New effect in nonlinear Born-Infeld electrodynamics

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A new experiment in which the coefficient a^2 of Born-Infeld nonlinear electrodynamics in a vacuum can be measured by using a ring laser is proposed. The dispersion equation and the frequencies of the generated electromagnetic waves are calculated for waves propagating in the ring laser towards each other when a vacuum is generated in one part of the contour and crossed constant fields $\vec{E}_0$ and $\vec{B}_0 \sim 10^5$ G are applied to this region. It is shown that by measuring the difference of frequencies of the generated electromagnetic waves in the ring laser, the coefficient a^2 of Born-Infeld nonlinear electrodynamics can be measured up to an accuracy of $a^2 \sim 10^{-32}$ G^{-2}. A comparison of the predictions of quantum electrodynamics and Born-Infeld electrodynamics concerning the results of the proposed experiment is carried out.

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I. INTRODUCTION

As is well known, nature is nonlinear. That is why nonlinear models of field theory, and especially their different observable consequences, are worthy of very serious attention. However, for a long time the nonlinear effects of a physical vacuum were the subject of study only for theoreticians and just in recent time has their investigation moved from theory to practice. The first of such experiments, as it is known [1], was the observation of multiphoton light-by-light scattering in which real photons transformed into electron-positron pairs. Thus, this experiment showed that electrodynamics in a vacuum constitutes a nonlinear theory.

This circumstance has again excited interest in studying nonlinear electrodynamic models. Nowadays, two such models are discussed, in general, in the scientific literature.

Theoretically, the most elaborated of them corresponds to the nonlinear electrodynamics which follows from quantum electrodynamics [2,3]. Its Lagrangian at first order of perturbation theory has the form

$$L=\frac{1}{8\pi}[\vec{E}^2-\vec{B}^2]+\frac{\alpha\{(\vec{B}^2-\vec{E}^2)^2+7(\vec{B}\vec{E})^2\}}{360\pi^2B_q^2}, \qquad (1)$$

where $\alpha=e^2/\hbar c \approx 1/137$ is the fine structure constant, and $B_q=m^2c^2/e\hbar \sim 4.41\times 10^{13}$ G is the characteristic quantum electrodynamic induction.

Another theory of nonlinear electrodynamics in a vacuum was proposed by Born and Infeld [4]. The Lagrangian of the electromagnetic field in this theory has the form

$$L=-\frac{1}{4\pi a^2}[\sqrt{1+a^2(\vec{B}^2-\vec{E}^2)-a^4(\vec{B}\vec{E})^2}-1], \qquad (2)$$

where a^2 is a certain constant.

It is convenient to express this constant through the characteristic quantum electrodynamic induction B_q and a dimensionless constant η: $a=\eta/B_q$.

The Born-Infeld electrodynamics possesses a whole series of interesting properties.

First, the energy of the electromagnetic field of a point charge is a finite quantity in the framework of this theory.

Second, though the velocity of an electromagnetic wave depends on the values of the fields B^2 and E^2 in this theory, it does not exceed the speed of light, c, in Maxwell's electrodynamics.

Finally, the ideology of this theory is very close to Einstein's idea [5] of introducing a nonsymmetric metric tensor $G_{ik}\neq G_{ki}$ with the symmetric part corresponding to the usual metric tensor g_{ik} and the antisymmetric part to the electromagnetic field tensor F_{ik}:

$$G_{ik}=g_{ik}+aF_{ik}.$$

By using the relations of tensor algebra [6] it is not difficult to show that

$$G=\det||G_{ik}||=g\left[1-\frac{a^2}{2}F_{(2)}-\frac{a^4}{4}F_{(4)}+\frac{a^4}{8}F_{(2)}^2\right],$$

where $F_{(2)}=F_{ik}F^{ki}$, $F_{(4)}=F_{ik}F^{km}F_{mn}F^{ni}$ are the invariants of the electromagnetic field and g is the determinant of the metric tensor g_{ik}.

In the absence of gravitational fields and by using the Cartesian coordinates of an inertial frame of reference, the quantities appearing in this relation have the form

$$g=-1, \quad F_{(2)}=2(\vec{E}^2-\vec{B}^2),$$

$$F_{(4)}=2(\vec{B}^2-\vec{E}^2)^2+4(\vec{B}\vec{E})^2.$$

Therefore, the Lagrangian (2) can be written as

$$L=-\frac{1}{4\pi a^2}[\sqrt{-G}-\sqrt{-g}].$$

It should be noted that Born-Infeld electrodynamics can be obtained from more general sypersymmetric theories [7].

Thus, Born-Infeld electrodynamics in many respects constitutes a distinguished theory.

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For B, $E \ll B_q$, the Lagrangian (2) differs from that of Maxwell's electrodynamics by nonlinear terms of higher order:

$$L = -\frac{1}{8\pi}(\vec{B}^2-\vec{E}^2)+\frac{a^2}{32\pi}[(\vec{B}^2-\vec{E}^2)^2+4(\vec{B}\vec{E})^2] + O(a^4B^6).$$

Comparing this formula with Eq. (1), it is easy to observe that there is no way of choosing the constant a^2 in order that the Lagrangians coincide. This means that nonlinear electrodynamics with Lagrangians (1) and (2) are essentially different theories. So it is interesting to check experimentally the predictions of these theories and to make clear the question about their adequacy to reality. However, with the values of the electromagnetic fields B, $E \sim 10^6$ G, which can be achieved in terrestrial conditions, the corrections to Maxwell's Lagrangian in both theories are so small that it is very difficult to observe nonlinear effects in vacuum electrodynamics.

At present, as a result of the successes of the spectroscopy of superhigh resolution, the realistic possibility of carrying out experiments which allow us to choose between these theories has arisen. This possibility has to do with the specific characteristics of the ring laser. In the ring laser, as is known, two electromagnetic waves propagating towards each other in a triangular, rectangular, or any closed contour will have different frequencies if the conditions of their propagation are nonidentical.

For this reason, the ring laser constitutes a very precise device for measuring different fine physical effects [8].

As we shall show below, nonidentical conditions for these waves arise, according to nonlinear electrodynamics, if a constant and homogeneous electromagnetic field is generated in one part of the contour. In this case the vectors $\vec{B}_0$ and $\vec{E}_0$ must be perpendicular to each other and to the wave vector of the electromagnetic wave. Therefore, if this part of the contour of the ring laser is separated from the rest by transparent and impermeable to gas partitions, and the above electromagnetic fields are generated there, then, according to nonlinear electrodynamics, the frequency of the electromagnetic waves propagating towards each other will be different depending on the form of the Lagrangian of the theory.

Since the modern level of experimental techniques allows us [9] to measure the difference of frequencies in ring lasers up to $\delta\nu = 1.4\times 10^{-7}$ Hz, then, nowadays, such an experiment is evidently one of the most prospective for checking the predictions of nonlinear electrodynamics with the Lagrangians (1) and (2).

II. THE DISPERSION EQUATION IN THE BORN-INFELD THEORY FOR AN ELECTROMAGNETIC WAVE PROPAGATING IN A VACUUM IN EXTERNAL ELECTROMAGNETIC FIELDS

As is known, the electromagnetic field equations of Born-Infeld electrodynamics are analogous to the equations of macroscopic electrodynamics,

$$\mathrm{rot}\ \vec{H} = \frac{1}{c}\frac{\partial \vec{D}}{\partial t}, \quad \mathrm{div}\ \vec{D}=0,$$

$$\mathrm{rot}\ \vec{E} = -\frac{1}{c}\frac{\partial \vec{B}}{\partial t}, \quad \mathrm{div}\ \vec{B}=0, \tag{3}$$

but the vectors $\vec{D}$ and $\vec{H}$ have a different meaning:

$$\vec{D} = 4\pi\frac{\partial L}{\partial \vec{E}} = \frac{\vec{E}+a^2(\vec{B}\vec{E})\vec{B}}{\sqrt{1+a^2(\vec{B}^2-\vec{E}^2)-a^4(\vec{B}\vec{E})^2}},$$

$$\vec{H} = -4\pi\frac{\partial L}{\partial \vec{B}} = \frac{\vec{B}-a^2(\vec{B}\vec{E})\vec{E}}{\sqrt{1+a^2(\vec{B}^2-\vec{E}^2)-a^4(\vec{B}\vec{E})^2}}. \tag{4}$$

Further, by supposing that $1+a^2(\vec{B}^2-\vec{E}^2)-a^4(\vec{B}\vec{E})^2 \neq 0$, and substituting the expression (4) in the first equation of the system (3), we get

$$[1+a^2(\vec{B}^2-\vec{E}^2)-a^4(\vec{B}\ \vec{E})^2]\left\{\mathrm{rot}\,\vec{B}-a^2(\vec{B}\vec{E})\mathrm{rot}\,\vec{E}+a^2[\vec{E}\vec{\nabla}(\vec{B}\vec{E})]-\frac{\partial}{\partial x^0}[\vec{E}+a^2(\vec{B}\vec{E})\vec{B}]\right\}+\frac{a^2}{2}[\vec{B}\vec{\nabla}(\vec{B}^2-\vec{E}^2)]-\frac{a^4}{2}(\vec{B}\vec{E})$$
$$\times[\vec{E}\vec{\nabla}(\vec{B}^2-\vec{E}^2)]-\frac{a^4}{2}[\vec{B}\vec{\nabla}(\vec{B}\vec{E})^2]+\frac{a^6}{2}(\vec{B}\vec{E})[\vec{E}\vec{\nabla}(\vec{B}\vec{E})^2]+\frac{1}{2}\{\vec{E}+a^2(\vec{B}\vec{E})\vec{B}\}\frac{\partial}{\partial x^0}[a^2(\vec{B}^2-\vec{E}^2)-a^4(\vec{B}\vec{E})^2]=0. \tag{5}$$

Let us assume that a constant and homogeneous electromagnetic field $\vec{B}_0=\mathrm{const}$, $\vec{E}_0=\mathrm{const}$, is created on a certain part of the ring laser's contour. Let us find the dispersion equation for the electromagnetic wave propagating in this field. For this purpose, we shall write the vectors $\vec{B}_w$ and $\vec{E}_w$ of the electromagnetic wave in the approximation of geometrical optics as

$$\vec{B}_w=\vec{b}\exp[-i(\omega t-\vec{k}\vec{r})], \quad \vec{E}_w=\vec{e}\exp[-i(\omega t-\vec{k}\ \vec{r})], \tag{6}$$

where ω is the frequency, $\vec{k}$ is the wave vector, and the vectors $\vec{b}$ and $\vec{e}$ are slowly varying functions of t and $\vec{r}$ in comparison with the function $\exp[i(\omega t-\vec{k}\vec{r})]$.

Then, in the linear approximation of the ''weak'' field of the electromagnetic wave, we have, from Eqs. (5),

$$[1+a^2(\vec{B}_0^2-\vec{E}_0^2)-a^4(\vec{B}_0\vec{E}_0)^2]\left\{[\vec{k}\vec{b}]-a^2(\vec{B}_0\vec{E}_0)[\vec{k}\vec{e}]+\frac{\omega}{c}\vec{e}-a^2\{(\vec{b}\vec{E}_0)+(\vec{B}_0\vec{e})\}[\vec{k}\vec{E}_0]\right.$$
$$\left.+\frac{\omega a^2}{c}[(\vec{b}\vec{E}_0)\vec{B}_0+(\vec{B}_0\vec{e})\vec{B}_0+(\vec{B}_0\vec{E}_0)\vec{b}]\right\}+\{a^2(\vec{b}\vec{B}_0)-a^2(\vec{e}\vec{E}_0)-a^4(\vec{B}_0\vec{E}_0)(\vec{b}\vec{E}_0)-a^4(\vec{B}_0\vec{E}_0)(\vec{B}_0\vec{e})\}$$
$$\times\left\{a^2(\vec{B}_0\vec{E}_0)[\vec{k}\vec{E}_0]-[\vec{k}\vec{B}_0]-\frac{\omega}{c}\vec{E}_0-\frac{\omega a^2}{c}(\vec{B}_0\vec{E}_0)\vec{B}_0\right\}=0. \tag{7}$$

Let us eliminate the vector $\vec{b}$ from this relation by substituting the expression (6) in the third equation of the system (3). Cancelling the exponential factor, we get

$$\frac{\omega}{c}\vec{b}=[\vec{k}\vec{e}]. \tag{8}$$

Now multiply Eq. (7) by ω/c and use the equality (8). As a result we obtain a homogeneous system of three linear algebraic equations for the three vector components of $e_\beta=(\vec{e})_\beta$:

$$A^{\alpha\beta}e_\beta=0, \tag{9}$$

where the three-dimensional tensor $A^{\alpha\beta}$ has the form

$$A^{\alpha\beta}=W_0\left\{k^\alpha k^\beta+\left(\frac{\omega^2}{c^2}-\vec{k}^2\right)\delta^{\alpha\beta}\right\}+a^2W_1\left\{N_E^\alpha N_E^\beta-\frac{\omega}{c}[B_0^\alpha N_E^\beta+N_E^\alpha B_0^\beta]+\frac{\omega^2}{c^2}B_0^\alpha B_0^\beta\right\}+a^2N_B^\alpha N_B^\beta+\frac{\omega a^2}{c}[E_0^\alpha N_B^\beta+N_B^\alpha E_0^\beta]$$
$$+\frac{\omega^2a^2}{c^2}E_0^\alpha E_0^\beta-a^4(\vec{B}_0\vec{E}_0)[N_E^\alpha N_B^\beta+N_B^\alpha N_E^\beta]+\frac{\omega a^4}{c}(\vec{B}_0\vec{E}_0)[B_0^\alpha N_B^\beta+N_B^\alpha B_0^\beta]-\frac{\omega a^4}{c}(\vec{B}_0\vec{E}_0)[E_0^\alpha N_E^\beta+N_E^\alpha E_0^\beta]$$
$$+\frac{\omega^2a^4}{c^2}(\vec{B}_0\vec{E}_0)[B_0^\alpha E_0^\beta+E_0^\alpha B_0^\beta]$$

and for convenience we have denoted

$$W_0=[1+a^2(\vec{B}_0^2-\vec{E}_0^2)-a^4(\vec{B}_0\vec{E}_0)^2],\quad \vec{N}_E=[\vec{k}\vec{E}_0],$$

$$W_1=[1+a^2(\vec{B}_0^2-\vec{E}_0^2)],\quad \vec{N}_B=[\vec{k}\vec{B}_0].$$

In order to avoid misunderstandings, it should be noted that we are using the Cartesian coordinates of an inertial frame of reference. Consequently, the role of the metric tensor of the three-dimensional space is played by the Kronecker symbol $\delta^{\alpha\beta}$. Thus, one does not need to distinguish between covariant and contravariant indices; this fact considerably simplifies the calculations.

Since we are interested in the nontrivial solution of Eq. (9), we must have

$$\det||A^{\alpha\beta}||=0. \tag{10}$$

In order to represent this equation in its explicit form, we shall use some tensor analysis formulas proved in [6,10]. The tensor $\Psi_{ik}^{(S)}(x)$ is formed by the product of S tensors $\Psi_{ik}(x)$, whose indices are contracted by the metric tensor $\delta_{\mu\nu}$ of the three-dimensional Euclidean space according to the rule

$$\Psi_{(S)}^{\alpha\beta}=\underbrace{\Psi^{\alpha\mu_1}\cdot\delta_{\mu_1\nu_1}\cdot\Psi^{\nu_1\mu_2}\ldots\delta_{\mu_{S-1}\nu_{S-1}}\cdot\Psi^{\nu_{S-1}\beta}}_{S}. \tag{11}$$

The tensor $\Psi_{ik}^{(S)}(x)$ is called the Sth power of the tensor $\Psi_{ik}(x)$. According to this definition, we put $\Psi_{(0)}^{\alpha\beta}=\delta^{\alpha\beta}$ for $S=0$.

By contracting the remaining indices in the expression (11) one gets the invariant

$$\Psi_{(S)}=\Psi_{(S)}^{\alpha\beta}\delta^{\alpha\beta}$$

of the Sth power of the tensor.

Using this notation, the determinant of the second rank tensor $\Psi^{\alpha\beta}$ in three-dimensional space can be written in terms of the following combination of its invariants $\Psi_{(1)}$, $\Psi_{(2)}$, and $\Psi_{(3)}$:

$$\det||\Psi^{\alpha\beta}||=\frac{1}{6}[2\Psi_{(3)}-3\Psi_{(2)}\Psi_{(1)}+\Psi_{(1)}^3].$$

Therefore, the condition (10) for a nontrivial solution, which is the dispersion equation, takes the form

$$2A_{(3)}-3A_{(2)}A_{(1)}+A_{(1)}^3=0.$$

By obtaining the powers of the tensor $A^{\alpha\beta}$, constructing its invariants according to the rule (11), and taking into account the trivial relations

$$\vec{N}_E^2=\vec{k}^2\vec{E}_0^2-(\vec{k}\vec{E}_0)^2, \quad \vec{N}_B^2=\vec{k}^2\vec{B}_0^2-(\vec{k}\vec{B}_0)^2,$$

$$(\vec{N}_B\vec{N}_E)=(\vec{B}_0\vec{E}_0)\vec{k}^2-(\vec{k}\vec{B}_0)(\vec{k}\vec{E}_0),$$

we obtain the following dispersion equation:

$$W_0\frac{\omega^2}{c^2}\left\{\frac{\omega^2}{c^2}[1+a^2\vec{B}_0^2]-\frac{2\omega}{c}a^2(\vec{k}[\vec{E}_0\vec{B}_0])-\vec{k}^2[1-a^2\vec{E}_0^2] -a^2[(\vec{k}\vec{E}_0)^2+(\vec{k}\vec{B}_0)^2]\right\}^2=0.$$

By solving this equation we find the frequency dependence of the weak electromagnetic wave propagating in the constant and homogeneous fields $\vec{B}_0=\mathrm{const}$, $\vec{E}_0=\mathrm{const}$ on the wave vector $\vec{k}$:

$$\omega=\frac{c}{[1+a^2\vec{B}_0^2]}(a^2(\vec{k}[\vec{E}_0\vec{B}_0])\pm[a^4(\vec{k}[\vec{E}_0\vec{B}_0])^2 +(1+a^2\vec{B}_0^2)\{(1-a^2\vec{E}_0^2)\vec{k}^2+a^2(\vec{k}\vec{E}_0)^2 +a^2(\vec{k}\vec{B}_0)^2\}]^{1/2}). \tag{12}$$

Thus, in the framework of Born-Infeld electrodynamics, the frequency of a weak electromagnetic wave propagating in the constant and homogeneous fields $\vec{B}_0=\mathrm{const}$, $\vec{E}_0=\mathrm{const}$ depends on the direction of the propagation (on the direction of the wave vector $\vec{k}$). Thus, the substitution of $\vec{k}$ by $-\vec{k}$ changes the frequency. Since $a^2B_0^2\ll 1$ and $a^2E_0^2\ll 1$, then the expansion of the expression (12) up to first order with respect to these small parameters is

$$\omega=c\left\{a^2(\vec{k}[\vec{E}_0\vec{B}_0])\pm k\left[1-\frac{a^2}{2}(\vec{B}_0^2+\vec{E}_0^2)+\frac{a^2}{2k^2}[(\vec{k}\vec{E}_0)^2 +(\vec{k}\vec{B}_0)^2]\right]\right\}. \tag{13}$$

By using the definition of group velocity $\vec{V}_{gr}=\partial\omega/\partial\vec{k}$, from the expression (13) we have

$$\vec{V}_{gr}=c\left\{a^2[\vec{E}_0\vec{B}_0]\pm\frac{\vec{k}}{k}\left[1-\frac{a^2}{2}(\vec{B}_0^2+\vec{E}_0^2)-\frac{a^2}{2k^2}(\vec{k}\vec{E}_0)^2 -\frac{a^2}{2k^2}(\vec{k}\vec{B}_0)^2\right]\pm\frac{a^2}{k}[(\vec{k}\vec{B}_0)\vec{B}_0+(\vec{k}\vec{E}_0)\vec{E}_0]\right\}.$$

Thus, according to Born-Infeld electrodynamics, the direction of the group velocity $\vec{V}_{gr}$ of a weak electromagnetic wave propagating in a constant and homogeneous electromagnetic field does not coincide with the direction of the wave vector $\vec{k}$.

From this expression it also follows that the two signs in Eq. (13) correspond to two different directions of propagation of a plane electromagnetic wave.

III. CALCULATION OF THE FREQUENCY OF THE WAVES GENERATED IN THE RING LASER

We shall denote the perimeter of the ring laser's contour as P. Let a part of this contour of length l_1 be separated from the rest of the contour by gas-impermeable transparent partitions. Let the external fields $\vec{B}_0$ and $\vec{E}_0$, perpendicular to the vector $\vec{k}$ of the electromagnetic wave, be generated in this part of the contour in vacuum.

As is known [11], in a ring laser, the active media intensify only the electromagnetic waves whose phase changes by $\delta\varphi=2\pi N$ after a round along the contour; here N is some integer number.

Thus, an elementary calculation which uses Eq. (13) shows that

$$\left\{P-l_1+\left[1+\frac{a^2}{2}(E_0^2+B_0^2)-a^2\left(\frac{\vec{k}}{k}[\vec{E}_0\vec{B}_0]\right)\right]l_1\right\}\nu_-=cN_-,$$

$$\left\{P-l_1+\left[1+\frac{a^2}{2}(E_0^2+B_0^2)+a^2\left(\frac{\vec{k}}{k}[\vec{E}_0\vec{B}_0]\right)\right]l_1\right\}\nu_+=cN_+, \tag{14}$$

where $\nu_\pm$ are the frequencies of the generated electromagnetic waves propagating in the ring laser along the clockwise and the counterclockwise directions, respectively; $N_\pm$ are integer numbers equal to the numbers of wavelengths $\lambda_\pm$ confined in the optical paths of the rays.

Since the expected value of the difference $\nu_+-\nu_-\sim 10^{-18}\nu_+$ is small, then it is evident that $N_+=N_-=N$. Therefore, it is not difficult to obtain, from the relation (14),

$$\Delta\nu=\nu_+-\nu_-=-\frac{2ca^2l_1}{P\lambda}\left(\frac{\vec{k}}{k}[\vec{E}_0\vec{B}_0]\right), \tag{15}$$

where we have taken into account that $N=P/\lambda$.

It is obvious that in the considered experiment the expected value of the difference of frequencies of the electromagnetic waves in the ring laser is very small. Thus, if we do not take all the corresponding steps, a possible pulling of frequencies can occur: when two waves with close frequencies ω_1 and ω_2 influence each other and, as a result, in the ring laser are generated waves with some average frequency ω_3 instead of waves with frequencies ω_1 and ω_2.

The standard way of eliminating this effect in optics is by creating some known starting difference of frequencies Ω for the waves propagating towards each other in the ring laser. Thus, the measured difference of frequencies of the generated waves in the proposed experiment will not be $\Delta\nu$, but $\Delta\nu+\Omega/(2\pi)$. Knowing the frequency of the starting splitting Ω, it is easy to determine $\Delta\nu$.

As was shown in Refs. [12,13], in the Born-Infeld theory the velocity of propagation of a weak electromagnetic wave in an external electromagnetic field does not depend on its polarization. So the polarization of electromagnetic waves propagating towards each other in the ring laser, according to this theory, can be arbitrary.

IV. NONLINEAR EFFECTS OF QUANTUM ELECTRODYNAMICS IN THE PROPOSED EXPERIMENT

In the scientific literature [12–15], nonlinear electrodynamic effects following from quantum electrodynamics have been studied from a theoretical point of view significantly wider than the Born-Infeld electrodynamic effects. Recently one of these effects, namely, the multiphoton light-by-light scattering in which real photons transform into electron-positron pairs, was experimentally verified [1].

That is why comparison of the predictions of this nonlinear electrodynamics with the predictions of Born-Infeld electrodynamics in the proposed experiment becomes interesting.

By using the Lagrangian (1) and performing analogous calculations to those of Secs. II and III, it is not hard to obtain the dispersion equation for an electromagnetic wave propagating in a constant and homogeneous field. This equation in the framework of the nonlinear theory with the Lagrangian (1) has two roots:

$$\omega_1(\vec{k})=ck\left\{1-\frac{2\alpha}{45\pi B_q^2k^2}\{[\vec{k}\ \vec{B}_0]^2+[\vec{k}\vec{E}_0]^2 -2k(\vec{k}[\vec{E}_0\ \vec{B}_0])\}\right\},$$

$$\omega_2(\vec{k})=ck\left\{1-\frac{7\alpha}{90\pi B_q^2k^2}\{[\vec{k}\vec{B}_0]^2+[\vec{k}\vec{E}_0]^2 -2k(\vec{k}[\vec{E}_0\ \vec{B}_0])\}\right\}.$$

These roots depend not only on the mutual orientation of the vectors $\vec{B}_0$ and $\vec{E}_0$, but also on the direction of propagation of the electromagnetic wave (exchange $\vec{k}$ with $-\vec{k}$). This means that, according to quantum electrodynamics, in a constant and homogeneous electromagnetic field two kinds of electromagnetic waves with different phase velocity and polarization can propagate in each direction.

Therefore we can take from the ring laser four waves with different frequencies that are, in general, linearly polarized in different planes.

By using standard optical methods a part of these waves can be suppressed and the planes of polarization of the remaining waves can be turned in order to direct them to the beam combiner. Depending on which waves we suppress and which we direct to the beam combiner, we can obtain a whole spectrum of the frequency differences predicted by quantum electrodynamics in the proposed experiment:

$$(\Delta\nu)_1=\nu_1(-\vec{k})-\nu_2(-\vec{k}) =\frac{\alpha c l_1}{30\pi\lambda P B_q^2k^2}\{[\vec{k}\vec{B}_0]^2+[\vec{k}\vec{E}_0]^2+2k(\vec{k}[\vec{E}_0\vec{B}_0])\},$$

$$(\Delta\nu)_2=\nu_1(\vec{k})-\nu_2(\vec{k}) =\frac{\alpha c l_1}{30\pi\lambda P B_q^2k^2}\{[\vec{k}\vec{B}_0]^2+[\vec{k}\vec{E}_0]^2-2k(\vec{k}[\vec{E}_0\vec{B}_0])\},$$

$$(\Delta\nu)_{11}=\nu_1(\vec{k})-\nu_1(-\vec{k})=\frac{8\alpha c l_1}{45\pi\lambda P B_q^2}\left(\frac{\vec{k}}{k}[\vec{E}_0\vec{B}_0]\right),$$

$$(\Delta\nu)_{22}=\nu_2(\vec{k})-\nu_2(-\vec{k})=\frac{14\alpha c l_1}{45\pi\lambda P B_q^2}\left(\frac{\vec{k}}{k}[\vec{E}_0\vec{B}_0]\right),$$

$$(\Delta\nu)_{12}=\nu_1(\vec{k})-\nu_2(-\vec{k}) =\frac{\alpha c l_1}{90\pi\lambda P B_q^2k^2}\{22k(\vec{k}[\vec{E}_0\vec{B}_0])+3[\vec{k}\vec{B}_0]^2 +3[\vec{k}\vec{E}_0]^2\},$$

$$(\Delta\nu)_{21}=\nu_2(\vec{k})-\nu_1(-\vec{k}) =\frac{\alpha c l_1}{90\pi\lambda P B_q^2k^2}\{22k(\vec{k}[\vec{E}_0\vec{B}_0])-3[\vec{k}\vec{B}_0]^2 -3[\vec{k}\vec{E}_0]^2\}. \tag{16}$$

Unlike the Born-Infeld electrodynamics, not all differences of frequency (16) predicted by quantum electrodynamics will become zero after ‘‘turning off’’ the electric field. Actually, by substituting $\vec{E}_0=0$ in Eq. (16) we obtain that in this case the differences of frequencies $(\Delta\nu)_{12}=(\Delta\nu)_1=(\Delta\nu)_2=-(\Delta\nu)_{21}=\alpha c l_1 B_0^2/(30\pi\lambda P B_q^2)$ are nonzero while $(\Delta\nu)_{11}$ and $(\Delta\nu)_{22}$ vanish.

V. DISCUSSION

Thus, the predictions of nonlinear Born-Infeld electrodynamics and quantum electrodynamics about the frequencies of electromagnetic waves generated in the ring laser on the part of which constant and homogeneous fields were generated in vacuum are significantly different. According to the Born-Infeld electrodynamic equations, only one electromagnetic wave with arbitrary polarization must propagate in each direction. The difference of frequencies of the waves moving in the ring laser towards each other will differ from zero only if the triple scalar product of $\vec{E}_0$, $\vec{B}_0$, and $\vec{k}$ does not vanish: $(\vec{k}[\vec{E}_0\vec{B}_0])\neq 0$.

From the equations of quantum electrodynamics it follows that two polarized (in mutually perpendicular planes) waves propagate in each direction with close frequencies under the same conditions in the ring laser. Therefore, depending on which waves have been mixed, it is possible to get a whole spectrum (16) of predicted differences of frequencies. An important difference between the predictions of this theory and those of Born-Infeld electrodynamics is the possibility of observing this effect without the electric field in the former theory. In this case $(\Delta\nu)_{11}=(\Delta\nu)_{22}=0$, but $(\Delta\nu)_{12}$, $(\Delta\nu)_1$, $(\Delta\nu)_2$, and $(\Delta\nu)_{21}$ do not vanish.

Hence, in the proposed experiment it is possible to determine which theory is adequate to nature.

Let us estimate the minimal values of the parameters a^2 and η in the Born-Infeld theory; these parameters can be measured in the proposed experiment. In the best case, when the vectors $\vec{B}_0$, $\vec{E}_0$, and $\vec{k}$ are mutually perpendicular, from the expression (15) it follows that the minimal value of the parameter a^2 which can be measured by using the ring laser is equal to

$$(a^2)_{min} = \frac{P\lambda}{2cl_1E_0B_0}(\Delta\nu)_{min}.$$

This expression depends significantly on the value of the product $E_0 \times B_0$, and the greater this product, the greater the chance to observe the given nonlinear effect. It is evident that the final judgement about the maximum value of this product which can be achieved experimentally must be made by specialists. Therefore, we shall make a preliminary estimation of this product from general considerations.

Modern hybrid superconducting magnets, as they are known, allow us to generate the field $B_0 = 2\times10^5$ G. If the considered part of the ring laser is situated between the two different poles of two magnets, then on the path of the rays will be generated a constant and homogeneous enough field $B_0 = 4\times10^5$ G.

The generation of the field E_0 is more complex since if its value is large enough, a series of phenomena hindering the experiment arises. Among these phenomena the most significant according to our viewpoint are the self-ionization of the residual gas in the ''vacuum'' region of the electric field and the electric puncture of the dielectric surrounding the vacuum region.

However, nowadays, there exist dielectrics with high values of the puncture voltage being reached, for example [16], for specially prepared specimens (potassium bromide, potassium chloride, melted quartz, and others), until $E_0 \sim 10^9$ V/cm $= 3\times10^6$ units (in the Gaussian system of units).

The self-ionization of the residual gas in the vacuum region of the electric field, as it is known, becomes significant for $E_0 = 6\times10^8$ V/cm $= 2\times10^6$ units.

These estimations show that nowadays it is technically possible, in the framework of the proposed experiment, to use a field of $E_0 = 6\times10^7$ V/cm $= 2\times10^5$ units which is an order smaller than the above-mentioned experimental values.

Then, by assuming that the constant electromagnetic fields $E_0 \sim 2\times10^5$, $B_0 \sim 4\times10^5$ G are generated in 1/4 of the contour in the path of the rays and substituting the characteristics [9] of the ring laser $(\Delta\nu)_{min} = 1.4\times10^{-7}$ Hz, $\lambda = 633.0$ nm we obtain

$$(a^2)_{min} = 7.4\times10^{-33}\ \mathrm{G}^{-2}.$$

Taking into account that $a = \eta/B_q$, we have

$$\eta_{min} = 4.8\times10^{-3}.$$

Thus, the minimum value of the numerical coefficient η which can be measured in this experiment is considerably smaller than 1.

Let us estimate now the values of the frequency differences predicted by quantum electrodynamics in the proposed experiment.

With mutually perpendicular vectors $\vec{B}_0$, $\vec{E}_0$, and $\vec{k}$, and setting $E_0 = 2\times10^5$ units and $B_0 = 4\times10^5$ G, from the expressions (16) we get

$$(\Delta\nu)_1 = 1.7\times10^{-6}\ \mathrm{Hz}, \quad (\Delta\nu)_2 = 1.9\times10^{-7}\ \mathrm{Hz},$$

$$(\Delta\nu)_{11} = 2.0\times10^{-6}\ \mathrm{Hz}, \quad (\Delta\nu)_{12} = 3.7\times10^{-6}\ \mathrm{Hz},$$

$$(\Delta\nu)_{21} = 1.8\times10^{-6}\ \mathrm{Hz}, \quad (\Delta\nu)_{22} = 3.5\times10^{-6}\ \mathrm{Hz}.$$

If in the proposed experiment only the magnetic field $B_0 = 4\times10^5$ G is used and $E_0 = 0$, then $(\Delta\nu)_{12} = (\Delta\nu)_1 = (\Delta\nu)_2 = -(\Delta\nu)_{21} = 7.5\times10^{-7}$ Hz.

Thus, with the electromagnetic fields attainable in the laboratory the differences of frequencies predicted by quantum electrodynamics and Born-Infeld electrodynamics differences of frequencies are of almost one order higher than the present frequency resolution in the ring laser.

For this reason, although the experiment we propose is complex from the technical point of view, it constitutes a solvable problem. This experiment is of great significance for physics since it clarifies which one of the above-considered nonlinear electrodynamic theories describes nature properly.

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