$\omega \rightarrow \rho\pi$ transition and $\omega \rightarrow 3\pi$ decay

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We evaluate the $\omega \rightarrow \rho\pi$ transition and the $\omega \rightarrow 3\pi$ decay using a quark level linear sigma model (QLσM). We obtain $g_{\omega\rho\pi}^{\mathrm{QL}\sigma\mathrm{M}} = (10.33 - 14.75)$ GeV^{-1} to be compared with other model dependent estimates averaging to $g_{\omega\rho\pi} = 16$ GeV^{-1}. We show that in the QLσM a contact term is generated for the $\omega \rightarrow 3\pi$ decay. Although the contact contribution by itself is small, the interference effects turn out to be important.

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I. INTRODUCTION

The problem of understanding low energy hadron dynamics is being supported by recent experimental data obtained at the Novosibirsk-VEPP-2M detector and by the DAΦNE facility which will soon produce large amounts of experimental data in the energy region around 1 GeV. This energy region is particularly interesting since the nonperturbative QCD effects that govern hadron dynamics are far from being completely understood.

This paper is concerned with the $\omega \rightarrow \rho\pi$ vertex and $\omega \rightarrow \pi\pi\pi$ decay. Existing experimental data seem to confirm the Gell-Mann–Sharp–Wagner (GSW) suggestion [1] that the $\omega \rightarrow \pi\pi\pi$ decay is dominated by the $\omega\rho\pi$ transition followed by the $\rho \rightarrow \pi\pi$ decay, although a $\omega\pi\pi\pi$ contact contribution cannot be excluded. The theoretical description of these problems has been considered by a number of authors using different techniques such as approximate SU(3) symmetry [2], vector meson dominance (VMD) [3], QCD sum rules [4–6], and effective chiral Lagrangians [7].

It is our purpose in this paper to consider the quark level linear sigma model (QLσM) predictions for the $\omega \rightarrow \rho\pi$ transition and the $\omega \rightarrow \pi\pi\pi$ decay. The model describes the $U(2)\times U(2)$ chiral invariant interactions of effective quarks with pseudoscalar and scalar mesons. Vector mesons are incorporated in the model as gauge bosons, even though in our work they do not appear in loops. In this model, in addition to the GSW mechanism, the $\omega \rightarrow \pi\pi\pi$ decay proceeds through quark box diagrams with the ω and three pions in the box vertices, which can be interpreted as a contact term. By itself the contact term is not important: it leads to a $\Gamma(\omega \rightarrow \pi\pi\pi) \cong 0.1$ MeV. However, its interference with the amplitude arising from the GSW mechanism leads to a sizable 25% effect in the decay rate.

The paper is organized as follows. In Sec. II we introduce the model and discuss the determination of the corresponding coupling constants. In Sec. III we compute $g_{\omega\rho\pi}$, working in the soft momentum limit. In the QLσM this transition is generated by loops of quarks. In Sec. IV we present calculations for $\omega \rightarrow \pi\pi\pi$ within QLσM. This includes the GSW mechanism and the contribution arising from a box of quarks. Comparison with the observed $\omega \rightarrow \pi\pi\pi$ decay rate is then made. In Sec. V we draw our conclusions.

II. THE MODEL

The quark level LσM describes the $U(2)\times U(2)$ chiral invariant interaction of mesons and effective quarks. We have chosen to work with a pseudoscalar rather than a derivative coupling which has the advantage that no anomalous interactions are required to describe one pion processes. Vector mesons are incorporated in the model as gauge bosons, even though in our work they are not involved in loop calculations. The QLσM Lagrangian is

$$\mathcal{L}_{\mathrm{int}} = \bar{\psi}[i\not{D} - M + \sqrt{2}g(S + i\gamma_5 P)]\psi + (D_\mu B D^\mu B^\dagger)/2 + \cdots, \tag{1}$$

where $B \equiv S + iP$ with S,P scalar and pseudoscalar fields, respectively [$P = (1/\sqrt{2})(\eta_0 + \vec{\tau}\cdot\vec{\pi})$, scalar and vector fields being defined in a similar way] and ψ denotes the quark isospinor. Vector fields are introduced as gauge fields through the covariant derivative:

$$D_\mu \psi \equiv \left(\partial_\mu + i\frac{g_V}{\sqrt{2}}V_\mu\right)\psi,$$

$$D_\mu B \equiv \partial_\mu B + i\frac{g_V}{\sqrt{2}}[V_\mu, B]. \tag{2}$$

The chiral structure of the theory has been extensively discussed in the literature [8]. The quark mass matrix M in Eq.

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(1) is generated by spontaneous breaking of chiral symmetry, quark masses being related to the πqq coupling g through the Goldberger-Trieman (GT) relation $m_q = g f_\pi$. The ellipsis in Eq. (1) refers to vector meson kinetic and mass terms and to scalar-pseudoscalar Yukawa interactions which are not relevant for the purposes of this work.

Using the explicit representation for S, P, and V above, from Eqs. (1), (2) we obtain the following interaction terms relevant to the calculations presented in this paper:

$$\mathcal{L}_{\rm int} = -g_V(\vec{\rho}_\mu \times \vec{\pi}) \cdot \partial^\mu \vec{\pi} - \frac{g_V}{2}\bar{\psi}\gamma^\mu \vec{\tau}\psi \cdot \vec{\rho}_\mu - \frac{g_V}{2}\bar{\psi}\gamma^\mu \psi \omega_\mu . \tag{3}$$

The model relates the ρqq, ωqq, and the $\rho\pi\pi$ couplings. These relations arise from the gauge principle which, in this context, is a statement of universality. As shown below, this principle is supported at the constituent quark level by the existing data. The gauge coupling g_V can be estimated from experimental data on the $\rho \to \pi\pi$ decay. An alternative determination of this coupling is achieved by assuming vector meson dominance *at the constituent quark level*. To this end we proceed as follows.

Vector meson dominance of the electromagnetic form factor of the pion [9] at zero momentum transfered leads to the following relation between the $\rho - \gamma (f_{\rho\gamma})$, and the $\rho\pi\pi$ $(g_{\rho\pi\pi})$ coupling:

$$f_{\rho\gamma} = \frac{e m_\rho^2}{g_{\rho\pi\pi}}. \tag{4}$$

Assuming that the electromagnetic form factor of the constituent quark is dominated by vector mesons, the following relations are obtained:

$$f_{\rho\gamma} = \frac{e m_\rho^2}{g_{\rho qq}}, \tag{5}$$

$$f_{\omega\gamma} = \frac{e m_\omega^2}{3 g_{\omega qq}}, \tag{6}$$

which in turn lead to

$$g_{\rho\pi\pi} = g_{\rho qq} = g_{\omega qq} \frac{m_\omega^2}{m_\rho^2} \frac{3 f_{\omega\gamma}}{f_{\rho\gamma}}. \tag{7}$$

The $\frac{1}{3}$ factor in Eq. (6) arises because the isoscalar contribution to the quark electric charge is proportional to the quark baryonic number.

The $f_{\rho\gamma}$ and $f_{\omega\gamma}$ couplings can be extracted from $\omega \to e^+e^-$ and $\rho \to e^+e^-$ decays. Existing data [10] on these processes implies $f_{\rho\gamma} \approx 3 f_{\omega\gamma}$ which supports the universality of g_V in Eq. (3). Deviations from this universal behavior are reflected on the actual values extracted from Eqs. (5), (6) and the data on vector mesons leptonic widths, namely, $g_V^{\rho ll} \equiv g_{\rho qq} = 5.03$ and $g_V^{\omega ll} \equiv g_{\omega qq} = 5.68$, whereas the $\rho \to \pi\pi$ decay leads to $g_V^{\rho\pi\pi} = 6.01$ (superindices in g_V indicate the process from which its value is extracted).

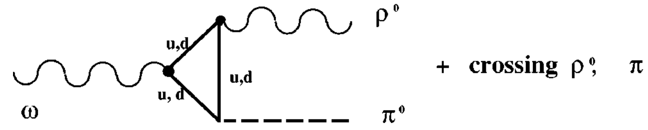

FIG. 1. Quark loop triangle contributions to the $\omega \to \rho^0 \pi^0$ transition.

Summarizing, for g_V in Eqs. (2), (3) we can use either $g_V^{\rho\pi\pi} = 6.01$, $g_V^{\rho ll} \equiv g_{\rho qq} = 5.03$ or $g_V^{\omega ll} \equiv g_{\omega qq} = 5.68$. In the rest of the paper we report numerical results for these values of g_V, even though, as we argue below, the most accurate determination comes from $\omega \to e^+e^-$ $(g_V^{\omega ll})$.

III. QLσM AND THE $\omega \to \rho\pi$ TRANSITION

It is conventional to define the $g_{\omega\rho\pi}$ coupling in terms of the amplitude:

$$M_{\omega\rho\pi} = g_{\omega\rho\pi} \epsilon_{\mu\nu\alpha\beta} P^\mu P'^\nu \epsilon^\alpha(\omega) \eta^\beta(\rho), \tag{8}$$

where $P(P')$ denote the $\omega(\rho)$ momentum and $\epsilon(\eta)$ are the respective polarization vectors. Although there is no phase space from which to measure an $\omega \to \rho\pi$ transition, this vertex can be extracted from many theoretical models. Very approximate SU(3) symmetry of the 1970s suggest [2] $g_{\omega\rho\pi} \approx 16$ GeV^{-1}, while QCD sum rules obtain [4] $g_{\omega\rho\pi} \approx (15-17)$ GeV^{-1}, and the analogue light cone sum rules method extracts [6] $g_{\omega\rho\pi} = 15$ GeV^{-1}. Recently QCD sum rules for the polarization operator in an external field concludes [5] $g_{\omega\rho\pi} \approx 16$ GeV^{-1}.

In the QLσM the $\omega\rho\pi$ vertex is described in terms of the quark loops of Fig. 1. A straightforward calculation yields

$$\mathcal{M}[\omega(Q,\eta) \to \rho(k,\varepsilon) + \pi(r)] = g_{\omega\rho\pi} \epsilon(k,r,\eta,\varepsilon),$$

where

$$g_{\omega\rho\pi} = -2 i N_c m_q g_V^2 g (I^a + I^b), \tag{9}$$

with

$$I^a \equiv \int \frac{d^4 l}{(2\pi)^4} \frac{1}{\nabla(l)\nabla(l-k)\nabla(l-k-r)}, \tag{10}$$

$$I^b = I^a(r \leftrightarrow k),$$

where $\nabla(p) \equiv p^2 - m_q^2$. Since we are interested in the $\omega\rho\pi$ vertex involved in $\omega \to \pi\pi\pi$ decay, we calculate the integrals in Eq. (10) in the soft pion momentum limit, i.e., $k,r \to 0$. This amounts to keeping the leading term in the expansion of the amplitude in terms of the external momenta k and r. In this approximation $I^a = I^b$, and we get for the $\omega\rho\pi$ coupling:

$$g_{\omega\rho\pi}^{\rm QL\sigma M} = \frac{g_V^2 N_c}{8\pi^2 f_\pi}. \tag{11}$$

Using the values of the coupling constants previously estimated we conclude, using $f_\pi = 93$ MeV and $N_c = 3$:

$$g_{\omega\rho\pi}^{\mathrm{QL}\sigma\mathrm{M}}=(10.33,13.19,14.75)\ \ \mathrm{GeV}^{-1}. \tag{12}$$

These values are obtained using in Eq. (11) $g_V=(g_V^{\rho ll},g_V^{\omega ll},g_V^{\rho\pi\pi})$, respectively. These QL$\sigma$M predictions for $g_{\omega\rho\pi}$ have consequences for some other processes [11].

As a byproduct of our analysis we report the predictions of the QLσM for the $\omega\to\pi^0\gamma$ and $\rho^0\to\pi^0\gamma$ decays. The corresponding calculations are very similar to the ones presented so far, the only difference being the appearance of the γqq coupling instead of the Vqq coupling. We obtain

$$\mathcal{M}[V(Q,\eta)\to\pi(r)\gamma(k,\varepsilon)]=g_{V\pi\gamma}^{\mathrm{QL}\sigma\mathrm{M}}\epsilon(k,r,\eta,\varepsilon), \tag{13}$$

where

$$g_{\rho\pi\gamma}^{\mathrm{QL}\sigma\mathrm{M}}=-i\frac{g_V e}{8\pi^2}\frac{g}{m_q}N_c(e_u+e_d),$$

$$g_{\omega\pi\gamma}^{\mathrm{QL}\sigma\mathrm{M}}=-i\frac{g_V e}{8\pi^2}\frac{g}{m_q}N_c(e_u-e_d). \tag{14}$$

Note that both $g_{\omega\rho\pi}$ and $g_{V\pi\gamma}$ as calculated in this work, agree with those derived from a chiral Lagrangian with vector mesons incorporated via the hidden scheme [12].

Using the GTR and $g_V=g_V^{\rho ll},g_V^{\omega ll},g_V^{\rho\pi\pi}$ we obtain, respectively,

$$|g_{\omega\pi\gamma}^{\mathrm{QL}\sigma\mathrm{M}}|=(0.622,0.703,0.743)\ \ \mathrm{GeV}^{-1},$$

$$|g_{\rho\pi\gamma}^{\mathrm{QL}\sigma\mathrm{M}}|=(0.207,0.234,0.247)\ \ \mathrm{GeV}^{-1}. \tag{15}$$

These numbers are to be compared with the experimental results $|g_{\omega\pi\gamma}^{exp}|=0.703\pm0.020\ \ \mathrm{GeV}^{-1}$, $|g_{\rho\pi\gamma}^{exp}|=0.29\pm0.037\ \ \mathrm{GeV}^{-1}$.

Notice that the QLσM predictions for the $\omega\to\pi\gamma$ transition are in good agreement with the experimental data when $g_V=g_V^{\omega ll}$—as extracted from the ω leptonic width—is used. On the other hand, predictions for $\rho\to\pi\gamma$ agree with experimental results within two standard deviations. These results and the fact that the reported decay rate [10] for $\omega\to\pi\gamma$ is more accurate than for $\rho\to\pi\gamma$ lead us to consider $g_V=5.68$ as the most confident value for g_V.

IV. QLσM AND THE $\omega\to\pi\pi\pi$ DECAY

In this section we work out the QLσM predictions for the $\omega\to\pi\pi\pi$ decay. Two mechanisms contribute to this process within the model. The first one is through an intermediate ρ in the s,t,u channels as depicted in Fig. 2, which involve the previously calculated $g_{\omega\rho\pi}$ and $g_{\rho\pi\pi}$. The second mechanism involves quark boxes, as shown in Fig. 3.

The ρ-mediated contribution leads to an $\omega\to\pi\pi\pi$ amplitude

$$\mathcal{M}^{\mathrm{GSW}}[\omega(Q,\eta)\to\pi^+(q)\pi^-(p)\pi^0(r)]=A^{\mathrm{GSW}}\varepsilon(\eta,p,q,r), \tag{16}$$

where

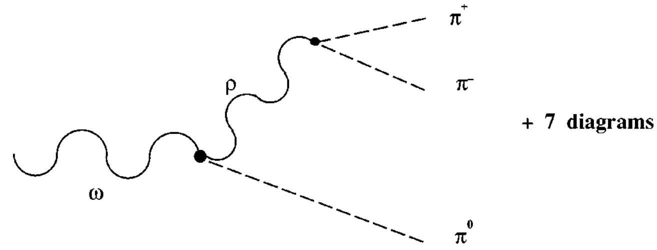

FIG. 2. Intermediate ρ contributions to the $\omega\to\pi^0\pi^+\pi^-$ decay.

$$A^{\mathrm{GSW}}=2g_{\omega\rho\pi}g_V\left(\frac{1}{s-m_\rho^2}+\frac{1}{t-m_\rho^2}+\frac{1}{u-m_\rho^2}\right). \tag{17}$$

The amplitude corresponding to the quark box contributions is

$$A^{\mathrm{box}}=-4g_Vg^3m_qIN_c, \tag{18}$$

where

$$I=\sum_{i=1}^{6}I^i$$

and

$$I^1=\int\frac{d^4l}{(2\pi)^4}[\nabla(l)\nabla(l-r)\nabla(l-r-p)\nabla(l-r-p-q)]^{-1}. \tag{19}$$

The five remaining integrals are obtained from I^1 by permutations of the pions momenta (p,q,r). Again, we consider the leading term for the expansion of this integral in terms of the pions momenta [higher order terms are expected to be suppressed by powers of $(m_\pi/m_q)^2$]. In this approximation we obtain

$$A^{\mathrm{box}}=-\frac{g_VN_c}{4\pi^2f_\pi^3}. \tag{20}$$

The decay rate $\Gamma(\omega\to\pi\pi\pi)$ is given by

$$\Gamma(\omega\to3\pi)=\frac{g_V^2g_{\omega\rho\pi}^2m_\omega^3}{768\pi^3}J, \tag{21}$$

where J stands for the phase space integral

$$J=\int_{4\beta}^{(1-\sqrt{\beta})^2}dx\int_{y_+}^{y_-}dy|f(x,y)|^2\Delta(x,y), \tag{22}$$

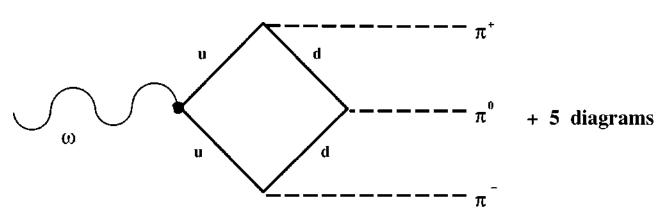

FIG. 3. Quark box contributions to $\omega\to\pi^+\pi^0\pi^-$.

with

$$y_{\pm}=\frac{1}{2}\Bigg[1+3\beta-x \mp\sqrt{\left(1-\frac{4\beta}{x}\right)[x-(1+\sqrt{\beta})^2][x-(1-\sqrt{\beta})]^2}\Bigg], \tag{23}$$

and

$$f(x,y)=f^{\mathrm{GSW}}(x,y)+f^{\mathrm{box}}(x,y),$$

where

$$f^{\mathrm{GSW}}(x,y)=\frac{1}{x-\alpha}+\frac{1}{y-\alpha}+\frac{1}{1+3\beta-x-y-\alpha},$$

$$f^{\mathrm{box}}(x,y)=-\frac{m_\omega^2 N_c}{8\pi^2 f_\pi^3 g_{\omega\rho\pi}}.$$

The Kibble determinant Δ in Eq. (22) is given by

$$\Delta(x,y)=xy(1+3\beta)-x^2y-xy^2-\beta(1-\beta)^2, \tag{24}$$

where

$$x=\frac{s}{m_\omega^2}, \qquad y=\frac{t}{m_\omega^2}, \qquad \beta=\frac{m_\pi^2}{m_\omega^2}, \qquad \alpha=\frac{m_\rho^2}{m_\omega^2}.$$

The phase space integral has been worked out by Thews [3] in the case of a constant matrix element. Since the numerical evaluation of the integral is straightforward, below we report the results according to an exact numerical calculation, which of course reproduces the results of Ref. [3] in the case of a constant matrix element.

The quark box contribution to the $\omega\to\pi\pi\pi$ decay rate is small:

$$\Gamma_{\mathrm{box}}(\omega\to 3\pi)\simeq 0.11 \text{ MeV}. \tag{25}$$

In Table I we summarize the numerical results for the $\omega\to 3\pi$ decay width for the three possible values of g_V. These results are to be compared with the experimental data:

$$\Gamma_{\mathrm{exp}}(\omega\to 3\pi)=7.5\pm 0.1 \text{ MeV}. \tag{26}$$

Notice that while the quark box contribution alone is small, the interference with the ρ mediated amplitude is important.

TABLE I. Numerical results for the $\omega\to 3\pi$ decay width for three possible values of g_V.

g_ρ	$g_{\omega\rho\pi}$ (GeV^{-1})	$\Gamma^{\mathrm{GSW}}(\omega\to 3\pi)$ (MeV)	$\Gamma^{\mathrm{tot}}(\omega\to 3\pi)$ (MeV)
5.03	10.33	2.72	3.79
5.68	13.19	5.66	7.39
6.01	14.75	7.92	10.06

In the QLσM the nonresonant contribution arising from quark box diagrams (Fig. 3) necessarily exists. From the numerical results reported in Table I we conclude that QLσM predictions are in agreement with experimental data within the 20% uncertainty associated with the determination of g_V. It is worth remarking that, as shown in Table I, the data on $\omega\to 3\pi$ seems to favor the value of g_V as extracted from the ω leptonic width, which also nicely reproduce the experimental data on $\omega\to\pi\gamma$ decay.

V. SUMMARY

We worked out in Sec. III the quark level linear sigma model predictions for the $\omega\rho\pi$ coupling constant. We obtained $g_{\omega\rho\pi}^{\mathrm{QL}\sigma\mathrm{M}}=(10.33,13.19,14.75)$ GeV^{-1}, these values corresponding to g_V as extracted from ρ leptonic width, ω leptonic width and $\rho\to\pi\pi$, respectively.

A nonresonant contribution to the $\omega\to 3\pi$ decay naturally arises within the QLσM. We evaluated the box contributions (Fig. 3) to the $\omega\to 3\pi$ decay in Sec. IV. We found an amplitude which leads to a small decay rate by itself, however, the interference between the box contributions and the ρ mediated amplitude leads to a sizable effect in the decay rate.

The amplitudes for the $\omega\to\pi\gamma(g_{\omega\pi\gamma})$ and $\rho\to\pi\gamma(g_{\rho\pi\gamma})$ were obtained as a by-product of the $g_{\omega\rho\pi}$ calculation. The agreement with experimental data is good for the $\omega\to\pi\gamma$ decay. In the case of the $\rho\to\pi\gamma$ decay, the QLσM predictions are within two standard deviations from the central value reported by the Particle Data Group (PDG) [10].

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