Regular bouncing cosmological solutions in effective actions in four dimensions

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(Received 30 June 1999; published 25 January 2000)

We study cosmological scenarios resulting from effective actions in four dimensions which are, under some assumptions, connected with multidimensional, supergravity and string theories. These effective actions are labeled by the parameters ω, the dilaton coupling constant, and n which establishes the coupling between the dilaton and a scalar field originating from the gauge field existing in the original theories. There is a large class of bouncing as well as Friedmann-like solutions. We investigate under which conditions bouncing regular solutions can be obtained. In the case of the string effective action, regularity is obtained through the inclusion of contributions from the Ramond-Ramond sector of superstring.

PACS number(s): 98.80.Hw, 04.50.+h, 04.65.+e, 98.80.Cq

Higher dimensional space-time, supersymmetry and strings are some of the most outstanding concepts employed currently for the construction of a theory unifying all physical interactions. The emergence of the idea of a string replacing point particles as the fundamental element in nature has permitted one to obtain a multiplet including the graviton together with massless spin 1 particles [1]. Supersymmetry is naturally included in it, completing the fermionic sector of the multiplet. The critical dimension of the bosonic string is 26, while its supersymmetric version, the superstring, has a critical dimension equal to 10. In this way, supergravity and Kaluza-Klein theories are, in some sense, inserted in the framework of this unification program. The true nature of superstring theories is revealed on very high energy levels, near the Planck scale, $E_p \sim 10^{19}$ GeV. Hence, the early Universe is the most viable laboratory to test this theory, unless some phenomenology can be obtained at a much smaller energy scale, which is not the case until now.

The structure of string theory is very complex, and in trying to study its cosmological consequences, it comes out more feasible to work with the effective theory in the low energy level [1,2]. Even at this level the theory presents a non-trivial field composition: essentially, there is the gravity sector, the dilaton and a three-form field. In the gravity sector, at one loop approximation, the Gauss-Bonnet term comes to scene. The dilaton field couples non-minimally to gravity. All these terms are written in ten dimensions and the reduction to four dimensions leads to the appearance of moduli fields via the compactification process.

These moduli fields appear also in multidimensional theories, not necessarily related to strings. In principle, a pure multidimensional theory must contain just geometry, the reduction to four dimensions producing gauge and scalar fields. But if we consider a supergravity theory in higher dimensions (the most famous ones being the eleven and ten dimensional supergravities), gravity, in its higher dimensional formulation, is generally coupled to gauge or scalar fields, composing the bosonic sector. The reduction to four dimensions leads to a non-trivial coupling between gravity and scalar fields and among scalar fields themselves, some of them coming from the compactification of the internal dimensions.

The cosmological consequences of these effective models in four dimensions has been extensively studied. Some examples are the string [3,6,4,5], multidimensional [7] and supergravities [8,9] effective models. In some cases, multidimensional effective models with gauge fields have been considered and, in particular, with a higher dimensional conformal gauge field theory [10]. The low energy superstring effective model is the basis of the so-called pre-big-bang [5] scenario, where there is a phase, prior to the radiative era, during which the Universe could be either in an expanding inflationary regime or in a contraction phase, without reaching the singularity.

The structure of the coupling between gravity and scalar fields in these effective theories in 4 dimensions can be, under certain assumptions to be specified later, represented by the following expression:

$$L=\sqrt{-g}\left[\left(\phi R-\omega\frac{\phi_{;\rho}\phi^{;\rho}}{\phi}-\phi^{n}\Psi_{;\rho}\Psi^{;\rho}\right)-\chi_{;\rho}\chi^{;\rho}\right]+L_{m}. \qquad (1)$$

The term L_m represents ordinary matter. Here, we will be interested in the radiative fluid only, since it seems more realistic when we have in mind the primordial Universe. As examples of this Lagrangian, we note that:

(1) It corresponds to pure multidimensional theories with $\Psi=\chi=$ constant and $\omega=(1-d)/d$, where d is the number of compactified dimensions (in this case, we consider the compactification on the torus).

(2) If we consider a two form-gauge field in higher dimensions we obtain $\Psi\neq$const and $n=-2/d+1$.

(3) A conformal gauge field, represented by a $(d+4)/2$-form, leads to $n=-2/d$.

(4) In string theory $\omega=-1$; moreover, the fact that in general the three-form field $H_{\mu\nu\lambda}$ has, in four dimensions,

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just one degree of freedom permits one to write $H^{\mu\nu\lambda} = \phi^{-1}\epsilon^{\mu\nu\lambda\sigma}\Psi_{;\sigma}$ which corresponds to $n=-1$ in (1) [11].

The χ term is the scalar component coming from another three-form field which is present in the Ramond-Ramond sector of type IIB superstring. Note that this field does not couple directly with the dilaton. Since we are interested in cosmological applications, we have set all gauge fields equal to zero in the effective model in four dimensions. Moreover, in all these cases, there are some simplifications in the process of obtaining the effective action. For string theory, for example, the moduli fields and the Gauss-Bonnet term are absent; in what concerns multidimensional models, the moduli fields are taken into account but the curvature of the internal space is zero.

The purpose of this article is to consider the Lagrangian (1) with ω and n arbitrary. In this way, we can map many possible cosmological scenarios resulting from the existence of supersymmetry and extra dimensions, with the assumptions specified above. Our interest is to verify the generality of bouncing and, in some cases, inflationary solutions. In a bouncing cosmological scenario the scale factor has initially an infinite value, decreasing then to a minimum value different from zero, after which it follows an expanding phase. Of course, a bouncing solution has necessarily an inflationary phase, since a minimum for the scale factor implies $\dot{a}=0$ and $\ddot{a}>0$ near it. The main question we try to answer is the conditions under which a bouncing or inflationary universe may be found in the realm of the effective theories described by Eq. (1). We remark, however, that the existence of a bouncing solution is not a sufficient condition for having singularity-free models, as will be shown later. We will analyze also the existence of complete regular solutions. In this sense, we must investigate divergences of the curvature invariants as well as, in the special case of string models, divergences in the loop expansion parameter.

In fact, if we restrict ourselves first to the string cosmology program, it is has already been shown [11,12] that the existence of an axion field in the tree level string effective action leads to the existence of a minimum for the scale factor, that is a bouncing solution. However, a singularity is still present in the beginning of the evolution of the Universe, even if the scale factor is non-zero. In this sense, the introduction of loop approximations is an essential ingredient of string cosmology since it permits one to avoid the singularity. However, these analyses were made considering $\omega=-1$ and $n=-1$, i.e. string cosmology with a particular coupling between the dilaton and axion fields. We will extend them here, showing that for $\omega=-1$ a singularity seems to be unavoidable. To avoid this singularity, we must have $n<1$ and $-\frac{3}{2}>\omega>-\frac{4}{3}$. In string cosmology, this implies considering moduli fields, whose analysis is more complex since a third scalar field, with non-trivial coupling with the other two, appears in Eq. (1). But we will show that the introduction of terms coming from the Ramond-Ramond sector of superstring can render the bouncing solutions regular even at the tree level.

We will study essentially the predictions of models described by Eq. (1) concerning the evolution of the scale factor in the primordial Universe. We will verify that bouncing solutions are most favored by the conditions $\omega<0$ and $n<1$, even if in some situations it can occur for other values of these parameters. Complete regular solutions can be obtained when $\omega=0$ and $n<1$ (which, on the other hand, seems to be unstable) or for $-\frac{3}{2}>\omega>-\frac{4}{3}$. We will perform our study in the vacuum case, where only those fundamental fields are present, and when they are coupled to a radiative fluid. The radiative fluid will be minimally coupled to the geometry in Jordan's frame, where dilaton is non-minimally coupled to gravity.

The field equations coming from Lagrangian (1) are

$$R_{\mu\nu}-\frac{1}{2}g_{\mu\nu}R=\frac{8\pi}{\phi}T_{\mu\nu}+\frac{\omega}{\phi^2}\left(\phi_{;\mu}\phi_{;\nu}-\frac{1}{2}g_{\mu\nu}\phi_{;\rho}\phi^{;\rho}\right)+\frac{1}{\phi}(\phi_{;\mu;\nu}-g_{\mu\nu}\Box\phi)+\phi^{n-1}\left(\Psi_{;\mu}\Psi_{;\nu}-\frac{1}{2}g_{\mu\nu}\Psi_{;\rho}\Psi^{;\rho}\right)+\frac{1}{\phi}\left(\chi_{;\mu}\chi_{;\nu}-\frac{1}{2}g_{\mu\nu}\chi_{;\rho}\chi^{;\rho}\right), \tag{2}$$

$$\Box\phi+\frac{1-n}{3+2\omega}\phi^n\Psi_{;\rho}\Psi^{;\rho}+\frac{\chi_{;\rho}\chi^{;\rho}}{3+2\omega}=\frac{8\pi}{3+2\omega}T, \tag{3}$$

$$\Box\Psi+n\frac{\phi_{;\rho}}{\phi}\Psi^{;\rho}=0, \tag{4}$$

$$\Box\chi=0, \tag{5}$$

$$T^{\mu\nu}{}_{;\mu}=0. \tag{6}$$

We insert in these equations the Friedmann-Lemaitre-Robertson-Walker (FLRW) metric

$$ds^2=dt^2-a(t)^2\left(\frac{dr^2}{1-\epsilon r^2}+r^2(d\theta^2+\sin^2\theta d\phi^2)\right). \tag{7}$$

A flat, open and closed spatial section implies $\epsilon=0,-1$ and 1, respectively. We consider also a barotropic perfect fluid

$$T^{\mu\nu}=(\rho+p)u^\mu u^\nu-pg^{\mu\nu},\ \ p=\alpha\rho-1\leqslant\alpha\leqslant 1. \tag{8}$$

As happens in general when we are treating gravity in the presence of scalar fields, it is more convenient to reparametrize the time coordinate such that

$$dt=a^3 d\theta. \tag{9}$$

In this new parametrization and with primes denoting derivatives with respect to θ, the equations of motion read

$$3\left(\frac{a'}{a}\right)^2+3\epsilon a^4=\frac{8\pi}{\phi}\rho a^6+\frac{\omega}{2}\left(\frac{\phi'}{\phi}\right)^2-3\frac{a'}{a}\frac{\phi'}{\phi}+\frac{\phi^{n-1}}{2}\Psi'^2+\frac{1}{2}\frac{\chi'^2}{\phi}, \tag{10}$$

$$\phi''+\frac{1-n}{3+2\omega}\phi^n\Psi'^2+\frac{\chi'^2}{3+2\omega}=\frac{8\pi}{\phi}(\rho-3p)a^6, \quad (11)$$

$$\Psi''+n\frac{\phi'}{\phi}\Psi'=0, \quad (12)$$

$$\chi''=0, \quad (13)$$

$$\rho'+3\frac{a'}{a}(\rho+p)=0. \quad (14)$$

Equations (12),(13),(14) are easily integrated, leading to

$$\Psi'=A\phi^{-n}, \;\; \chi=B\theta+C, \;\; \rho=\rho_0 a^{-3(1+\alpha)}, \quad (15)$$

where A, B, C and ρ_0 are integration constants. The integration of the other two equations depends on whether $\chi=$ constant or not and on the presence of ordinary matter. We will consider separately these different cases.

(I) Vacuum case, $\rho=0$.

(Ia) $\chi=$const. In this case, Eq. (11) admits a first integral:

$$\frac{\phi'^2}{2}+\frac{A^2}{3+2\omega}\phi^{1-n}=\frac{D}{2}, \quad (16)$$

where D is an integration constant. From Eqs. (9),(16) we obtain a transcendental relation for ϕ as function of θ, through hypergeometric functions

$$\theta=-\frac{2}{(n-1)\sqrt{D}}\phi_2F_1\left(\frac{1}{1-n},\frac{1}{2},\frac{2-n}{1-n},(r\phi)^{1-n}\right), \quad (17)$$

where $r=[(3+2\omega)D/2A^2]^2$. In order to integrate Eq. (10), we redefine the scale factor $a=\phi^{-1/2}b$, obtaining the equation

$$3\left(\frac{b'}{b}\right)^2+3\epsilon\frac{b^4}{\phi^2}=\frac{3+2\omega}{4}\left(\frac{\phi'}{\phi}\right)^2+\frac{A^2}{2}\phi^{-1-n}. \quad (18)$$

We will first study the cases where $n\neq1$; the case $n=1$ will be studied separately. Using Eq. (16), Eq. (18) becomes

$$\frac{b'}{b}=\left(\frac{3+2\omega}{12}D-\epsilon b^4\right)^{1/2}\frac{1}{\phi}. \quad (19)$$

Eliminating again $d\theta$ through Eq. (16), one obtains the integral relation

$$\int\frac{dy}{y\sqrt{1-\epsilon y^2}}=\frac{2}{1-n}\sqrt{\frac{3+2\omega}{3}}\int\frac{dx}{x\sqrt{1-x^2}}, \quad (20)$$

where

$$x=\sqrt{\frac{2A^2}{(3+2\omega)D}}\phi^{(1-n)/2}$$

and

$$y=\sqrt{\frac{12}{(3+2\omega)D}}b^2.$$

In order to impose that $x^2<1$, we define $x=\sin\xi$. Integrating Eq. (20) and using the definitions given above, we obtain the following solution for a:

$$a=a_0(\sin\xi)^{1/(n-1)}\left[\frac{\tan^p\frac{\xi}{2}}{1+\epsilon k^2\tan^{2p}(\xi/2)}\right]^{1/2}, \quad (21)$$

where k is another integration constant and

$$p=\frac{2}{1-n}\sqrt{1+\frac{2}{3}\omega}.$$

The cosmic time is connected with the variable ξ through the relation

$$t=\frac{L}{1-n}\int a^3\sin^{(1+n)/(1-n)}\xi d\xi, \quad (22)$$

where

$$L=2\left(\frac{2A^2}{(3+2\omega)D}\right)^{1/(1-n)}.$$

The integrand of this relation being always positive, t is a monotonic function of ξ.

For $\epsilon=0$, the asymptotic behavior for $\xi\to0$ is $a\propto\xi^{[1/(1-n)](-1+\sqrt{1+(2/3)\omega})}$, while for $\xi=\pi-z$, $z\to0$, the asymptotic behavior reads $a\propto z^{[-1/(1-n)](1+\sqrt{1+(2/3)\omega})}$. From these expressions, we can classify the possible scenario: for $n<1$, $\omega<0$, Eq. (21) represents a bouncing solution; for $n>1$ and $\omega<0$, we find a big bang followed by a big crunch, in spite of the fact that the spatial section is flat; all other cases represent an expanding universe.

Following the same asymptotic analysis we find bouncing solutions for $\epsilon\neq0$ only when $\omega<0$ and $n<1$. For the other cases, the corresponding Friedmann-like scenarios are recovered.

A special case is $\omega=0$. Then, the solution (21) takes the form

$$a=a_0\frac{1}{\sqrt{\cos^{4/(1-n)}(\xi/2)+\epsilon k^2\sin^{4/(1-n)}(\xi/2)}}. \quad (23)$$

In this case, if $n<1$, we are at the boundary of two different behaviors and this special solution reflects this fact. For $\epsilon=0$, $n<1$, Eq. (23) is a sequence of bouncing universes. If $\epsilon=-1$, we find again a bouncing universe. When $\epsilon=1$ the universe oscillates, the scale factor never reaching zero. This last case is an example of a complete, eternal, regular universe. In particular, when $k=1$, we find a static universe even if the scalar fields evolve with time. If $n>1$, on the other hand, Eq. (23) represents, for $\epsilon=0,1$, an oscillating universe, the minimum value of the scale factor being zero. For $\epsilon=-1$ we find again a bouncing universe.

There exists also the limit case $n=1$ for which the scalar field behaves as $\phi=E\theta$, E being a constant. The equation for the scale factor can be integrated in a similar way as before, leading to the following expressions:

$$\epsilon=1,-1,\quad a=a_0\theta^{-1/2}\sqrt{\frac{\theta^{\pm q}}{1+\epsilon\theta^{\pm 2q}}}, \tag{24}$$

$$\epsilon=0,\quad a=a_0 t^{(q\pm 1)/(3q\pm 1)}, \tag{25}$$

where

$$q=\sqrt{1+\tfrac{2}{3}\omega+A^2}. \tag{26}$$

These solutions represent an expanding universe with an initial singularity, for any value of the curvature constant ϵ. There are inflationary expanding solutions when $-\frac{3}{2}-A^2<\omega<-\frac{4}{3}-A^2$.

(Ib) $\chi\neq$const. This case seems to admit an exact solution only for $n=-1$. The first integrals for Ψ and χ are

$$\Psi'=A\phi,\quad \chi=E\theta. \tag{27}$$

Using these first integrals, the solution for the scalar field ϕ reads

$$\phi=C\sin\kappa\theta-\frac{E^2}{2A^2},\qquad \kappa=\sqrt{\frac{2A^2}{3+2\omega}}, \tag{28}$$

while the equation for the scale factor can be reduced to

$$\int\frac{dy}{y\sqrt{1-y^2}}=\frac{2D}{\sqrt{3}}\int\frac{d\theta}{\phi}, \tag{29}$$

where $a=\phi^{-1/2}b$, $y^2=(\sqrt{3\epsilon/D^2})b^4$. The final solutions are

$$\epsilon=1,\quad a=a_0\frac{1}{\sqrt{\sin\kappa\theta-s}}\cosh^{-1/2}f(\theta), \tag{30}$$

$$\epsilon=0,\quad a=a_0\frac{1}{\sqrt{\sin\kappa\theta-s}}\exp[f(\theta)], \tag{31}$$

$$\epsilon=-1,\quad a=a_0\frac{1}{\sqrt{\sin\kappa\theta-s}}\sinh^{-1/2}f(\theta), \tag{32}$$

where

$$f(\theta)=\frac{4D}{\sqrt{3}C\kappa}\frac{1}{\sqrt{|s^2-1|}}\arctan\left[\frac{1-s\tan(\kappa\theta/2)}{\sqrt{|s^2-1|}}\right],\quad s=\frac{E^2}{2A^2C}. \tag{33}$$

These solutions are valid for $s<1$, which is the most interesting case. The solutions (30),(31),(32) represent a bouncing universe for any value of ω. Moreover, there is a local maximum for the scale factor when $\epsilon=1$ and $\theta=(2/\kappa)\arctan(1/s)$.

(II) Radiative fluid case and $\chi=$const.

The integration of the equations follows the same procedure as before. We note that for the radiative case $T^{\rho}{}_{\rho}=T=0$. Hence the equations for the scalar fields, and their corresponding first integrals, keep their form. However, for the scale factor, and after redefining $a=\phi^{-1/2}b$, we obtain

$$\frac{b'}{b}=\left(\frac{3+2\omega}{12}D+mb^2-3\epsilon b^4\right)^{1/2}\frac{1}{\phi}, \tag{34}$$

where we have written $m=8\pi\rho_0$. The first integral of Eq. (11) permits us again to employ the reparametrization $\phi^{1-n}=\sin\xi$. We can then integrate the equations for b, reconstructing the solutions for a. We will present the solution for the case $\epsilon=0$. It reads

$$a=a_0(\sin\xi)^{1/(n-1)}\left\{\frac{\tan^{p/2}(\xi/2)}{1-\tan^{p}(\xi/2)}\right\}, \tag{35}$$

where p is defined as before. This solution exhibits bouncing solutions for $\omega<0$ and $n<1$.

The different scenarios exposed here have many interesting features that deserve a more detailed analysis, mainly through the construction of realistic models and testing them against observations. We studied the behavior of cosmological models inspired in string, supergravities and Kaluza-Klein theories. These different effective models can be recast in a unified form, labeled essentially by the two parameters n and ω. In what concerns the effective model coming from string there are two important simplifications: the Gauss-Bonnet term is not included; the compactification from ten to four dimensions is performed on the torus, with a constant internal scale factor. Concerning effective models coming from multidimensional theories, one of the most important restrictions is the absence of curvature in the internal space.

The solutions obtained reveal bouncing, in general, for $n<1$ and $\omega<0$. The presence of the term χ, originating from the Ramond-Ramond sector, leads in general to the presence of a bounce. However, we must remark that the fact that the scale factor goes to an infinite value in the initial state of the Universe does not mean an absence of singularity. To verify this, we investigate the behavior of a curvature scalar, like R, the Ricci scalar. It is written as

$$R=-6\left[\frac{\ddot{a}}{a}+\left(\frac{\dot{a}}{a}\right)^2\right], \tag{36}$$

and when $a\propto t^m$ (what can be an approximation for the solutions founded before for small intervals of time), $R\propto t^{-2}$. Hence, $R\to\infty$ if the initial states occurs at $t=0$. It is regular if $t\to-\infty$, meaning a singularity-free universe. For example, in the vacuum and radiative cases, with $\chi=$const and $\epsilon=0$, the Universe is completely regular for $n<1$ and $-\frac{3}{2}<\omega<-\frac{4}{3}$. When $\chi\neq$const, with $n=-1$, the solutions are always regular. In all other cases, even if the scale factor never goes to zero, the curvature invariants diverge at $t=0$ in spite of the fact that the scale factor has an infinite value. In order to have a bouncing regular solution, where the Universe begins and ends in a Minkowskian state, in the context of string

($\omega=-1$), the field χ must be considered. Otherwise, there is a singular initial state and the corrections coming from the one loop approximation must be taken into account. The question of the conditions of validity of the string effective model, with respect to curvature divergences, has been considered in [13] where the moduli fields are taken into account.

In the special case of string cosmology, there is also a singularity in the expansion parameter. This parameter can be related to the dilaton field Φ by the expression $g_s^2=e^{\Phi}$. Since in our case we have the relation $\phi=e^{-\Phi}$, when ϕ goes to zero, $g_s^2\rightarrow\infty$. Hence, the loop expansion has no meaning anymore. However, the type IIB superstring theory, from which comes the field χ considered in this paper, has an *s* duality, where the strong coupling regime is mapped onto the weak coupling regime. This duality is verified, for example, in a flat background; for curved backgrounds, as is the case here, it is not straightforward to verify whether the *s* duality can be employed or not. This question is not a trivial one, but due to the *s*-duality properties of type IIB superstring the model considered here might make sense even with the divergence in the expansion parameter. In the study performed before, this remark is restricted to the case $\omega=-1$, which correspond to the string effective action.

Regular bouncing solutions are not so common in the literature. In principle, one may think that the existence of a bounce by itself can lead to singularity-free cosmological models. But this is not true, as has been shown in [10,11] for example. However, such kinds of cosmological models, if they are regular everywhere, represent an interesting alternative, or amending, to the standard one since (1) they are free of an initial singularity, which is one of the most important problems in the standard cosmological model; (2) they contain naturally an inflationary phase, which ends at a given moment after the beginning of the expansion phase; (3) they can join the radiative phase of the standard model, keeping all of its advantages from the observational point of view. In principle, a bouncing universe must constitute a primordial scenario, and perhaps we must consider the possibility of a decaying of the primordial scalar fields, originating ordinary matter, from where a smooth transition to the ordinary radiative phase can be obtained. But this leads us to consider a more complicated scenario (from technical and conceptual point of view) of baryogenesis.

The introduction of a radiative fluid may permit one to obtain some observational constraints on these models. In this case, it is possible to associate a temperature that varies as the inverse of the scale factor, as usual. Hence, the bouncing universe has a cold origin and the temperature mounts to a maximum value, decreasing from that moment on. This can leave traces in the primordial nucleosynthesis with specific predictions for the relative abundances of light elements and on the anisotropy of the cosmic microwave background. The determination of these observational traces is outside the scope of the present work, but evidently it will constraint the parameters ω and n.

Another good criterion to select viable models is the stability against small perturbations. Because of the complexity of the background solutions, this is a very difficult analysis to be performed. However, we note that the field ϕ plays the role of a variable gravitational constant and in general there is an instability when there is an anti-gravity phase, i.e., when ϕ takes negative values during a certain period [9,14] or when it takes a zero value in a finite proper time. In the solutions determined before, the gravitational coupling is always positive and this kind of problem is absent, excepting for some values of the parameter n when $\omega=0$, where the gravitational coupling can become negative or zero during the evolution of the Universe. Hence, the vacuum case with $\omega=0$ must be unstable.

It is important to notice that in the case of a bouncing universe, there is a primordial phase where not only the strong energy condition can be violated (leading to an inflationary phase), but also in some cases the weak energy condition [15]. However, there are many examples of physical systems that can violate the weak energy condition under certain circumstances [16] and this can not be viewed as a drawback of the models exhibiting a bounce, mainly when these models concern a primordial phase.

Finally, we must stress the fact that we have worked here in the Jordan frame. However, the solutions can also be written in the Einstein frame, since the redefinition $a=\phi^{-1/2}b$ employed to solve the equations is just a conformal transformation that transports the action from one frame to another. It is important to remark that, in the Einstein frame, all solutions have a singular initial state. This leads us to the problem of what is the physical frame [17–19]. Our choice was to work in the Jordan frame since the effective model appears naturally in it.

We thank K. Bronnikov and Loriano Bonora for useful discussions. We also thank CNPq and CAPES (Brazil) for financial support.

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