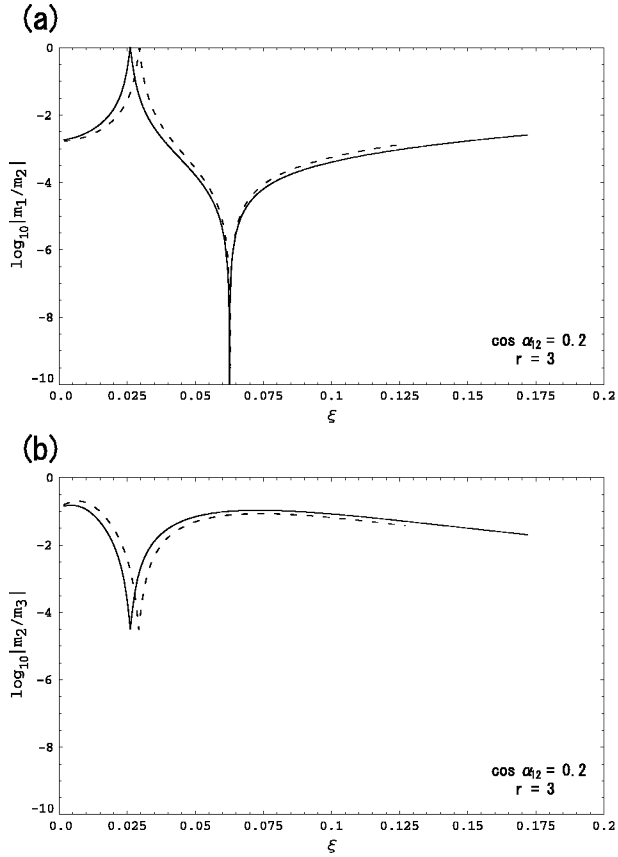

FIG. 12. The dependence of $\log_{10}|m_1/m_2|$ (a) and $\log_{10}|m_2/m_3|$ (b) on ξ for $\cos\alpha_{12}=0.2$ and $r=3$. The dotted line (solid line) shows the ξ dependence for $\cos\alpha_{23}\geqslant 0$ $(\cos\alpha_{23}\leqslant 0)$ in each diagram. Both lines terminate at the points from where $|\cos\alpha_{23}|\geqslant 1$ or $|\cos\beta_{23}|\geqslant 1$ as are seen from Fig. 9 and Fig. 11. The singular behaviors of (a) and (b) come from those of m_1 and m_2 (see Fig. 2).

D_ν in Eq. (6) are determined, from Eqs. (36), (37), and (38), as functions of $\cos\alpha_{12}$ and r except for the common overall factor $s'/(r'^2\gamma)$ as

$$M_\nu=\begin{pmatrix}0 & A_\nu & 0\\ A_\nu & B_\nu & C_\nu\\ 0 & C_\nu & D_\nu\end{pmatrix}$$

$$=-s'/(r'^2\gamma)\begin{pmatrix}0 & A_D & 0\\ A_D & B_D & C_D\\ 0 & C_D & D_D\end{pmatrix}$$

$$\times\begin{pmatrix}0 & A_{S'} & 0\\ A_{S'} & B_{S'} & C_{S'}\\ 0 & C_{S'} & D_{S'}\end{pmatrix}^{-1}\begin{pmatrix}0 & A_D & 0\\ A_D & B_D & C_D\\ 0 & C_D & D_D\end{pmatrix}, \qquad (39)$$

$$A_\nu=-\frac{s'}{r'^2\gamma}\left(\frac{A_D^2}{A_{S'}}\right),$$

$$B_\nu=-\frac{s'}{r'^2\gamma}\left\{\frac{A_DB_D}{A_{S'}}+C_D\left(\frac{C_D}{D_{S'}}-\frac{A_DC_{S'}}{A_{S'}D_{S'}}\right)\right.$$

$$\left.+A_D\left(\frac{B_D}{A_{S'}}-\frac{C_DC_{S'}}{A_{S'}D_{S'}}-\frac{A_D(-C_{S'}{}^2+B_{S'}D_{S'})}{A_{S'}{}^2D_{S'}}\right)\right\},$$

$$C_\nu=-\frac{s'}{r'^2\gamma}\left\{\frac{A_DC_D}{A_{S'}}+D_D\left(\frac{C_D}{D_{S'}}-\frac{A_DC_{S'}}{A_{S'}D_{S'}}\right)\right\},$$

$$D_\nu=-\frac{s'}{r'^2\gamma}\left(\frac{D_D^2}{D_{S'}}\right). \qquad (40)$$

Therefore, the neutrino mass ratios m_1/m_2 and m_2/m_3 and hence MNS lepton mixing matrix elements are also determined as functions of $\cos\alpha_{12}$ and r. The common overall factor $s'/(r'^2\gamma)$ is determined by the Δm^2 data from neutrino oscillation experiments. The light Majorana neutrino masses are obtained by diagonalizing M_ν as

$$O_\nu^T\begin{pmatrix}0 & A_\nu & 0\\ A_\nu & B_\nu & C_\nu\\ 0 & C_\nu & D_\nu\end{pmatrix}O_\nu=\begin{pmatrix}m_1 & 0 & 0\\ 0 & m_2 & 0\\ 0 & 0 & m_3\end{pmatrix}. \qquad (41)$$

For the case in which $B_\nu, C_\nu, D_\nu \gg A_\nu$ is satisfied, the neutrino masses are approximately expressed in terms of A_ν, B_ν, C_ν, and D_ν as

$$m_1 \simeq -\frac{D_\nu A_\nu^2}{B_\nu D_\nu - C_\nu^2},$$

$$m_2 \simeq \frac{1}{2}\{B_\nu + D_\nu - \sqrt{(B_\nu+D_\nu)^2 - 4(B_\nu D_\nu - C_\nu^2)}\} + \frac{D_\nu^2 + C_\nu^2 - \frac{1}{2}D_\nu\{B_\nu + D_\nu - \sqrt{(B_\nu+D_\nu)^2 - 4(B_\nu D_\nu - C_\nu^2)}\}}{(B_\nu D_\nu - C_\nu^2)\sqrt{(B_\nu+D_\nu)^2 - 4(B_\nu D_\nu - C_\nu^2)}} A_\nu^2, \tag{42}$$

$$m_3 \simeq \frac{1}{2}\{B_\nu + D_\nu + \sqrt{(B_\nu+D_\nu)^2 - 4(B_\nu D_\nu - C_\nu^2)}\} - \frac{D_\nu^2 + C_\nu^2 - \frac{1}{2}D_\nu\{B_\nu + D_\nu + \sqrt{(B_\nu+D_\nu)^2 - 4(B_\nu D_\nu - C_\nu^2)}\}}{(B_\nu D_\nu - C_\nu^2)\sqrt{(B_\nu+D_\nu)^2 - 4(B_\nu D_\nu - C_\nu^2)}} A_\nu^2.$$

The orthogonal matrices O_ν which diagonalize M_ν are expressed in terms of the diagonalized masses m_1, m_2, and m_3 and the matrix components A_ν, B_ν, C_ν, and D_ν as [14]

$$O_\nu = \begin{pmatrix} \frac{A_\nu}{m_1}(O_\nu)_{21} & \frac{A_\nu}{m_2}(O_\nu)_{22} & \frac{A_\nu}{m_3}(O_\nu)_{23} \\ (O_\nu)_{21} & (O_\nu)_{22} & (O_\nu)_{23} \\ \frac{C_\nu}{m_1 - D_\nu}(O_\nu)_{21} & \frac{C_\nu}{m_2 - D_\nu}(O_\nu)_{22} & \frac{C_\nu}{m_3 - D_\nu}(O_\nu)_{23} \end{pmatrix}, \tag{43}$$

with

$$(O_\nu)_{21}^2 = \frac{1}{\left(\frac{A_\nu}{m_1}\right)^2 + 1 + \left(\frac{C_\nu}{m_1 - D_\nu}\right)^2},$$

$$(O_\nu)_{22}^2 = \frac{1}{\left(\frac{A_\nu}{m_2}\right)^2 + 1 + \left(\frac{C_\nu}{m_2 - D_\nu}\right)^2}, \tag{44}$$

$$(O_\nu)_{23}^2 = \frac{1}{\left(\frac{A_\nu}{m_3}\right)^2 + 1 + \left(\frac{C_\nu}{m_3 - D_\nu}\right)^2}.$$

It should be remarked that the light neutrino mass matrix M_ν itself is out of type I via the seesaw mechanism and that the MNS lepton mixing matrix is obtained from Eqs. (43) and (17). Since O_e is almost diagonal, the magnitudes of off-diagonal elements are predominated by Eq. (44). Thus the seesaw mechanism changes the form of the lepton mixing matrix from that of the CKM matrix given by Eq. (16).

Now, by changing the values of the free parameters in our model, we proceed to find the solutions which are consistent with the recent following findings that (i) the atmospheric neutrino oscillation experiment indicates the ν_μ-ν_τ large mixing ($0.28 \lesssim |U_{23}|^2 \lesssim 0.72$ [26]) with $\Delta m_{23}^2 = m_3^2 - m_2^2 = (1.5\sim 6)\times 10^{-3} \simeq 3.5\times 10^{-3}$ eV^2, and (ii) the solar neutrino experiments imply the MSW small mixing angle solution [27] with $\Delta m_{12}^2 = m_2^2 - m_1^2 = (4-10)\times 10^{-6}$ eV^2 and $\sin^2 2\theta_{12} = (2-10)\times 10^{-3}$, or the large mixing angle solution [27] with $\Delta m_{12}^2 = m_2^2 - m_1^2 \simeq (8-30)\times 10^{-6}$ eV^2 and $\sin^2 2\theta_{12} = (0.5-1)$. In the following analysis, we transform A_e, B_e, C_e, and D_e in Eq. (6) into $-A_e$, $-B_e$, $-C_e$, and $-D_e$, respectively by rephasing of the right-handed charged lepton fields.

First assuming that the mass matrices M_u, M_d, and M_e are all of type I, we calculate numerically the MNS lepton mixing matrix U using the central values for the running quarks and charged leptons masses at $\mu = m_Z$ [28]:

$$m_u(m_Z) = 2.33^{+0.42}_{-0.45}\ \text{MeV},$$

$$m_c(m_Z) = 677^{+56}_{-61}\ \text{MeV}, \quad m_t(m_Z) = 181 \pm 13\ \text{GeV},$$

$$m_d(m_Z) = 4.69^{+0.60}_{-0.66}\ \text{MeV},$$

$$m_s(m_Z) = 93.4^{+11.8}_{-13.0}\ \text{MeV}, \quad m_b(m_Z) = 3.00 \pm 0.11\ \text{GeV}, \tag{45}$$

$$m_e(m_Z) = 0.487\ \text{MeV}, \quad m_\mu(m_Z) = 103\ \text{MeV},$$

$$m_\tau(m_Z) = 1.75\ \text{GeV}.$$

Since the recent atmospheric neutrino oscillation data indicates large value of (2,3) element of U, ($0.28 \lesssim |U_{23}|^2 \lesssim 0.72$ [26]), we obtain the allowed region of the parameters space, $\cos\alpha_{12}$ vs r space which reproduces a large $|U_{23}|$. The result is given in Fig. 1. In this allowed parameter region, $r \simeq 3$ is automatically satisfied without any fine tuning. However, we have a serious problem that in this allowed parameter space we cannot accommodate the overall factor $s'/(r'^2\gamma)$ simultaneously to the data $\Delta m_{12}^2 = m_2^2 - m_1^2 \lesssim 10^{-4}$ eV^2 (here we have adopted a rather conservative value; we accept more restrictive ones later) from solar neutrino oscillation experiments and the data $\Delta m_{23}^2 = m_3^2 - m_2^2 \simeq 3.5\times 10^{-3}$ eV^2 from atmospheric neutrino oscillation experiments. Taking deviations from the central values [28] for quarks and charged leptons masses does not resolve this problem. This difficulty is resolved by abandoning the above assumption that the mass matrices M_u, M_d, and M_e are all of type I. Let us assume that M_e deviates from type I although M_u and M_d are of type I. Then, we can accommodate the overall factor $s'/(r'^2\gamma)$ simultaneously to both Δm^2 data from solar and atmospheric neutrino oscillation experiments.

Next we discuss this new scenario and show that there are solutions consistent with the data. First we represent the deviation from type I as $B_e = m_\mu(1+\xi)$. In this case, the entries of the mass matrix M_e for charged leptons in Eq. (6) are given, in the unit of eV, as

$$A_e = \sqrt{\frac{m_e m_\mu m_\tau}{m_\tau - \xi m_\mu - m_e}} \simeq 7.1\times 10^6,$$

$$B_e = (1+\xi)m_\mu \simeq 1.02\times 10^8(1+\xi),$$

$$C_e = \sqrt{(m_e+\xi m_\mu)(m_\tau-(1+\xi)m_\mu - m_e)\left(\frac{m_\tau - \xi m_\mu}{m_\tau - \xi m_\mu - m_e}\right)} \simeq \sqrt{8.1\times 10^{14} + 1.7\times 10^{17}\xi},$$

$$D_e = -m_e - \xi m_\mu + m_\tau \simeq 7.1\times 10^9, \tag{46}$$

and the expressions for ϵ, α and the elements of the matrices S and S' are given by

$$\epsilon \simeq -\frac{(-r(1+\xi)m_\mu + 3m_s)m_t - m_c(3m_b - rm_\tau)}{r((1+\xi)m_b m_\mu - m_s m_\tau)},$$

$$\alpha \simeq -\frac{r((1+\xi)m_b m_\mu - (1+\xi)m_d m_\mu - m_s m_\tau)}{m_b m_c + (-rm_\mu - r\xi m_\mu - m_s)m_t - m_c(m_d - rm_\tau)},$$

$$S_{12} \simeq ((rm_\mu + m_s)[\cos\alpha_{12}\sqrt{m_d m_s}(-3m_b m_c - r(1+\xi)m_\mu m_t) + \cos\alpha_{12}\sqrt{m_d m_s}(3m_s m_t + rm_c m_\tau)$$
$$- r(-m_b m_\mu + m_s m_\tau)\sqrt{m_c m_u}])/(4rm_s(m_b m_\mu - m_s m_\tau)),$$

$$S_{22} \simeq -\frac{(-rm_\mu - m_s)(-r(1+\xi)m_\mu + 3m_s)m_t}{4r(m_b m_\mu - m_s m_\tau)},$$

$$S_{23} \simeq (-\cos\alpha_{23}\sqrt{m_b m_d}(-rm_\mu - m_s)m_t[-3m_b m_c - r(1+\xi)m_\mu m_t + 3m_s m_t + rm_c m_\tau]$$
$$- r(-rm_\mu - m_s)m_t[(1+\xi)m_b m_\mu - m_s m_\tau]\sqrt{m_t m_u})/[r^2 m_\tau[m_b m_c m_\mu + m_s(\xi m_\mu m_t - m_c m_\tau)]$$
$$- r(3m_b^2 m_c m_\mu + 4m_s^2 m_t m_\tau - m_b m_s[(4+\xi)m_\mu m_t + 3m_c m_\tau])],$$

$$S_{33} \simeq -\frac{(-rm_\mu - m_s)m_t(3m_b - rm_\tau)}{4r(m_b m_\mu - m_s m_\tau)},$$

$$S'_{12} \simeq \frac{(\cos\alpha_{12}\sqrt{m_d m_s}(rm_\mu + m_s)m_t - r(m_b m_\mu - m_s m_\tau)\sqrt{m_c m_u})}{4m_s m_t},$$

$$S'_{22} \simeq \frac{1}{4}(rm_\mu + m_s),$$

$$S'_{23} \simeq -\left(\cos\alpha_{23}\sqrt{m_b m_d} + \frac{r(m_b m_\mu - m_s m_\tau)m_u}{(-rm_\mu - m_s)\sqrt{m_t m_u}}\right) \Bigg/ \left(-1 + \frac{(m_b m_\mu - m_s m_\tau)([-r(1+\xi)m_\mu + 3m_s]m_t - m_c(3m_b - rm_\tau))}{(-rm_\mu - m_s)m_t((1+\xi)m_b m_\mu - m_s m_\tau)}\right),$$

$$S'_{33} \simeq \frac{1}{4}(m_b - m_d + rm_\tau). \tag{47}$$

The point $r\simeq 3$ is rather singular in the following sense. As is seen from Eq. (35), ϵ becomes small at $r\simeq 3$, hence we cannot neglect ξ in this region. That is, ϵ is sensitive to small ξ. For instance, substituting Eq. (47) into Eqs. (25) we obtain M_D and M_R and therefore M_ν through Eq. (39). The behaviors of the elements in the neutrino mass matrix M_ν are depicted in Fig. 2. The large ν_μ-ν_τ neutrino mixing appears under the condition that $B_\nu\simeq D_\nu$ which is realized at $\xi\simeq 0.02$. By changing ξ freely with fixed $r(=3)$, we can well reproduce the experimental data as shown in Figs. 3–5, in which the constraints from $|U_{23}|$, $\Delta m^2_{12}/\Delta m^2_{23}$, and the both are satisfied, respectively. It is seen from Fig. 5 that by deviating B_e a little bit from type I ($\xi\sim 0.01$), we can well reproduce the experimental data for the solar neutrino oscillation and atmospheric neutrino deficit. If we relax the condition $r=3$ and change r freely as well as $\cos\alpha_{12}$ and ξ around the values of the above solutions, we have the larger allowed region as is shown in Fig. 6. In the above allowed regions shown in Figs. 1–5, we have used only the conservative condition for Δm^2_{12} from the solar neutrino experiments, that is, we have not used the constraints of the mixing angle from the solar neutrino oscillation experiments. When we take them into account in addition to the constraints from Δm^2_{12}, we obtain more restrictive allowed region than that of Fig. 6. Under the condition of the small mixing angle solution for solar neutrino experiments, the larger region of $|\cos\alpha_{12}|$ in Fig. 6 is eliminated and we have the allowed region as is shown in Fig. 7. On the other hand, under the condition of the large mixing angle solution, the smaller region of $|\cos\alpha_{12}|$ is eliminated and the allowed region is given in Fig. 8. It should be noted that as seen in Figs. 6–8 our model not only satisfies the experimental observations in the lepton sector but also provides the restriction on the CP violation phase, $\cos\alpha_{12}$, from the neutrino oscillation experiments. Of course, we can also restrict the other CP violation phases, $\cos\alpha_{23}$, $\cos\beta_{12}$ and $\cos\beta_{23}$, which are respectively depicted in Figs. 9–11. Also it follows from Eq. (39) that the neutrino mass ratios $|m_1/m_2|$ and $|m_2/m_3|$ become sensitive functions of ξ, as are shown in Fig. 12 taking typical values of r and $\cos\alpha_{12}$. The common overall factor $s'/(r'^2\gamma)$ in Eq. (39) is determined to be of order 10^{-13} by the Δm^2 data from the solar and atmospheric neutrino oscillation experiments.

Finally we discuss the entries of the Cabibbo-Kobayashi-Maskawa (CKM) quark mixing matrix which are given by Eqs. (16) with (35). In our model, all the elements of the CKM mixing matrix are also functions of two free parameters $\cos\alpha_{12}$ and ξ. The parameters determined so far from the lepton sector do not give rise to any inconsistency with the data in quark sector.

V. SUMMARY

In this paper we have presented and discussed a model of texture four zero quark-lepton mass matrices in the context of SO(10). The consistent fitting of the free parameters to the data for neutrino oscillation experiments has forced us to use the charged lepton mass matrix which slightly deviates from purely type I form ($\xi\sim 0.01$). Using this deviated type of mass matrix for the charged leptons and the mass matrices for quarks of type I, we have been able to reproduce four entries in the CKM quark mixing matrix and to predict six entries in the MNS lepton mixing matrix and three Majorana neutrino masses which are consistent with the experimental data. The model has also given the restrictions on the CP violating phases which came from the neutrino oscillation experiments. Remarkably enough the parameter r fixed from data fitting is coincident with the value $r\simeq(2\text{–}3)$ obtained from the renormalization equation [24]. So it is attractive to expect that the above deviation ($\xi\sim 0.01$) from type I form can be obtained by taking the evolution equation of Yukawa coupling fully. (In this paper we have considered the loop correction of gauge boson in the evolution equation.) Though the detail calculations will be developed in the forthcoming paper, we will roughly outline our idea. That is, (charged lepton) mass matrix is exactly of type I at some scale. However, they change their form due to the evolutionary equation of the Yukawa coupling Y_a until the corresponding Higgs field acquires the vacuum expectation value [28]

$$\frac{dY_a}{dt}=\frac{1}{16\pi^2}(T^f-G^f+H^f), \tag{48}$$

where T^f, G^f, H^f are the vertex corrections due to the fermion, the gauge boson and the Higgs boson, respectively. After that, each mass furthermore changes its value according to the mass renormalization equation. The evolution equation of Yukawa coupling is very sensitive to the Higgs potentials and the initial conditions. One such sensitivity has been found in the behavior of ξ. The details will be given in a forthcoming paper.

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