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$SU(2)_L \times U(1)_Y \times S_3 \times D$ model for atmospheric and solar neutrino deficits

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Motivated by the recent Super-Kamiokande experiment on atmospheric and solar neutrinos we propose a see-saw model of three generations of neutrinos based on the gauge group $SU(2)_L \times U(1)_Y$ with the discrete symmetries $(S_3 \times D)$ and three right handed singlet neutrinos so that this model can accommodate the recent Super-Kamiokande data on atmospheric and solar neutrino oscillations. The model predicts maximal mixing between ν_μ and ν_τ with $\sin^2 2\theta_{\mu\tau}=1$ as required by the atmospheric neutrino data and small mixing between ν_e and ν_μ with $\sin^2 2\theta_{e\mu} \sim (10^{-2}-10^{-3})$ as a possible explanation of the solar neutrino deficit through the MSW mechanism. The model admits two mass scales of which one breaks the electroweak symmetry and the other is responsible for the breaking of the lepton number symmetry at GUT scale leading to a small Majorana mass of the left handed doublet neutrinos.

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I. INTRODUCTION

Recent results from the Super-Kamiokande experiment on the atmospheric neutrino anomaly [1] and solar neutrino deficit [2] have supported neutrino flavor oscillation as a possible explanation of these effects. Atmospheric neutrino data are consistent with $\nu_\mu \rightarrow \nu_\tau$ oscillations with $\Delta m^2{}_{\mu\tau} = (0.5-6)\times 10^{-3}$ eV^2 and a nearly maximal mixing $\sin^2 2\theta_{\mu\tau} \geqslant 0.82$. The data are equally consistent if ν_τ is replaced by a sterile neutrino and several authors have considered models with an extra light singlet neutrino, in additon to the usual three heavy right handed singlet neutrinos [3].

Furthermore, Super-Kamiokande data on solar neutrino deficit allow a solution in terms of ν_e disappearence vacuum oscillation [4] with $\Delta m^2{}_{e\mu} \sim 10^{-10}$ eV^2 and a nearly maximal mixing angle $\sin^2 2\,\theta_{e\mu} \sim 1.0$. Also consistent with these data are the small angle matter enhanced solution with $\Delta m^2{}_{e\mu} \sim (0.5-1)\times 10^{-5}$ eV^2 and $\sin^2 2\theta_{e\mu} \sim 10^{-2}-10^{-3}$ [5] as well as matter enhanced large angle solution with $\Delta m^2{}_{e\mu} \sim (10^{-5}-10^{-4})$eV^2 and $\sin^2 2\theta_{e\mu} \sim 1.0$ [6] although the small angle is more likely at the moment. The recent CHOOZ [7] reactor result excludes the large mixing angle neutrino oscillation of $\nu_\mu \rightarrow \nu_e$ as far as $\Delta m^2{}_{\mu e} \geqslant 9\times 10^{-4}$ eV^2. While the results of CHORUS [8] and NOMAD [9] on ν_μ oscillations are awaited and the Liquid Scintillation Neutrino Detector (LSND) [10] data, which are disfavoured by the KARMEN [11] experiment, needs confirmation from future experiments, several authors have proposed the strategy of fitting the Super-Kamiokande data on atmospheric and solar neutrinos discarding the LSND data to discuss the problem of neutrino flavor oscillations within the framework of three generations of neutrinos. Such studies have shown that two mass squared differences $\Delta m^2_{\rm solar}$ and $\Delta m^2_{\rm atmos}$ are relevant and the hierarchy of light neutrino masses such as $m_1, m_2 \gg m_3$ or $m_1, m_2 \ll m_3$ is adequate to explain the recent Super-Kamiokande data on neutrino oscillations. Another type of study [12] investigating the impact of neutrino oscillation on the texture of neutrino mass matrices in a model independent way led to maximal mixing and appropriate mass squared difference, which are necessary to explain the Super-Kamiokande data.

Recently several authors [13,14] have studied see-saw models based on the extension of the standard model including two singlet neutrinos with an extra $U(1)'$ gauge group corresponding to a newly defined gauge charge, the gauge charge being $(B-3L_e)$ for Ref. [13] and $(B-3/2[L_\mu + L_\tau])$ for Ref. [14]. Assuming the $U(1)'$ symmetry breaking scale to be of the order of $(\sim 10^{12}-10^{16})$ GeV, the model naturally accounts for the large (small) mixing solutions to the atmospheric (solar) neutrino oscillations and explains the hierarchy of the left handed light neutrino masses. This has motivated us to propose an $SU(2)_L \times U(1)_Y$ model with the $(S_3 \times D)$ discrete symmetries and three right handed singlet neutrinos to explain the results of Super-Kamiokande experiment on atmospheric and solar neutrinos. Our model differs from those in Refs. [13] and [14] in not having extra gauge symmetry and in the choice of discrete symmetries. The present model contains three right handed singlet neutrinos $\nu_{eR}, \nu_{\mu R}, \nu_{\tau R}$ along with three doublet Higgs fields ϕ_1, ϕ_2, ϕ_3 and three singlet Higgs fields χ_1, χ_2, χ_3. The doublet Higgs fields are responsible for generating the Dirac mass terms of the neutrinos whereas right handed singlet neutrinos acquire their heavy masses through the singlet Higgs fields. We have imposed an extra global symmetry in our model to start with to ensure lepton number conservation by making appropriate lepton number assignments for the Higgs fields. In our model each singlet Higgs field is assigned with a (-2) lepton number. As soon as χ_1, χ_2, χ_3 develop their nonzero VEVs, the lepton number symmetry is broken spontaneously at the grand unified theory (GUT) scale $(\sim 10^{16})$ GeV yielding three heavy right handed Majorana neutrino masses (M_1, M_2, M_3) through the well-known see-saw mechanism, which induce small hierarchical masses to the left handed doublet neutrinos $(\nu_e, \nu_\mu, \nu_\tau)$. The $(S_3 \times D)$ discrete symmetries facilitate appropriate mixing be-

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tween the left handed neutrino flavors in order to explain atmospheric and solar neutrino data.

We organize our paper as follows. Section II contains the model. Section III describes the Higgs potential, the neutrino mass matrix and its analysis. Conclusions are presented in Sec. IV.

II. THE MODEL

In our present model the lepton and the Higgs fields have the following representation contents under the gauge group $SU(2)_L \times U(1)_Y$:

$$l_{iL}(2,-1)_1, e_{iR}(1,-2)_1, \nu_{eR}(1,0)_1, \nu_{\mu R}(1,0)_1, \nu_{\tau R}(1,0)_1, \tag{1}$$

where $i=1,\ 2,\ 3,$

$$\phi_i(2,1)_0, \chi_i(1,0)_{-2}. \tag{2}$$

The subscripts in the parenthesis correspond to the lepton number L $(=L_e+L_\mu+L_\tau)$.

We consider the following vacuum expectation values (VEVs) of the Higgs fields:

$$\langle \phi \rangle = \begin{pmatrix} \langle \phi_i^+ \rangle \\ \langle \phi_i^0 \rangle \end{pmatrix} = \begin{pmatrix} 0 \\ v_i \end{pmatrix},$$

$$\langle \chi_i^0 \rangle = k_i. \tag{3}$$

The following discrete symmetries have been incorporated in the model in order to generate the required pattern of Majorana neutrino mass matrix as well as the mixing.

(i) S_3 symmetry:

$$l_{1L} \to 1, l_{2L} \to 1, l_{3L} \to 1, (e_R, \tau_R) \to 2, \mu_R, \to 1, (\nu_{eR}, \nu_{\tau R}) \to 2, \nu_{\mu R} \to 1,$$

$$(\phi_1, \phi_3) \to 2, \phi_2 \to 1, \chi_1 \to 1, \chi_2 \to 1, \chi_3 \to 1. \tag{4}$$

(ii) D symmetry:

$$l_{1L} \to i\omega^* l_{1L}, l_{2L} \to -i\omega^* l_{2L}, l_{3L} \to -i\omega^* l_{3L}, e_R \to -i\omega^* e_R, \mu_R \to \mu_R,$$

$$\tau_R \to \omega^* \tau_R, \nu_{eR} \to -i\omega^* \nu_{eR}, \nu_{\mu R} \to \nu_{\mu R}, \nu_{\tau R} \to \omega^* \nu_{\tau R},$$

$$\phi_1 \to -i\phi_1, \phi_2 \to i\omega\phi_2, \phi_3 \to i\phi_3, \chi_1 \to -\omega^2\chi_1, \chi_2 \to \chi_2,$$

$$\chi_3 \to \omega^2 \chi_3, \tag{5}$$

where $\omega = e^{2\pi i/3}$.

The choice of the discrete symmetries $S_3 \times D$ allow ν_{eL} to couple with $\nu_{\tau R}$ only through ϕ_1 Higgs field. But similar couplings of ν_{eL} with other two singlet neutrinos are prohibited. However, $\nu_{\mu L}$ and $\nu_{\tau L}$ couple with the singlet neutrinos through the Higgs fields ϕ_2 and ϕ_3. This facilitates small mixing between ν_e and ν_μ as required for explanation of the solar neutrino deficit and maximal mixing between ν_μ and ν_τ in order to explain atmospheric neutrino anomaly with appropriate choice of the VEVs of the Higgs fields: $\langle \phi_1 \rangle \sim 1$ GeV, $\langle \phi_2 \rangle = \langle \phi_3 \rangle \sim 10^2$ GeV. The small VEV of the Higgs field ϕ_1 is achieved by choosing positive mass term of the ϕ_1 Higgs field in the Higgs potential. The nonzero but small VEV of ϕ_1 arises due to the presence of quartic coupling term in the Higgs potential [15].

III. HIGGS POTENTIAL AND NEUTRINO MASS MATRIX

We focus our attention on the relevant terms of the Higgs potential, which yield small value of the VEV of ϕ_1 and, hence, write explicitly the following ϕ_1 dependent terms in the potential:

$$\begin{aligned} V_{\phi_1} = & m_1^2(\phi_1^\dagger\phi_1) + \lambda_1(\phi_1^\dagger\phi_1)(\phi_2^\dagger\phi_2) + \lambda_2(\phi_1^\dagger\phi_1)(\phi_3^\dagger\phi_3) \\ & + \lambda_3(\phi_1^\dagger\phi_1)(\chi_1^*\chi_1) + \lambda_4(\phi_1^\dagger\phi_1)(\chi_2^*\chi_2) \\ & + \lambda_5(\phi_1^\dagger\phi_1)(\chi_3^*\chi_3) + \lambda'(\phi_1^\dagger\phi_3\chi_1^*\chi_3 + \chi_3^*\chi_1\phi_3^\dagger\phi_1). \end{aligned} \tag{6}$$

After substituting the VEVs of the Higgs fields in the above potential and minimizing with respect to $\langle \phi_1 \rangle = v_1$, we obtain

$$v_1 = -\frac{\lambda' v_3 k_1 k_3}{M_{\phi_1}^2}, \tag{7}$$

where

$$M_{\phi_1}^2 = m_1^2 + \lambda_1 v_2^2 + \lambda_2 v_3^2 + \lambda_3 k_1^2 + \lambda_4 k_2^2 + \lambda_5 k_3^2 \tag{8}$$

is the physical mass of the ϕ_1 Higgs field.

Assuming m_1^2 to be much greater than all other terms in Eq. (8) and putting $M_{\phi_1}^2 \sim m_1^2$, which does not affect the essential results obtained in our model, we get

$$v_1 = \frac{-\lambda' v_3 k_1 k_3}{m_1^2}. \tag{9}$$

We require the value of the VEV of $\langle \phi_1 \rangle \sim 1$ GeV. This would correspond to $\lambda' \sim 1, v_3 \sim 10^2$ GeV $k_1 \sim k_3$ and $m_1 \sim 10 k_3$.

Turning to the neutrino sector, the most general discrete symmetry invariant Yukawa interaction is given by

$$\begin{aligned} L_y^\nu = & f_1 \overline{l}_{1L} \nu_{\tau R} \tilde{\phi}_1 + f_2 \overline{l}_{2L} \nu_{\mu R} \tilde{\phi}_2 + f_3 \overline{l}_{2L} \nu_{\tau R} \tilde{\phi}_3 + f_4 \overline{l}_{3L} \nu_{\mu R} \tilde{\phi}_2 \\ & + f_1 \overline{l}_{3L} \nu_{\tau R} \tilde{\phi}_3 + g_1 \overline{\nu_{eR}^c} \nu_{eR} \chi_1^0 + g_2 \overline{\nu_{\mu R}^c} \nu_{\mu R} \chi_2^0 \\ & + g_3 \overline{\nu_{\tau R}^c} \nu_{\tau R} \chi_3^0 + \text{H.c.} \end{aligned} \tag{10}$$

Substituting the VEVs of the Higgs fields in Eq. (10) we obtain the following form of (6×6) Majorana neutrino mass matrix in the basis $(\nu_{eL}, \nu_{\mu L}, \nu_{\tau L}, \nu_{eR}^c, \nu_{\mu R}^c, \nu_{\tau R}^c)$:

$$M_\nu=\begin{pmatrix}0 & D^T\\ D & M\end{pmatrix}=\begin{pmatrix}0&0&0&0&0&c_3\\0&0&0&0&a_2&a_3\\0&0&0&0&b_2&b_3\\0&0&0&M_1&0&0\\0&a_2&b_2&0&M_2&0\\c_3&a_3&b_3&0&0&M_3\end{pmatrix},\tag{11}$$

where D and M denote the (3×3) Dirac and Majorana neutrino mass matrices, respectively. The matrix elements of Eq. (11) are given by

$$c_3=f_1v_1, a_2=f_2v_2, a_3=f_3v_3, b_2=f_4v_2,$$

$$b_3=f_5v_3, M_1=g_1k_1, M_2=g_2k_2, M_3=g_3k_3.\tag{12}$$

We see from the (6×6) mass matrix that the right handed Majorana neutrino mass matrix M becomes diagonal with mass eigenvalues M_1, M_2, and M_3 due to the imposition of the $(S_3\times D)$ discrete symmetries. While all the heavy right handed neutrinos acquire their masses at the lepton number symmetry breaking scale which is of the order of $\sim10^{16}$ GeV, we assume the following hierarchy of masses:

$$\frac{M_2}{M_3}=\frac{1}{10}\tag{13}$$

in order to arrive at the required mass ratio for the left handed doublet neutrinos.

The induced (3×3) mass matrix for the left handed neutrinos (ν_e,ν_μ,ν_τ) can be calculated from Eq. (11) using the see-saw formula in the basis of a diagonal Majorana mass matrix as

$$m_{ij}=\frac{D_{1i}D_{1j}}{M_1}+\frac{D_{2i}D_{2j}}{M_2}+\frac{D_{3i}D_{3j}}{M_3},\tag{14}$$

where i and j denote the three neutrino flavors and D_{1i}, D_{2i}, and D_{3i} refer to the respective Dirac mass matrix elements in Eq. (11). Thus we get

$$m_\nu=\begin{pmatrix}\frac{c_3^2}{M_3} & \frac{c_3a_3}{M_3} & \frac{c_3b_3}{M_3}\\ \frac{c_3a_3}{M_3} & \frac{a_2^2}{M_2}+\frac{a_3^2}{M_3} & \frac{a_2b_2}{M_2}+\frac{a_3b_3}{M_3}\\ \frac{c_3b_3}{M_3} & \frac{a_2b_2}{M_2}+\frac{a_3b_3}{M_3} & \frac{b_2^2}{M_2}+\frac{b_3^2}{M_3}\end{pmatrix}.\tag{15}$$

With the redefinition of the matrix elements as

$$\frac{c_3}{\sqrt{M_3}}\to C_3,\quad \frac{a_3}{\sqrt{M_3}}\to A_3,\quad \frac{b_3}{\sqrt{M_3}}\to B_3,$$

$$\frac{a_2}{\sqrt{M_2}}\to A_2,\quad \frac{b_2}{\sqrt{M_2}}\to B_2.$$

Equation (15) is reduced to the following simple form:

$$m_\nu=\begin{pmatrix}C_3^2 & C_3A_3 & C_3B_3\\ C_3A_3 & A_2^2+A_3^2 & A_2B_2+A_3B_3\\ C_3B_3 & A_2B_2+A_3B_3 & B_2^2+B_3^2\end{pmatrix}.\tag{16}$$

We now proceed to calculate the masses and mixing angles of the three light left handed neutrinos by diagonalizing the mass matrix in Eq. (16). It can be shown that the determinant of m_ν is zero implying that one of the eigenvalues of m_ν is zero.

We then assume the approximation, $A_2,B_2\gg A_3,B_3\gg C_3$, which will be true for our parameter space of interest. After diagonalization of the (3×3) induced neutrino mass matrix we get the following mass eigenvalues:

$$m_1\sim A_2^2+B_2^2,$$

$$m_2\sim\frac{(A_2B_3-A_3B_2)^2}{A_2^2+B_2^2},$$

$$m_3=0,\tag{17}$$

where $m_2/m_1\sim M_2/M_3$.

Finally, we investigate the mixing matrix connecting the three flavor eigenstates (ν_e,ν_μ,ν_τ) to the mass eigenstates (ν_3,ν_2,ν_1) in increasing order of mass:

$$\begin{pmatrix}\nu_e\\ \nu_\mu\\ \nu_\tau\end{pmatrix}=\begin{pmatrix}1 & \frac{-C_3\sqrt{(A_2^2+B_2^2)}}{A_2B_3-A_3B_2} & -1\\ \frac{C_3B_2}{A_2B_3-A_3B_2} & \frac{B_2}{\sqrt{A_2^2+B_2^2}} & \frac{C_3A_2}{A_2A_3+B_2B_3}\\ \frac{-C_3A_2}{A_2B_3-A_3B_2} & \frac{-A_2}{\sqrt{A_2^2+B_2^2}} & \frac{C_3B_2}{A_2A_3+B_2B_3}\end{pmatrix}\times\begin{pmatrix}\nu_3\\ \nu_2\\ \nu_1\end{pmatrix}.\tag{18}$$

In our model the mixing angle between ν_μ and ν_τ is given by ratio of the two Yukawa couplings as follows:

$$\tan\theta_{\mu\tau}\sim\frac{A_2}{B_2}\sim\frac{f_2}{f_4}.\tag{19}$$

Assuming these Yukawa couplings to be equal, we obtain $\theta_{\mu\tau}=45^0$, i.e., $\sin^2 2\theta_{\mu\tau}=1$. This agrees with the Super-Kamiokande data on atmosoheric neutrinos, which suggest maximal mixing between ν_μ and ν_τ. Now, for the solar neutrino oscillation $\nu_e \to \nu_\mu$, the mixing angle between ν_e and ν_μ in our model is given by

$$\sin\theta_{e\mu} \sim \frac{C_3\sqrt{(A_2^2+B_2^2)}}{A_2B_3 - A_3B_2} \sim \frac{f_1 v_1}{f_{3,5} v_3}. \tag{20}$$

If the couplings are assumed to be of the same order, we get

$$\sin\theta_{e\mu} \sim \frac{v_1}{v_3} \sim 10^{-2}. \tag{21}$$

This is in good agreement with the recent Super-Kamiokande solar neutrino data.

We shall now estimate the two nonzero light neutrino masses in our model. Assuming $v_3 = 1.5\times 10^2$ GeV and $M_2 \sim 10^{15}$ GeV and $M_3 \sim 10^{16}$ GeV, we get $m_{\nu_\mu} \sim 0.0023$ eV and $m_{\nu_\tau} \sim 0.045$ eV. Thus our model leads to $\Delta m^2_{\mu\tau} \sim 2\times 10^{-3}\,\mathrm{eV}^2$ and $\Delta m^2_{e\mu} \sim 0.53\times 10^{-5}\,\mathrm{eV}^2$, which are consistent with the experimental results of Super-Kamiokande.

IV. CONCLUSIONS

In the present paper we have focused our attention on the Super-Kamiokande data on atmospheric and solar neutrinos without taking into account the LSND result. We demonstrate that the evidence of atmospheric and solar neutrino oscillations provided by Super-Kamiokande experiments can be accommodated in an extension of the standard electroweak gauge model based on the gauge group $SU(2)_L \times U(1)_Y$ with appropriate discrete symmetries $(S_3 \times D)$ and Higgs fields along with the three right handed singlet neutrinos. The model can simultaneously reconcile the maximal mixing between ν_μ and ν_τ as required by the atmospheric neutrino data as well as matter enhanced Mikheyev-Smirnov-Wolfenstein (MSW) solution to solar neutrino problem with small mixing angle between ν_e and ν_μ. The desired small masses of the left handed neutrinos are generated by the well known see-saw mechanism with reasonable choice of the model parameters. Apart from the electroweak symmetry breaking scale, the model admits a lepton number symmetry breaking scale at $\sim 10^{16}$ GeV.

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[1] Super-Kamiokande Collaboration, Y. Fukuda *et al.*, Phys. Rev. Lett. **81**, 1562 (1998); Phys. Lett. B **436**, 33 (1998); **433**, 9 (1998); T. Kajita, Talk presented at Neutrino 98, Takayama, Japan, 1998.

[2] Super-Kamiokande Collaboration, Y. Fukuda *et al.*, Phys. Rev. Lett. **81**, 1158 (1998); Y. Suzuki, Talk presented at Neutrino 98, Takayama, Japan, 1998.

[3] E. Ma and P. Roy, Phys. Rev. D **52**, R4780 (1995); N. Gaur, A. Ghosal, E. Ma, and P. Roy, Report No. UCRHEP-T233, MRI-PHY/980444, 1998.

[4] V. Barger, R. J. N. Phillips, and K. Whisnant, Phys. Rev. D **24**, 538 (1981); S. L. Glashow and L. M. Krauss, Phys. Lett. B **190**, 199 (1987); A. Acker, S. Pakvasa, and J. Pantaleone, Phys. Rev. D **43**, 1754 (1991); P. I. Krastev and S. T. Petkov, Phys. Lett. B **285**, 85 (1992); Phys. Rev. Lett. **72**, 1960 (1994); Phys. Rev. D **53**, 1665 (1996).

[5] N. Hata and P. G. Langacker, Phys. Rev. D **56**, 6107 (1997).

[6] J. N. Bahcall, P. J. Krastev, and A. Yu. Smirnov, Phys. Rev. D **58**, 096016 (1998).

[7] M. Appollonio *et al.*, Phys. Lett. B **420**, 397 (1998).

[8] CHORUS Collaboration, E. Eskut *et al.*, Phys. Lett. B **424**, 202 (1998).

[9] NOMAD Collaboration, J. Altegoer *et al.*, Report No. CERN-EP/98-57.

[10] LSND Collaboration, C. Anthassopoulas *et al.*, Phys. Rev. Lett. **75**, 2650 (1995); Phys. Rev. C **54**, 2685 (1996); Phys. Rev. Lett. **77**, 3082 (1996); Phys. Rev. C **58**, 2489 (1998); Phys. Rev. Lett. **81**, 1774 (1998).

[11] KARMEN Collaboration, R. Armbruster *et al.*, Phys. Rev. C **57**, 3414 (1998); Phys. Lett. B **423**, 15 (1998); K. Eitel *et al.*, Nucl. Phys. B (Proc. Suppl.) **77**, 212 (1999).

[12] G. Altarelli and F. Feruglio, Phys. Lett. B **439**, 112 (1998); M. Bando, T. Kugo, and K. Yoshioka, Phys. Rev. Lett. **80**, 3004 (1998).

[13] E. Ma, D.P. Roy, and U. Sarkar, Report No. UCRHEP-T236, 1998.

[14] E. Ma and D. P. Roy, Report No. DTP/98/78; UCRHEP-T210, 1998.

[15] A. K. Ray and S. Sarkar, Phys. Rev. D **58**, 055010 (1998).