High-energy forward scattering and the Pomeron: Simple pole versus unitarized models

J. R. Cudell,[1,*] V. Ezhela,[2,†] K. Kang,[3,‡] S. Lugovsky,[2,§] and N. Tkachenko[2,‖]
[1]*Institut de Physique, Université de Liège, Bât. B-5, Sart Tilman, B4000 Liège, Belgium*
[2]*COMPAS Group, IHEP, Protvino, Russia*
[3]*Department of Physics, Brown University, Providence, Rhode Island 02912*
and School of Physics, Korea Institute for Advanced Study, Seoul 130-012, Korea
(Received 3 August 1999; published 12 January 2000)

Using the largest data set available, we determine the best values that the data at $t=0$ (total cross sections and real parts of the hadronic amplitudes) give for the intercepts and couplings of the soft Pomeron and of the ρ/ω and a/f trajectories. We show that these data cannot discriminate between a simple-pole fit and asymptotic $\log^2 s$ and $\log s$ fits, and hence are not sufficient to reveal the ultimate nature of the Pomeron. However, we evaluate the existing evidence (factorization, universality, quark counting) favoring the simple-pole hypothesis. We also examine the range of validity in energy of the fits, and show that one cannot rely on such fits in the region $\sqrt{s}<9$ GeV. We also establish bounds on the odderon and the hard Pomeron.

PACS number(s): 13.85.−t, 11.55.−m, 12.40.Nn, 13.60.Hb

I. INTRODUCTION

The description of forward scattering by universal fits has been an open question for the last 20 years. The data from the DESY ep collider HERA, which now extend the measurement of off-shell cross sections to very low values of Q^2, have revived interest in this problem, as it can shed some light on the nature of the Pomeron. Because of the presence of large logarithms of the center-of-mass energy $\sqrt{s}$, perturbative QCD predicts an explosive increase of the cross sections with energy. Whether this prediction is stable remains to be seen, but such a sharp rise is qualitatively present in the deep inelastic scattering (DIS) data from HERA. However, this is in marked contrast to the observation of on-shell hadronic total cross sections, which have a very slow rise with s.

Two schools of thought exist regarding this puzzle. The first one starts from the simplest assumption within Regge theory: that this rise with s is the result of the presence of a glueball trajectory, for which there are at present strong candidates [1]. This trajectory is called the Pomeron, and has an intercept slightly larger than 1. This assumption leads to the prediction of a universal rise with s, and of factorization. The further hypothesis, that the Pomeron couples to constituent quarks, leads further to the prediction of quark-counting rules. Moreover, simple refinements have enabled Donnachie and Landshoff (DL) to push these ideas further [2], and to reproduce qualitatively well all soft data for the scattering of on-shell particles, even at nonzero t. The problem with this approach is that it cannot be automatically extended to off-shell particles and, in particular, to DIS. The only possible hypothesis [3] would be that an extra trajectory enters the problem, and that this trajectory decouples at $Q^2=0$. The possibility of such a stable trajectory is phenomenologically viable, and is confirmed, to some extent, by the Dokshitzer-Gribov-Lipatov-Altarelli-Parisi (DGLAP) evolution.

The other school of thought starts from perturbative QCD, and assumes that unitarization changes the fierce rise observed at large Q^2 to something compatible with the Froissart bound. This approach suffers from the fact that, despite recent progress [4], no one has reliably unitarized a QCD cross section. However, it is clear that such a unitarization will involve the exchange of a very large number of gluons between the quarks. Hence, the details of the quark structure—the hadronic wave function—should matter, and this means that the simple Regge factorization property would be lost, as well as quark counting and even strict universality [5]. Furthermore, it is expected that such a unitarization would lead to a cut singularity instead of a pole and to a power behavior in $\log s$.

The question we want to address here is whether one can distinguish between these two approaches by studying soft data. In order to maximize the number of data points, we shall consider the full hadronic amplitude, i.e., both the total cross section, giving the imaginary part, and the ρ parameter, giving the ratio of the real part to the imaginary part. As we shall see, the consideration of the χ^2 alone surprisingly does not discriminate between the different hypotheses, but leads one to refine the description of lower trajectories and to define a minimum energy below which none of these fits work. The only discrimination that the soft data can bring in lies in the confirmation of the properties that suggest that the Pomeron is a simple pole coupled to the constituent quarks, i.e., universality, factorization, and quark counting.

This study complements and expands the results of a recent letter [6], where two of us (J.R.C. and K.K.) with Kim presented a detailed statistical analysis of the parameters of the DL model, as well as the analysis subsequently presented (by V.E., S.L. and N.T.) in the 1998 Review of particle physics [7].

This paper is organized as follows. In Sec. II, we describe the data sample and the hypothesis-testing procedure. In Sec. III, we concentrate on the simple-pole fit, and study first the

*Email address: JR.Cudell@ulg.ac.be
†Email address: ezhela@mx.ihep.su
‡Email address: kang@het.brown.edu
§Email address: lugovsky@mx.ihep.su
‖Email address: tkachenkon@mx.ihep.su

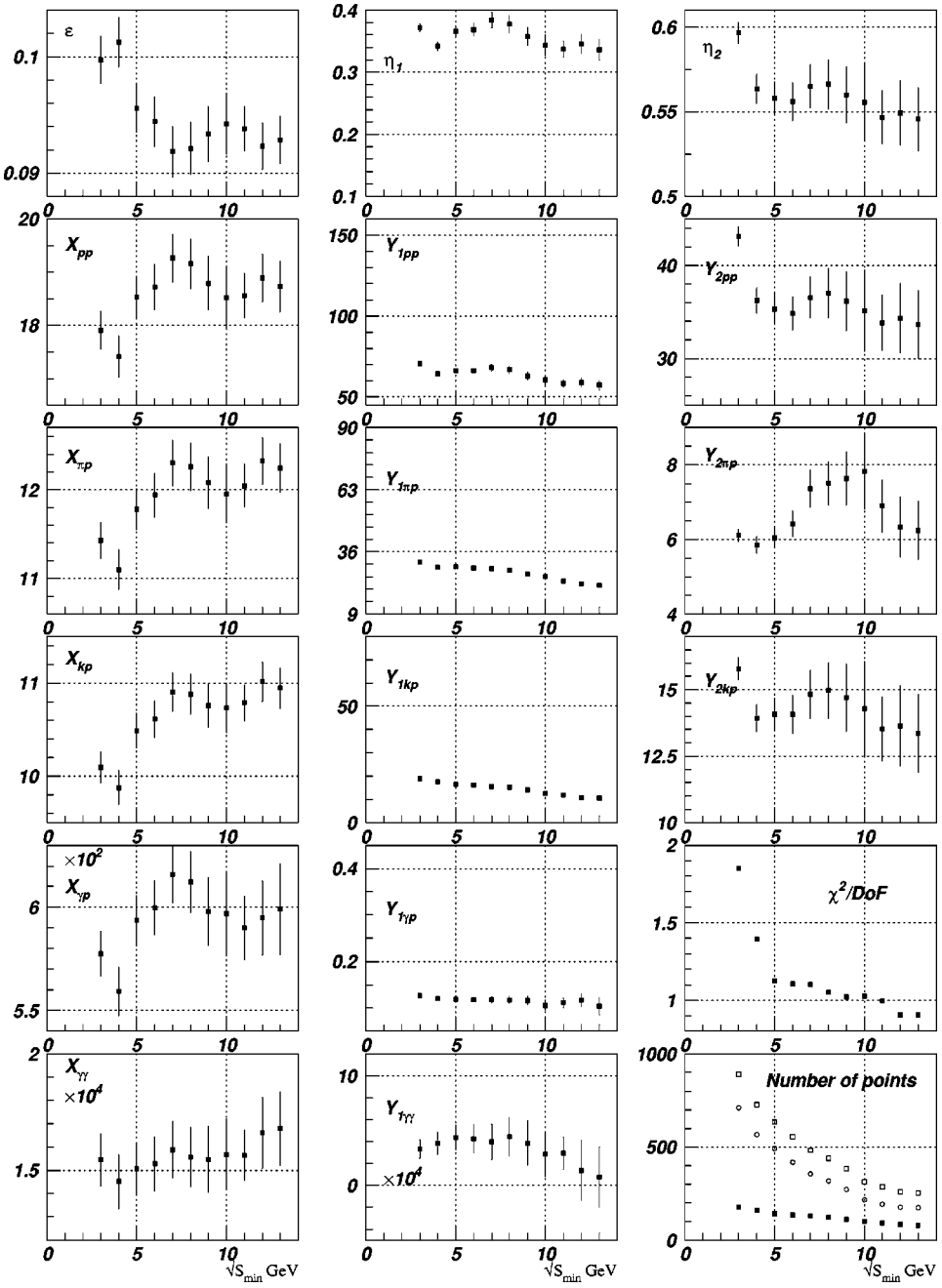

FIG. 1. Parameters of the RRP model as functions of the minimum energy considered in the fit.

changes one has to introduce in order to describe the low-energy data reliably. In Sec. IV, we present the evidence for simple-pole behavior, and in Sec. V we consider alternative (unitary) forms for the Pomeron-exchange term. In Sec. VI, we mention several attempts to extend the fit to the low-energy region. In Sec. VII, we use our data set to place bounds on other trajectories, and we present some predictions for cross sections.

II. DATA SET AND STATISTICAL PROCEDURE

A. Data set

Three of us (V.E., S.L., and N.T.) have prepared a complete and maintained [8] set of published data for the cross sections and real-to-imaginary part ratios for the processes pp, $\bar{p}p$, $\pi^{\pm}p$, and $K^{\pm}p$, as well as for the total cross sections of γp and $\gamma\gamma$ scattering. Some superseded points have been removed, and typos have been corrected. It was found in [6] that irrespectively of the models used, the χ^2/[degrees of freedom (DOF)] was large due to bad data points at ISR energies. Once about 10% of the ISR points were removed, an acceptable χ^2/DOF was achieved, leading to reliable estimates of the parameters and their errors. As it will turn out, the new data set does not necessitate such a filtering procedure, and thus seems more coherent. The data set contains 2747 (303) data points for total cross sections (respectively, real-to-imaginary ratios), and the number of points used in the fits is given as a function of energy in Fig. 1. It is our

TABLE I. The values of the parameters of the hadronic amplitude in model RRP (3), (5), corresponding to a cutoff $\sqrt{s} \geqslant 9$ GeV and the values of the individual χ^2 of the various processes together with the number of points N.

ϵ	η_1	η_2	χ^2/DOF	Statistics	
0.0933 ± 0.0024	0.357 ± 0.015	0.560 ± 0.017	1.02	383	
	pp	πp	Kp	$\gamma p \times 10^{-2}$	$\gamma\gamma \times 10^{-4}$
X (mb)	18.79 ± 0.51	12.08 ± 0.29	10.76 ± 0.23	5.98 ± 0.17	1.55 ± 0.14
Y_1 (mb)	63.0 ± 2.3	26.2 ± 0.74	14.08 ± 0.57	11.64 ± 0.88	3.9 ± 2.0
Y_2 (mb)	36.2 ± 3.2	7.63 ± 0.72	14.7 ± 1.3		
Process	χ^2/N, σ_{tot} (N)	χ^2/N, ρ (N)	Process	χ^2/N, σ_{tot} (N)	χ^2/N, ρ (N)
pp	1.01 (75)	1.27 (59)	K^+p	0.539 (22)	0.635 (7)
$\bar{p}p$	1.24 (35)	0.518 (11)	K^-p	0.837 (28)	1.99 (5)
π^+p	0.562 (24)	2.21 (7)	γp	0.624 (25)	
π^-p	1.14 (47)	0.953 (23)	$\gamma\gamma$	0.324 (15)	

hope that this data set will become the standard reference when studying the validity of models for forward quantities.

B. Definition of χ^2 and of the errors

As the data set is quite large and has no substantial inconsistencies, the conventional definition of χ^2 is used. Note, however, that as the most interesting quantities are sensitive to the highest-s region, where the data are scarce, our definition may not be the best suited for determining the soft Pomeron intercept and other definitions giving more weight to the highest-energy data are possible. One may also worry about whether one should consider only total cross sections or the full amplitude. The best data are certainly the measurements of the total cross section, and one might wonder whether the interference between Pomeron exchange and the Coulomb cross sections can be reliably calculated.

Despite these two worries, we see no fundamental reason for rejecting part of the data, or using a nonconventional χ^2, as was done in, e.g., [3,9], where equal weights were given to the $\bar{p}p$ and the pp data sets, while not fitting to the other cross sections or to the ρ parameters.

Choosing a conventional χ^2 definition and weighting all the points with inverse squares of their total errors enables us to define errors through the usual definition[1] of a change of χ^2 of 1 unit for acceptable fits with a χ^2/DOF of order 1. In case of bad fits, we shall sometimes give an estimate of the error, which corresponds to a change of χ^2 of χ^2_{min}/DOF; in other words, we shall then dilate the errors by the Birge factor. This definition also allows us to reject models or parameters corresponding to values of the χ^2/DOF appreciably larger than 1. Note that for the total error, we have added the statistical and the systematic errors in quadrature.

One further problem is linked to the fact that the fits considered below are asymptotic: it is clear that smooth functions cannot describe the resonance region; hence the fits can be trusted only above a certain energy $\sqrt{s_{min}}$ which is a parameter in itself and could, in principle, be process dependent. We demand for the fits to be trusted that the value of the parameters remains stable with respect to $\sqrt{s_{min}}$, and that the χ^2/DOF. be less than 1. This criterion implies that our determination of the parameters describing the Pomeron is stable or, equivalently, that the low-energy data are not of primary importance.

III. REGGE FITS AND LOWER TRAJECTORIES

First we discuss the Regge-pole parametrization of the data. It is based on the idea that the cross sections should be reproduced by the simplest singularities in the complex J plane, i.e., simple poles, corresponding to the exchange of bound-state trajectories. The imaginary part of the hadronic amplitude is then given by

$$\mathrm{Im}\, A_{h_1 h_2}(s,t) = \sum_i (\pm 1)^{S_i} C_{h_1 h_2}(t) \left(\frac{s}{s_0}\right)^{\alpha_i(t)}, \qquad (1)$$

with S_i the signature of the exchange. The total cross section is then equal to

$$\sigma_{tot}(s) = \mathrm{Im}\, A(s,0)/s. \qquad (2)$$

The trajectories $\alpha_i(t)$ are universal, and the process (and mass) dependence is present only in the constants $C_{h_1 h_2}(0)$ (which absorb the scale s_0). The highest trajectory, responsible for the rise of cross sections, is that of the soft Pomeron. The others are those of the mesons, and, in principle, they are numerous. However, once the energy is high enough, only those with the largest intercept, $\alpha_i(0)$ of order

[1]Note that the errors in this paper are smaller than those of [6] because there a change of 5 units was considered, and because we have now a larger data set.

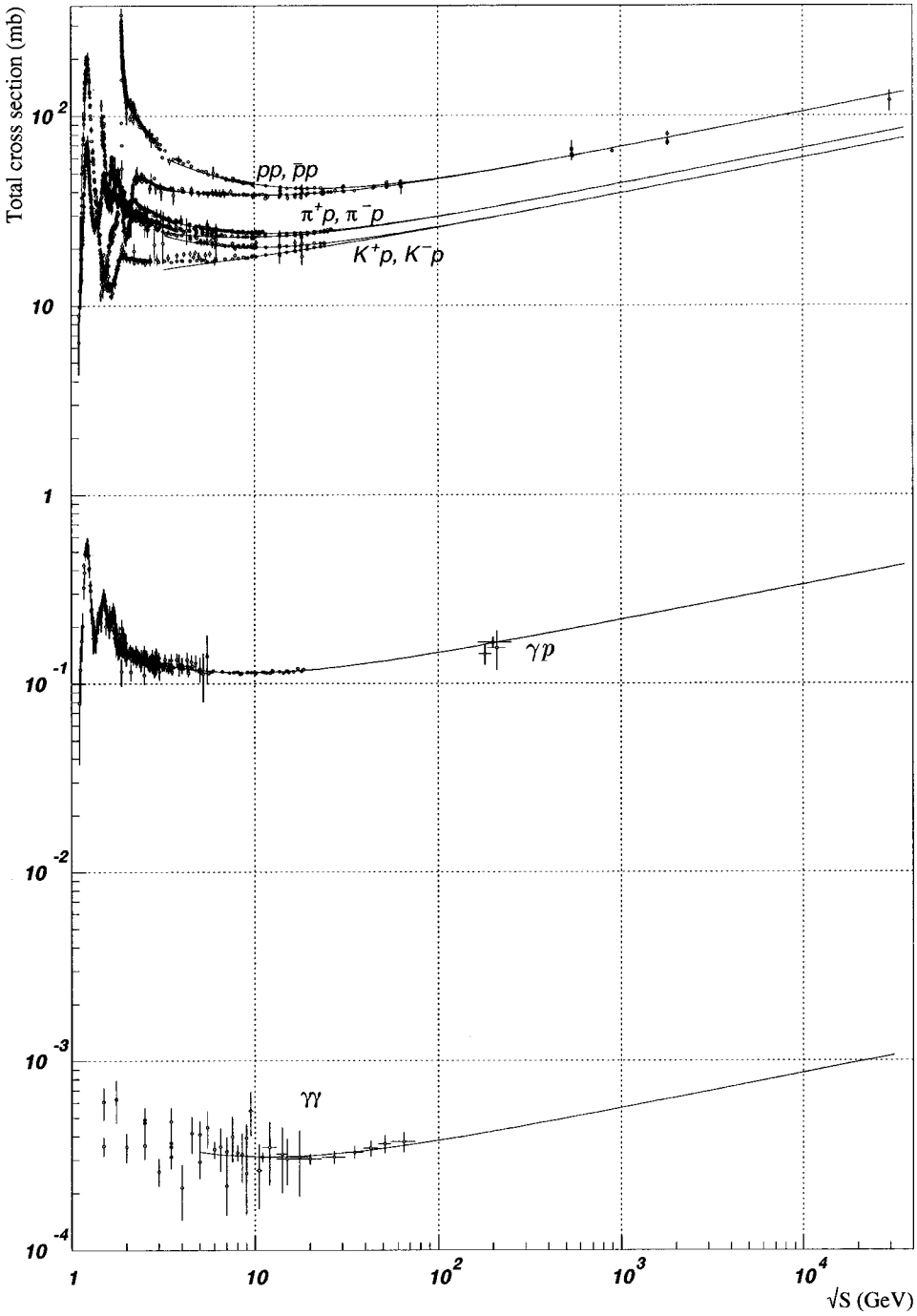

FIG. 2. The fit to the total cross sections from the parametrization RRP.

1/2, will contribute at $t=0$. The four highest meson trajectories can be clearly seen in a M^2 vs J plot of the meson data. They correspond to the ρ, the a, the ω, and the f resonant states.

The simplest assumption, which would result from a simple string model of the mesons, is that these trajectories are degenerate, which implies that they have the same intercepts [9]. However, the results of a previous fit [6] show that an exchange-degenerate meson trajectory fails to satisfy the proposed criteria: the χ^2/DOF is large (of order 1.3), the parameters and their errors are unstable when the model is fitted to the total cross sections and the ρ parameter, and, in fact, the assumption of exchange degeneracy for $C=\pm 1$ meson trajectories is not supported even by fits to total cross sections only.

This situation persists for other parametrizations of the Pomeron term, and in the following we shall keep the low-energy model of cross sections presented here and resulting from the exchange of two nondegenerate $C=+1$ and $C=-1$ meson trajectories.

Furthermore, the situation remains identical when one considers the additional data presented here. Hence, we shall

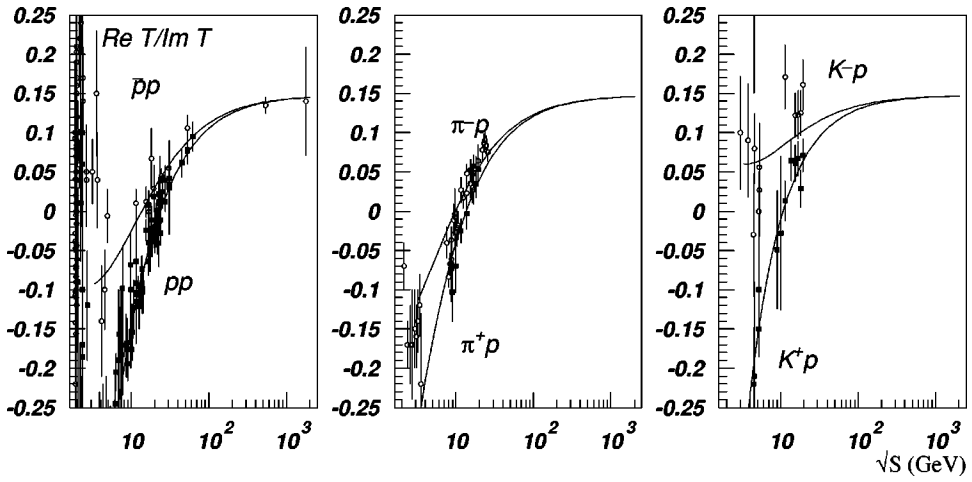

FIG. 3. The fit to the ρ values from the parametrization RRP.

adopt the simple generalization proposed in [6], which we shall call the RRP (Reggeon-Reggeon-Pole) model, and which assumes independent $C=+1$ (a/f) and $C=-1$ (ρ/ω) intercepts. Hence, the formula (1) for the total cross section becomes

$$\frac{\mathrm{Im}\,A_{h_1h_2}(s)}{s} = X^{h_1h_2}s^{\epsilon} + Y_1^{h_1h_2}s^{-\eta_1} \mp Y_2^{h_1h_2}s^{-\eta_2}, \qquad (3)$$

with the intercepts given by

$$\alpha_P = 1+\epsilon,$$

$$\alpha_{(C=+1)} = 1-\eta_1,$$

$$\alpha_{(C=-1)} = 1-\eta_2. \qquad (4)$$

The sign of the Y_2 term flips when fitting $h_1\overline{h_2}$ data compared to h_1h_2 data. The real parts of the forward elastic amplitudes are calculated from analyticity (see, for example [10–12]):

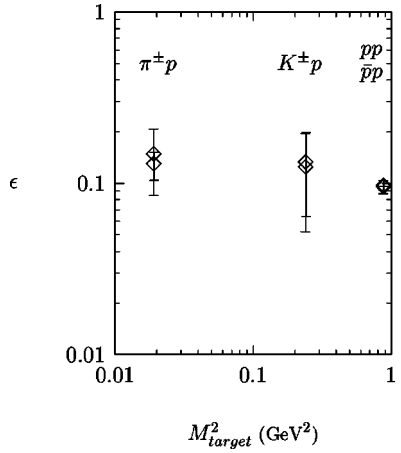

FIG. 4. The value of the Pomeron intercept for three different processes.

$$\frac{\mathrm{Re}\,A_{h_1h_2}(s)}{s} = -X^{h_1h_2}s^{\epsilon}\cot\left(\frac{1+\epsilon}{2}\pi\right) - Y_1^{h_1h_2}s^{-\eta_1}\cot\left(\frac{1-\eta_1}{2}\pi\right) \mp Y_2^{h_1h_2}s^{-\eta_2}\tan\left(\frac{1-\eta_2}{2}\pi\right), \qquad (5)$$

where the upper (lower) sign refers to a proton scattering with a negatively (positively) charged particle.

We now study the stability of the fit, changing $\sqrt{s_{min}}$ from 3 to 13 GeV. The number of points and the resulting χ^2/DOF are shown in Fig. 1. Clearly, the fit is bad for small energies. This is expected, as there is no reason then for neglecting the effect of lower trajectories. As in [6], we need $C=\pm 1$ meson trajectories that are nondegenerate, primarily because of the constraints coming from fitting the ρ parameters. We also see that values of 1 or smaller for the χ^2/DOF can be achieved for $\sqrt{s_{min}}=9$ GeV. Hence, Regge fits are not to be trusted below that energy.

The problem with such a high value of the minimum energy is that the Pomeron is reasonably determined, while the lower trajectories are much more poorly fixed. Clearly, if the fit is physically meaningful past a certain energy, its parameters cannot depend any longer on its starting point. In fact, one can see from Fig. 1 that the Pomeron intercept and couplings are stable with respect to $\sqrt{s_{min}}$ once we are above 8 GeV or so. However, this is not the case for the lower trajectories: although the $C=\pm 1$ intercepts and the $C=-1$ couplings are stable, within large errors, the $C=+1$ couplings do depend on the minimum energy.

It is to be noted that this problem has to do with the definition of the error bars, as all the values of the couplings above 9 GeV shown in Fig. 1 would lead to a χ^2/DOF smaller than 1. Furthermore, the parameters of the $C=+1$ trajectory are highly correlated to those of the Pomeron.[2] A

[2]The unrounded parameter values, the corresponding dispersions,

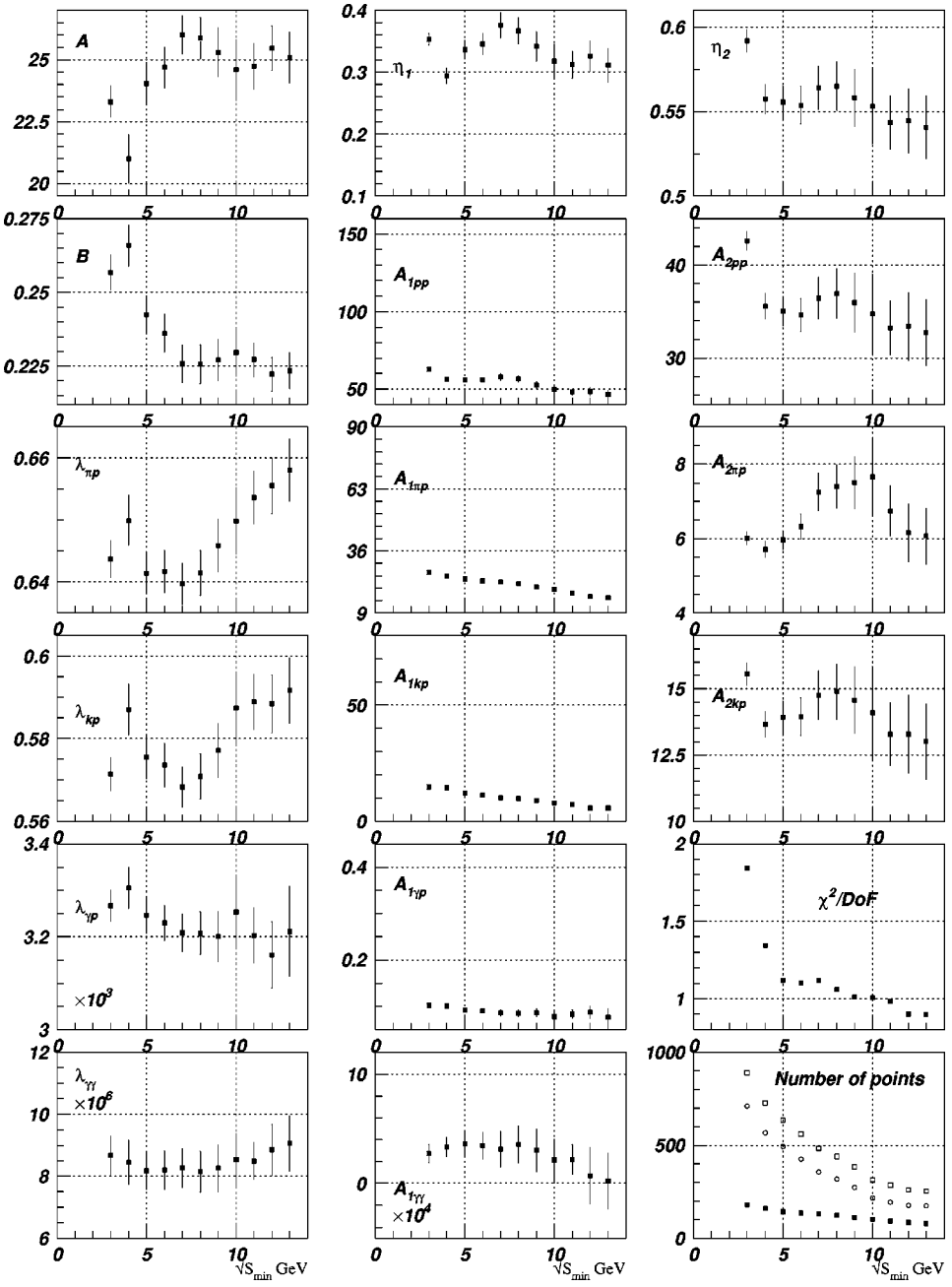

FIG. 5. Parameters of the RRL2 model, as functions of the minimum energy considered in the fit.

small change in the latter can produce a large variation in the former once one is at high energy. The bottom line, however, is that we cannot reliably determine the couplings of the a/f trajectory through the fitting procedure outlined here. This situation may change once photon cross sections are more precisely measured at HERA and at the CERN e^+e^- collider LEP.

The best we can do is to quote the values that we obtain for $\sqrt{s_{min}}=9$ GeV, with the above caveats. These values are given in Table I.

We also show the χ^2 per data points and the number of data points for each process fitted to. One can see that, as in [6], the χ^2 is high for some of the subprocesses. We have shown in [6] that this has nothing to do with the model, but rather with the dispersion of the data. Filtering the data for these two processes did not change the determination of the parameters. As the global χ^2/DOF is good, we do not resort here to such a procedure, as it is likely to bias the analysis slightly. In Sec. V, we shall demonstrate in another way that this is probably due to inconsistencies within the data, by comparing with other parametrizations of the Pomeron, and that these high values of a few χ^2 do not affect our conclusions.

The fits for the total cross sections and ρ parameters for

and the correlation matrices for each fit can be obtained by request from tkachenkon@mx.ihep.su

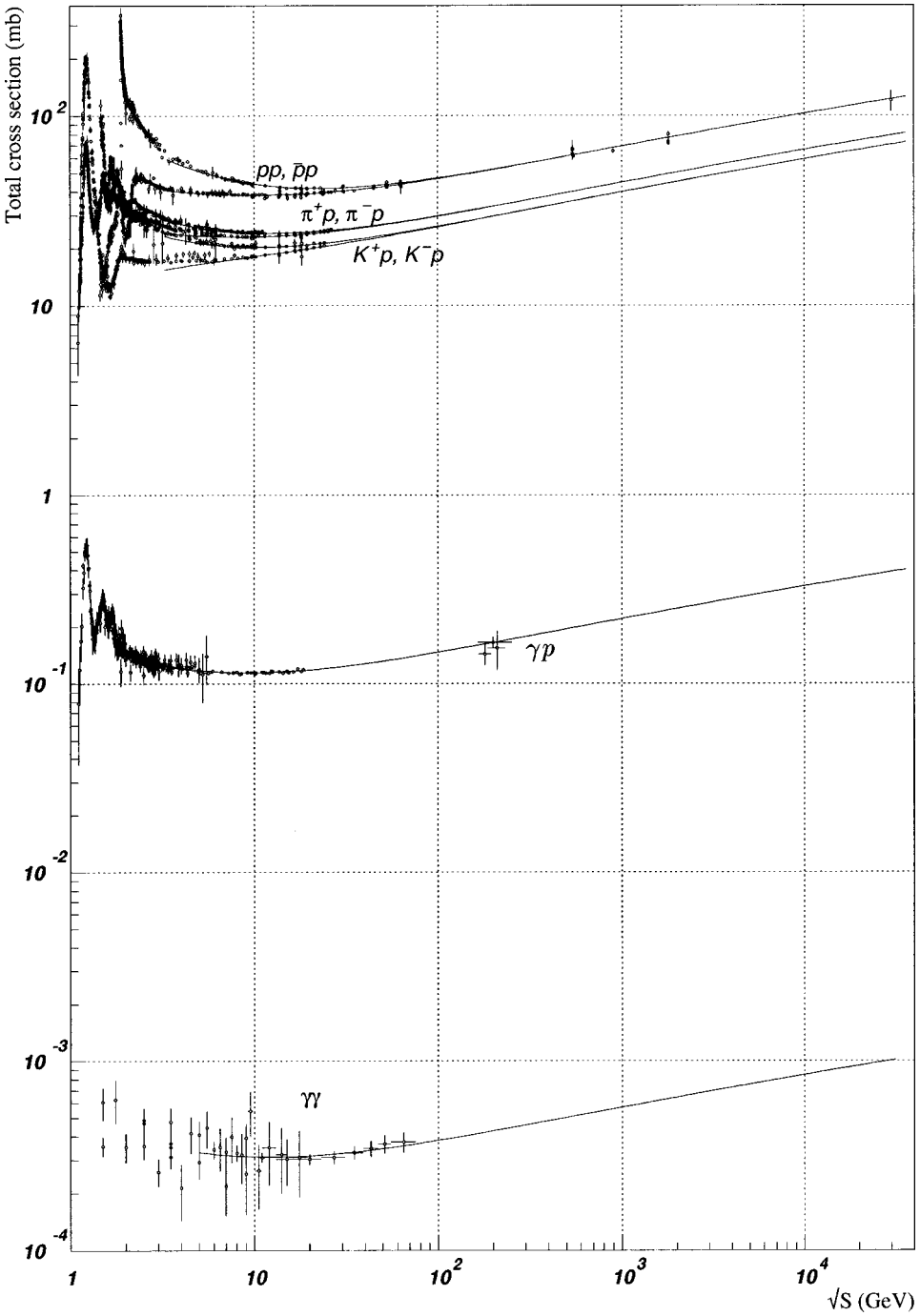

FIG. 6. The fit to the total cross sections from the parametrization RRL2.

$\sqrt{s} \geqslant 9$ GeV, extrapolated to $\sqrt{s} \geqslant 5$ GeV, are shown in Figs. 2 and 3. Although the value of χ^2/DOF is bad in the low-energy region (it goes above 2), and thus is statistically unacceptable, the fits look deceptively satisfactory. This shows the need for a careful statistical analysis with physically sound criteria imposed on.

IV. THE CURRENT EVIDENCE FOR THE SIMPLE POLE ANSATZ

It is not possible either to favor or to reject the simple-pole nature of the Pomeron from fits to the data. On the one hand, it is clear that the above fit is as good as it can be once the energy is large enough, given its χ^2/DOF. On the other hand, as we shall see in Sec. V, other fits fare as well. Hence, the belief that the Pomeron may be a simple pole is based on other evidence.

A. Universality

The first requirement from Regge theory is that the singularities be universal, whether poles or cuts. Hence, the s dependence of the data has to be a combination of parts which rise or fall with energy in a process-independent manner.

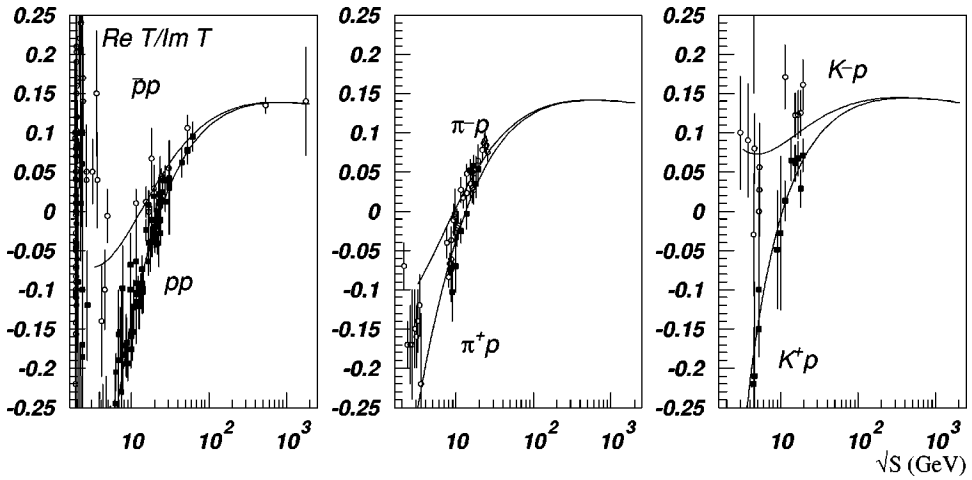

FIG. 7. The fit to the ρ values from the parametrization RRL2.

Note that, in general, this does not have to be exactly obeyed by a diagrammatic expansion of perturbative QCD (PQCD), as the hadronic wave functions come into the calculation of the various terms in the perturbative expansion, and one could *a priori* have a small deviation from universality [5]. The question of whether the intercepts are universal is also linked to the study of F_2 at HERA [13]. There, for photons with negative masses squared, it is observed that the effective Pomeron intercept, defined as the power of $1/x$ in $F_2(x)/x$, seems to depend on $Q^2=-M^2_{\gamma^*}$. It is of interest to check whether such a behavior is seen on the other side of $M^2=0$.

The problem here is that very little can be said in general. One can achieve for each process values of the χ^2/DOF much smaller than 1 if one fits to that process only. Hence, the usual definition of errors is meaningless. For instance, pion data have a very low sensitivity to the Pomeron intercept. If we accepted all fits with a χ^2/DOF smaller than 1, then the error bars on the various parameters would be much too large to reach any sensible conclusion. We choose here to do a partial fit, fixing the $C=\pm 1$ meson intercepts, and letting all other parameters free, and thus deriving errors on the Pomeron intercept. As the χ^2/DOF is still small, the errors should correspond to a change in the χ^2 such as the χ^2/DOF becomes equal to 1. In order to minimize the errors, we have chosen to include both cross sections and real parts in each fit, and kept $\sqrt{s_{min}}=9$ GeV. We show the results of such an analysis in Fig. 4. We have not included the γp data, as there is some uncertainty regarding these. They would lead to an intercept of order 0.075 with large error bars. Note also that in the pp and $\bar{p}p$ cases, the χ^2/DOF is larger than 1; hence the errors correspond there to a change of 1 unit in the χ^2/DOF. We see that the soft Pomeron intercept may be universal, and may be independent of the target mass, but the evidence is not overwhelming.

B. Factorization and quark counting

The couplings of Regge exchanges are expected to factorize into the product of two couplings, one for each interacting hadron. One further and much more stringent assumption is that the Pomeron couples to single quarks as a $C=+1$ photon. The Pomeron being an extended object, this assumption is only viable for constituent quarks, and it strongly suggests that the Pomeron is a simple pole, as, otherwise, the cuts would feel the hadronic wave function. The results shown in Table I can be rewritten:

$$\frac{X_{pp}/X_{\pi p}}{3/2}=1.04\pm 0.11, \tag{6}$$

$$X_{Kp}/X_{\pi p}=0.89\pm 0.05, \tag{7}$$

TABLE II. The values of the parameters of the hadronic amplitude in model RRL2 (11), (12), corresponding to a cutoff $\sqrt{s}\geqslant 9$ GeV.

A (mb)	B (mb)	s_0	η_1	η_2	χ^2/DOF
25.29 ± 0.98	0.2271 ± 0.0071	1 (fixed)	0.341 ± 0.024	0.558 ± 0.017	1.01
	pp	πp	Kp	$\gamma p\times 10^{-2}$	$\gamma\gamma\times 10^{-4}$
λ	1	0.6459 ± 0.0043	0.5772 ± 0.0065	0.3201 ± 0.0055	0.083 ± 0.076
Y_1 (mb)	52.6 ± 2.2	20.17 ± 0.62	9.00 ± 0.75	8.65 ± 0.87	(3.0 ± 2.0)
Y_2 (mb)	36.0 ± 3.2	7.50 ± 0.71	14.6 ± 1.3		

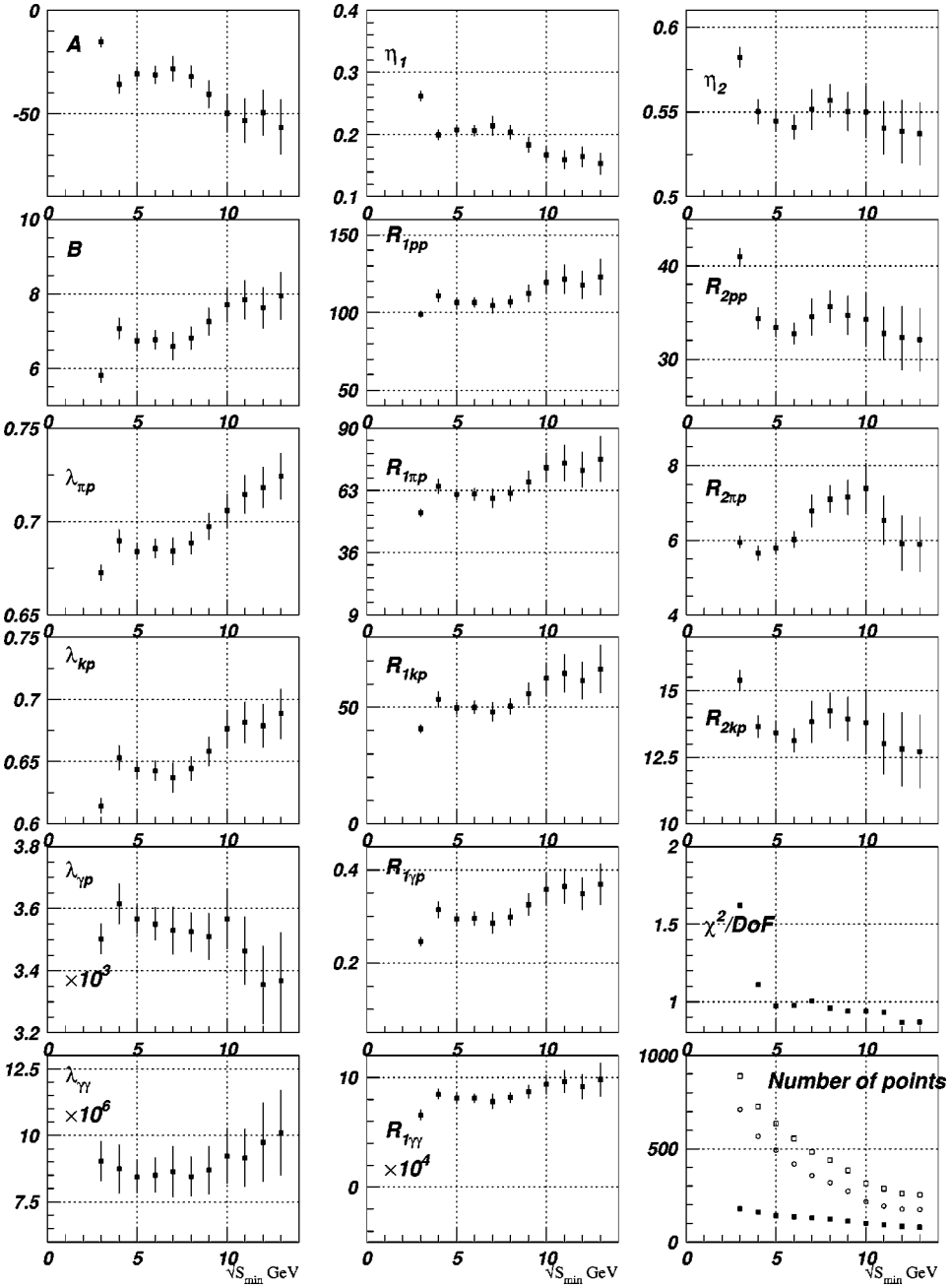

FIG. 8. Parameters of the RRL1 model, as functions of the minimum energy considered in the fit.

$$\frac{X_{\gamma p}}{g_{elm}^2\left[\frac{1}{f_\rho^2}+\frac{1}{f_\omega^2}+\frac{1}{f_\Phi^2}\right](1+\delta)X_{\pi p}} \approx \frac{213.9X_{\gamma p}}{X_{\pi p}}$$

$$= 1.06 \pm 0.04, \tag{8}$$

$$\frac{X_{pp}X_{\gamma\gamma}}{X_{\gamma p}^2} = 0.78 \pm 0.15. \tag{9}$$

The first and second relations illustrate quark counting, the third comes from factorization and generalized vector-meson dominance (GVMD) [14], where the contribution of off-diagonal terms δ is expected to be about 15%, and the fourth is an example of factorization. Hence, the properties of factorization and quark counting seem to hold within 10%. However, it is clear that data from other targets would need to be collected at sufficiently high energy before any firm conclusion can be reached.

The quark-counting property can be summarized by rewriting the Pomeron couplings to single quarks as $1/\Lambda_u$ and $1/\Lambda_s$ for light and strange quarks, so that, for instance, we write $X_{pp}=(3/\Lambda_u)^2$. The values of the scales thus obtained are

$$\Lambda_u \approx 0.43 \ \text{GeV},$$
$$\Lambda_s \approx 0.60 \ \text{GeV}. \tag{10}$$

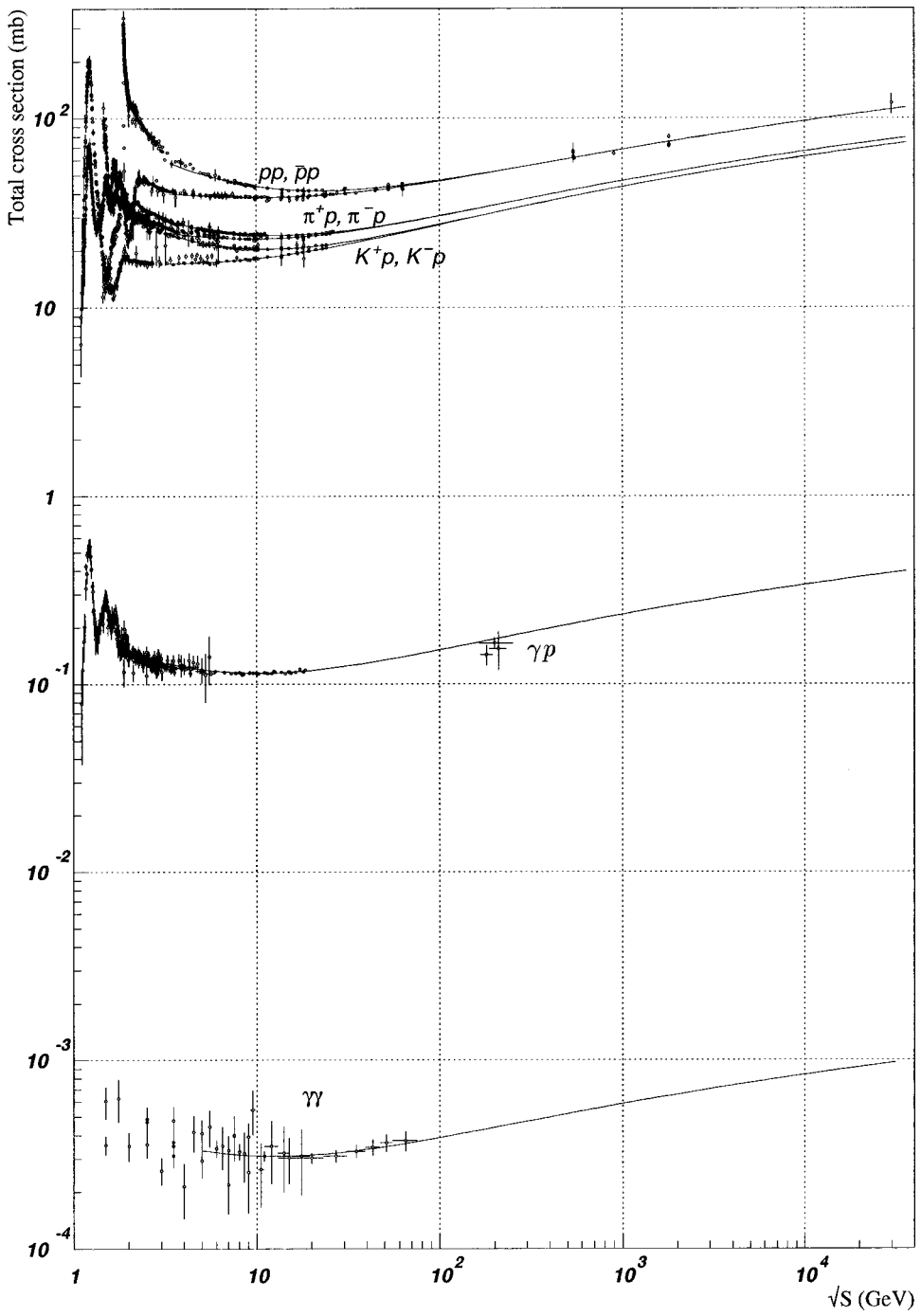

FIG. 9. The fit to the total cross sections from the parametrization RRL1.

It is to be noted that quark counting fails to be present for the other trajectories. Hence, these exchanges have to probe multiquark configuration, whereas the Pomeron seems to be coupled mainly to single quarks.

V. THE QUESTION OF UNITARIZATION AND ALTERNATIVE MODELS

It has been known for a long time that simple poles cannot be the only singularities of the hadronic amplitudes, and that their existence implies that of cuts in the complex J plane. These arise through multiple exchanges and restore unitarity (and the Froissart-Martin bound). These multiple exchanges are expected to play a significant role at the highest energies, but it is not clear whether present data require them. The problem in studying these is that, although one knows qualitatively what their effect will be, nobody knows the precise form that they will take in hadronic interactions.

For instance, Donnachie and Landshoff have proposed to consider the exchange of two Pomerons as a measure of the strength of unitarization effects; others [15] have used eikonal forms, or N/D methods [16]. We shall not consider all

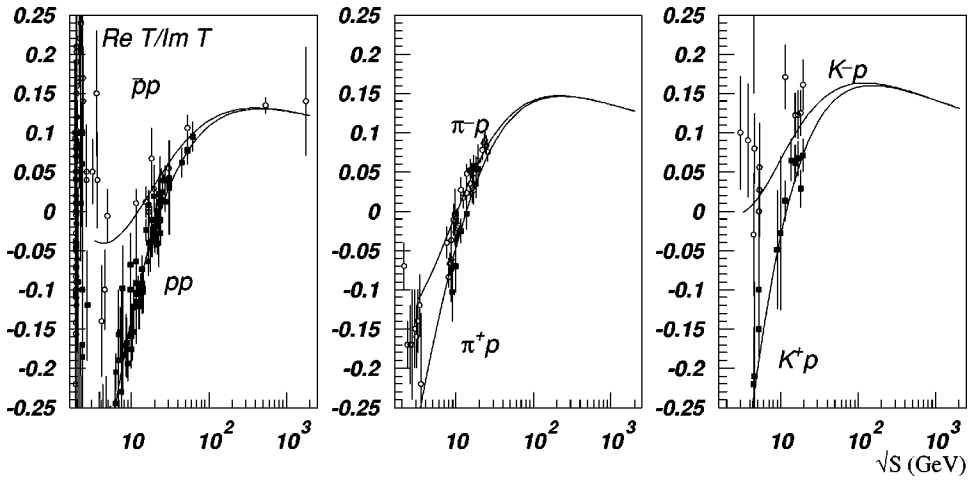

FIG. 10. The fit to the ρ values from the parametrization RRL1.

the possibilities, as we shall show that an ansatz based on anexplicitly unitary answer is indistinguishable from the simple pole fit.

Indeed, one may assume that the simple pole ansatz is strongly unitarized even at low s, and that multiple exchanges occur very early and turn the s dependence of the cross section into $\log^2 s$ [11], which would saturate unitarity, and this for energies as low as 10 GeV. Such a form can be obtained, e.g., through an eikonal formalism, although there is no justification for it in a QCD context. Following the above logic, we have considered an amplitude of the form

$$\frac{\mathrm{Im}\ A_{h_1h_2}(s)}{s}=\lambda_{h_1h_2}\left[A+B\log^2\left(\frac{s}{s_0}\right)\right]+Y_1^{h_1h_2}s^{-\eta_1} \mp Y_2^{h_1h_2}s^{-\eta_2}, \tag{11}$$

$$\frac{\mathrm{Re}\ A_{h_1h_2}(s)}{s}=\pi\lambda_{h_1h_2}B\log\left(\frac{s}{s_0}\right) - Y_1^{h_1h_2}s^{-\eta_1}\cot\left(\frac{1-\eta_1}{2}\pi\right) \mp Y_2^{h_1h_2}s^{-\eta_2}\tan\left(\frac{1-\eta_2}{2}\pi\right). \tag{12}$$

In order to simplify our discussion, and to have the same number of parameters for both fits, we set[3] $s_0=1$ GeV^2. This form, which we shall call the RRL2 (Reggeon-Reggeon-log$^2 s$) amplitude, leads to the results shown in Fig. 5. In a manner entirely similar to the RRP case, the fit is bad in the region $\sqrt{s}<9$ GeV, and again the $C=+1$ couplings are not stable. Furthermore, it is interesting to note that this parametrization leads to fits which are indistinguishable from the simple-pole case. Hence, all forms of partial unitarization which lead to something between a simple pole and a $\log^2 s$ fit cannot be distinguished on the basis of $t=0$ data alone. We show in Figs. 6 and 7 the result of such a fit for the cross sections and ρ parameters. The parameters corresponding again to $\sqrt{s_{min}}=9$ GeV are given in Table II.

Finally, it is also conceivable that unitarity is not saturated—e.g., multiple exchanges of a Pomeron with $\epsilon=0$ may lead to a slower rise—in $\log s$ [11,17],

$$\frac{\mathrm{Im}\,A_{h_1h_2}(s)}{s}=\lambda_{h_1h_2}\left[A+B\log\left(\frac{s}{s_0}\right)\right]+Y_1^{h_1h_2}s^{-\eta_1} \mp Y_2^{h_1h_2}s^{-\eta_2}, \tag{13}$$

and the real part is again obtained through analyticity:

$$\frac{\mathrm{Re}\,T_{h_1h_2}(s)}{s}=\frac{\pi}{2}\lambda_{h_1h_2}B-Y_1^{h_1h_2}s^{-\eta_1}\cot\left(\frac{1-\eta_1}{2}\pi\right) \mp Y_2^{h_1h_2}s^{-\eta_2}\tan\left(\frac{1-\eta_2}{2}\pi\right). \tag{14}$$

The scale s_0 can be reabsorbed into A and will be set to 1 in the following.

This fit leads to a slightly better χ^2/DOF than the two previous ones and to stable parameters for $\sqrt{s}\geqslant 5$ GeV. We show the details of this fit, which we shall refer to as RRL1 (Reggeon-Reggeon-log s), in Figs. 8–10, and the best values of the parameters in Table III. As mentioned by the E811 Collaboration [18], a logarithmic fit favors their new measurement. However, for our purpose, we must point out that, despite the fact that the fit seems better, the Pomeron contribution becomes negative for $\sqrt{s}<12$ GeV. Hence, one encounters another low-energy problem, and we do not favor such a fit over the other ones for this reason.

VI. THE LOW-ENERGY REGION

Clearly, all the fits presented here become valid only above 10 GeV or so. Extending their region of validity

[3]It is possible to get slightly better fits below $\sqrt{s}=9$ GeV if one lets this parameter free, but it reaches unphysical values of the order of 100 MeV^2 or smaller, and the stability of the fit is not improved.

TABLE III. The values of the parameters of the hadronic amplitude in model RRL1 (13), (14) corresponding to a cutoff $\sqrt{s} \geqslant 5$ GeV.

A (mb)	B (mb)	s_0	η_1	η_2	χ^2/DOF
-30.8 ± 3.6	6.74 ± 0.22	1 (fixed)	0.2078 ± 0.0079	0.545 ± 0.0063	0.97
	pp	πp	Kp	$\gamma p \times 10^{-2}$	$\gamma\gamma \times 10^{-4}$
λ	1	0.6839 ± 0.0045	0.6439 ± 0.0073	0.3566 ± 0.0048	0.0845 ± 0.0061
Y_1 (mb)	106.3 ± 2.9	61.2 ± 2.4	49.7 ± 2.5	29.4 ± 1.3	8.1 ± 3.5
Y_2 (mb)	33.36 ± 0.96	5.78 ± 0.16	13.42 ± 0.38		

would be an important progress, as one would be able to compare them with the other processes, mainly measured at lower energies, and hence to test factorization better.

We have first tried [19] to modify the energy variable in the fits, and used $\tilde{s} = (s-u)/2$, which is the variable predicted by Regge theory [20]. Note that the use of $\tilde{s}$ instead of s makes no difference from $\sqrt{s} = 9$ GeV onwards. It only produces significantly better fits below 9 GeV, but those fits are still statistically unacceptable.

We have also tried to implement thresholds more rigorously, and found the same situation as in the $\tilde{s}$ case. We have also attempted to introduce lower trajectories, but the number of parameters involved then is too high to obtain any convincing answer. Hence, the question of the extension of the fits to lower energies remains open.

VII. THE OTHER TRAJECTORIES

A. Hard Pomeron

As we already mentioned, one of the possibilities is that the sharp rise observed at HERA is due to the presence of another singularity, so far undetected, which would correspond to a new kind of Pomeron, called the hard Pomeron. Assuming that this is a simple pole [3], one can get beautiful fits to DIS data. However, one would expect such an object to have some kind of manifestation in soft interactions. Our procedure enables us to place the following 2σ bounds on the hard Pomeron, assuming a hard intercept of 0.4:

$$\frac{X^{pp}_{hard}}{X^{pp}_{soft}} < 2 \times 10^{-6},$$

TABLE IV. Predictions of the pp and $\bar{p}p$ total cross sections and ρ parameter values for future and present machines.

σ^{ab}_{tot} [mb]		RHIC 200 [GeV]	RHIC 500 [GeV]	Tevatron 2000 [GeV]	LHC 14000 [GeV]
RRP	pp	52.45 ± 0.18	61.87 ± 0.38	80.56 ± 0.92	117.77 ± 2.33
	$\bar{p}p$	52.64 ± 0.18	61.94 ± 0.38	80.57 ± 0.92	117.77 ± 2.33
RRL2	pp	52.96 ± 0.18	62.81 ± 0.38	81.52 ± 0.83	114.86 ± 1.72
	$\bar{p}p$	53.15 ± 0.18	62.88 ± 0.38	81.54 ± 0.83	114.86 ± 1.72
RRL1	pp	52.41 ± 0.11	61.25 ± 0.23	76.76 ± 0.53	101.11 ± 1.13
	$\bar{p}p$	52.63 ± 0.12	61.32 ± 0.23	76.78 ± 0.53	101.12 ± 1.13
Re/Im		RHIC 200 [GeV]	RHIC 500 [GeV]	Tevatron 2000 [GeV]	LHC 14000 [GeV]
RRP	pp	0.132 ± 0.002	0.145 ± 0.003	0.153 ± 0.003	0.155 ± 0.004
	$\bar{p}p$	0.135 ± 0.002	0.146 ± 0.003	0.153 ± 0.003	0.155 ± 0.004
RRL2	pp	0.137 ± 0.002	0.147 ± 0.003	0.145 ± 0.002	0.131 ± 0.002
	$\bar{p}p$	0.140 ± 0.003	0.145 ± 0.003	0.145 ± 0.002	0.131 ± 0.002
RRL1	pp	0.126 ± 0.001	0.130 ± 0.002	0.121 ± 0.002	0.101 ± 0.002
	$\bar{p}p$	0.129 ± 0.001	0.131 ± 0.002	0.121 ± 0.002	0.101 ± 0.002

$$\frac{X^{\pi p}_{hard}}{X^{\pi p}_{soft}} \approx \frac{X^{Kp}_{hard}}{X^{Kp}_{soft}} < 3\times 10^{-2},$$

$$\frac{X^{\gamma p}_{hard}}{X^{\gamma p}_{soft}} < 10^{-4}. \qquad (15)$$

Hence, it seems that if the hard Pomeron is a simple pole, it must decouple at $Q^2 \leqslant 0$.

B. Odderon

The exchange of a $C=-1$ trajectory [21] with intercept close to 1 is needed within the Donnachie-Landshoff model to reproduce the large-t dip in elastic scattering. Such an object does not seem to be present at $t=0$. Again, we can place bounds similar to the above, but this time we allow the intercept to be as low as 1. We then obtain the 2σ bounds

$$\left|\frac{X^{pp}_{odd}}{X^{pp}_{soft}}\right| < 2\times 10^{-3}, \qquad (16)$$

$$\left|\frac{X^{\pi p}_{odd}}{X^{\pi p}_{soft}}\right| < 10^{-3},$$

$$\left|\frac{X^{Kp}_{odd}}{X^{Kp}_{soft}}\right| < 2\times 10^{-3}.$$

VIII. PREDICTIONS

Finally, for each fit, we present the 1σ limits that one gets on σ_{tot} and ρ at current or future hadronic machines (see Table IV). Clearly, one will have to wait until the results from the LHC to discriminate among the various possibilities presented here. It is not even totally clear whether the LHC will be able to measure total cross sections sufficiently well within its presently approved program.

We cannot unfortunately make any firm statement on γp and $\gamma\gamma$ cross sections, as both are linked. A reliable measurement of either of these would enable us (through factorization) to predict the other one. At present, the published data are not consistent or precise enough to reach a firm conclusion. Equation (9) shows that factorization would imply higher $\gamma\gamma$ and/or lower γp cross sections.

IX. CONCLUSION

We have shown that a simple-pole model for the soft Pomeron produces very good fits to $t=0$ data, once the energy is bigger than 9 GeV. From our updated compilation of data points, and from the 264 points above 9 GeV, we determined the Pomeron intercept to be 1.093 ± 0.003, in agreement with the conclusions of [6]. We have shown that the lower $C=\pm 1$ trajectories are nondegenerate, and have intercepts given in Table I. The determination of these parameters is stable and reliable, as is that of the Pomeron couplings. We have also explained that the interplay between $C=+1$ contributions makes the determination of the couplings of these trajectories problematic. Further stabilization of these is needed.

Finally, we have indicated that $t=0$ data are not sufficient to rule out other models of forward scattering amplitudes, but the factorization and quark counting properties, which are well respected, are difficult to understand outside the context of simple poles.

ACKNOWLEDGMENTS

We thank P. Gauron for his careful check and confirmation of our fit results for $\sqrt{s_{\rm mm}}=9$ GeV and for pointing out several misprints and mistakes in Table I. We also thank B. Nicolescu for his careful reading and for his kind proposals which made our references correct. K.K. was supported in part by DOE Grant DE-FG02-91ER40688-Task A. The work of the COMPAS Group is supported in part by RFBR Grants RFBR-96-07-89230 and RFBR-99-07-90356.

[1] WA102 Collaboration, D. Barberis *et al.*, Phys. Lett. B **432**, 436 (1998); WA91 Collaboration, S. Abatzis *et al.*, *ibid.* **324**, 509 (1994).
[2] A. Donnachie and P. V. Landshoff, Nucl. Phys. **B244**, 322 (1984); **B267**, 690 (1986); P. D. B. Collins, F. D. Gault, and A. Martin, *ibid.* **B85**, 141 (1975).
[3] A. Donnachie and P. V. Landshoff, Phys. Lett. B **437**, 408 (1998); J. R. Cudell, A. Donnachie, and P. V. Landshoff, *ibid.* **448**, 281 (1999).
[4] R. Dib, J. Khoury, and C. S. Lam, Phys. Rev. D **60**, 036001 (1999).
[5] J. R. Cudell and B. U. Nguyen, Nucl. Phys. **B420**, 669 (1994).
[6] J. R. Cudell, K. Kang, and S. K. Kim, Phys. Lett. B **395**, 311 (1997).
[7] Particle Data Group, C. Caso *et al.*, Eur. Phys. J. C **3**, 1 (1998), http://pdg.lbl.gov.
[8] Data used were extracted from the PPDS accessible at http://wwwppds.ihep.su:8001/ppds.html. Computer readable data files are also available at http://pdg.lbl.gov.
[9] A. Donnachie and P. V. Landshoff, Phys. Lett. B **296**, 227 (1992).
[10] K. Kang, Nucl. Phys. B (Proc. Suppl.) **12**, 64 (1990); K. Kang, P. Valin, and A. R. White, in Proceedings of the Vth Blois Workshop, Providence, Rhode Island, 1993.
[11] See, for instance, K. Kang, P. Valin, and A. R. White, Nuovo Cimento A **107**, 2103 (1994); K. Kang and B. Nicolescu, Phys. Rev. D **11**, 2461 (1975).
[12] M. M. Block and R. N. Cahn, Rev. Mod. Phys. **57**, 563 (1985).
[13] H1 Collaboration, C. Adloff *et al.*, Nucl. Phys. **B497**, 3 (1997); ZEUS Collaboration, J. Breitweg *et al.*, Eur. Phys. J. C **7**, 609 (1998).
[14] G. Shaw, Phys. Rev. D **47**, 3676 (1993); Phys. Lett. B **228**, 129 (1989); P. Ditsas and G. Shaw, Nucl. Phys. **113**, 246 (1976); J. J. Sakurai and D. Schildknecht, Phys. Lett. **40B**, 121

(1972).
[15] C. Bourrely, J. Soffer, and T. Wu, Phys. Lett. B **339**, 322 (1994).
[16] A. P. Contogouris and F. Lebessis, Mod. Phys. Lett. A **7**, 2415 (1992).
[17] M. Block, K. Kang, and A. R. White, Int J. Mod. Phys. **7**, 4449 (1992).
[18] E811 Collaboration, C. Avila *et al.*, Phys. Lett. B **445**, 419 (1998).
[19] K. Kang *et al.*, , presented at the 4th Workshop on Quantum Chromodynamics, Paris, France, 1998 and presented at the 4th Workshop on Small x and Diffractive Physics, Batavia, Illinois, 1998, in Batavia 1998, Small-x and diffractive physics, pp. 203–211, hep-ph/9812429.
[20] P. D. B. Collins, *An Introduction to Regge Theory and High Energy Physics* (Cambridge University Press, Cambridge, England, 1977).
[21] L. Lukaszuk and B. Nicolescu, Lett. Nuovo Cimento **8**, 405 (1973); G. Bialkowski, K. Kang, and B. Nicolescu, *ibid.* **13**, 401 (1975); Kang and Nicolescu [11].