Conserved cosmological structures in the one-loop superstring effective action

Jai-chan Hwang

Department of Astronomy and Atmospheric Sciences, Kyungpook National University, Taegu, Korea

Hyerim Noh

Korea Astronomy Observatory, San 36-1, Whaam-dong, Yusung-gu, Daejon, Korea

A generic form of a low-energy effective action of superstring theories with one-loop quantum corrections is well known. Based on this action we derive the complete perturbation equations and general analytic solutions in cosmological spacetime. Using the solutions we identify the conserved quantities characterizing the perturbations, the amplitude of the gravitational wave and the perturbed three-space curvature in the uniform-field gauge both in the large-scale limit, and the angular momentum of rotational perturbation are *conserved independently of changing gravity sector*. The implications for calculating perturbation spectra generated in the inflation era based on the string action are presented.

PACS number(s): 98.80.Cq, 04.30.Db, 11.25.Mj, 11.27.+d

I. INTRODUCTION

Superstring theory is often regarded as the leading candidate for unifying gravity with the other fundamental forces and for quantum theory of gravity [1]. If it is the correct theory, it may have important consequences in the early history of our universe. The low-energy effective action of string theory differs from Einstein gravity, and the differences may leave distinct cosmological evidence which can be probed by astronomical observations of cosmic microwave background radiation and large-scale cosmic structures. The gravitational aspect of string theory can be studied using the low-energy effective action of string theory with loop and string tension (α') expansions. Although the effective action probes the full string theory only perturbatively, it may show some generic features of the full theory, and is expected to be applicable in the low-energy limit before the full quantum gravitational effect becomes important. The generic form of the effective action of the four-dimensional superstring model with a one-loop correction is known in the literature [2]. Many studies have been made on the effects of this action with some new results found in black hole physics and comology. On the cosmological side, the studies are often concerned with the possibility of realizing the nonsingular universe or the inflation mechanism [3]. Recent cosmological studies (based on the paradigm of inflation-generated-cosmic-structures) show, however, that the quantitative aspects of observational constraints on physics in the inflation era are available through cosmic structure formation processes [4]. The results of such analyses based on the low-energy effective action can be found in [5,6].

In this paper we consider the evolution of linear stage cosmic structures based on sigma-model one-loop corrected string action. We start with a general action which includes both the one-loop string effective action and Einstein gravity. On a conventional cosmological model we apply the most general perturbations. Complete sets of equations are derived and the analytic form solutions with important cosmological implications are found. Based on the general solutions we identify some quantities characterizing complete cosmic structures which remain conserved even under the changes of the underlying gravity. In the following we do not consider any specific cosmological scenario; instead, we derive general results which are applicable to any such a scenario based on the cosmological background. In order to make this paper self-contained we present useful equations in the Appendix.

II. GRAVITY AND PERTURBED WORLD MODEL

We consider the following general action:

$$S = \int d^4x \sqrt{-g} \left[\frac{1}{2} f(\phi, R) - \frac{1}{2} \omega(\phi) \phi^{;a} \phi_{,a} - V(\phi) \right.$$
$$\left. - \frac{1}{8} \xi(\phi) R_{GB}^2 + L_m \right], \tag{1}$$

where $f(\phi, R)$ is an algebraic function of a scalar field ϕ and the scalar curvature R, and $\omega(\phi)$, $V(\phi)$, and $\xi(\phi)$ are general functions of ϕ; $R_{GB}^2 \equiv R^{abcd} R_{abcd} - 4 R^{ab} R_{ab} + R^2$, and L_m is a Lagrangian of additional energy-momentum content. The field equation and the equation of motion are presented in Eqs. (A1),(A2). Einstein gravity with a minimally coupled scalar field is a case with $f = R$, $\omega = 1$, and $\xi = 0$. The low-energy effective action of string theories is a case with $f = e^{-\phi} R$, $\omega = -e^{-\phi}$, $V = 0$, and with $\xi \propto e^{-\phi}$ from the one-loop string correction. In a conformally transformed Einstein frame we have the theory with $f = R$, $\omega = 1$, $V = 0$, and with $\xi = \xi(\phi)$ from the one-loop string correction. Equation (1) also includes Brans-Dicke theory, nonminimally coupled scalar field, induced gravity, R^2-gravity, etc. Studies of Eq. (1) without the R_{GB}^2 term have been made in [7–9]. Notice that our perturbation analyses in the following will be applicable for the general theory in Eq. (1). An important advantage of such a unified analysis will be explained in Sec. VI.

We consider as the metric a spatially homogeneous, isotropic model with the most general perturbations

$$ds^2 = -a^2(1+2\alpha)d\eta^2 - a^2(\beta_{,\alpha}+B_\alpha)d\eta dx^\alpha$$
$$+ a^2[g^{(3)}_{\alpha\beta}(1+2\varphi)+2\gamma_{,\alpha|\beta}+2C_{\alpha|\beta}+2C_{\alpha\beta}]dx^\alpha dx^\beta, \tag{2}$$

where $a(t)$ is the cosmic scale factor with $dt \equiv a d\eta$. $\alpha(\mathbf{x},t)$, $\beta(\mathbf{x},t)$, $\varphi(\mathbf{x},t)$, and $\gamma(\mathbf{x},t)$ characterize the scalar-type perturbation. $B_\alpha(\mathbf{x},t)$ and $C_\alpha(\mathbf{x},t)$ are tracefree ($B^\alpha_{|\alpha}=0$) and correspond to the vector-type perturbation. $C_{\alpha\beta}(\mathbf{x},t)$ is transverse and tracefree ($C^\beta_{\alpha|\beta}=0=C^\alpha_\alpha$), and corresponds to the tensor-type perturbation. Indices are based on $g^{(3)}_{\alpha\beta}$ as the metric, and a vertical bar indicates a covariant derivative based on $g^{(3)}_{\alpha\beta}$. We decompose the energy-momentum tensor and the scalar field as $T^a_b(\mathbf{x},t) = \bar{T}^a_b(t)+\delta T^a_b(\mathbf{x},t)$ and $\phi(\mathbf{x},t) = \bar{\phi}(t)+\delta\phi(\mathbf{x},t)$; an overbar indicates a background order quantity and will be omitted unless necessary. The equations for the background cosmological model are presented in Eqs. (A3)–(A6). The three types of perturbations decouple from each other due to the symmetry in the background world model and the linearity of the structures we are assuming. Thus, we can handle them individually. The complete sets of equations for three perturbation types in a spatially *flat* model are presented in Eqs. (A7)–(A19).

III. SCALAR-TYPE PERTURBATION

Equations (A7)–(A15) are presented in a gauge-ready form [10]. Thus, we still have a right to choose one temporal gauge condition; all variables used in the equations are spatially gauge-invariant. Some choices are the following: the synchronous gauge ($\alpha\equiv0$), the uniform-curvature gauge ($\varphi\equiv0$), the uniform-expansion gauge ($\kappa\equiv0$), the zero-shear gauge ($\chi\equiv0$), the uniform-field gauge ($\delta\phi\equiv0$), the uniform-F gauge ($\delta F\equiv0$), etc. Except for the synchronous gauge, each one of the other gauge conditions completely fixes the temporal gauge condition; a variable in such a gauge condition is equivalent to a gauge-invariant combination of the variable concerned and the variable used in the gauge condition. A proper choice of the gauge condition is often essential for a convenient handling of the problem which is the case in our situation.

We take the uniform-field gauge which sets $\delta\phi\equiv0$, thus $\delta\xi = \xi_{,\phi}\delta\phi = 0$. Equivalently, we can set $\delta\phi=0$ and replace each variable with its corresponding gauge-invariant combination with $\delta\phi$. For example, φ in the uniform-field gauge is the same as the gauge-invariant combination using φ and $\delta\phi$ which is $\varphi - H\delta\phi/\dot\phi \equiv \varphi_{\delta\phi}$. *Assuming* $F=F(\phi)$ we also have $\delta F=0$. We *assume* $\delta T^{(s)a}_b=0$. From Eqs. (A7),(A9) we can express α in terms of $\dot\varphi$. From Eqs. (A8),(A7) we can express κ and χ in terms of φ and $\dot\varphi$. Thus, using either Eq. (A10) for κ or Eq. (A11) for χ we can derive a closed form second-order differential equation for $\varphi_{\delta\phi}$ as [11]

$$\frac{1}{a^3 Q}(a^3 Q \dot\varphi_{\delta\phi})^\cdot - s(t)\frac{\Delta}{a^2}\varphi_{\delta\phi}=0, \tag{3}$$

where

$$Q \equiv \frac{\omega\dot\phi^2 + \dfrac{3}{2}\dfrac{(\dot F - H^2\dot\xi)^2}{F-H\dot\xi}}{\left(H+\dfrac{1}{2}\dfrac{\dot F - H^2\dot\xi}{F-H\dot\xi}\right)^2},$$

$$s(t) \equiv 1 + \dot\xi\frac{\dfrac{1}{2}\left(\dfrac{\dot F - H^2\dot\xi}{F-H\dot\xi}\right)^2\left(\dfrac{\ddot\xi}{\dot\xi}-H-4\dot H\dfrac{F-H\dot\xi}{\dot F - H^2\dot\xi}\right)}{\omega\dot\phi^2 + \dfrac{3}{2}\dfrac{(\dot F - H^2\dot\xi)^2}{F-H\dot\xi}}.$$

Equation (3) can be written in the following form:

$$\psi'' + (sk^2 - z''/z)\psi = 0,$$
$$\psi \equiv z\varphi_{\delta\phi}, \quad z \equiv a\sqrt{Q}, \tag{4}$$

where a prime indicates a time derivative based on a conformal time η, $ad\eta \equiv dt$, and we introduced a comoving wave number using $\Delta \to -k^2$. In the large-scale limit, $sk^2 \ll z''/z$; thus ignoring the Laplacian term in Eq. (3) we have an exact solution

$$\varphi_{\delta\phi}(\mathbf{x},t) = C(\mathbf{x}) + D(\mathbf{x})\int_0^t \frac{1}{a^3 Q}dt, \tag{5}$$

where $C(\mathbf{x})$ and $D(\mathbf{x})$ indicate the coefficients of the growing and decaying solutions, respectively. Thus, ignoring the transient solution (which is higher order in the large-scale expansion compared with the solutions in the other gauges [7]), $\varphi_{\delta\phi}$ is *conserved* in the large-scale limit. Solutions for the other variables (even in the other gauge conditions) can be easily derived from our complete set of gauge-ready form equations in Eqs. (A7)–(A15).

IV. ROTATION

From Eqs. (A16)–(A18), using notations in Eq. (A20), we have

$$\frac{k^2}{2a^2}(F-H\dot\xi)\Psi = (\mu+p)v_\omega,$$

$$\frac{1}{a^4}[a^4(\mu+p)\times v_\omega]^\cdot = -\frac{k}{2a}p\pi_T. \tag{6}$$

If we ignore the anisotropic stress, $p\pi_T$, the angular momentum of the fluid is conserved as

$$a^4(\mu+p)\times v_\omega(\mathbf{x},t) \sim L(\mathbf{x}). \tag{7}$$

V. GRAVITATIONAL WAVE

From Eq. (A19) we have

$$\frac{1}{a^3 Q_g}(a^3 Q_g \dot C^\alpha_\beta)^\cdot - s_g(t)\frac{\Delta}{a^2}C^\alpha_\beta = \frac{1}{Q_g}\delta T^{(t)\alpha}_\beta, \tag{8}$$

where

$$Q_g \equiv F - H\xi, \quad s_g(t) \equiv \frac{F - \ddot{\xi}}{F - H\dot{\xi}}. \tag{9}$$

Equation (8) was studied in the string frame [13] and in the Einstein frame [12]. Equation (8) can be written in the following form:

$$\psi_g'' + (s_g k^2 - z_g''/z_g)\psi_g = 0,$$

$$\psi_g \equiv z_g C_\beta^\alpha, \quad z_g \equiv a\sqrt{Q_g}. \tag{10}$$

In the large-scale limit, $s_g k^2 \ll z_g''/z_g$; thus ignoring the Laplacian term in Eq. (8), and ignoring the anisotropic stress, we have an exact solution

$$C_\beta^\alpha(\mathbf{x},t) = C_{g\beta}^\alpha(\mathbf{x}) + D_{g\beta}^\alpha(\mathbf{x})\int_0^t \frac{dt}{a^3 Q_g}, \tag{11}$$

where $C_{g\beta}^\alpha(\mathbf{x})$ and $D_{g\beta}^\alpha(\mathbf{x})$ indicate the coefficients of the growing and decaying solutions, respectively. Ignoring the transient solution C_β^α is *conserved* in the large-scale limit.

VI. DISCUSSIONS

We have shown that the nontransient solutions of $\varphi_{\delta\phi}$ and C_β^α both in the large-scale limit and the angular momentum are generally *conserved*: see the solutions in Eqs. (5),(7),(11). Remarkably, these conservation properties are valid considering generally time varying $V(\phi)$, $\xi(\phi)$, $\omega(\phi)$, and $F(\phi,R)$ [$F(\phi)$ or $f(R)$ for $\varphi_{\delta\phi}$], thus are valid *independently of changes in underlying gravity theory*. The unified analyses of Eq. (1) is crucially important to make this point: that is, since the solutions and the conservation properties are valid considering general f, ω, V, and ξ, the quantities $\varphi_{\delta\phi}$, C_β^α and the angular momentum remain conserved independently of changing gravity sector. As an example, since Eq. (1) includes both the string theory (possibly including the one-loop correction term) and Einstein gravity, the conservation properties are valid while the underlying gravity changes from the former to the latter one. Indeed, this is a powerful result in the context of an inflation added early universe scenario. Under this scenario, the observationally relevant large-scale cosmic structures are supposed to be generated from the quantum fluctuations (of the field and the metric) and are pushed outside the horizon during the inflation. During the inflation-to-radiation transition phase the observationally relevant scales stay in the super-horizon scale, and in such a case our conservation properties of $\varphi_{\delta\phi}$ and C_β^α are applicable. In fact, *it does not matter how the transition can be realized in reality: as long as there occur transitions while the relevant scale is in the large-scale limit, we have the quantities conserved.* Meanwhile, Eqs. (3),(6),(8) are the exact equations valid in general scale.

Compared with our previous publications in [7–9], in this paper we have included the Gauss-Bonnet coupling term and have shown that we still have similar conserved quantities. This extension would be particularly useful if we have an inflation scenario where the Gauss-Bonnet coupling term has

a role during the inflation. In such a scenario, if we can calculate the quantum fluctuations (based on the vacuum expectation values) of the metric and field variables when the scale is already pushed outside the Hubble horizon (such calculations are usually available in the many known inflation scenarios, see [5,8,9,15]) the structures can be described by the conserved quantities derived in the present work. In order to make an application we need a specific inflation model based on the action in Eq. (1) with the R_{GB} term. Applications to specific inflation scenarios are made in Einstein gravity with a minimally coupled scalar field [14], in a nonminimally coupled scalar field [15], in the low-energy effective action of string theory [5,9], etc.

We expect our solutions and the general formulations made in this paper will be useful in probing the observationally relevant consequences of the superstring effective theory with one-loop correction. In this paper, however, we have considered the roles of a dilaton field together with a Gauss-Bonnet type of sigma-model one-loop correction term. We can also consider other stringy contributions from moduli fields and the antisymmetric tensor fields (axion) and the dual product of Riemann tensors [2,3,16]. The roles of the axion coupling term, $\nu(\phi)R\tilde{R}$ with $R\tilde{R} \equiv \eta^{abcd}R_{ab}{}^{ef}R_{cdef}$, have been recently considered in [17]: this term vanishes in the homogeneous-isotropic background world model, has no contributions to the scalar- and vector-type perturbations, and the gravitational wave is again described by conserved quantities which depend on the polarization states.

Apparently, by considering these additional fields we will have the multicomponent situation which will especially affect the scalar-type perturbation: in the n-component situation, generally, we will have a coupled $2n$th order differential equation and in general we do not expect a conserved quantity in such a situation. However, we expect the rotation (of the fluid components) and the gravitational wave will not be affected by the presence of the additional fields in the sense that the conservation properties of such perturbation will remain valid. We would like to investigate the roles of these additional contributions in the process of cosmological structure formation in future occasions.

APPENDIX

A. The field equation and the equation of motion: From variations of Eq. (1) we have

$$FG_b^a = \omega\left(\phi^{;a}\phi_{,b} - \frac{1}{2}\delta_b^a \phi^{;c}\phi_{,c}\right) - \frac{1}{2}\delta_b^a(RF - f + 2V) + F^{;a}{}_b$$

$$- \delta_b^a \Box F + (R^a{}_{cbd} - R_{cd}\delta_b^a + R_{cb}\delta_d^a)\xi^{;cd} + G_c^a \xi^{;c}{}_b$$

$$- G_b^a \Box \xi + T_b^a, \tag{A1}$$

$$\Box\phi+\frac{1}{2\omega}\left(\omega_{,\phi}\phi^{;a}\phi_{,a}+f_{,\phi}-2V_{,\phi}-\frac{1}{4}\xi_{,\phi}R^2_{GB}\right)=0,$$

(A2)

where $F\equiv\partial f/\partial R$, and T_{ab} is an additional energy momentum tensor defined as $\delta(\sqrt{-g}L_m)\equiv\frac{1}{2}\sqrt{-g}T^{ab}\delta g_{ab}$.

B. Background equations: From Eqs. (A1),(A2),(2), and $T^b_{0;b}=0$ we have

$$H^2+\frac{K}{a^2}=\frac{1}{F-H\dot\xi}\left[\frac{1}{6}(\omega\dot\phi^2+RF-f+2V)-H\dot F-\frac{1}{3}T^0_0\right],$$

(A3)

$$\dot H-\frac{K}{a^2}=\frac{1}{2(F-H\dot\xi)}\left[-\omega\dot\phi^2-\ddot F+H\dot F+(\ddot\xi-H\dot\xi)\right.$$
$$\left.\times\left(H^2+\frac{K}{a^2}\right)+T^0_0-\frac{1}{3}T^\alpha_\alpha\right],$$

(A4)

$$\ddot\phi+3H\dot\phi+\frac{1}{2\omega}\left[\omega_{,\phi}\dot\phi^2-f_{,\phi}+2V_{,\phi}+6\xi_{,\phi}(\dot H+H^2)\right.$$
$$\left.\times\left(H^2+\frac{K}{a^2}\right)\right]=0,$$

(A5)

$$\dot T^0_0+3HT^0_0=HT^\alpha_\alpha,$$

(A6)

where an overdot indicates the time derivative based on t and K is the sign of three-space curvature; $H\equiv\dot a/a$ and $R=6(\dot H+2H^2+K/a^2)$. Equation (A5) also follows from Eqs. (A3),(A4),(A6).

C. Perturbation equations: From Eqs. (A1),(A2),(2) we can derive the following set of equations (we assume a spatially flat model, thus $K=0$ and $g^{(3)}_{\alpha\beta}=\delta_{\alpha\beta}$):

$$\kappa\equiv-3(\dot\varphi-H\alpha)-\frac{\Delta}{a^2}\chi,$$

(A7)

$$\delta T^{(s)0}_0=2(F-H\dot\xi)\left(H\kappa+\frac{\Delta}{a^2}\varphi\right)-\omega\dot\phi^2\alpha+(\dot F-H^2\dot\xi)$$
$$\times(\kappa+3H\alpha)+\omega\dot\phi\delta\dot\phi+\frac{1}{2}(\omega_{,\phi}\dot\phi^2-f_{,\phi}+2V_{,\phi})\delta\phi$$
$$-3H\delta\dot F+\left(3\dot H+3H^2+\frac{\Delta}{a^2}\right)\delta F$$
$$+H^2\left(3H\delta\xi-\frac{\Delta}{a^2}\delta\xi\right),$$

(A8)

$$\delta T^{(s)0}_\alpha=\frac{1}{a}\left[-\frac{2}{3}(F-H\dot\xi)\left(\kappa+\frac{\Delta}{a^2}\chi\right)-(\dot F-H^2\dot\xi)\alpha+\omega\dot\phi\delta\phi\right.$$
$$\left.+\delta\dot F-H\delta F-H^2(\delta\dot\xi-H\delta\xi)\right]_{,\alpha},$$

(A9)

$$\delta T^{(s)\gamma}_\gamma-\delta T^{(s)0}_0=2(F-H\dot\xi)\left(\dot\kappa+2H\kappa+3\dot H\alpha+\frac{\Delta}{a^2}\alpha\right)$$
$$+[\dot F+H^2\dot\xi-2(H\dot\xi)^{\cdot}](\kappa+3H\alpha)$$
$$+3(\dot F-H^2\dot\xi)\dot\alpha+2(2\omega\dot\phi^2+3\ddot F)$$
$$-3H^3\dot\xi)\alpha-2(\ddot\xi-H\dot\xi)\frac{\Delta}{a^2}\varphi$$
$$-4\omega\dot\phi\delta\dot\phi-(2\omega_{,\phi}\dot\phi^2+f_{,\phi}-2V_{,\phi})\delta\phi$$
$$-3\delta\ddot F-3H\delta\dot F+\left(6H^2+\frac{\Delta}{a^2}\right)\delta F$$
$$+3H^2\delta\ddot\xi+(2\dot H+H^2)\left(3H\delta\xi-\frac{\Delta}{a^2}\delta\xi\right),$$

(A10)

$$\delta T^{(s)\alpha}_\beta-\frac{1}{3}\delta T^{(s)\gamma}_\gamma\delta^\alpha_\beta=-\frac{1}{a^2}\left(\nabla^\alpha\nabla_\beta-\frac{1}{3}\delta^\alpha_\beta\Delta\right)[(F-H\dot\xi)$$
$$\times(\varphi+\alpha-\dot\chi-H\chi)-(F-H\dot\xi)^{\cdot}\chi$$
$$-(\ddot\xi-H\dot\xi)\varphi+\delta F-(\dot H+H^2)\delta\xi],$$

(A11)

$$\delta\dot T^{(s)0}_0+H(3\delta T^{(s)0}_0-\delta T^{(s)\alpha}_\alpha)+(3T^0_0-T^\alpha_\alpha)\dot\varphi-\frac{1}{a}\delta T^{(s)0|\alpha}_\alpha$$
$$+\frac{1}{a^2}(T^0_0\Delta\chi-T^\alpha_\beta\chi^{|\beta}{}_\alpha)=0,$$

(A12)

$$\dot T^{(s)0}_\alpha+4HT^{(s)0}_\alpha+\frac{1}{a}T^{(s)\beta}_{\alpha,\beta}+\frac{1}{a}(\alpha+3\varphi)_{,\beta}T^\beta_\alpha-\frac{1}{a}(T^0_0\alpha$$
$$+T^\beta_\beta\varphi)_{,\alpha}=0,$$

(A13)

$$\delta\ddot\phi+\left(3H+\frac{\omega_{,\phi}}{\omega}\dot\phi\right)\delta\dot\phi+\left[\left(\frac{\omega_{,\phi}}{\omega}\right)_{,\phi}\frac{\dot\phi^2}{2}+\left(\frac{-f_{,\phi}+2V_{,\phi}}{2\omega}\right)_{,\phi}\right.$$
$$\left.+3H^2(\dot H+H^2)\left(\frac{\xi_{,\phi}}{\omega}\right)_{,\phi}-\frac{\Delta}{a^2}\right]\delta\phi$$
$$=\dot\phi(\kappa+\dot\alpha)$$
$$+\left(2\ddot\phi+3H\dot\phi+\frac{\omega_{,\phi}}{\omega}\dot\phi^2\right)\alpha$$
$$+\frac{1}{2\omega}(F_{,\phi}-H^2\xi_{,\phi})\delta R+\frac{2}{\omega}\xi_{,\phi}\dot H\left(H\kappa+\frac{\Delta}{a^2}\varphi\right),$$

(A14)

$$\delta R = -2\left[\dot{\kappa} + 4H\kappa + 3\dot{H}\alpha + \frac{\Delta}{a^2}(2\varphi + \alpha)\right];$$

$$\delta R_{GB}^2 = 4H^2\delta R - 16\dot{H}\left(\kappa + \frac{\Delta}{a^2}\varphi\right), \tag{A15}$$

$$\delta T_\alpha^{(v)0} = -\frac{1}{2}(F - H\dot{\xi})\frac{\Delta}{a^2}(B_\alpha + a\dot{C}_\alpha), \tag{A16}$$

$$\delta T_\beta^{(v)\alpha} = \frac{1}{2a^3}\{a^2(F - H\dot{\xi})[(B_\beta{}^{|\alpha} + B^\alpha{}_{,\beta})$$

$$+ a(C_\beta{}^{|\alpha} + C^\alpha{}_{,\beta})^{\cdot}]\}^{\cdot}, \tag{A17}$$

$$\dot{T}_\alpha^{(v)0} + 4HT_\alpha^{(v)0} + \frac{1}{a}\delta T^{(v)\beta}{}_{\alpha,\beta} = 0, \tag{A18}$$

$$\delta T_\beta^{(t)\alpha} = (F - H\dot{\xi})\ddot{C}_\beta^\alpha + [(F - H\dot{\xi})^{\cdot} + 3H(F - H\dot{\xi})]\dot{C}_\beta^\alpha$$

$$- (F - \ddot{\xi})\frac{\Delta}{a^2}C_\beta^\alpha, \tag{A19}$$

where Δ is a Laplacian based on $\delta_{\alpha\beta}$. Equations (A7)–(A15), Eqs. (A16)–(A18), and Eq. (A19) completely de-

scribe the scalar-, vector-, and tensor-type perturbations, respectively; we decomposed the perturbed energy-momentum tensor as $\delta T_b^a = \delta T_b^{(s)a} + \delta T_b^{(v)a} + \delta T_b^{(t)a}$, where superscripts (s), (v), and (t) indicate the three perturbation types. Equations (A12),(A13),(A18) follow from $T_{a;b}^b = 0$. Some useful quantities for deriving the equations can be found in our study of $R^{ab}R_{ab}$ gravity in [18].

For the scalar-type perturbation we introduced a spatially gauge-invariant combination $\chi \equiv a(\beta + a\dot{\gamma})$. In the above set of equations we have not fixed any gauge condition; thus the equations are written in a gauge-ready form. Equation (A14) also follows from Eqs. (A7)–(A10),(A12) and Eq. (A4). For the vector-type perturbation we introduce the following variables:

$$T_\alpha^{(v)0} \equiv (\mu + p)v_\omega Y_\alpha, \quad \delta T_\beta^{(v)\alpha} \equiv p\pi_T Y_\beta^\alpha,$$

$$B_\alpha + a\dot{C}_\alpha \equiv \Psi Y_\alpha; \quad \Delta Y_\alpha \equiv -k^2 Y_\alpha,$$

$$Y_{\alpha\beta} \equiv -\frac{1}{2k}(Y_{\alpha|\beta} + Y_{\beta|\alpha}), \tag{A20}$$

where $\mu \equiv -T_0^0$ and $p \equiv \frac{1}{3}T_\gamma^\gamma$; Y_α is a vector-type ($Y^\alpha{}_{|\alpha} = 0$) harmonic function with a wave number k.

---

[1] M. Green, J. Schwarz, and E. Witten, *Superstring Theory* (Cambridge University Press, Cambridge, England, 1987), Vols. 1 and 2; J. Polchinski, *String Theory* (Cambridge University Press, Cambridge, 1998), Vols. I and II.

[2] C. G. Callan, D. Friedan, E. J. Martinec, and M. J. Perry, Nucl. Phys. **B262**, 593 (1985); B. Zwiebach, Phys. Lett. **156B**, 315 (1985); S. Deser and A. N. Redlich, Phys. Lett. B **176**, 350 (1986); D. J. Gross and J. H. Sloan, Nucl. Phys. **B291**, 41 (1987).

[3] I. Antoniadis, J. Rizos, and K. Tamvakis, Nucl. Phys. **B415**, 497 (1994); J. Rizos and K. Tamvakis, Phys. Lett. B **326**, 57 (1994); R. Easther and K. Maeda, Phys. Rev. D **54**, 7252 (1996); P. Kanti, J. Rizos, and K. Tamvakis, *ibid.* **59**, 083512 (1999).

[4] J. E. Lidsey, *et al.*, Rev. Mod. Phys. **69**, 373 (1997); D. H. Lyth and A. Riotto, Phys. Rep. **314**, 1 (1999).

[5] J. Hwang, Astropart. Phys. **8**, 201 (1998).

[6] E. J. Copeland, J. E. Lidsey, and D. Wands, Phys. Lett. B **443**, 97 (1998); G. Veneziano, hep-th/9902097.

[7] J. Hwang, Phys. Rev. D **53**, 762 (1996); J. Hwang and H. Noh, *ibid.* **54**, 1460 (1996); J. Hwang, Class. Quantum Grav. **14**, 1981 (1997).

[8] J. Hwang, Class. Quantum Grav. **14**, 3327 (1997).

[9] J. Hwang and H. Noh, Class. Quantum Grav. **15**, 1387 (1998); J. Hwang, *ibid.* **15**, 1401 (1998).

[10] J. Hwang, Astrophys. J. **375**, 443 (1991).

[11] The authors of [12] tried to analyze the scalar-type equations in the Einstein frame without success. In order to derive this equation, our choice of the uniform-field gauge is essentially important to make the analyses simple.

[12] S. Kawai, M. Sakagami, and J. Soda, Phys. Lett. B **437**, 284 (1998); gr-qc/9901065; J. Soda, M. Sakagami, and S. Kawai, gr-qc/987056; S. Kawai, ''Cosmological perturbation on modulus driven non-singular cosmological model,'' report (1998).

[13] M. Gasperini, Phys. Rev. D **56**, 4815 (1997).

[14] D. H. Lyth and E. D. Stewart, Phys. Lett. B **274**, 168 (1992); J. Hwang, Phys. Rev. D **48**, 3544 (1993); E. D. Stewart and D. H. Lyth, Phys. Lett. B **302**, 171 (1993).

[15] J. Hwang and H. Noh, Phys. Rev. Lett. **81**, 5274 (1998); Phys. Rev. D **60**, 123001 (1999).

[16] E. Witten, Phys. Lett. B **149**, 351 (1984); K. Choi, Phys. Rev. D **56**, 6588 (1997).

[17] K. Choi, J. Hwang, and K. W. Hwang, hep-ph/9907244.

[18] H. Noh and J. Hwang, Phys. Rev. D **55**, 5222 (1997); J. Hwang and H. Noh, *ibid.* **57**, 2617 (1998); H. Noh and J. Hwang, *ibid.* **59**, 047501 (1999).