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Constraining isocurvature fluctuations with the Planck Surveyor

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We consider the detection possibilities of isocurvature fluctuations in the future CMB satellite experiments MAP and Planck for different cosmological reference models. We present a simultaneous 10 parameter fit (8 for the case of open model) to determine the correlations between the cosmological parameters, including the isocurvature cold dark matter contribution to the anisotropy. Assuming that polarization information can be fully exploited, we find that an isocurvature perturbation can be detected by the Planck Surveyor if it contributes more than 4% to the angular power spectrum at large scales. In the absence of polarization data, the signal from isocurvature perturbations can be confused with tensor perturbations or early re-ionization effects, and the limit is larger by almost an order of magnitude.

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I. INTRODUCTION

A second generation of satellite experiments, the Microwave Anisotropy Probe (MAP) [1] and Planck [2], will soon provide highly detailed temperature maps of the cosmic microwave background (CMB). From the measured spectrum of temperature fluctuations one may infer the properties of the cosmic fluid, such as its density or the rate by which it comoves with the expansion of the Universe, and thus determine most cosmological parameters to an accuracy of a few percent [3–5]. In addition, the spectrum of temperature fluctuations allows us to study and constrain physics at very small distance scales, corresponding to energies much higher than can be achieved in any particle accelerator in the foreseeable future. This is so because the features on the temperature map depend on the nature of the primordial density fluctuation.

Cosmic inflation is currently the most popular explanation for the origin of the primordial perturbations, and it typically predicts a near scale invariant spectrum with Gaussian fluctuations. The perturbations are adiabatic with the number density proportional to the entropy density so that $\delta(n/s) = 0$. This is so because the quantum fluctuations of the field responsible for inflation, called the inflaton, directly give rise to perturbations in the energy density of the inflaton field.

However, the inflaton may not be the only field which is subject to quantum fluctuations during inflation. In fact, any effectively massless scalar field will fluctuate by virtue of the nearly constant energy density of the inflation era with a root mean square amplitude $H/(2\pi)$, where H is the Hubble parameter during inflation. Such fluctuations will contribute to the perturbations in the microwave background. They may be adiabatic, in which case it is difficult to entangle their effect from the inflaton fluctuations. The other possibility is isocurvature fluctuations, which are fluctuations in the number rather than the energy density so that $\delta(n/s)\neq 0$. Examples of particle physics models giving rise to isocurvature fluctuations include axions [6,7], inflation with more than one inflaton field [8], and supersymmetric theories with flat directions in the potential [9].

Isocurvature perturbations [10,11] in a given particle species have $\delta\rho=0$ with the overdensities balanced by perturbations in other particle species, such as radiation. At the last scattering surface (LSS) the compensation for the isocurvature perturbations can be maintained only for scales larger than the horizon, effectively generating extra power to photon perturbations at small angular multipoles l. As a consequence, the spectrum of isocurvature perturbations differs a great deal from adiabatic perturbations, and a purely isocurvature cold dark matter (CDM) perturbation spectrum is in fact already ruled out [12] on the basis of the Cosmic Background Explorer (COBE) normalization and σ_8, the amplitude of the rms mass fluctuations in an $8h^{-1}$ Mpc^{-1} sphere. (There are suggestions that a decaying CDM model could sustain purely isocurvature perturbations [13]; however, such models are not motivated from a particle physics point of view.) A small isocurvature contribution might, however, be beneficial in the sense that it could help to improve the fit to the power spectrum in $\Omega_0=1$ CDM models with a cosmological constant [14,6].

The simplest possibility is that there is a single CDM which has both adiabatic and isocurvature perturbations. An example of such a situation is obtained in supersymmetric models with the so-called Affleck-Dine (AD) fields [15], which also provide a basis for baryogenesis. The AD fields are superpositions of squark and slepton fields corresponding to the flat directions. During inflation they will fluctuate along the flat directions forming a condensate. They are complex fields and, in the currently favored D-term inflation models [16], are effectively massless during inflation. The condensate does not correspond to the state of lowest energy but fragments into non-topological solitons [17], which carry baryonic (and sometimes leptonic) number [18] and are called B-balls (L-balls). The fluctuations in the AD field are

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both adiabatic and isocurvature [9], and will be inherited by the B-balls, which are either stable or decay later into lightest supersymmetric particles, which form the CDM. In the latter case there will be a lower bound on the amplitude of the isocurvature perturbation.

Adiabatic perturbations [19–21] are characterized by the gauge-invariant quantity ζ, which equals $\delta\rho/(\rho+p)$ when the perturbation exits or enters the horizon. Outside the horizon, ζ does not change with time. Isocurvature perturbations [10,20,21] are characterized by the gauge-invariant entropy perturbation $S\equiv\delta(n_c/s_\gamma)=\delta_c-(3/4)\delta_\gamma$, where n_c is the number density of CDM, s_γ is the entropy density associated with photons, and δ_c and δ_γ are the relative overdensities in the CDM and photon energy densities. During radiation domination $S\sim\delta_c\equiv\delta n_c/n_c$. Outside the horizon, S does not change with time. ''Initial'' scale-free ($n=1$) adiabatic and isocurvature perturbation spectra are given by

$$P_\zeta(k)\equiv\langle|\zeta_k|^2\rangle=Ak^{n-4}=Ak^{-3},$$

$$P_S(k)\equiv\langle|S_k|^2\rangle=Bk^{n-4}=Bk^{-3}. \tag{1}$$

Following these remarks, we assume in this paper that there is just a single species of CDM, and that it has both adiabatic and isocurvature fluctuations. As is natural when both fluctuations have their origin in inflation, we also assume that their power spectra are similar. The goal of the paper is then to study how well particle physics models with isocurvature fluctuations can be tested by the future CMB satellite experiments, and at what level isocurvature perturbation can be detected. In Sec. II we define the Fisher information matrix needed for the cosmological parameter error estimates, and describe the expected performance of MAP and Planck. In Sec. III we discuss parametrization of isocurvature fluctuation and possible cosmological models. In Sec. IV we present the results of a numerical study for MAP and Planck, while Sec. V contains a discussion of the results.

II. ACCURACY OF PARAMETER DETERMINATION

For a CMB temperature map with Gaussian fluctuations, all statistical information is contained in the angular power spectrum

$$C_{Tl}\equiv\langle|a^T_{lm}|^2\rangle. \tag{2}$$

This spectrum contains a wealth of cosmological information. The major cosmological parameters can be determined from a high-quality measurement of the spectrum, with an unprecedented accuracy. This is because the CMB anisotropy reflects conditions at the early universe, as well as the geometry out to the present horizon, the microwave photons having travelled through essentially the whole observable universe as well as most of its history [22]. There are, however, degeneracies with respect to some cosmological parameters: different effects causing similar features in the power spectrum. Also the information from the large angular scales is severely limited by cosmic variance.

Important additional information can be obtained by the measurement of the polarization of the CMB [23–26]. Since polarization of the CMB is caused by scattering only, polarization carries information about the LSS only (including late scattering due to reionization), eliminating confusion with line-of-sight effects. Polarization is especially useful in separating out the effects of tensor fluctuations, isocurvature fluctuations, and re-ionization, all of which produce excess power at large scales.

Unfortunately the polarization is expected to be of rather small amplitude, at least an order of magnitude below the anisotropy signal, making it difficult to detect [24]. The contamination with the foreground may also be a worse problem than for the temperature anisotropy, since there are polarized foreground sources, e.g., galactic synchrotron emission. The MAP satellite is expected to provide a definite detection of CMB polarization. The higher sensitivity of the Planck Surveyor will then help in getting a reasonable power spectrum for the polarization.

The polarization pattern on the sky can be separated into two components, the E-mode and the B-mode [23]. Thus an experiment which also measures the polarization will produce three maps: temperature, E-mode polarization, and B-mode polarization. The B-mode polarization of the CMB is uncorrelated with the temperature or the E-mode polarization. Thus the statistical information is carried by three angular power spectra

$$C_{Tl}\equiv\langle|a^T_{lm}|^2\rangle,$$

$$C_{El}\equiv\langle|a^E_{lm}|^2\rangle,$$

$$C_{Bl}\equiv\langle|a^B_{lm}|^2\rangle, \tag{3}$$

and one correlation function

$$C_{Cl}\equiv\langle a^{T*}_{lm}a^E_{lm}\rangle, \tag{4}$$

giving the correlation between the temperature and the E-mode polarization [23–27]. The different spectra are demonstrated in Fig. 1.

These four spectra can be calculated for different values of the cosmological parameters s_i and compared to data from CMB experiments. This way the true values of the cosmological parameters can be determined.

Assuming that the actual values of the parameters are $\overline{s_i}$, the information obtained on these parameters by observation is described by the Fisher information matrix [3,28,23,4,5,27]

$$F_{ij}=\sum_l\sum_{X,Y}\frac{\partial C_{Xl}}{\partial s_i}\bigg|_{\overline{s_i}}\mathrm{Cov}^{-1}(C_{Xl},C_{Yl})\frac{\partial C_{Yl}}{\partial s_j}\bigg|_{\overline{s_j}}, \tag{5}$$

where $C_{Xl}=C_{Tl},C_{El},C_{Bl}$, or C_{Cl}.

The covariance matrices, whose nonzero components are

$$\mathrm{Cov}(C_{Tl}C_{Tl})=N_l\left(C_{Tl}+\left[\sum_c w^c_T B^2_{l,c}\right]^{-1}\right)^2,$$

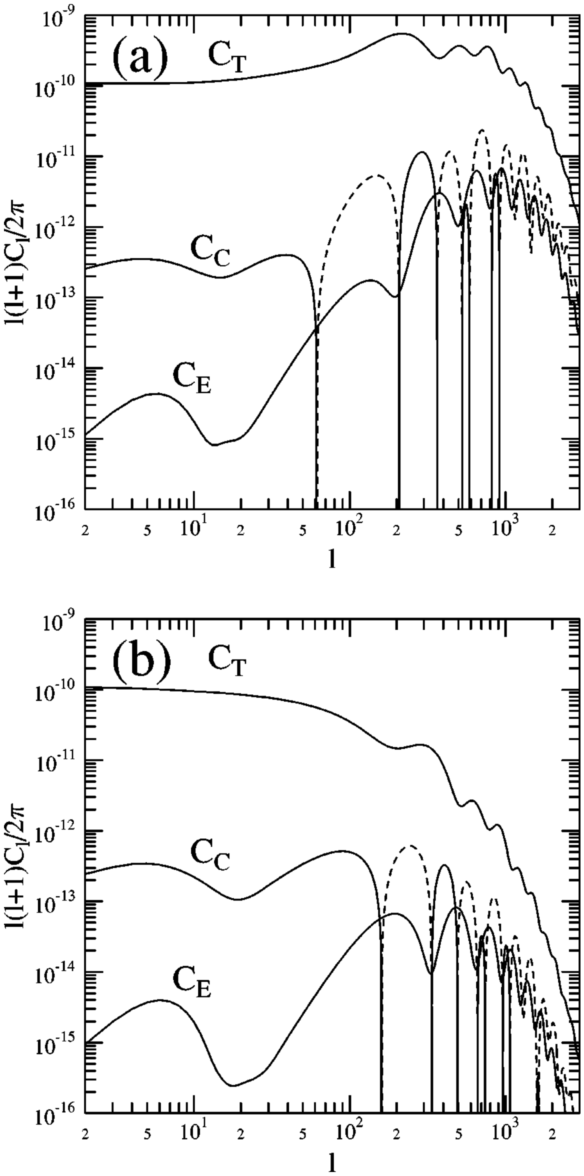

FIG. 1. The angular power spectra for SCDM (see Table II) adiabatic (a) and isocurvature (b) perturbations. These scalar perturbations produce E-mode polarization only; thus no B-mode spectra are shown. The dashed lines indicate negative values (for C_{Cl}).

$$\mathrm{Cov}(C_{El}C_{El}) = N_l\left(C_{El} + \left[\sum_c w_P^c B_{l,c}^2\right]^{-1}\right)^2,$$

$$\mathrm{Cov}(C_{Bl}C_{Bl}) = N_l\left(C_{Bl} + \left[\sum_c w_P^c B_{l,c}^2\right]^{-1}\right)^2,$$

$$\mathrm{Cov}(C_{Cl}C_{Cl}) = N_l\left\{C_{Cl}^2 + \left(C_{Tl} + \left[\sum_c w_T^c B_{l,c}^2\right]^{-1}\right)\times\left(C_{El} + \left[\sum_c w_P^c B_{l,c}^2\right]^{-1}\right)\right\},$$

$$\mathrm{Cov}(C_{Tl}C_{El}) = N_l C_{Cl}^2,$$

TABLE I. The beamwidths and sensitivities of the satellite experiments [1,2].

Instrument	Frequency	θ_{FWHM}	σ_T	σ_P
MAP	40 GHz	0.47°	35 μK	$\sqrt{2}\times 35$ μK
	60 GHz	0.35°	35 μK	$\sqrt{2}\times 35$ μK
	90 GHz	0.21°	35 μK	$\sqrt{2}\times 35$ μK
LFI	44 GHz	23′	2.4×10^{-6}	$\sqrt{2}\times 2.4\times 10^{-6}$
	70 GHz	14′	3.6×10^{-6}	$\sqrt{2}\times 3.6\times 10^{-6}$
	100 GHz	10′	4.3×10^{-6}	$\sqrt{2}\times 4.3\times 10^{-6}$
HFI	100 GHz	10.7′	1.7×10^{-6}	
	143 GHz	8.0′	2.0×10^{-6}	3.7×10^{-6}
	217 GHz	5.5′	4.3×10^{-6}	8.9×10^{-6}

$$\mathrm{Cov}(C_{Tl}C_{Cl}) = N_l C_{Cl}\left(C_{Tl} + \left[\sum_c w_T^c B_{l,c}^2\right]^{-1}\right),$$

$$\mathrm{Cov}(C_{El}C_{El}) = N_l C_{Cl}\left(C_{El} + \left[\sum_c w_P^c B_{l,c}^2\right]^{-1}\right), \tag{6}$$

depend on the sensitivity and angular resolution of the instrument with

$$w_T^c = \frac{1}{\sigma_{T,c}^2\theta_{\mathrm{FWHM},c}^2},$$

$$w_P^c = \frac{1}{\sigma_{P,c}^2\theta_{\mathrm{FWHM},c}^2}, \tag{7}$$

and

$$B_{l,c}^2 = e^{-l(l+1)\theta_{\mathrm{FWHM},c}^2/8\ln 2} \tag{8}$$

where σ_c is the pixel noise (sensitivity) and $\theta_{\mathrm{FWHM},c}$ is the beamwidth (full width at half maximum) of instrument channel c.

The prefactor is given by

$$N_l = \frac{2}{(2l+1)f_{\mathrm{sky}}}, \tag{9}$$

where f_{sky} takes into account the fact that the whole sky cannot be mapped because of foreground contamination. We adopt $f_{\mathrm{sky}} = 0.65$, i.e., assume that foreground can be completely removed from 65% of the sky and the rest of the sky is unusable.

The Planck Surveyor carries two instruments, HFI and LFI. For MAP and LFI we use the three highest frequency channels, for HFI the three lowest frequency channels for our estimates (see Table I).

The 1σ error on parameter s_i is

$$\Delta s_i = \sqrt{(F^{-1})_{ii}}, \tag{10}$$

when all parameters are estimated from the same data. [If all the other parameters were somehow known *a priori*, the error would be $\Delta s_i=(F_{ii})^{-1/2}$.] This is a lower limit for the error, but when the error is small it is usually a good approximation for the error [28,5].

III. PARAMETERS AND MODELS

We consider a simultaneous fit of 10 parameters, C_2, $\Omega_0\equiv\Omega_\Lambda+\Omega_m$, $\Omega_\Lambda-\Omega_m$, $\Omega_b h^2$, $\Omega_{\rm HDM}$, H_0,τ, n_S, r, and α. Here τ is the optical depth to LSS and $r\equiv C_2^T/C_2^S$ is the tensor/scalar ratio. We assume three matter components, $\Omega_m=\Omega_{\rm CDM}+\Omega_{\rm HDM}+\Omega_b$. Note that we have defined Ω_0 to include the vacuum energy; it is the parameter which determines the curvature of space. We assume $T_0=2.728$ K, $Y_{\rm He}=0.24$, and $N_\nu=3$. For hot dark matter we assume that all 3 neutrinos have equal masses.

The isocurvature contribution to anisotropy is given by the parameter α, which gives the fraction of C_2^S due to isocurvature perturbations. Thus

$$\alpha\equiv C_2^{\rm iso}/(C_2^{\rm iso}+C_2^{\rm ad}). \tag{11}$$

Our α is the same as $1-\alpha$ of Stompor *et al.* [14]. The isocurvature fluctuations are assumed to be primordial density fluctuations in CDM, initially balanced by radiation $+$baryons$+$neutrinos to make the total energy density homogeneous. The isocurvature spectral index is assumed to be the same as for adiabatic fluctuations; in principle, they could be different, but if both fluctuations originate from inflation, and in particular from massless field fluctuations during inflation, then their power spectra are guaranteed to be equal.

The large-scale anisotropy due to isocurvature perurbations is [21]

$$\frac{\delta T}{T}=-\frac{1}{3}S+\frac{1}{3}\Phi, \tag{12}$$

where

$$\Phi=-\frac{1}{5}S \tag{13}$$

is the Newtonian gravitational potential [29,20,22], whereas for adiabatic perturbations we have just

$$\frac{\delta T}{T}=\frac{1}{3}\Phi, \tag{14}$$

where the potential is related to the gauge-invariant ζ by [30]

$$\Phi=-\frac{3}{2}\zeta. \tag{15}$$

Thus the ''observational'' isocurvature parameter α is related to the amplitudes of the initial isocurvature and adiabatic spectra by

$$\frac{\alpha}{1-\alpha}=\frac{(-\delta T/T)^2_{\bf iso}}{(-\delta T/T)^2_{\bf ad}}=\left(\frac{4S}{5\zeta}\right)^2=\frac{16}{25}\left(\frac{B}{A}\right). \tag{16}$$

TABLE II. The cosmological models considered.

	Ω_0	Ω_Λ	Ω_b	$\Omega_{\rm HDM}$	H_0	τ	n_S	r	α
SCDM[a]	1.0	0	0.05	0	50	0.05	1	0	0
ΛCDM[b]	1.0	0.7	0.06	0	65	0.1	1	0	0
OCDM[c]	0.4	0	0.06	0	65	0.05	1	0	0

[a]Standard CDM.
[b]CDM with a cosmological constant.
[c]Open CDM.

In a given particle physics model A and B are known in terms of the model parameters, and Eq. (17) tells how these parameters are related to the observed isocurvature parameter α.

We have chosen to consider three different reference cosmological models (see Table II), which are the same as discussed by Wang *et al.* [31]: the standard CDM model with $\Omega_0=1$, a CDM model with a cosmological constant $\Omega_\Lambda=0.7$ and $\Omega_0=1$, and an open CDM model with $\Omega_0=0.4$.

To find the derivatives $\partial C_{Xl}/\partial s_i$ we used CMBFAST [32] to run variations around these three reference models, with steps Δs_i or $\Delta s_i/s_i$ between 0.01 and 0.05.

We COBE-normalized [33] C_2 for the three reference models. Note that as C_2 is one of the parameters to be determined from the data, we do not COBE-renormalize it for the variations around these reference models.

Wang *et al.* [31] considered 7-parameter fits. Our additional parameters are $\Omega_{\rm HDM}$, r, and α. For OCDM we considered 8-parameter fits only, keeping $r\equiv0$ and $\Omega_{\rm HDM}\equiv0$, since CMBFAST does not allow nonzero values for these parameters in open models.

IV. RESULTS

We are interested in how accurately the isocurvature contribution can be determined and what is the smallest contribution that can still be detected. In Tables III, IV, and V) we give the 1σ error on the isocurvature parameter α for the three experiments, MAP, Planck LFI, and Planck HFI, and an ideal experiment with zero noise and infinitely fine resolution. Full sky coverage would reduce these errors by a factor of $\sqrt{0.65}$.

We see that the addition of the two parameters, $\Omega_{\rm HDM}$ and r, increased the error in α determination significantly for the

TABLE III. $\Delta\alpha$ from 10-parameter fits.

	MAP	LFI	HFI	Ideal
Without polarization				
SCDM	0.37	0.26	0.23	0.14
ΛCDM	0.38	0.23	0.20	0.17
With polarization				
SCDM	0.23	0.058	0.038	0.015
ΛCDM	0.21	0.067	0.045	0.019

TABLE IV. $\Delta\alpha$ from 8-parameter fits ($\Omega_{HDM}\equiv 0, r\equiv 0$).

	MAP	LFI	HFI	Ideal
Without polarization				
SCDM	0.35	0.13	0.12	0.10
ΛCDM	0.22	0.16	0.15	0.15
OCDM	0.27	0.13	0.064	0.053
With polarization				
SCDM	0.21	0.056	0.037	0.015
ΛCDM	0.16	0.062	0.044	0.018
OCDM	0.14	0.046	0.033	0.012

case without polarization data, but had only a small effect for the case with polarization. This is because the temperature spectra from tensor and isocurvature fluctuations resemble each other, resulting in a degeneracy between the parameters r and α. (Adding Ω_{HDM} as a free parameter changes $\Delta\alpha$ at most by 10%.) Because only tensor fluctuations have B-mode polarization, polarization measurements break this degeneracy.

Note that the tensor and isocurvature parameters, r and α, are non-negative by definition. It is quite likely that their true values are smaller than the errors in their determination, so that we will only obtain an upper limit. For the case without polarization information, there will be a degeneracy between these two parameters, such that their sum can be obtained with a higher accuracy than the parameters themselves (see Fig. 2). Since the upper limit to the sum is also an upper limit to the individual parameters, we can then obtain a tighter upper limit to α than the $\Delta\alpha$ given in Table III. This upper limit is essentially the $\Delta\alpha$ given in Table IV, where we have made the restriction $r\equiv 0$.

For MAP, there are also other significant degeneracies among the 10 parameters considered here. For Planck without polarization, the only other important degeneracy is with the optical depth τ due to re-ionization. Polarization breaks this degeneracy, since early reionization produces a polarization signal at large angular scales which cannot be mimicked by other parameter combinations [34]. For Planck with polarization there are no significant degeneracies between α and the other parameters (except, of course, a degeneracy between C_2 and α), and so the errors would not become much smaller even if all the other parameters were assumed known (see Table V).

TABLE V. $\Delta\alpha$ from 2-parameter fits (C_2 and α). This table is shown to illustrate, by comparison to Table IV, the effect of degeneracies with other parameters. Because the best determinations of many of the other parameters have to come from the same CMB data, α cannot in reality be determined with this accuracy.

	MAP	LFI	HFI	Ideal
Without polarization				
SCDM	0.053	0.049	0.047	0.046
ΛCDM	0.055	0.052	0.051	0.050
OCDM	0.041	0.038	0.036	0.035
With polarization				
SCDM	0.053	0.039	0.030	0.013
ΛCDM	0.054	0.044	0.036	0.016
OCDM	0.041	0.030	0.026	0.010

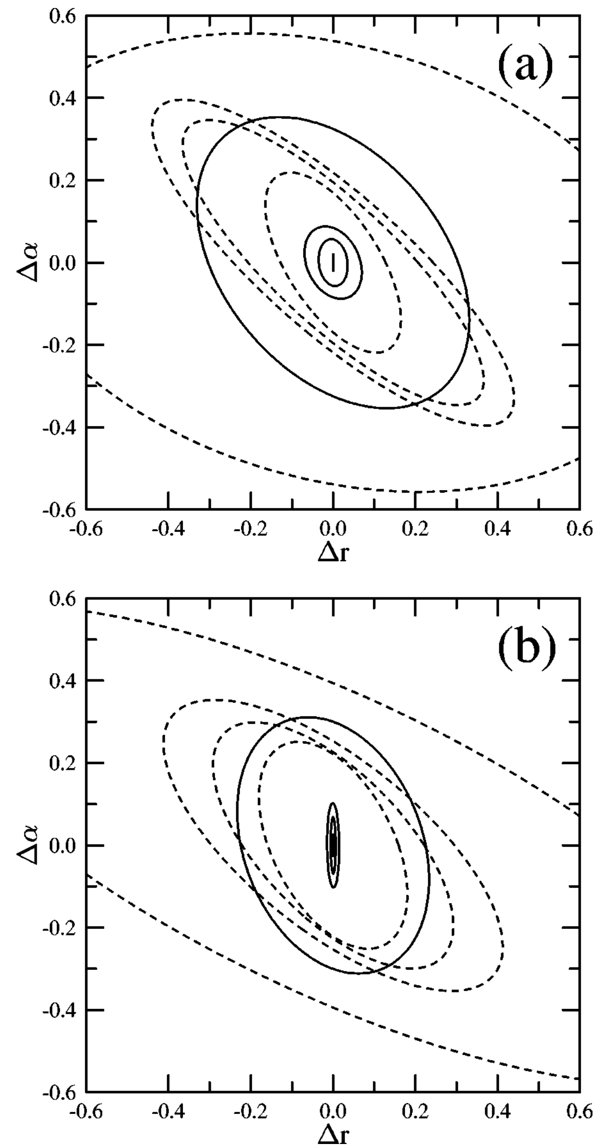

FIG. 2. The error ellipses in the (r,α) plane for (a) SCDM and (b) ΛCDM. The dashed lines are for the case without polarization measurement, the solid lines with polarization. The four ellipses from the outside inward are for MAP, Planck LFI, Planck HFI, and the ideal experiment.

V. DISCUSSION

From Tables III and IV we find that the (1σ) detection limit of isocurvature fluctuations is $\alpha\sim 0.04$ for the Planck Surveyor. This corresponds to a ratio $B/A\sim 0.07$ between the amplitudes of the initial isocurvature and adiabatic perturbation spectra. This is at the level predicted by certain B-ball models [9], which thus could be tested by Planck. In B-ball models the isocurvature perturbation arises because of fluctuations of the phase of the AD field. The size of the fluctuation is correlated with the adiabatic fluctuation of the AD field, which, in addition to the inflaton field, thus contributes to the adiabatic part of the power spectrum. However, a more detailed study of the correlation between the adiabatic and isocurvature fluctuation is required before any firm prediction can be made.

In models where axions are the main ingredient for CDM the situation is more straightforward [6]. Like in the AD field (i.e. B-ball) case, the spectral indices of the isocurvature and adiabatic perturbations are equal. COBE normalization and data on mass fluctuations at the scale of $8h^{-1}$ Mpc^{-1} sphere implies that the axion coupling constant F_a should be larger by some three orders of magnitude compared with its standard cosmological upper bound. Axions may not yet be ruled out on the basis of this since in more complicated string-based models F_a could be quite large. However, isocurvature measurements by MAP and Planck will definitely further constrain axion models.

In other models, such as two-field inflation models [8], the isocurvature and adiabatic perturbations may enter with different spectral indices. In such cases a direct comparison of A and B is not sufficient and instead one has to perform a more detailed analysis.

It is likely that other particle physics models can also be constrained in a significant way on the basis (or the absence) of isocurvature fluctuation in CMB. However, to reach this sensitivity polarization measurements are crucial, as is evident from Tables III and IV.

Without polarization information, the contribution from isocurvature fluctuations to temperature anisotropies could be confused with tensor perturbations or early re-ionization effects. Polarization measurements break this degeneracy, since these other two effects both have a unique polarization signal.

Whether such sensitivity can be realized is still unclear; the foreground contamination for the polarization spectra may turn out to be a more severe problem than in the case of the temperature angular power spectrum. However, optimizing the polarization information should have a high priority, in particular for Planck; because of its higher sensitivity, it is in this respect that Planck is truly a superior instrument compared with MAP. CMB polarization measures both tensor and isocurvature contributions, which are of paramount interest to inflation model builders.

Finally, the cosmic variance sets an absolute lower theoretical limit, at about $\alpha \sim 0.01$, to how small isocurvature fluctuations can be detected from the CMB anisotropy by any experiment.

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