Curvature dependence of running gauge coupling and confinement in AdS-CFT correspondence

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We construct a type IIB supergravity (viewed as dilatonic gravity) background with a nontrivial dilaton and with curved four-dimensional space. Such a background may describe another vacuum of maximally supersymmetric Yang-Mills theory or strong coupling regime of (non)supersymmetric gauge theory with (power-like) running gauge coupling which depends on curvature. A curvature dependent quark-antiquark potential is calculated where the geometry type of hyperbolic (or de Sitter universe) shows (or does not show) the tendency of the confinement. A generalization of the type IIB supergravity background with a nonconstant axion is presented. The quark-antiquark potential, being again curvature dependent, has the possibility to produce the standard area law for large separations.

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I. INTRODUCTION

AdS conformal field theory (CFT) correspondence [1] may provide new insight into the understanding of nonperturbative QCD. For example, in frames of type 0 string theory the attempts [2] have been done to reproduce such well-known QCD effects as running gauge coupling and possibly confinement. It is among the first problems to get the description of well-known QCD phenomena from bulk or boundary correspondence.

In another approach one can consider a type IIB supergravity (SG) vacuum which describes the strong coupling regime of a nonsupersymmetric gauge theory. This can be achieved by the consideration of deformed type IIB SG vacuums, for example, with a nonconstant dilaton which breaks conformal invariance and supersymmetry (SUSY) of boundary supersymmetric Yang-Mills (YM) theory. Such a background will be the perturbation of $AdS_5\times S_5$ vacuum. The background of such a sort (with a nontrivial dilaton) which interpolates between AdS (UV) and flat space with singular dilaton (IR) has been found in Ref. [3] where also conformal dimensions for (dilaton coupled) scalar have been found.

This solution of IIB SG [3] has been used in Ref. [4] with the interpretation of it as the one describing the running gauge coupling (via exponent of dilaton). It has been shown that running gauge coupling has a power law behavior with an ultraviolet (UV) stable fixed point and a quark-antiquark potential [5] has been calculated. QCD properties of such a background have been discussed in detail in Refs. [6]. Modifications of the IIB SG solution with nonconstant dilaton [3] due to the presence of an axion [7], constant self-dual vector [8], or world volume scalar [9] give further proof of the possible confinement and asymptotic freedom of the boundary non-SUSY gauge theory. Unfortunately, the situation is very complicated here due to the double role of type IIB SG backgrounds. From one side they may indeed correspond to IR gauge theory (deformation of the initial SUSY YM theory). At the same time such a background may simply describe another vacuum of the same maximally supersymmetric YM theory with a nonzero vacuum expectation value (VEV) of some operator. Because of the fact that operators corresponding to deformation to another gauge theory are not known, it is unclear what the case under discussion is (interpretation of SG background). Only some indirect arguments as below may be given. As we see these arguments indicate that the type IIB SG background discussed in this work most probably corresponds to another vacuum of the super-YM theory under consideration. Then renormalization group (RG) flow is induced in the theory via giving a nonzero (VEV) to some operator.

In the present paper, we continue the study of running dilaton and confinement from type IIB supergravity backgrounds with nontrivial dilaton. We generalize the solution of Ref. [3] for nonzero curvature of d-dimensional space. As a result, the type IIB supergravity background is changed drastically. The running dilaton (gauge coupling) depends explicitly on the four-dimensional curvature. The structure of quark-antiquark potential is modified. In a sense, confinement would become the characteristic of the Universe.

Let us remark on the AdS-CFT interpretation of the type IIB SG background. Choosing the coordinates in the asymptotically AdS_5 spacetime as

$$ds^2=d\sigma^2+S(\sigma)\sum_{i,j=0}^{3}\eta_{ij}dx^i dx^j, \tag{1}$$

let us assume the scalar field λ, e.g., dilaton, axion, or other fields, obeys the following equation:

$$\frac{d^2\lambda}{d\sigma^2}+4\frac{d\lambda}{d\sigma}=M^2\lambda \tag{2}$$

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near the boundary. Here M^2 is the "mass" of λ and $\sigma\to 0$ corresponds to the boundary of AdS. Then λ is associated with the operator $\mathcal{O}_\lambda$ with conformal dimension $\Delta=2+\sqrt{4+M^2}$. The solution of Eq. (2) is given by

$$\lambda=A\mathrm{e}^{-(4-\Delta)\sigma}+B\mathrm{e}^{-\Delta\sigma}. \tag{3}$$

The solution corresponding to A is not normalizable but the solution to B is normalizable. According to the argument in Ref. [19], the non-normalizable solution would be associated with the deformation of the $\mathcal{N}=4$ theory by $\mathcal{O}_\lambda$ but the normalizable solution would be associated with a different vacuum where $\mathcal{O}_\lambda$ has a nonzero vacuum expectation value. The behavior of the dilaton found in this paper is normalizable and seems to be associated with the dimension 4 operator, say $\mathrm{tr}\,F^2$. Then the argument in Ref. [19] would indicate that the solution found in this paper should correspond to another vacuum of $\mathcal{N}=4$ theory. Nevertheless, there might still be the possibility that the solution corresponds to nonsupersymmetric gauge theory. Since there occurs the condensation of $\mathrm{tr}\,F^2$ in the usual nonsupersymmetric QCD, however, the solution given here would describe some features typical for the nonsupersymmetric theory.

The situation is even more complicated due to limits of validity of dual SG description. In order that the classical supergravity description is valid, the curvature should be small and the string coupling should be also small. If the curvature is large, the α' corrections from string theory would appear. In the AdS-CFT correspondence, the radius R_s of the curvature is given by

$$R_s=(4\pi g_s N)^{1/4}. \tag{4}$$

Here g_s is the string coupling and N is the number of the coincident D-branes. Therefore we should require

$$g_s N \gg 1. \tag{5}$$

On the other hand, the classical picture works when the string coupling is small:

$$g_s \ll 1. \tag{6}$$

In the solution given in this paper, there appears the curvature singularity and g_s depends on the coordinates since the dilaton is nontrivial. If we concentrate on the behavior near the boundary, which is asymptotically AdS and is far from the singularity, the solution would be reliable and SG description would be trusted.

The work is organized as follows. In the next section we give the type IIB supergravity background with nonconstant dilaton and nonflat four-dimensional space. Via AdS-CFT it gives the curvature dependent (powerlike) running gauge coupling and quark-antiquark potential where hyperbolic geometry seems to support confinement. In Sec. III we generalize the background of Sec. II for the case when axion presents. (Curvature dependent) quark-antiquark potential is found. It is shown that inflationary Universe (de Sitter) with axion might predict confinement. Some outlook is given in the last section. Additional solutions of type IIB supergravity are presented in two Appendixes.

II. SOLUTION, RUNNING GAUGE COUPLING AND QUARK-ANTIQUARK POTENTIAL

We start from the following action of dilatonic gravity in $d+1$ dimensions:

$$S=-\frac{1}{16\pi G}\int d^{d+1}x\sqrt{-G}(R-\Lambda-\alpha G^{\mu\nu}\partial_\mu\phi\partial_\nu\phi). \tag{7}$$

In the following, we assume $\lambda^2\equiv-\Lambda$ and α to be positive. The action (7) is very general. It contains the effective action of type IIB string theory. The type IIB supergravity, which is the low energy effective action of the type IIB string theory, has a vacuum with only a nonzero metric and the anti-self-dual five form. The latter is given by the Freund-Rubin-type ansatz

$$F_{\mu\nu\rho\kappa\lambda}=-\frac{\sqrt{\Lambda}}{2}\epsilon_{\mu\nu\rho\kappa\lambda}, \quad \mu,\nu,\ldots=0,1,\ldots,4,$$

$$F_{ijkpq}=-\frac{\sqrt{\Lambda}}{2}\epsilon_{ijkpq}, \quad i,j,\ldots=5,\ldots,9. \tag{8}$$

The vacuum has the topology of $\mathrm{AdS}_5\times S^5$. Since AdS_5 has a four-dimensional Minkowski space as a subspace, especially on its boundary, AdS_5 has the four-dimensional Poincaré symmetry ISO(1,3). S^5 has, of course, SO(6) symmetry.

As an extension, we can consider the solution where the dilaton is nontrivial but the anti-self-dual five form is the same as in Eq. (8). Furthermore if we require the solution has the symmetry of $\mathrm{ISO}(1,3)\times\mathrm{SO}(6)$, the metric should have the following form:

$$ds^2=G_{\mu\nu}dx^\mu dx^\nu+g_{mn}dx^m dx^n, \tag{9}$$

where g_{mn} is the metric of S^5 and Ref. [3]:

$$G_{\mu\nu}dx^\mu dx^\nu=f(y)dy^2+y\sum_{i,j=0}^{d-1}\eta_{ij}dx^i dx^j. \tag{10}$$

In order to keep the symmetry of $\mathrm{ISO}(1,3)\times\mathrm{SO}(6)$, the dilaton field ϕ can only depend on y. Then by integrating five coordinates on S^5, we obtain the effective five-dimensional theory, which corresponds to $d=4$ and $\alpha=\frac{1}{2}$ case in Eq. (7). We keep working with above dilatonic gravity as it will be easy to come to type IIB supergravity ($d=4$, $\alpha=\frac{1}{2}$) at any step.

From the variation of the action (7) with respect to the metric $G^{\mu\nu}$, we obtain[1]

[1]The conventions of curvatures are given by $R=G^{\mu\nu}R_{\mu\nu}$, $R_{\mu\nu}=-\Gamma^\lambda_{\mu\lambda,\kappa}+\Gamma^\lambda_{\mu\kappa,\lambda}-\Gamma^\eta_{\mu\lambda}\Gamma^\lambda_{\kappa\eta}+\Gamma^\eta_{\mu\kappa}\Gamma^\lambda_{\lambda\eta}$, $\Gamma^\eta_{\mu\lambda}=\frac{1}{2}G^{\eta\nu}(G_{\mu\nu,\lambda}+G_{\lambda\nu,\mu}-G_{\mu\lambda,\nu})$.

$$0=R_{\mu\nu}-\frac{1}{2}G_{\mu\nu}R+\frac{\Lambda}{2}G_{\mu\nu}-\alpha \times\left(\partial_\mu\phi\partial_\nu\phi-\frac{1}{2}G_{\mu\nu}G^{\rho\sigma}\partial_\rho\phi\partial_\sigma\phi\right) \tag{11}$$

and from that of dilaton ϕ

$$0=\partial_\mu(\sqrt{-G}G^{\mu\nu}\partial_\nu\phi). \tag{12}$$

We assume that ϕ depends only on one of the coordinates, say $y\equiv x^d$ as in type IIB supergravity solution with the symmetry of ISO(1,3)$\times$SO(6) and we also assume, as a generalization of Eq. (10), that $G_{\mu\nu}$ has the following form:

$$ds_{d+1}^2=\sum_{\mu,\nu=0}^{d}G_{\mu\nu}dx^\mu dx^\nu=f(y)dy^2+y\sum_{i,j=0}^{d-1}g_{ij}dx^i dx^j. \tag{13}$$

Here g_{ij} is the metric in the Einstein manifold, which is defined by

$$r_{ij}=kg_{ij}. \tag{14}$$

Here r_{ij} is the Ricci tensor given by g_{ij} and k is a constant, especially $k>0$ for sphere and $k=0$ for the flat Minkowski space and $k<0$ for hyperboloid. Such a solution generalizes the previous solution of Ref. [3] (where $k=0$) as boundary gauge QFT lives now in four-dimensional curved spacetime. The case of $k=1$ is especially interesting as it corresponds to gauge theory in de Sitter (inflationary) Universe.

The equations of motion (11) and (12) take the following forms:

$$0=\frac{1}{2}\frac{rf}{y}-\frac{d(d-1)}{8}\frac{1}{y^2}+\frac{\lambda^2}{2}f+\frac{\alpha}{2}(\phi')^2, \tag{15}$$

$$0=-\left(r_{ij}-\frac{1}{2}rg_{ij}\right)\frac{f}{y}+\left\{\frac{d-1}{4}\frac{f'}{fy}-\frac{(d-1)(d-4)}{8}\frac{1}{y^2}+\frac{\lambda^2}{2}f-\frac{\alpha}{2}(\phi')^2\right\}g_{ij}, \tag{16}$$

$$0=\left(\sqrt{\frac{y^d}{f}}\phi'\right)'. \tag{17}$$

Here the prime expresses the derivative with respect to y and $r\equiv g^{ij}r_{ij}=kd$. Equation (15) corresponds to $(\mu,\nu)=(d,d)$ in Eq. (11) and Eq. (16) to $(\mu,\nu)=(i,j)$. The case of $(\mu,\nu)=(0,i)$ or $(i,0)$ is identically satisfied. Integrating Eq. (17), we find

$$\phi'=c\sqrt{\frac{f}{y^d}}. \tag{18}$$

Substituting Eq. (18) into Eq. (15), we can solve it algebraically with respect to f:

$$f=\frac{d(d-1)}{4y^2\lambda^2(1+\alpha c^2/\lambda^2y^d+kd/\lambda^2y)}. \tag{19}$$

Then we find from Eqs. (18) and (19),

$$\phi=c\int dy\sqrt{\frac{d(d-1)}{4y^{d+2}\lambda^2(1+\alpha c^2/\lambda^2y^d+kd/\lambda^2y)}}. \tag{20}$$

When y is small, $f(y)$ in Eq. (19) behaves as

$$f(y)\sim\frac{d(d-1)y^{d-2}}{4\alpha c^2}, \tag{21}$$

which makes a curvature singularity at $y=0$. The scalar curvature behaves when $y\sim0$ as

$$R\sim\alpha c^2y^{-d}. \tag{22}$$

The curvature singularity would be generated by the singular behavior of the dilaton ϕ when $y\sim0$:

$$\phi(y)\sim\mathrm{sgn}(c)\sqrt{\frac{d(d-1)}{4\alpha}}\ln y. \tag{23}$$

Here sgn(c) expresses the sign of c:

$$\mathrm{sgn}(c)=\begin{cases}+1 & \text{if}\quad c>0,\\ -1 & \text{if}\quad c<0.\end{cases} \tag{24}$$

The curvature singularity tells there should appear the α' correction from the string theory and the supergravity description would break down when $y\sim0$. Conversely and hopefully, the curvature singularity might be apparent and vanish when we can include full string corrections. In any case, the solution would be valid if we investigate the behavior near the boundary $(y\to+\infty)$.

We also note that the dilaton field behaves near the boundary $(y\to+\infty)$ as

$$\phi\sim\phi_0-c\sqrt{\frac{d-1}{d\lambda^2y^d}}+\cdots. \tag{25}$$

The term of $\mathcal{O}(1/y^{d/2})$ might tell that the solution given here would correspond to the condensation of the dimension d operator, say, $\mathrm{tr}\,F^2$. In the usual non-(or lower-)supersymmetric QCD, it is widely believed that there would occur the condensation of $\mathrm{tr}\,F^2$. Therefore not depending on that the solution given here corresponds the real deformation from the $\mathcal{N}=4$ theory or the deformation of the vacuum, the solution would possibly reflect the structure of nonsupersymmetric QCD.

If we change the coordinate y by ρ, which is defined by

$$\rho \equiv -\int dy \sqrt{\frac{f(y)}{y}} = -\int dy \sqrt{\frac{d(d-1)}{4y^3\lambda^2(1+\alpha c^2/\lambda^2 y^d + kd/\lambda^2 y)}}, \quad (26)$$

the metric in Eq. (13) has the following form:

$$G_{\mu\nu}dx^\mu dx^\nu = \Omega^2(\rho)\left(d\rho^2 + \sum_{i,j=0}^{d-1} g_{ij}dx^i dx^j\right). \quad (27)$$

Here $\Omega^2(\rho)$ is given by solving y in Eq. (26) with respect to ρ: $\Omega^2(\rho)=y(\rho)$. When ρ is small, y is large and the structure of the spacetime becomes AdS asymptotically. From Eq. (26), we find

$$\rho = \frac{\sqrt{d(d-1)}}{\lambda y^{1/2}}[1+\mathcal{O}(y^{-1})]. \quad (28)$$

Therefore we find

$$\Omega^2(\rho)=y(\rho)=\frac{R_s^2}{\rho^2}[1+\mathcal{O}(\rho^2)],$$

$$R_s \equiv \frac{\sqrt{d(d-1)}}{\lambda}. \quad (29)$$

We can compare the above behavior with that of the previous $AdS_5 \times S^5$ solution in type IIB supergravity [4]. The AdS_5 part in the solution has the form of

$$ds^2_{AdS_5} = (4\pi g_s N)^{1/2}\frac{1}{\rho^2}\left(d\rho^2 + \sum_{i,j=0}^{d-1}\eta_{ij}dx^i dx^j\right). \quad (30)$$

Therefore we find

$$R_s = (4\pi g_s N)^{1/4}, \quad (31)$$

where g_s is the string coupling and N is the flux of the five-form F in Eq. (8) through S^5, which is produced by the N coincident D3-branes. Using the definition of R_s in Eq. (29), the solutions (19) and (20) have the following form:

$$f = \frac{R_s^2}{4y^2(1+c^2R_s^2/2d(d-1)y^d + k/(d-1)y)},$$

$$\phi = c\int dy \sqrt{\frac{R_s^2}{4y^{d+2}(1+c^2R_s^2/2d(d-1)y^d + k/(d-1)y)}}. \quad (32)$$

Here we put $\alpha=\frac{1}{2}$ and $d=4$ in order to get explicitly IIB supergravity background. On the other hand, if we change the coordinate by

$$\sigma = \int dy\sqrt{f(y)}, \quad (33)$$

the metric in Eq. (13) has the following form:

$$G_{\mu\nu}dx^\mu dx^\nu = d\sigma^2 + S(\sigma)\sum_{i,j=0}^{d-1} g_{ij}dx^i dx^j, \quad (34)$$

where $S(\sigma)$ is given by solving y in Eq. (33) with respect to σ: $S(\sigma)=y(\sigma)$.

We now consider the case $k<0$. First let the dilaton field to be constant or small. Then from Eq. (19), when y decreases from the positive infinity, the function f increases and diverges at a finite value of y : $y=y_0$ and after that the signature of the metric seems to change. This is not, however, real but apparent. Near $y=y_0$, the function $f(y)$ behaves as

$$f(y) \sim \frac{f_0}{y-y_0}, \quad (35)$$

where f_0 is a constant. When we introduce a new coordinate u by

$$y-y_0=u^2, \quad (36)$$

the metric has the following form when $y\sim y_0$:

$$ds^2_{d+1} \sim 4f_0 du^2 + y_0\sum_{i,j=0}^{d-1}\eta_{ij}dx^i dx^j. \quad (37)$$

The metric in Eq. (37) is regular even when $u\sim 0$ ($y\sim y_0$) and there is no curvature singularity. The change of coordinates in Eq. (36) tells that y increases again as u increases when $u>0$. Then when we write the solution by the coordinate u, the solution connects two boundaries at $u=-\infty$ and $u=+\infty$. The structure of the spacetime, however, changes when the dilaton becomes large. Let us write $f(y)$ in the following form:

$$f(y)=\frac{d(d-1)}{4y^2\lambda^2 h(y)}, \quad h(y)\equiv 1+\frac{\alpha c^2}{\lambda^2 y^d}+\frac{kd}{\lambda^2 y}. \quad (38)$$

We now investigate the condition $h(y)$ vanishes or $f(y)$ diverges and changes its sign. The minimum $h_{\min}$ of $h(y)$ can be found by the equation $dh(y)/dy=0$, which can be solved as follows:

$$y=y_0\equiv\left(-\frac{\alpha c^2}{k}\right)^{1/(d-1)} \quad (39)$$

and we find

$$h_{\min}=1+\frac{k(d-1)}{\lambda^2}\left(-\frac{\alpha c^2}{k}\right)^{-1/(d-1)}. \quad (40)$$

Therefore $h(y)$ does not vanish if $h_{\min}>0$, that is,

$$c^2>c_0^2\equiv-\frac{k}{\alpha}\left(-\frac{\lambda^2}{k(d-1)}\right)^{1-d}. \quad (41)$$

When $c^2 > c_0^2$, the solution connects the boundary at $y=\infty$ with the singular boundary at $y=0$ as in the $k=0$ and $k>0$ cases.

We now consider the running of the gauge coupling. Usually the AdS string coupling, which is the square of the coupling in $\mathcal{N}=4$ SU(N) super-Yang-Mills when $d=4$, is proportional to an exponential of the dilaton field ϕ, which we assume in the following. From Eq. (20), when y is large and $d>2$, we find that the dilaton field behaves as

$$\phi=\phi_0+c\frac{\sqrt{d(d-1)}}{2\lambda}\times\left\{-\frac{2y^{-d/2}}{d}+\frac{2}{d+2}\frac{kd}{2\lambda^2}y^{-d/2-1}+\cdots\right\}. \quad (42)$$

Here the ellipsis expresses the higher order terms of $1/y$. We now assume the gauge coupling has the following form [5–9] (of course, other ways to define running gauge coupling might be possible):

$$g=g_s e^{2\beta\sqrt{\alpha/d(d-1)}(\phi-\phi_0)}=g_s\left\{1-\frac{2\beta c\sqrt{\alpha}}{d\lambda}y^{-d/2}+\frac{kd\beta c\sqrt{\alpha}}{(d+2)\lambda^3}y^{-d/2-1}+\cdots\right\}. \quad (43)$$

In the case of type IIB supergravity ($\alpha=\frac{1}{2}$),

$$\beta=\sqrt{\frac{d(d-1)}{2}} \quad (44)$$

and using the definition of R_s in Eq. (29), we find

$$g=g_s\left\{1-\frac{cR_s}{d}y^{-d/2}+\frac{kcR_s^3}{2(d+2)(d-1)}y^{-d/2-1}+\cdots\right\}. \quad (45)$$

The next-to-leading order term is proportional to k if $k\neq 0$. This changes the renormalization group equations drastically. If we multiply $N^{1/2}$ with g, we obtain the 't Hooft coupling $g_H=gN^{1/2}$. If we define a new coordinate U by

$$y=U^2, \quad (46)$$

U expresses the scale on the (boundary) d dimensional space (due to holography [10]). Following the correspondence between long-distances and high-energy in the AdS-CFT scheme, U can be regarded as the energy scale of the boundary field theory. Then from Eq. (43), we obtain the following renormalization group equation:

$$\beta(U)\equiv U\frac{dg}{dU}=-d(g-g_s)-\frac{2kd\beta c\sqrt{\alpha}}{(d+2)\lambda^3}\times\left(-\frac{d\lambda}{2\beta c\sqrt{\alpha}}\right)^{2/d+1}\frac{(g-g_s)^{2/d+1}}{g_s^{2/d}}+\cdots. \quad (47)$$

The leading behavior is identical with the previous works [4,6–8] but the next to leading term contains the fractional power of $(g-g_s)$ although the square of $(g-g_s)$ appears for $k=0$ case. We should note that the qualitative behavior does not depend on β which appears in the coupling (43).

Hence, we found that beta function explicitly depends on the curvature of four-dimensional manifold. Of course, curvature dependence is not yet logarithmic as it happens with usual quantum field theories (QFTs) (perturbative consideration) in curved spacetime [11]. The powerlike running of gauge coupling is much stronger than in $k=0$ case. Note that previous discussion of powerlike running includes grand unified theories (GUTs) with large internal dimensions [12]. In the case under investigation we get the gauge coupling beta function as an expansion on fractional powers of gauge coupling.

We now consider the static potential between ''quark'' and ''antiquark'' [5]. We evaluate the following Nambu-Goto action:

$$S=\frac{1}{2\pi}\int d\tau d\sigma\sqrt{\det(g^s_{\mu\nu}\partial_\alpha x^\mu\partial_\beta x^\nu)}, \quad (48)$$

with the ''string'' metric $g^s_{\mu\nu}$, which could be given by multiplying a dilaton function $k(\phi)$ to the metric tensor in Eq. (9). Especially we choose $k(\phi)$ by

$$k(\phi)=e^{2\gamma\sqrt{\alpha/d(d-1)}(\phi-\phi_0)}=1-\frac{2\gamma c\sqrt{\alpha}}{d\lambda y^{d/2}}+\cdots. \quad (49)$$

In the case of type IIB supergravity,

$$\gamma=\beta=\sqrt{\frac{d(d-1)}{2}}. \quad (50)$$

We consider the static configuration $x^0=\tau$, $x^1\equiv x=\sigma$, $x^2=x^3=\cdots=x^{d-1}=0$, and $y=y(x)$. We also choose the coordinates on the boundary manifold so that the line given by $x^0=$const, $x^1\equiv x$ and $x^2=x^3=\cdots=x^{d-1}=0$ is geodesic and $g_{11}=1$ on the line. Substituting the configuration into Eq. (48), we find

$$S=\frac{T}{2\pi}\int dx k(\phi(y))y\sqrt{\frac{f(y)}{y}(\partial_x y)^2+1}. \quad (51)$$

Here T is the length of the region of the definition of τ. The orbit of y can be obtained by minimizing the action S or solving the Euler-Lagrange equation $\delta S/\delta y-\partial_x(\delta S/\delta(\partial_x y))=0$. The Euler-Lagrange equation tells that

$$E_0=\frac{k(\phi(y))y}{\sqrt{[f(y)/y](\partial_x y)^2+1}} \tag{52}$$

is a constant. If we assume y has a finite minimum y_0, where $\partial_x y|_{y=y_0}=0$, E_0 is given by

$$E_0=k[\phi(y_0)]y_0. \tag{53}$$

Introducing a parameter t, we parametrize y by

$$y=y_0\cosh t. \tag{54}$$

Then we find

$$\frac{dx}{dt}=\frac{y_0^{-1/2}}{A}\cosh^{-3/2}t\{1+B\cosh^{-1}ty_0^{-1}+\cdots\},$$

$$A\equiv\frac{2\lambda}{\sqrt{d(d-1)}},\quad B\equiv-\frac{kd}{2\lambda^2}. \tag{55}$$

Here we assume that y_0 is large enough and the orbit of the string does not approach to the singularity at $y=0$, where the supergravity description breaks down. Taking $t\to+\infty$, we find the distance L between ''quark'' and ''antiquark'' is given by

$$L=\frac{C_{3/2}y_0^{-1/2}}{A}+\frac{BC_{5/2}y_0^{-3/2}}{A}+\cdots,$$

$$C_a\equiv\int_{-\infty}^{\infty}dt\cosh^{-a}t=\frac{2^{(a-1)}\Gamma(a/2)^2}{\Gamma(a)}. \tag{56}$$

We should note that the large y_0 corresponds to small L. As one sees the next-to-leading correction to distance depends on the curvature of spacetime.

Equation (56) can be solved with respect to y_0 and we find

$$y_0=\left(\frac{C_{3/2}}{AL}\right)^2\left\{1+\frac{2BC_{5/2}}{C_{3/2}}\left(\frac{AL}{C_{3/2}}\right)^2+\cdots\right\}. \tag{57}$$

Using Eqs. (52), (54), and (56), we find the following expression for the action S:

$$S=\frac{T}{2\pi}E(L),$$

$$E(L)=\int_{-\infty}^{\infty}dt\frac{dx}{dt}\frac{k(\phi(y(t)))^2y(t)^2}{k(\phi(y_0))y_0}. \tag{58}$$

Here $E(L)$ expresses the total energy of the ''quark-antiquark'' system. The energy $E(L)$ in Eq. (58), however, contains the divergence due to the self-energies of the infinitely heavy quark and antiquark. The sum of their self-energies can be estimated by considering the configuration $x^0=\tau$, $x^1=x^2=x^3=\cdots=x^{d-1}=0$, and $y=y(\sigma)$ (note that x_1 vanishes here) and the minimum of y is y_D where brane would lies. We divide the region for y to two ones, $\infty>y>y_0$ and $y_0>y>y_D$. Using the parametrization of Eq. (54) and identifying t with σ $(t=\sigma)$ for the region $\infty>y>y_0$, we find the following expression of the sum of self-energies:

$$E_{\rm self}=\int_{-\infty}^{\infty}dt\,k(\phi(y(t)))y(t)\sqrt{\frac{f(\phi(y(t)))[\partial_t y(t)]^2}{y}}+2\int_{y_D}^{y_0}dy\,k(\phi(y))\sqrt{yf(y)}. \tag{59}$$

Then the finite potential between quark and antiquark is given by

$$E_{q\bar q}(L)\equiv E(L)-E_{\rm self}=\frac{1}{A}\left(\frac{C_{3/2}}{AL}\right)\left\{D_0+B\left(\frac{C_{5/2}D_0}{C_{3/2}}+D_2\right)\left(\frac{AL}{C_{3/2}}\right)^2+\cdots\right\},$$

$$D_d\equiv2\int_0^{\infty}dt\cosh^{-(d+1)2}t\,e^{-t}+\frac{4}{d-1}=\frac{2^{(d-3)/2}\Gamma((d-1)/4)^2}{\Gamma((d-1)/2)}. \tag{60}$$

Here we neglected the L independent terms. Note that leading and next-to-leading term does not depend on the parameter γ in Eq. (49). The leading behavior is consistent with the previous works and attractive since $D_0=-2.39628\cdots$ but we should note that next-to-leading term is linear in L (for $k=0$ it was cubic), which does not depend on the dimension d. Since $C_{5/2}D_0/C_{3/2}+D_2=3.49608>0$ and B is negative if k is positive and vice versa from Eq. (55). Therefore the linear potential term in Eq. (60) is repulsive if $k>0$ (sphere, i.e., gauge theory in de Sitter Universe) and attractive if $k<0$ (hyperboloid).

Of course, the confinement depends on the large L behavior of the potential. When L is large, however, the orbit of the string would approach to the curvature singularity at $y=0$, where the supergravity description would break down. Despite of this, it might be interesting to investigate the large L behavior. Since the behavior of $f(y)$ and the dilaton ϕ when y is small is given by Eqs. (21) and (23), the integrand in Eq. (51) behaves as

$$k(\phi(y))y\sqrt{\frac{f(y)}{y}(\partial_x y)^2+1}\sim y^{\mathrm{sgn}(c)\sqrt{d(d-1)/2}+1}\sqrt{\frac{d(d-1)}{4\alpha c^2}y^{d-3}(\partial_x y)^2+1}$$
$$=\sqrt{\frac{d(d-1)}{4\alpha c^2[\mathrm{sgn}(c)\sqrt{d(d-1)/2}+(d+1)/2]^2}(\partial_x\tilde{U})^2+(\tilde{U})^{\gamma_0}}. \tag{61}$$

Here

$$\tilde{U}\equiv y^{\mathrm{sgn}(c)\sqrt{d(d-1)/2}+(d+1)/2}, \quad \gamma_0\equiv\frac{2\,\mathrm{sgn}(c)\sqrt{d(d-1)/2}+2}{\mathrm{sgn}(c)\sqrt{d(d-1)/2}+(d+1)/2}. \tag{62}$$

When $d=4$, $0<\gamma_0<2$ when $c>0$ and $\gamma<0$ when $c<0$. According to the analysis in Ref. [19], the orbit of the string goes straight to the region $y\sim 0$ when $c>0$ $(0<\gamma_0<2)$ and the potential becomes independent of L. In this case, however, the potential would receive the α' correction from the string theory. On the other hand, when $c<0$ $(\gamma<0)$, there is an effective barrier which prevents the orbit of string from approaching into the curvature singularity and the potential would not receive the α' correction so much and the supergravity description would be reliable. Furthermore $c<0$ $(\gamma<0)$ case predicts the confinement.

We can also evaluate the potential between monopole and antimonopole by using the Nambu-Goto action for D-string instead of Eq. (48) (see Ref. [13]):

$$S=\frac{1}{2\pi}\int d\tau d\sigma\frac{1}{k(\phi)^2}\sqrt{\det(g^s_{\mu\nu}\partial_\alpha x^\mu\partial_\beta x^\nu)}. \tag{63}$$

For the static configuration $x^0=\tau$, $x^1\equiv x=\sigma$, $x^2=x^3=\cdots=x^{d-1}=0$, and $y=y(x)$, we find, instead of Eq. (51)

$$S=\frac{T}{2\pi}\int dx\frac{y}{k(\phi(y))}\sqrt{\frac{f(y)}{y}(\partial_x y)^2+1}. \tag{64}$$

We should note that $k(\phi)$ is replaced by $1/k(\phi)$ compared with the quark-antiquark case (51), which corresponds to the replacement of $\gamma\to-\gamma$. As the potential in Eq. (60) does not depend on γ in the given order, we find the monopole antimonopole potential $E_{m\bar{m}}$ is identical with $E_{q\bar{q}}$ when L is small:

$$E_{m\bar{m}}(L)=E_{q\bar{q}}(L). \tag{65}$$

If we consider, however, large L behavior as in Eq. (61), we find

$$\frac{y}{k(\phi(y))}\sqrt{\frac{f(y)}{y}(\partial_x y)^2+1}\sim y^{-\mathrm{sgn}(c)\sqrt{d(d-1)/2}+1}\sqrt{\frac{d(d-1)}{4\alpha c^2}y^{d-3}(\partial_x y)^2+1}$$
$$=\sqrt{\frac{d(d-1)}{4\alpha c^2(-\mathrm{sgn}(c)\sqrt{d(d-1)/2}+(d+1)/2)^2}(\partial_x\tilde{U}^{(m)})^2+(\tilde{U}^{(m)})^{\gamma_0^{(m)}}}. \tag{66}$$

Here

$$\tilde{U}^{(m)}\equiv y^{-\mathrm{sgn}(c)\sqrt{d(d-1)/2}+(d+1)/2},$$

$$\gamma_0^{(m)}\equiv\frac{-2\,\mathrm{sgn}(c)\sqrt{d(d-1)/2}+2}{-\mathrm{sgn}(c)\sqrt{d(d-1)/2}+(d+1)/2}. \tag{67}$$

We should note that $\mathrm{sgn}(c)$ in Eqs. (61) and (62) is replaced by $-\mathrm{sgn}(c)$ in (66) and (67). Therefore the behavior of the potential between monopole and antimonopole for large L is changed from that of the potential between quark and antiquark, that is, monopole and antimonopole would be confined for $c>0$ but would not be confined for $c<0$.

It is not difficult to study the curvature dependence in more detail, for example, numerically for different choices of parameters and regions. Nevertheless, we do not do this as most qualitative features are clear.

III. AXIONIC BACKGROUND WITH NONZERO CURVATURE AND NONCONSTANT DILATON

Let us now present the generalization of the above type IIB SG background with nontrivial dilaton when nonconstant axion is included into the action. Such a study for the case of flat four-dimensional space has been presented earlier in Ref. [7] (for the effects of additional scalars, see also Ref. [9]).

We include the axion field χ into the action of type IIB supergravity $(\alpha=\frac{1}{2})$ in Eq. (7), following Ref. [14]:

$$S=-\frac{1}{16\pi G}\int d^{d+1}x\sqrt{-G}\times\left(R+\lambda^2-\frac{1}{2}G^{\mu\nu}\partial_\mu\phi\partial_\nu\phi+\frac{1}{2}e^{2\phi}G^{\mu\nu}\partial_\mu\chi\partial_\nu\chi\right). \tag{68}$$

We work in the coordinate choice (13) and we assume that the d-dimensional manifold is curved (14) and χ only depends on y. Then, instead of Eqs. (15)–(17), we obtain

$$0=\frac{1}{2}\frac{rf}{y}-\frac{d(d-1)}{8}\frac{1}{y^2}+\frac{\lambda^2}{2}f+\frac{1}{4}(\phi')^2-\frac{1}{4}e^{2\phi}(\chi')^2, \tag{69}$$

$$0=-\left(r_{ij}-\frac{1}{2}rg_{ij}\right)\frac{f}{y}+\left\{\frac{d-1}{4}\frac{f'}{fy}-\frac{(d-1)(d-4)}{8}\frac{1}{y^2}+\frac{\lambda^2}{2}f-\frac{1}{4}(\phi')^2+\frac{1}{4}e^{2\phi}(\chi')^2\right\}g_{ij}, \tag{70}$$

$$0=\left(\sqrt{\frac{y^d}{f}}\phi'\right)'+\sqrt{\frac{y^d}{f}}e^{2\phi}(\chi')^2, \tag{71}$$

$$0=\left(\sqrt{\frac{y^d}{f}}e^{2\phi}\chi'\right)'. \tag{72}$$

Equation (72) can be integrated to give

$$\sqrt{\frac{y^d}{f}}e^{2\phi}\chi'=c_\chi. \tag{73}$$

Using Eq. (73), we can delete χ in Eq. (71) and obtain

$$0=\sqrt{\frac{y^d}{f}}\left(\sqrt{\frac{y^d}{f}}\phi'\right)'+e^{-2\phi}c_\chi^2. \tag{74}$$

Equation (74) gives another integral:

$$c_\phi=\frac{y^d}{f}(\phi')^2-c_\chi^2e^{-2\phi}. \tag{75}$$

By using Eqs. (73) and (75), we can delete χ' and ϕ' in Eq. (69):

$$0=\frac{1}{2}\frac{rf}{y}-\frac{d(d-1)}{8}\frac{1}{y^2}+\frac{\lambda^2}{2}f+\frac{c_\phi f}{4y^d}, \tag{76}$$

which can be solved algebraically with respect to $f(y)$:

$$f=\frac{d(d-1)}{4y^2\lambda^2(1+c_\phi/2\lambda^2y^d+kd/\lambda^2y)}. \tag{77}$$

The obtained metric is identical to that in Eq. (19), where the axion vanishes, if we replace c_ϕ in Eq. (77) with $\alpha c^2/2$. Therefore there appears the curvature singularity at $y=0$ again and the supergravity description would break down when $y\sim 0$. Note that as we work with type IIB SG we assume that $d=4$.

We now introduce a new coordinate η by

$$\eta=-\int dy\sqrt{\frac{f}{y^d}}=\int dy\sqrt{\frac{d(d-1)}{4y^{d+2}\lambda^2(1+c_\phi/2\lambda^2y^d+kd/\lambda^2y)}}, \tag{78}$$

Equations (73) and (75) can be written as follows:

$$c_\chi=e^{2\phi}\frac{d\chi}{d\eta}, \tag{79}$$

$$c_\phi=\left(\frac{d\phi}{d\eta}\right)^2-c_\chi^2e^{-2\phi}. \tag{80}$$

Equation (80) can be integrated to give

$$e^\phi=\frac{c_\chi}{\sqrt{c_\phi}}\sinh[\sqrt{c_\phi}(\eta-\eta_0)]. \tag{81}$$

Here η_0 is a constant of the integration. Substituting Eq. (81) into Eq. (79) and integrating it, we find

$$\chi=\chi_0-\frac{\sqrt{c_\phi}}{c_\chi}\coth(\sqrt{c_\phi}(\eta-\eta_0)). \tag{82}$$

Here χ_0 is a constant of the integration. Axion describes the running theta angle.

When $y\to+\infty$, the geometry of the spacetime approaches to AdS_5 asymptotically. Then Eq. (78) can be integrated perturbatively

$$\eta=\frac{1}{\lambda}\sqrt{\frac{d-1}{d}}\left(\frac{1}{y^{d/2}}-\frac{kd}{2(d+2)\lambda^2y^{d/2+1}}+\cdots\right). \tag{83}$$

Here we have chosen the constant of the integration so that η vanishes when y goes to positive infinity. When η vanishes, ϕ and χ behave as

$$e^\phi\to-\frac{c_\chi}{\sqrt{c_\phi}}\sinh(\eta_0\sqrt{c_\phi}),$$

$$\chi\to\chi_0+\frac{\sqrt{c_\phi}}{c_\chi}\coth(\eta_0\sqrt{c_\phi}). \tag{84}$$

We should note that k dependence does not appear in ϕ and χ if we use the coordinate η because it is hidden in this coordinate. If we choose $\eta_0=0$, $e^\phi\to 0$. Since $4\pi e^\phi$ can be regarded as the Yang-Mills coupling constant and $\rho\to 0$ ($y\to+\infty$) corresponds to the ultraviolet fixed point from the viewpoint of AdS-CFT correspondence, the theory can be regarded as asymptotically free.

We now compare the above results with those in Ref. [7] for $k=0$ and $d=4$. We introduce a new coordinate r by

$$e^{-\eta\sqrt{2c_\phi/3}}=\tanh\left(\frac{\lambda}{\sqrt{3}}(r-r_0)\right). \tag{85}$$

The coordinate transformation (85) can be given in terms of y when $k=0$ and $d=4$ by using Eqs. (78) and (83),

$$y^2=K^4(r)\equiv\sqrt{\frac{c_\phi}{2\lambda^2}}\sinh\left(\frac{2\lambda}{\sqrt{3}}(r-r_0)\right). \tag{86}$$

Then the metric in Eq. (13) for $k=0$ has the following form:

$$ds_{d+1}^2=dr^2+K^2(r)\sum_{i,j=0}^{d-1}\eta_{ij}dx^i dx^j. \tag{87}$$

By using Eq. (85), the dilaton and axion fields in Eqs. (81) and (82) can be rewritten as follows:

$$e^{\phi}=\frac{c_\chi}{2\sqrt{c_\phi}}\left\{\left[\coth\left(\frac{\lambda}{\sqrt{3}}(r-r_0)\right)\right]^{\sqrt{3/2}}e^{-\eta_0\sqrt{c_\phi}}-\left[\tanh\left(\frac{\lambda}{\sqrt{3}}(r-r_0)\right)\right]^{\sqrt{3/2}}e^{-\eta_0\sqrt{c_\phi}}\right\},$$

$$\chi=\chi_0-\frac{\sqrt{c_\phi}}{c_\chi}\frac{\{\coth((\lambda/\sqrt{3}(r-r_0))\}^{\sqrt{3/2}}+1}{\{\coth(\lambda/\sqrt{3}(r-r_0))^{\sqrt{3/2}}-1}. \tag{88}$$

Then the solution in Ref. [7] seems to be a special case corresponding to $\eta_0=\chi_0=0$.

Let us consider the potential between quark and antiquark. As we are interested in the case of asymptotically free theory, we put $\eta_0=0$ in Eq. (81) and $d=4$. Then we find

$$k(\phi(y))\equiv e^{\phi}=\frac{c_\phi R_s}{4c_\chi y^2}\left(1-\frac{k}{3\lambda^2 y}+\cdots\right),$$

$$f(y)=\frac{R_s^2}{4y^2}\left(1-\frac{4k}{\lambda^2 y}+\cdots\right). \tag{89}$$

Then in a way similar to the discussion in the second section where axion is not present instead of Eqs. (55) and (56), we find

$$\frac{dx}{dt}=\frac{R_s}{\sqrt{2y_0}}\cosh^{-3/2}t\left\{1+\frac{2k}{\lambda^2 y_0}\left(-\frac{1}{\cosh t}-\frac{\cosh t}{3(\cosh t+1)}\right)+\cdots\right\}, \tag{90}$$

$$L=\frac{R_s}{\sqrt{2y_0}}\left\{C_{3/2}+\frac{2k}{\lambda^2 y_0}\left(-C_{5/2}-\frac{E_{1/2}}{3}\right)+\cdots\right\}, \tag{91}$$

$$E_a\equiv\int_{-\infty}^{\infty}dt\frac{\cosh^{-a}t}{\cosh t+1}.$$

Equation (91) can be solved with respect to y_0 as follows:

$$y_0=\frac{1}{2}\left(\frac{C_{3/2}}{R_s L}\right)^2\left\{1+\frac{8k}{\lambda^2 C_{3/2}}\left(-C_{5/2}-\frac{E_{1/2}}{3}\right)\left(\frac{C_{3/2}}{R_s L}\right)^{-2}+\cdots\right\}. \tag{92}$$

Here we assume again that y_0 is large and L is small and not to break the supergravity description. Then using Eq. (58), we obtain the following expression for $E(L)$:

$$E(L)=\frac{R_s}{2}\left(\frac{C_{3/2}}{R_s L}\right)\left\{C_{7/2}+\frac{k}{\lambda^2}\left(\frac{C_{3/2}}{R_s L}\right)^{-2}\left(-\frac{16}{3}C_{9/2}+\frac{4}{3}E_{7/2}-\frac{4}{C_{3/2}}\left(C_{5/2}+\frac{E_{1/2}}{3}\right)\right)+\cdots\right\}. \tag{93}$$

Note that the integral is finite before subtraction the self energy of quark and antiquark. We should note that the linear potential appears in the next-to-leading term. The coefficient

$$\left[-\frac{16}{3}C_{9/2}+\frac{4}{3}E_{7/2}-\frac{4}{C_{3/2}}\left(C_{5/2}+\frac{E_{1/2}}{3}\right)\right]$$

of the next-to-leading term is negative, since $C_{3/2}$, $C_{5/2}$, and $E_{1/2}$ are positive and $-\frac{16}{3}C_{9/2}+\frac{4}{3}E_{7/2}$ is negative, which can be easily found is

$$\begin{aligned}-\frac{16}{3}C_{9/2}+\frac{4}{3}E_{7/2}&=-\frac{4}{3}\int_{-\infty}^{\infty}dt\cosh^{-9/2}t\left(4-\frac{\cosh t}{\cosh t+1}\right)\\&<-\frac{4}{3}\int_{-\infty}^{\infty}dt\cosh^{-9/2}t(4-1)\\&<0.\end{aligned} \tag{94}$$

Therefore the linear potential in the next-to-leading term becomes attractive if $k<0$ and repulsive if $k>0$. The result is consistent to the potential without axion in Eq. (60). We should note that Eqs. (78) and (81) tell that the dilaton field behaves as

$$\phi\sim-\sqrt{\frac{d(d-1)}{2}}\ln y, \tag{95}$$

which corresponds $c<0$ case in the pure dilaton case in Eq. (23). Since the behavior of $f(y)$ in Eq. (77) is essential identical with the pure dilaton case in Eq. (19), the supergravity description would be valid even for large L and the confinement for quarks would be predicted (and monopoles would not be confined).

We now investigate the supersymmetric background. For $k=0$ it has been found in Ref. [7]. We look for its k-dependent generalization. Since we consider the background where the fermion fields, that is, dilatino ξ and gravitino ψ_μ vanish, if the variation under some of the supersymmetry transformations of these fermionic fields vanishes, the corresponding supersymmetries are preserved. The supersymmetry transformations of these fields are given by [14]

$$\delta\xi = -\frac{1}{2}(e^{\phi}\partial_\mu\chi - \partial_\mu\phi)\gamma^\mu\epsilon^*,$$

$$\delta\xi^* = -\frac{1}{2}(e^{\phi}\partial_\mu\chi + \partial_\mu\phi)\gamma^\mu\epsilon,$$

$$\delta\psi_\mu = \left(\nabla_\mu + \frac{1}{4}e^{\phi}\partial_\mu\chi - \frac{\lambda}{4\sqrt{3}}\gamma_\mu\right)\epsilon,$$

$$\delta\psi_\mu^* = \left(\nabla_\mu - \frac{1}{4}e^{\phi}\partial_\mu\chi - \frac{\lambda}{4\sqrt{3}}\gamma_\mu\right)\epsilon^*. \tag{96}$$

When substituting the solution in Eqs. (81) and (82) into $\delta\xi$ and $\delta\xi^*$, we find

$$\delta\xi = -\frac{\sqrt{c_\phi}}{2}\left(\frac{1-\cosh(\sqrt{c_\phi}(\eta-\eta_0))}{\sinh(\sqrt{c_\phi}(\eta-\eta_0))}\right)\gamma^\eta\epsilon^*,$$

$$\delta\xi^* = -\frac{\sqrt{c_\phi}}{2}\left(\frac{1+\cosh(\sqrt{c_\phi}(\eta-\eta_0))}{\sinh(\sqrt{c_\phi}(\eta-\eta_0))}\right)\gamma^\eta\epsilon. \tag{97}$$

Therefore all the supersymmetries break down in general since $\delta\chi$ and $\delta\chi^*$ do not vanish. In the limit of $c_\phi\to 0$, however, we find

$$\delta\xi\to 0,$$

$$\delta\xi^* = -\frac{1}{\eta-\eta_0}\gamma^\eta\epsilon. \tag{98}$$

Therefore there is a possibility that half of the supersymmetries corresponding to ϵ^* survives in this limit. It should be noted that, in the limit, $f(y)$ in Eq. (77) becomes

$$f = \frac{d(d-1)}{4y^2\lambda^2(1+kd/\lambda^2 y)}, \tag{99}$$

which tells that the metric of the spacetime becomes nothing but the metric of $AdS_5\times S^5$ although the dilaton and the axion fields are nontrivial. Then if we choose the spinor parameter ϵ^* by using the Killing spinor ζ in $AdS_5\times S^5$ as follows [14,7]:

$$\epsilon^* = e^{\phi/4}\zeta \to c_\chi^{1/4}(\eta-\eta_0)^{1/4}\zeta, \tag{100}$$

$\delta\psi_\mu^*$ vanishes in the limit of $c_\phi\to 0$, which tells that half of the supersymmetry corresponding to ϵ^*, in fact, survives in this limit. This situation does not depend on k. Such a solution corresponds to some vacuum of maximally supersymmetric YM theory where supersymmetry is broken to $\mathcal{N}=2$. (Note that deformations of $\mathcal{N}=4$ super YM theory which flow to fixed points as in Refs. [15,16] may also define running gauge coupling). In the limit of $c_\phi\to 0$, the solution in Eqs. (81) and (82) has the following form:

$$e^{\phi}\to c_\chi(\eta-\eta_0),$$

$$\chi\to\chi_0 - \frac{1}{c_\chi(\eta-\eta_0)}. \tag{101}$$

Even in the limit, the theory becomes asymptotically free when $\eta_0=0$ since the coupling is assumed to be given by e^{ϕ} vanishes in the ultraviolet limit corresponding to $\eta=0$. We should also note that the potential ($\eta_0=0$ case) between quark and antiquark in Eq. (93) is not changed in the leading and next-to-leading orders since c_ϕ is not included to the corresponding expression.

IV. DISCUSSION

In summary, we found the background of type IIB supergravity with nonconstant dilaton, nonzero curvature of four-dimensional space-time and with (or without) nontrivial axion. By assuming the coupling is given by the exponential of the dilaton field ϕ, AdS-CFT interpretation of such a solution gives the (powerlike) running gauge coupling and predicts its curvature dependence. In the presence of axion, background may have half of the supersymmetries unbroken. In all cases, we calculated quark-antiquark potential and showed that the term linear on distance L explicitly depends on the curvature. Hence, there is the possibility that curvature of Universe might predict the confinement.

The complete interpretation of type IIB SG background via AdS-CFT correspondence is not yet clear. We gave the arguments that our background most probably corresponds to another vacuum of maximally supersymmetric YM theory with some nonzero VEV operator. However, the possibility that it may be deformation of theory to another less symmetric (super) YM theory is not yet completely ruled out. The only possibility for understanding it now is to investigate all properties of SG background and compare it with properties of corresponding QFT.

For example, it would be really interesting to find further development of such a scenario in order to present more realistic (logarithmic) behavior for running gauge coupling. Clearly, major modifications of background are necessary. Note in this respect the recent paper [17] where it was shown that AdS orbifolds may describe the running gauge coupling.

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APPENDIX A

In this appendix, we point out that there are many kinds of Einstein manifolds which satisfy Eq. (14). The Einstein equations are given by

$$R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R + \frac{1}{2}\Lambda g_{\mu\nu} = T_{\mu\nu}^{\mathrm{matter}}. \tag{A1}$$

Here $T_{\mu\nu}^{\mathrm{matter}}$ is the energy-momentum tensor of the matter fields. If we consider the vacuum solution where $T_{\mu\nu}^{\mathrm{matter}}=0$, Eq. (A1) can be rewritten as

$$R_{\mu\nu}=\frac{\Lambda}{2}g_{\mu\nu}. \tag{A2}$$

If we put $\Lambda=2k$, Eq. (A2) is nothing but the equation for the Einstein manifold (14). The Einstein manifolds are not always homogeneous manifolds like flat Minkowski, (anti–)de Sitter space or Nariai space but they can be some black hole solutions such as the Schwarzschild black hole,

$$ds_4^2\equiv\sum_{\mu,\nu=0}^{3}g_{\mu\nu}dx^\mu dx^\nu=-\left(1-\frac{r_0}{r}\right)dt^2+\frac{dr^2}{(1-r_0/r)}+r^2d\Omega^2, \tag{A3}$$

or Kerr one for[2] $k=0$ or Schwarzschild (anti–)de Sitter black hole

$$ds_4^2=-\left(1-\frac{\mu}{x}-\frac{2k}{3}x^2\right)dt^2+\frac{dr^2}{\left[1-\frac{\mu}{x}-(2k/3)x^2\right]}+r^2d\Omega^2, \tag{A4}$$

for $k\neq0$. In these solutions, the curvature singularity at $r=0$ has a form of line penetrating AdS_5 and the horizon makes a tube surrounding the singularity. This configuration seems to express D-string whose boundary lies on the boundary of AdS_5 or possibly D3-brane. Especially in case of a Kerr or Kerr-(anti–)de Sitter solution, the object corresponding to the singularity has an angular momentum.

We should note that the dilaton depends on the geometry of the boundary manifold only through k as in Eq. (20). Therefore the behavior of the running coupling or renormalization group equation is irrelevant with the existence of the black hole singularity.

APPENDIX B

In this appendix we present one more solution of type IIB supergravity with two timelike signatures of the metric. The physical interpretation of this solution is not quite clear as well as its dual interpretation.

It was already a few times mentioned that AdS radial coordinate plays the role of energy coordinate via holographic correspondence. It is also known that in general relativity there were attempts to identify the energy with time flow. Then the following interesting question appears: Can the same sort of AdS solution be reinterpreted as the one depending from extra time coordinates? In a sense one then has a new IIB SG solution with a few timelike signatures. There was some discussion of solutions with a few timelike signatures in various gravitational theories.

In order to get the time dependent solution and consider a kind of AdS cosmology, we perform the analytic continuation in the solution in Eqs. (19) and (20) with $k=0$ as follows:

$$c^2\to-c^2,\quad \phi_0\to\phi_0-\frac{1}{2}\sqrt{\frac{(d-1)}{d\alpha}}\ln(-1). \tag{B1}$$

Then we obtain the following metric and the dilaton field:

$$ds_{d+1}^2=f(y)dy^2+y\sum_{i,j=0}^{d-1}\eta_{ij}dx^idx^j, \tag{B2}$$

$$f=\frac{d(d-1)}{4y^2(\lambda^2-\alpha c^2/y^d)}, \tag{B3}$$

$$\phi=\phi_0+\frac{1}{2}\sqrt{\frac{(d-1)}{d\alpha}}\ln\left\{\frac{-2\alpha c^2}{\lambda^2y^d}+1\mp\sqrt{\left(\frac{2\alpha c^2}{\lambda^2y^d}+1\right)^2-1}\right\}. \tag{B4}$$

We can directly check that the solutions (B2)–(B4) satisfy Eqs. (11) and (12). When $\lambda^2-\alpha c^2/y^d<0$, dilaton field ϕ is real and $f(y)$ becomes negative, which tells that y can be regarded as another time coordinate (AdS time) besides the physical time coordinate in d-dimensional Minkowski space corresponding to η_{ij} in (B2). We have unusual signature of the metric with two timelike coordinates. Changing the coordinate y by

$$y=\left(\frac{\alpha c^2}{\lambda^2}\right)^{1/d}\sin^{2/d}t, \tag{B5}$$

we obtain the following metric and the dilaton field:

$$ds_{d+1}^2=-\frac{d-1}{d\lambda^2}dt^2+\left(\frac{\alpha c^2}{\lambda^2}\right)^{1/d}\sin^{2/d}t\sum_{i,j=0}^{d-1}\eta_{ij}dx^idx^j, \tag{B6}$$

$$\phi=\phi_0+\frac{1}{2}\sqrt{\frac{(d-1)}{d\alpha}}\ln\left(\frac{1\mp\cos t}{\sin t}\right)^2. \tag{B7}$$

Note that $t=0,\pi$ corresponds to $y=0$. Therefore there is a curvature singularity there. This indicates that α' expansion in string theories becomes unreliable and we need to exclude the region $t\sim0,\pi$. Equation (B7) indicates that the coupling becomes t-dependent, especially in the case of type IIB supergravity we find

$$g=g_se^{\phi-\phi_0}=g_s\left(\frac{1\mp\cos t}{\sin t}\right)^{\sqrt{2-2/d}}. \tag{B8}$$

If we change the coordinate t by τ as

[2]This type of solutions for $k=0$ case has been considered in Ref. [18].

$$\tau = \left(\frac{d-1}{dc\sqrt{\alpha}} \right) \int \frac{dt}{\sin^{1/d} t}, \tag{B9}$$

we have the metric in the following form:

$$ds^2_{d+1} = \Theta(\tau) \left(-d\tau^2 + \sin^{2/d} t \sum_{i,j=0}^{d-1} \eta_{ij} dx^i dx^j \right), \tag{B10}$$

where

$$\Theta(\tau) \equiv \left(\frac{\alpha c^2}{\lambda^2} \right)^{1/d} \sin^{2/d} t(\tau). \tag{B11}$$

Note that t is solved with respect to τ by using Eq. (B9). It follows from the above speculation that one can understand running of gauge coupling also as dependence on ''second time'' (AdS time). It would be interesting to understand if such a picture may have any physical meaning.

The conclusion drawn from such an interpretation is that AdS solution may contain a few times. Then the possibility of a kind of phase transition between these times should be considered (this is, of course, highly speculative). The physical time should be naturally defined by observer living in such a world. One possibility may be to introduce potential depending on angles defining the sort of signature of any particular dimension. Then the minimum of this potential may probably define the real physical time. In any case, the interpretation of the type IIB SG solution considered in this appendix could be understood simply as one more type IIB SG solution.

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