Topological structure of a chiral QCD vacuum

V. Gogohia*
HAS, CRIP, RMKI, Department of Theoretical Physics, P.O.B. 49, H-1525, Budapest 114, Hungary and Research Center for Nuclear Physics (RCNP), Osaka University, Mihogaoka 10-1, Ibaraki, Osaka 567-0047, Japan

H. Toki
Research Center for Nuclear Physics (RCNP), Osaka University, Mihogaoka 10-1, Ibaraki, Osaka 567-0047 Japan
(Received 9 August 1999; published 12 January 2000)

Using the trace anomaly relation, low-energy theorem, and Witten-Veneziano formula, we have developed an analytical formalism which allows one to calculate the gluon condensate, the topological susceptibility, and the mass of the η' meson in the chiral limit as functions of the nonperturbative vacuum energy density. It is used for numerical evaluation of the chiral QCD topology within the QCD vacuum model consisting mainly of the quantum component given by the recently proposed zero modes enhancement model and the classical component given by the random instanton liquid model. We sum up both contributions into the total, nonperturbative vacuum energy density. A very good agreement with the phenomenological values of the topological susceptibility, the mass of the η' meson in the chiral limit, and the gluon condensate has been obtained. This puts the above-mentioned QCD vacuum model on a firm phenomenological ground.

PACS number(s): 11.15.Tk, 12.38.Lg

I. INTRODUCTION

The nonperturbative QCD vacuum has a very rich dynamical and topological structure [1]. It is a very complicated medium and its dynamical and topological complexity means that its structure can be organized at various levels (quantum, classical) and it can contain many different components and ingredients which may contribute to the vacuum energy density, one of the main characteristics of the QCD ground state. The quantum part of the vacuum energy density is determined by the effective potential approach for composite operators introduced by Cornwall, Jackiw, and Tomboulis (CJT) [2] (see also Ref. [3]). It allows us to investigate the nonperturbative QCD vacuum, since in the absence of external sources the effective potential is nothing but the vacuum energy density. It gives the vacuum energy density in the form of the loop expansion where the number of the vacuum loops (consisting of the confining quarks and nonperturbative gluons properly regularized with the help of ghosts) is equal to the power of the Plank constant, $\hbar$.

In our previous work [4] we have formulated a new, quantum model of the QCD ground state (its nonperturbative vacuum), the so-called zero modes enhancement (ZME) model. It is based on the existence and importance of such kind of nonperturbative, topologically nontrivial quantum excitations of the gluon field configurations (due to the self-interactions of massles gluons only) which can be effectively, correctly described by the q^{-4}-type behavior of the full gluon propagator in the deep infrared domain. It allows one to calculate the nonperturbative vacuum energy density from first principles using the CJT approach for composite operators [2]. We have also formulated the method of how to determine numerically the finite part of the vacuum energy density. We propose to minimize the effective potential at a fixed scale as a function of a parameter which has a clear physical meaning. When it is zero then only the perturbative phase remains in our model. Equivalently one can minimize the corresponding auxiliary effective potential as a function of the ultraviolet (UV) cutoff itself. The nonperturbative chiral QCD vacuum is found stable since its main characteristic—the vacuum energy density—has no imaginary part and it is always negative.

Within the ZME quantum model of the QCD ground state [4], the vacuum energy density depends on a scale at which the nonperturbative effects become important. If QCD itself is a confining theory, such a characteristic scale should certainly exist. The quark part of the vacuum energy density depends in addition on the constant of integration of the corresponding Schwinger-Dyson (SD) equation. The numerical value of the nonperturbative scale as well as the above mentioned constant of integration is obtained from the bounds

$$87.2 \leqslant F_\pi^o \leqslant 93.3 \ \ (\mathrm{MeV}), \tag{1.1}$$

for the pion decay constant in the chiral limit by implementing a physically well-motivated scale-setting scheme [4]. We have obtained the following numerical results for the nonperturbative vacuum energy density, $\epsilon = \epsilon_g + N_f \epsilon_q$:

$$\epsilon = -(0.01425 - N_f 0.00196) \ \ \mathrm{GeV}^4, \tag{1.2}$$

$$\epsilon = -(0.01087 - N_f 0.00150) \ \ \mathrm{GeV}^4, \tag{1.3}$$

where, obviously, the first and second values are due to upper and lower bounds in Eq. (1.1), respectively. Let us recall that these numerical values have been obtained by approximating the full gluon propagator by its nonperturbative term in the whole range, i.e., it has been assumed that the perturbative contribution has been already subtracted. Let us recall that here and further on below N_f is the number of light

*Email address: gogohia@rcnp.osaka-u.ac.jp and gogohia @rmki.kfki.hu

TABLE I. The gluon condensate in the strong coupling limit.

F^0_π (MeV)	$\overline{G}_2$ (GeV^4)	$N_f=0$	$N_f=1$	$N_f=2$	$N_f=3$
87.2		0.043	0.037	0.031	0.025
93.3		0.057	0.049	0.041	0.033

flavors and in what follows we will use $\epsilon=\epsilon_g\equiv\epsilon_{YM}$ in Eqs. (1.2), (1.3) in the case of pure Yang-Mills (YM) fields, $N_f=0$.

On the other hand, many models of the QCD vacuum involve some extra classical color field configurations (such as randomly oriented domains of constant color magnetic fields, background gauge fields, averaged over spin and color, stochastic colored background fields, etc.) and ingredients such as color-magnetic and Abelian-projected monopoles (see Refs. [1,5] and references therein). The relevance of center vortices to QCD by both lattice [6] and analytical method [7] was recently investigated as well. However, the most elaborated classical models are the random and interacting instanton liquid models (RILM and IILM) of the QCD vacuum [8]. They are based on the existence of the topologically nontrivial instanton-type fluctuations of gluon fields, which are solutions to the classical equations of motion in Euclidean space [8] (and references therein).

In this paper we treat the chiral QCD vacuum as consisting mainly of the two components, the classical one given by RILM [8] and the quantum one given by ZME [4], by summing up their contributions into the total, nonperturbative vacuum energy density. The main purpose of this paper is to show that this model of the nonperturbative QCD vacuum is in fair agreement with phenomenology. For example, it exactly reproduces the phenomenological value of the topological susceptibility. In Secs. II, III, and IV using the trace anomaly relation [9], low-energy theorem [10,11] and Witten-Veneziano (WV) formula [12] we develop an analytical formalism which allows us to calculate the gluon condensate, the topological susceptibility and the mass of the η' meson in the chiral limit as functions of the total, nonperturbative vacuum energy density (the bag constant, apart from the sign, by definition). In Sec. V we present our estimate of the nonperturbative vacuum energy density in the chiral limit due to instantons. Section VI is devoted to discussion and our conclusions are given in Sec. VII. The numerical results are shown in Tables I–IX.

II. THE GLUON CONDENSATE IN THE STRONG COUPLING LIMIT

The vacuum energy density is important in its own right as the main characteristic of the nonperturbative vacuum of QCD. Furthermore it assists in estimating such an important phenomenological parameter as the gluon condensate, introduced in the QCD sum rules approach to resonance physics [13]. The famous trace anomaly relation [9] in the general case (nonzero current quark masses m_f^0) is

$$\Theta_{\mu\mu}=\frac{\beta(\alpha_s)}{4\alpha_s}G^a_{\mu\nu}G^a_{\mu\nu}+\sum_f m_f^0\overline{q}_f q_f, \tag{2.1}$$

where $\Theta_{\mu\mu}$ is the trace of the energy-momentum tensor and $G^a_{\mu\nu}$ being the gluon field strength tensor while $\alpha_s=g^2/4\pi$. Sandwiching Eq. (2.1) between vacuum states and on account of the obvious relation $\langle 0|\Theta_{\mu\mu}|0\rangle=4\epsilon_t$, one obtains

$$\epsilon_t=\frac{1}{4}\langle 0|\frac{\beta(\alpha_s)}{4\alpha_s}G^a_{\mu\nu}G^a_{\mu\nu}|0\rangle+\frac{1}{4}\sum_f m_f^0\langle 0|\overline{q}_f q_f|0\rangle, \tag{2.2}$$

where ϵ_t is the sum of all possible independent, nonperturbative contributions to the vacuum energy density (the total vacuum energy density) and $\langle 0|\overline{q}_f q_f|0\rangle$ is the quark condensate. From this equation in the chiral limit ($m_f^0=0$), one obtains

$$\langle\overline{G^2}\rangle\equiv-\left\langle\frac{\beta(\alpha_s)}{4\alpha_s}G^2\right\rangle\equiv-\langle 0|\frac{\beta(\alpha_s)}{4\alpha_s}G^a_{\mu\nu}G^a_{\mu\nu}|0\rangle=-4\epsilon_t, \tag{2.3}$$

where we need to introduce a new quantity, namely the gluon condensate in the strong coupling limit, i.e., not using in general the weak coupling limit solution to the β function (see Sec. V below). If confinement happens then the β function is always in the domain of attraction (i.e., always negative) without IR stable fixed point [14]. Thus the nonperturbative gluon condensate $\langle\overline{G^2}\rangle$, defined in Eq. (2.3), is always positive as it should be. Saturating ϵ_t by our values (1.2), (1.3) which are relevant in the strong coupling limit, one obtains

$$\overline{G}_2\equiv\langle\overline{G^2}\rangle=-4(\epsilon_g+N_f\epsilon_q), \tag{2.4}$$

which gives the gluon condensate in the strong coupling limit as a function of N_f. The numerical results are shown in Table I.

III. THE TOPOLOGICAL SUSCEPTIBILITY

One of the main characteristics of the QCD nonperturbative vacuum is the topological density operator (topological susceptibility) in gluodynamics ($N_f=0$)

$$\chi_t=-\lim_{q\to 0} i\int d^4x\, e^{iqx}\langle 0|T\{q(x)q(0)\}|0\rangle, \tag{3.1}$$

where $q(x)$ is the topological charge density, defined as $q(x)=(\alpha_s/8\pi)G(x)\tilde{G}(x)=(\alpha_s/8\pi)G^a_{\mu\nu}(x)\tilde{G}^a_{\mu\nu}(x)$ and $\tilde{G}^a_{\mu\nu}(x)=(1/2)\epsilon^{\mu\nu\rho\sigma}G^a_{\rho\sigma}(x)$ is the dual gluon field strength tensor. In the definition of the topological susceptibility (3.1) it is assumed that the corresponding regularization and subtraction of the perturbative contribution have been already done in order (3.1) to stand for the renormalized, finite and the nonperturbative topological susceptibility (see Refs. [10–12,15]). Precisely this quantity measures the fluctuation of the topological charge in the nonperturbative vacuum.

As it was shown in Refs. [10,11], the topological susceptibility can be related to the nonperturbative gluon condensate via the low energy theorem in gluodynamics as follows:

TABLE II. ZME model values for $\chi_t^{1/4}$ and $m_{\eta'}$ in MeV units.

F_π^0	$\chi_t^{1/4}$ (NSVZ)	(HZ)	$m_{\eta'}^0$ (NSVZ)	(HZ)
87.2	159.6	129.8	710	473.3
93.3	170	139	759.7	506.5

$$\lim_{q\to 0} i\int d^4x\, e^{iqx}\langle 0|T\left\{\frac{\alpha_s}{8\pi}G\tilde{G}(x)\frac{\alpha_s}{8\pi}G\tilde{G}(0)\right\}|0\rangle = \xi^2\left\langle\frac{\beta(\alpha_s)}{4\alpha_s}G^2\right\rangle. \tag{3.2}$$

There exist two proposals to fix the numerical value of the coefficient ξ. The value $\xi = 2/b,\ b=11$ was suggested a long time ago by Novikov, Schifman, Vanshtein, and Zakharov (NSVZ), who used the dominance of self-dual fields hypothesis in the YM vacuum [10]. A second one, $\xi = 4/3b$, was advocated very recently by Halperin and Zhitnitsky (HZ), using a one-loop connection between the conformal and axial anomalies in the theory with auxiliary heavy fermions [11] (and references therein). However, in our numerical calculations we will use both values for the ξ parameter. Using the trace anomaly relation (2.3) and saturating the total vacuum energy density by its part at $N_f=0$, ϵ_{YM}, the topological susceptibility (3.1), on account of (3.2), can be easily expressed as follows:

$$\chi_t = -(2\xi)^2\epsilon_{YM}. \tag{3.3}$$

The significance of this formula is that it gives the topological susceptibility as a function of the nonperturbative vacuum energy density for pure gluodynamics, ϵ_{YM}. For numerical calculations, however, it is much more convenient (see below, next section) to use the topological susceptibility in the WV form [12], which is related to Eq. (3.3) as

$$\chi_t^{WV} = \left(\frac{2}{N_c}\right)^2\chi_t = -\left(\frac{4\xi}{N_c}\right)^2\epsilon_{YM}, \tag{3.4}$$

where N_c is the number of different colors. In what follows all our numerical results for this quantity stand for χ_t^{WV} and not for χ_t, shown in Eq. (3.3), however the superscript ''WV'' will be omitted for simplicity. It is easy to show that one obtains the same expression for the topological susceptibility (3.3) or (3.4) as a function of the nonperturbative YM vacuum energy density if one would use the weak coupling limit solution to the β function from the very beginning (see Sec. V below). The numerical results due to ZME model are shown in Table II.

In conclusion, let us note that there exists an interesting relation between the HZ and NSVZ values for the ξ parameter, namely $\xi_{HZ} = (2/N_c)\xi_{NSVZ}$, $N_c=3$, which in principle may be traced back to the different definitions of anomaly equations.

IV. THE $U(1)$ PROBLEM

The topological susceptibility (3.1) assists in the resolution of the $U(1)$ problem[1] [16] via the WV formula for the mass of the η' meson [12]. Within our notations it is expressed as $f_{\eta'}^2 m_{\eta'}^2 = (16N_f/N_c^2)\chi_t$, where $f_{\eta'}$ is the η' residue defined as $\langle 0|\Sigma_{q=u,d,s}\bar{q}\gamma_\mu\gamma_5 q|\eta'\rangle = i\sqrt{N_f}f_{\eta'}p_\mu$ and $\langle 0|N_f(\alpha_s/4\pi)G\tilde{G}|\eta'\rangle = (N_c\sqrt{N_f}/2)f_{\eta'}m_{\eta'}^2$. So, following Witten [12], the anomaly equation is $\partial_\mu J_5^\mu = 2N_f(2/N_c)(\alpha_s/8\pi)G\tilde{G}$. Using also the normalization relation $f_{\eta'} = \sqrt{2}F_\pi^0$, one finally obtains

$$F_\pi^2 m_{\eta'}^2 = \frac{8N_f}{N_c^2}\chi_t = 2N_f\chi_t^{WV}, \tag{4.1}$$

where the topological susceptibility is introduced in a useful WV form for numerical calculations, as it was mentioned above in Eq. (3.4). In previous expression we omit for simplicity the superscript ''0'' in the pion decay constant as well as in $m_{\eta'}^2$. In the numerical evaluation of the expression (4.1), we will put, of course, $N_f = N_c = 3$, while the topological susceptibility will be evaluated at $N_f = 0$ as it should be by definition. Using then Eq. (3.4), one obtains

$$m_{\eta'}^2 = -2N_f\left(\frac{4\xi}{F_\pi N_c}\right)^2\epsilon_{YM}, \tag{4.2}$$

which expresses the mass of the η' meson as a function of the nonperturbative vacuum energy density. This allows one to easily calculate it in the chiral limit within our formalism (see again Table II).

It is instructive to reproduce the WV formula (4.1) in the nonchiral case as well, namely

$$m_{\eta'}^2 = \frac{2N_f}{F_\pi^2}\chi_t^{WV} + \Delta, \tag{4.3}$$

where $\Delta = 2m_K^2 - m_\eta^2$. The precise validity of the WV formula (4.3) is, of course, not completely clear, nevertheless, let us regard it (for simplicity) as exact. Using now experimental values of all physical quantities entering this formula, one obtains that the phenomenological (''experimental'') value of the topological susceptibility is

$$\chi_t^{phen} = 0.001058\ \text{GeV}^4 = (180.36\ \text{MeV})^4. \tag{4.4}$$

In the chiral limit $\Delta = 0$ since $K^\pm$ and η particles are Nambu-Goldstone (NG) bosons. Omitting formally this contribution from the right hand side of Eq. (4.3) and on account of (1.1), one is able to derive an upper and absolute lower bounds for the mass of the η' meson in the chiral limit

$$854 \le m_{\eta'}^0 \le 913.77\ \ (\text{MeV}), \tag{4.5}$$

[1] We are going to consider here only one aspect of this problem, namely the large mass of the η' meson.

which should be compared with its experimental value $m_{\eta'}^{expt}=957.77$ MeV. One can conclude that the mass of η' meson remains large even in the chiral limit. It is worth noting that neither the numerical value of the topological susceptibility nor the mass of the η' meson in the chiral limit cannot exceed their phenomenological and experimental values. So the WV formula (4.3) in the chiral limit provides an absolute lower bound for the pion decay constant in this case, namely $F_\pi^0 \geqslant 83.2$ MeV.

In order to directly apply this formalism to RILM we need the realistic estimate of the corresponding chiral vacuum energy density in this model.

V. THE VACUUM ENERGY DENSITY DUE TO INSTANTONS

The instanton-type topological fluctuations, being a classical phenomena, nevertheless also contributed to the vacuum energy density through a tunneling effect which was known to lower the energy of the ground state [8]. It can be estimated as follows. Let us consider again the trace anomaly relation (2.2) in the chiral limit, i.e., $\epsilon_t = (1/4)\langle 0|(\beta(\alpha_s)/4\alpha_s)G^a_{\mu\nu}G^a_{\mu\nu}|0\rangle$. Using the weak coupling limit solution to the β function now

$$\beta(\alpha_s) = -b\frac{\alpha_s^2}{2\pi} + O(\alpha_s^3), \quad b = 11 - \frac{2}{3}N_f, \tag{5.1}$$

one obtains

$$\epsilon_t = -\frac{b}{4}\times\frac{1}{8}\langle 0|\frac{\alpha_s}{\pi}G^a_{\mu\nu}G^a_{\mu\nu}|0\rangle. \tag{5.2}$$

The phenomenological analysis of QCD sum rules [13] for the gluon condensate implies

$$G_2 \equiv \langle G^2\rangle \equiv \left\langle \frac{\alpha_s}{\pi}G^2\right\rangle \equiv \langle 0|\frac{\alpha_s}{\pi}G^a_{\mu\nu}G^a_{\mu\nu}|0\rangle \simeq 0.012 \ \ \text{GeV}^4, \tag{5.3}$$

which can be changed within a factor of two [13]. From the phenomenological estimate (5.3), one easily can calculate $(1/8)\langle 0|(\alpha_s/\pi)G^a_{\mu\nu}G^a_{\mu\nu}|0\rangle \simeq 0.0015 \ \ \text{GeV}^4 \simeq 1.0 \ \text{fm}^{-4}$. Having in mind this and assuming that the gluon condensate in the weak coupling limit is determined by the instanton-type fluctuations only, Shuryak [8] (see also references therein) has concluded that the ''average separation'' between instantons was $R\simeq 1.0$ fm, so the corresponding density of the instanton-type fluctuations should be $n \simeq 1.0 \ \ \text{fm}^{-4}$. Let us note that the second parameter of the instanton liquid model of the QCD vacuum, the instanton size $\rho_0 \simeq 1/3$, was chosen to reproduce standard (also as gluon condensate) phenomenologically estimated from QCD sum rules [13]) value of the quark condensate. This contribution to the vacuum energy density via the trace anomaly relations (2.1), (2.2) vanishes in the chiral limit. However, due to all reasonable estimates of light quark masses, numerically its contribution is at 20% and thus comparable with the systematic error in the determination of the gluon condensate itself [13,17].

Saturating the total vacuum energy in the weak coupling limit by instanton component ϵ_I and using the above mentioned estimate, from Eq. (5.2) for dilute ensemble, one finally obtains

$$\epsilon_I = -\frac{b}{4}n = -\frac{b}{4}\times 1.0 \ \ \text{fm}^{-4} = -(0.00417 - N_f 0.00025) \ \ \text{GeV}^4. \tag{5.4}$$

Thus instanton contribution to the vacuum energy density was not calculated independently but was postulated via the trace anomaly relation using the phenomenological value of the gluon condensate (5.3) as well as weak coupling limit solution to the β function (5.1). It is well known that density of instanton-type fluctuations is suppressed in the chiral limit and is again restored because of dynamical breakdown of chiral symmetry [8] (and references therein). In any case it cannot be large in the chiral limit, so the functional dependence of the vacuum energy density on the instanton density, established in Eq. (5.4) due to dilute gas approximation, seems to be justified in this case. The only problem is the numerical value of the instanton density itself, which can be taken either from phenomenology or from lattice simulations.

In Ref. [10] it was argued that the gluon condensate in the chiral limit is approximately two times less than the above mentioned phenomenological (empirical) value (5.3), i.e., $\langle G^2\rangle_{ch} \simeq 0.5\langle G^2\rangle_{phen}$. This means that in this case instanton density $n\simeq 0.5 \ \ \text{fm}^{-4}$ and the vacuum energy density due to instantons approximately two times less than Eq. (5.4). However, it has been already pointed out [18] that QCD sum rules substantially underestimate the value of the gluon condensate. The most recent phenomenological calculation of the gluon condensate is given by Narison in Ref. [19], where a brief review of many previous calculations is also presented. His analysis leads to the update average value as

$$\langle 0|\frac{\alpha_s}{\pi}G^a_{\mu\nu}G^a_{\mu\nu}|0\rangle = (0.0226 \pm 0.0029) \ \ \text{GeV}^4. \tag{5.5}$$

This means that instanton density is approximately two times bigger than it was estimated by Shuryak for instanton liquid model [8], but in the chiral limit we are again left with Eq. (5.4).

In lattice QCD situation with instanton density and their sizes is also ambigious. In quenched $(N_f=0)$ lattice QCD by using the so-called ''cooling'' method the role of the instanton-type fluctuations in the QCD vacuum was investigated [20]. In particular, it was found that the instanton density should be $n=(1+\delta) \ \ \text{fm}^{-4}$, where $\delta \simeq 0.3$–0.6 depending on cooling steps. Moreover, by studying the topological content of the vacuum of $SU(2)$ pure gauge theory using a method of RG mapping [21], it is concluded that the average radius of an instanton is about 0.2 fm, at a density of about 2 fm^{-4}. However, in Ref. [22] the topological content of the

TABLE III. Chiral topology due to instantons.

n (fm^{-4})	0.5	1.0
ϵ_I (GeV^4)	-0.002085	-0.00417
G_2 (GeV^4)	0.006	0.012
$\chi_t^{1/4}$ (MeV)		
(NSVZ)	104	125
(HZ)	86	102
$m^0_{\eta'}$ (MeV)		
$F^0_\pi = 87.2$ MeV		
(NSVZ)	311	439.7
(HZ)	207.3	293
$m^0_{\eta'}$ (MeV)		
$F^0_\pi = 93.3$ MeV		
(NSVZ)	290.6	411
(HZ)	193.7	274

$SU(3)$ vacuum was studied using the same method as for $SU(2)$ gauge theory earlier and was obtained a fair agreement with Shuryak's phenomenologically estimated numbers for the instanton liquid model. At the same time, in Refs. [23,24] considerably larger values were reported. Thus at this stage it is rather difficult to choose some well-justified numerical value of the instanton-type contribution to the nonperturbative vacuum energy density. In any case, in what follows we will consider Eq. (5.4) as a realistic upper bound for the instanton contribution to the vacuum energy density in the chiral limit. If the instanton number density is about $n \simeq 2$ fm^{-4}, then in the chiral limit we again are left with Eq. (5.4), but if it is about $n \simeq 1$ fm^{-4}, we will be left with half of Eq. (5.4). Then the instanton contributions to the topological susceptibility and the mass of the η' meson in the chiral limit are to be calculated via Eqs. (3.4) and (4.2), respectively, on account of the substitution $\epsilon_{YM} \to \epsilon_I(N_f = 0)$, where ϵ_I is given in Eq. (5.4) with the two different values for instanton number densities in the chiral limit, $n = 0.5$ fm^{-4}, 1.0 fm^{-4}. The numerical results are shown in Table III. In conclusion, we note that for densities $n > 2$ fm^{-4} (which means $n > 1$ fm^{-4} in the chiral limit) the applicability of the dilute gas approximation becomes, apparently, doubtful.

VI. DISCUSSION

A. The gluon condensate

It becomes almost obvious that we must distinguish the two types of gluon condensates, both of which are the nonperturbative quantities. The first one is determined by Eq. (2.3) and is the one which is relevant in the strong coupling limit. In this case the total vacuum energy is mainly saturated by the ZME component as it is precisely shown in Eq. (2.4). In the weak coupling limit, saturating ϵ_t by ϵ_I, from Eqs. (5.2)–(5.4) one obtains

$$\left\langle \frac{\alpha_s}{\pi} G^2 \right\rangle = -\frac{32}{b}\epsilon_t \simeq -\frac{32}{b}\epsilon_I = 8n; \tag{6.1}$$

TABLE IV. Chiral QCD topology ($F^0_\pi = 87.2$ MeV).

	$\epsilon_{ZME}+\epsilon_I(0.5$ fm$^{-4})$	$\epsilon_{ZME}+\epsilon_I(1.0$ fm$^{-4})$
ϵ_t (GeV^4)	-0.013	-0.015
$\chi_t^{1/4}$ (MeV)		
(NSVZ)	166.2	172.3
(HZ)	135.7	140.7
$m^0_{\eta'}$ (MeV)		
(NSVZ)	776.4	834
(HZ)	517.6	556

i.e., the gluon condensate in the weak coupling limit does not explicitly depend on N_f. As was mentioned above, precisely this gluon condensate was introduced long ago [13]. This unphysical situation takes place because in instanton calculus [8] there is no other way to calculate the vacuum energy density than the trace anomaly relations (2.1), (2.2) which becomes finally Eq. (6.1) as it was described above. In this case it is preferable to have the N_f dependent vacuum energy density than the gluon condensate since the former is the main characteristic of the nonperturbative vacuum. Contrast to this, we have calculated the vacuum energy density completely independently from the trace anomaly relation. We use it only to calculate the gluon condensate in the strong coupling limit. That is why in our case both quantities are N_f dependent functions.

Our bounds for full QCD ($N_f = 3$) gluon condensate in the strong coupling limit

$$0.025 \leqslant -\langle 0 | \frac{\beta(\alpha_s)}{4\alpha_s} G^a_{\mu\nu} G^a_{\mu\nu} | 0 \rangle \leqslant 0.033 \;\; (\mathrm{GeV}^4), \tag{6.2}$$

are comparable with recent phenomenological determination of the standard gluon condensate by Narison (5.5). The parametrization (the left hand side) of the two types of the gluon condensate may be, of course, the same but their numerical values (the right hand sides) are not to be the same. This difference is not only due to different physical observables as was noticed in Ref. [8]. Though both quantities are the nonperturbative phenomena, nevertheless this difference reflects different underlying physics. Our gluon condensate (2.3) is the strong coupling limit result and reflects the nontrivial topology of the true QCD vacuum where quantum excitations of gluon fields play an important role. As was

TABLE V. Chiral QCD topology ($F^0_\pi = 93.3$ MeV).

	$\epsilon_{ZME}+\epsilon_I(0.5$ fm$^{-4})$	$\epsilon_{ZME}+\epsilon_I(1.0$ fm$^{-4})$
ϵ_t (GeV^4)	-0.016	-0.018
$\chi_t^{1/4}$ (MeV)		
(NSVZ)	175	180.3
(HZ)	1423	147.2
$m^0_{\eta'}$ (MeV)		
(NSVZ)	805	854
(HZ)	536.6	569.2

TABLE VI. The bag constant ($F_\pi^0=87.2$ MeV, $n=0.5$ fm^{-4}).

$B=-\epsilon_t$	$N_f=0$	$N_f=1$	$N_f=2$	$N_f=3$
GeV^4	0.013	0.01133	0.097	0.008
MeV^4	$(337.7)^4$	$(326.25)^4$	$(313.8)^4$	$(300)^4$
GeV/fm^3	1.7	1.47	1.26	1.04

shown in our preceding papers [4,25] precisely these type of gluon field configurations are mainly responsible for quark confinement and DCSB. At the same time, the standard gluon condensate (5.3) is the weak coupling phenomenon due to classical instanton-type fluctuations in the true QCD vacuum which by themselves do not confine quarks [21,26–28].

Concluding let us note that in the lattice simulations there already exist calculations of the gluon condensate which are one order of magnitude bigger than the standard value, namely $G_2 \simeq 0.1046$ GeV^4 for $SU(3)$ in Ref. [29] and $G_2 \simeq 0.1556$ GeV^4 for $SU(2)$ in Ref. [30] (see also review [8]). In phenomenology also there exist large values, namely

$$0.04 \leqslant \langle 0| \frac{\alpha_s}{\pi} G^a_{\mu\nu} G^a_{\mu\nu} |0\rangle \leqslant 0.105 \ \ (\mathrm{GeV}^4), \tag{6.3}$$

which were recently derived from the families of J/Ψ and Υ mesons in Ref. [31].

B. Topology of chiral QCD vacuum

Our numerical results for the quantum part of the nonperturbative vacuum energy density and for the topological susceptibility with the mass of the η' meson in the chiral limit are presented in Eqs. (1.2), (1.3) and in Table II, respectively. In general our values for the vacuum energy density are an order of magnitude bigger than RILM can provide at all in various modifications (see Table III). That is why the quantum part of the nonperturbative vacuum energy density saturates the phenomenological value of the topological susceptibility and the mass of the η' meson in the chiral limit much better than the classical part given by instantons (compare Tables II and III). Especially this is obvious for the NSVZ value of the ξ parameter, introduced in the low-energy theorem, Eq. (3.2). The instanton contribution substantially underestimates the phenomenological value of the topological susceptibility and therefore cannot account for the large mass of the η' meson in the chiral limit alone (see Table III).

TABLE VII. The bag constant ($F_\pi^0=93.3$ MeV, $n=0.5$ fm^{-4}).

$B=-\epsilon_t$	$N_f=0$	$N_f=1$	$N_f=2$	$N_f=3$
GeV^4	0.016	0.01426	0.01216	0.01
MeV^4	$(355.65)^4$	$(345.56)^4$	$(332)^4$	$(316)^4$
GeV/fm^3	2.0	1.8	1.58	1.30

TABLE VIII. The bag constant ($F_\pi^0=87.2$ MeV, $n=1$ fm^{-4}).

$B=-\epsilon_t$	$N_f=0$	$N_f=1$	$N_f=2$	$N_f=3$
GeV^4	0.015	0.0133	0.01154	0.0098
MeV^4	$(350)^4$	$(339.6)^4$	$(327.75)^4$	$(314.6)^4$
GeV/fm^3	1.95	1.73	1.5	1.27

However, the total vacuum energy density, ϵ_t, is, in principle, the sum of all possible independent, the nonperturbative contributions. Thus, at least it is the sum of the two well-established contributions, quantum $\epsilon \equiv \epsilon_{ZME}$ and classical ϵ_I, i.e., $\epsilon_t = \epsilon_{ZME} + \epsilon_I + \cdots$, where the dots denote other possible independent contributions. In this case an excellent agreement with phenomenology is achieved indeed (see Tables IV and V). The numerical values of the bag constant B, defined as the difference between the perturbative and nonperturbative vacua are given now by the relation $B = -\epsilon_t$ and can be explicitly evaluated using Eqs. (1.2), (1.3) for ϵ_{ZME} and Eq. (5.4) for ϵ_I on account of the above mentioned two different instanton number densities (see Tables VI–IX). For the readers convenience the bag constant (and consequently the total, nonperturbative vacuum energy density) is given in often used different physical units.

VII. CONCLUSIONS

In summary, using the trace anomaly relation, NSVZ and HZ low-energy theorem and Witten-Veneziano formula, we have developed an analytical formalism which allows one to calculate the gluon condensate, the topological susceptibility and the mass of the η' meson in the chiral limit as functions of the nonperturbative vacuum energy density. It was immediately used for numerical investigation of the chiral QCD nonperturbative vacuum topology within the recently proposed ZME quantum model. We have explicitly shown that precisely our values for the nonperturbative vacuum energy density (1.2), (1.3) are of the necessary order of magnitude in order to saturate the large mass of the η' meson in the chiral limit. We have obtained good approximation to the phenomenological value of the topological susceptibility as well. The NSVZ value of the ξ parameter, introduced in the low-energy theorem (3.2), especially nicely saturates them (for all results mentioned above; see Table II). At the same time, it is clear that instanton-induced contribution should be added to our values in order to achive an excellent agreement with phenomenology. Indeed, from Table V it follows that

$$\chi_t(NSVZ) = (180.3 \ \ \mathrm{MeV})^4 \tag{7.1}$$

TABLE IX. The bag constant ($F_\pi^0=93.3$ MeV, $n=1$ fm^{-4}).

$B=-\epsilon_t$	$N_f=0$	$N_f=1$	$N_f=2$	$N_f=3$
GeV^4	0.01842	0.0162	0.014	0.0118
MeV^4	$(368.4)^4$	$(356.76)^4$	$(344)^4$	$(330)^4$
GeV/fm^3	2.4	2.1	1.82	1.53

and consequently (as it should be)

$$m^0_{\eta'}(NSVZ)=854 \ \text{MeV}, \tag{7.2}$$

which are in fair agreement with Eq. (4.4) and the lower bound in Eq. (4.5), respectively. Let us recall that these numbers have been obtained when the pion decay constant was precisely approximated by its experimental value. The above displayed excellent agreement with phenomenological values of the corresponding quantities is achieved by summing up our contribution and instanton-induced contribution into the total, nonperturbative vacuum energy density, i.e., the summation was done purely phenomenologically by simply summing up the two well-established contributions (classical instanton's and quantum ZME's). How to take into account 't Hooft's instanton-induced interaction [32] at the fundamental quark level within our approach is not completely clear for us, though see paper [33] (and references therein).

Let us also make a few things perfectly clear. At low energies QCD is governed by $SU_L(N_f)\times SU_R(N_f)$ chiral symmetry and its dynamical breakdown in the vacuum to the corresponding vectorial subgroup [34]. The chiral limit is not a physical one but nevertheless remains a very important theoretical limit since to understand the chiral limit physics means to correctly understand the dynamical structure of low-energy QCD as well as the topological properties of its ground state. So a realistic calculation of various physical quantities as well as chiral properties of its vacuum becomes important. In particular, any model of the QCD vacuum should pass the chiral limit test in order to be justified for further extrapolation to the realistic (nonchiral) case. In our previous publication [4] ZME quantum model was formulated. Here we have explicitly shown its important and novel feature, namely it itself passes the chiral limit test justified thereby for use in the nonchiral case as well. Complemented by instanton-induced contribution it is in a fair agreement with phenomenology.

In conclusion a few remarks are in order. It is well known that instanton-type fluctuations require the topological charge to be integer (± 1) and the vacuum angle, θ, nonzero, which violates P and CP invariance of strong interactions [10,16]. The nonperturbative q^{-4}-type quantum excitations do not require the introduction of the vacuum angle, θ, at all. Moreover, it is quite possible that topological charge in this case is not restricted to integer values. Crewther [35] has explicitly shown that fractional (noninteger) topological charge configurations are required to resolve the $U(1)$ problem (see also review [16]). However, the θ dependence of the QCD nonperturbative vacuum energy remains an important problem. For recent developments of this problem in the large N_c limit of four-dimensional gauge theories see papers [36]. In particular, in Refs. [37,38] it has been discussed that the picture of its dependence in QCD for finite N_c might be more complicated than that predicted by the large N_c values. And, finally, let us emphasize once more indisputable simplicity of our analytical calculation of the topological susceptibility (7.1) in comparison with the indisputable complexity of its calculation by lattice method [21–24,30,39–41].

One of the authors (V.G.) is grateful to the late Professor V.N. Gribov for many interesting discussions and remarks on nonperturbative QCD. He also would like to thank I. Halperin for correspondence and A. Vainshtein for useful remarks.

[1] *Confinement, Duality and Nonperturbative Aspects of QCD*, edited by P. van Baal (NATO Advanced Study Institute, Series B: Physics), Vol. 368; Prog. Theor. Phys. Suppl. 131 (1998); V. N. Gribov, LUND report, LU TP 91-7 (1991), pp. 1–51.

[2] J. M. Cornwall, R. Jackiw, and E. Tomboulis, Phys. Rev. D **10**, 2428 (1974).

[3] A. Barducci *et al.*, Phys. Rev. D **38**, 238 (1988); R. W. Haymaker, Riv. Nuovo Cimento **14**, 1 (1991); K. Higashijima, Suppl. Prog. Theor. Phys. **104**, 1 (1991).

[4] V. Gogohia, Gy. Kluge, H. Toki, and T. Sakai, Phys. Lett. B **453**, 281 (1999); V. Gogohia, H. Toki, T. Sakai, and Gy. Kluge, hep-ph/9810516.

[5] J. Hosek and G. Ripka, Z. Phys. A **354**, 177 (1996).

[6] P. de Forcrand and M. D'Elia, Phys. Rev. Lett. **82**, 4582 (1999).

[7] J. M. Cornwall, Phys. Rev. D **59**, 125015 (1999).

[8] T. Schafer and E. V. Shuryak, Rev. Mod. Phys. **70**, 323 (1998).

[9] R. J. Crewther, Phys. Rev. Lett. **28**, 1421 (1972); M. S. Chanowitz and J. Ellis, Phys. Rev. D **7**, 2490 (1973); J. C. Collins, A. Duncan, and S. D. Joglecar, *ibid.* **16**, 438 (1977).

[10] V. A. Novikov, M. A. Shifman, A. I. Vainshtein, and V. I. Zakharov, Nucl. Phys. **B191**, 301 (1981).

[11] I. Halperin and A. Zhitnitsky, Nucl. Phys. **B539**, 166 (1999).

[12] E. Witten, Nucl. Phys. **B156**, 269 (1979); G. Veneziano, *ibid.* **B159**, 213 (1979).

[13] M. A. Shifman, A. I. Vainshtein, and V. I. Zakharov, Nucl. Phys. **B147**, 385 (1979); **B147**, 448 (1979).

[14] W. Marciano and H. Pagels, Phys. Rep., Phys. Lett. **36C**, 1 (1978).

[15] G. Gabadadze, Phys. Rev. D **58**, 094015 (1998).

[16] G. A. Christos, Phys. Rep. **116**, 251 (1984).

[17] M. Schaden, Phys. Rev. D **58**, 025016 (1998).

[18] J. S. Bell and R. A. Bertlmann, Nucl. Phys. **B177**, 218 (1981); R. A. Bertlmann *et al.*, Z. Phys. C **39**, 231 (1988).

[19] S. Narison, Phys. Lett. B **387**, 162 (1996).

[20] M.-C. Chu, J. M. Grandy, S. Huang, and J. W. Negele, Phys. Rev. D **49**, 6039 (1994).

[21] T. DeGrand, A. Hasenfratz, and T. G. Kovacs, Nucl. Phys. **B505**, 417 (1997).

[22] A. Hasenfratz and C. Nieter, Nucl. Phys. B (Proc. Suppl.) **73**, 503 (1999).

[23] D. A. Smith and M. J. Teper, Phys. Rev. D **58**, 014505 (1998).

[24] P. de Forcrand *et al.*, Nucl. Phys. B (Proc. Suppl.) **63A-C**, 549 (1998); P. de Forcrand *et al.*, hep-lat/9802017.

[25] V. Gogohia, Gy. Kluge, and M. Prisznyak, Phys. Lett. B **368**, 221 (1996).

[26] S. Coleman, in *The Whys of Subnuclear Physics*, International School of Subnuclear Physics, Erice, 1977, edited by A. Zichichi (Plenum, New York, 1979), p. 805.
[27] R. C. Brower, D. Chen, J. W. Negele, and E. Shuryak, Nucl. Phys. B (Proc. Suppl.) **73**, 512 (1999); J. W. Negele, *ibid.* **73**, 92 (1999).
[28] C. G. Callan, Jr., R. Dashen, D. J. Gross, F. Wilczek, and A. Zee, Phys. Rev. D **18**, 4684 (1978).
[29] M. Campostrini, A. Di Giacomo, and Y. Gunduc, Phys. Lett. B **225**, 393 (1989).
[30] B. Alles, M. Campostrini, A. Di Giacomo, Y. Gunduc, and E. Vicari, Phys. Rev. D **48**, 2284 (1993).
[31] B. V. Geskenbein and V. L. Morgunov, Yad. Fiz. **58**, 1980 (1995) [Phys. At. Nucl. **58**, 1873 (1995)].
[32] G. 't Hooft, Phys. Rev. D **14**, 3432 (1976).
[33] E. Klempt, B. C. Metsch, C. R. Munz, and H. R. Petry, Phys. Lett. B **361**, 160 (1995).
[34] H. Pagels, Phys. Rep., Phys. Lett. **16C**, 219 (1975); J. Gasser and H. Leutwyler, Phys. Rep. **87**, 77 (1982).
[35] R. J. Crewther, Nucl. Phys. **B209**, 413 (1982).
[36] E. Witten, Phys. Rev. Lett. **81**, 2862 (1998); M. Shifman, Phys. Rev. D **59**, 021501 (1998).
[37] I. Halperin and A. Zhitnitsky, Phys. Lett. B **440**, 77 (1998); I. Halperin and A. Zhitnitsky, Phys. Rev. Lett. **81**, 4071 (1998).
[38] G. Gabadadze, Nucl. Phys. **B552**, 194 (1999).
[39] E. Meggiolaro, Phys. Rev. D **58**, 085002 (1998).
[40] B. Alles, M. D'Elia, and A. Di Giacomo, Nucl. Phys. **B494**, 281 (1997).
[41] R. Narayanan and R. L. Singleton, Jr., Nucl. Phys. B (Proc. Suppl.) **63A-C**, 555 (1998).