Using heavy quark spin symmetry in semileptonic B_c decays

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The form factors parametrizing the B_c semileptonic matrix elements can be related to a few invariant functions if the decoupling of the spin of the heavy quarks in B_c and in the mesons produced in the semileptonic decays is exploited. We compute the form factors as an overlap integral of the meson wave functions obtained using a QCD relativistic potential model, and give predictions for semileptonic and nonleptonic B_c decay modes. We also discuss possible experimental tests of the heavy quark spin symmetry in B_c decays.

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I. INTRODUCTION

The discovery of the B_c^+ meson by the Collider Detector at Fermilab (CDF) Collaboration at the Fermilab Tevatron [1] opens up some interesting investigations concerning the structure of strong and weak interactions in the quarkonium-like $\bar{b}c$ hadronic system. The studies will be further developed at the hadronic machines currently under construction, such as the Large Hadron Collider (LHC) accelerator at CERN, where a copious production of B_c meson and of its radial and orbital excitations is expected [2,3]; at these experimental facilities, together with the measurement of the mass of the particles belonging to the $\bar{b}c$ ($b\bar{c}$) family, it will be possible to observe the decay chains reaching the $^1S_0$ ground state B_c which decays weakly.

A peculiarity of the B_c decays, with respect to the decays of the $B_{u,d}$ and B_s mesons, is that both the quarks are involved in the weak decay process with analogous probability. The weak decays of the charm quark, whose mass is lighter than the b quark mass, are mainly governed by the Cabibbo-Kobayashi-Mashawa (CKM) matrix element V_{cs} which is larger than V_{cb} mainly controlling the b quark transitions; the result is that both the quark decay processes contribute on a comparable footing to the B_c decay width. Another peculiar aspect is that the $\bar{b}c$ annihilation amplitude, proportional to V_{cb}, is enhanced with respect to the analogous amplitude describing the B^+ annihilation mode.

The above considerations have inspired several theoretical analyses [4–8] aimed at predicting the B_c lifetime. Namely, a QCD analysis [7], based on the OPE expansion in the inverse mass of the heavy quarks and on the assumption of quark-hadron duality, provides for τ_{B_c} a prediction in agreement (at least within the current experimental accuracy) with the CDF measurement $\tau(B_c)=0.46^{+0.18}_{-0.16}$ (stat)± 0.03 (syst)10^{-12} s [1]. The agreement supports the overall picture of the inclusive B_c decays.

The calculation of the B_c exclusive decay modes can be carried out either using QCD-based methods, such as lattice QCD or QCD sum rules, or adopting some constituent quark model. So far, lattice QCD has only been employed to calculate the B_c purely leptonic width [9]. As for QCD sum rules [10], the B_c leptonic constant, as well as the matrix elements relevant for the semileptonic decays, were computed in Refs. [11,6,12]. These analyses identified a difficulty in correctly considering the Coulomb pole contribution in the three-point functions needed for the calculation of the semileptonic matrix elements. Attempts aimed at taking this correction into account are described in [13]; however, the problem of including the contribution of the Coulomb pole for all the values of the squared momentum transfer t to the lepton pair has not been solved yet. Extending to all values of t the expression of the Coulomb contribution valid at t_{max} only allows us to conclude that it represents a large correction to the lowest order quark spectral functions.

It is worth looking at the outcome of constituent quark models which, although less established on the QCD theoretical ground, can nevertheless provide us with significant information to be compared to the experimental results.

The models in Refs. [14,15] have been used in the past [4,16] to estimate the semileptonic B_c decay rates. More recently, different versions of the constituent quark model have been used to analyze the decays induced both by the $b\to c(u)$ and $c\to s(d)$ transitions [17,18]. It is noticeable that the calculations can be put on a firmer theoretical ground if some dynamical features of the B_c decays are taken into account. Such features are mainly related to the decoupling of the spin of the heavy quarks of the B_c meson, as well as of the meson produced in the semileptonic decays, i.e., mesons belonging to the $\bar{c}c$ family ($\eta_c, J/\psi$, etc.) and mesons containing a single heavy quark ($B_s^{(*)}$, $B_d^{(*)}$, $D^{(*)}$). The decoupling occurs in the heavy quark limit ($m_b, m_c \gg \Lambda_{QCD}$), and produces a symmetry, the heavy quark spin symmetry, allowing us to relate the form factors governing the B_c decays into a 0^- and 1^- final meson to a few invariant functions [19]. The main consequence is that the number of form factors parametrizing the matrix elements is reduced, and the description of the semileptonic transitions is greatly simplified.

However, at odds of the heavy quark flavor symmetry, holding for heavy-light mesons, spin symmetry does not fix the normalization of the form factors at any point of the phase space. The normalization, as well as the functional dependence near the zero-recoil point, must be computed by some nonperturbative approach.

So far, the ''universal'' form factors of semileptonic B_c decays have been estimated using nonrelativistic meson wave functions [19] and employing the ISGW model at the zero-recoil point [20]. An analysis in the framework of a different quark model is described in [17].

In this paper we present a calculation based on a constituent quark model which has been used to describe several aspects of the heavy meson phenomenology [21]. The peculiar features of the model are related to the interquark potential, which follows general QCD properties, such as scalar flavor-independent confinement at large distances, and asymptotically free QCD Coulombic behavior at short distances. Moreover, the use of the relativistic form of the quark kinematics allows us to describe heavy-light as well as heavy-heavy mesons, and to account for deviations from the nonrelativistic limit. As a result, the B_c form factors can be written as overlap integrals of meson wave functions, obtained by solving the wave equation defining the model. As discussed in the following, the representation as overlap integral of meson wave functions allows us to predict, in the heavy quark limit, the normalization of the invariant functions at the zero-recoil point and to obtain, for example, the suppression factor between the form factors of the B_c transitions into heavy-light mesons with respect to the corresponding functions governing the decays $B_c \to \eta_c l \nu$ and $B_c \to J/\psi l \nu$.

The calculation of the overlap integrals and of the B_c semileptonic form factors is presented in Sec. III, after having reviewed in Sec. II the consequences of the heavy quark spin symmetry in B_c decays. In Sec. IV, using the obtained invariant functions, we analyze the semileptonic decay modes, and in Sec. V, assuming the factorization ansatz, we estimate several nonleptonic B_c decay rates. Section VI is devoted to the conclusions.

II. HEAVY QUARK SPIN SYMMETRY

Heavy quark spin symmetry amounts to assume the decoupling between the spin of the heavy quarks in the B_c meson, since the $\bar{b}c$ spin-spin interaction vanishes in the infinite heavy quark mass limit, as well as the vanishing of the heavy quark-gluon vertex. This symmetry has been invoked in [19] to work out relations among the semileptonic matrix elements between B_c and other heavy mesons (both heavy-heavy and heavy-light). The main difference with respect to the most well known case of the heavy-light systems is that in the latter case one can exploit heavy quark flavor symmetry, which also holds in the heavy quark limit and allows us to relate B to D form factors.

In order to apply spin symmetry to B_c decays one should distinguish decays due to charm transitions from b quark transitions. To the first category belong processes such as $B_c \to (B_s, B_s^*) l \nu$ and $B_c \to (B_d, B_d^*) l \nu$, induced at the quark level by the transitions $c \to s$ and d, respectively. Since $m_c \ll m_b$, the energy released in such decays to the final hadronic system is much less than m_b, and therefore the b quark remains almost unaffected. As a consequence, the final B_a meson [a is a light $SU(3)_F$ index] keeps the same B_c four-velocity v, apart from a small residual momentum q. The initial and final meson momenta can then be written as $p_{B_c} = M_{B_c} v$ and $p_{B_a} = M_{B_a} v + q$, with $v \cdot q = O(1/m_Q)$. The relation between the residual momentum q and the momentum k transferred to the lepton pair is

$$k^\mu = p^\mu_{B_c} - p^\mu_{B_a} = (M_{B_c} - M_{B_a}) v^\mu - q^\mu. \tag{2.1}$$

In this kinematic situation, exploiting the decoupling of the spin of the heavy quarks in the mesons, several relations can be worked out among the semileptonic B_c form factors. A straightforward way to derive such relations is to use the trace formalism [22,23].[1] This has been done in Ref. [19], and we repeat here the derivation for the sake of completeness.

One introduces a 4×4 matrix $H^{c\bar{b}}$ describing the doublet (B_c, B_c^*) of $c\bar{b}$ mesons of four-velocity v [19]:

$$H^{c\bar{b}} = \frac{(1+\not{v})}{2} [B_c^{*\mu} \gamma_\mu - B_c \gamma_5] \frac{(1-\not{v})}{2}, \tag{2.2}$$

where $B_c^{*\mu}$ and B_c annihilate a vector B_c^* and a pseudoscalar B_c meson of four-velocity v. Under spin rotations of the heavy quarks, $H^{c\bar{b}}$ transforms as $H^{c\bar{b}} \to S_c H^{c\bar{b}} S_{\bar{b}}^\dagger$.

On the other hand, for heavy-light B_a and B_a^* mesons, the analogous 4×4 matrix describing the (B_a, B_a^*) spin multiplet reads

$$H_a = \frac{(1+\not{v})}{2} [B_a^{*\mu} \gamma_\mu - B_a \gamma_5]; \tag{2.3}$$

all the fields in Eqs. (2.2),(2.3) contain a factor $\sqrt{M_{B_{c,a}}}$ and have therefore dimension 3/2.

Applying the trace formalism, one gets that the hadronic matrix elements relative to the decays $B_c \to B_a^{(*)} l \nu$ have the following general form, compatible with heavy quark spin symmetry:

$$\langle B_a^{(*)}, v, q | \bar{q}_a \Gamma c | B_c, v \rangle = -\sqrt{M_{B_c} M_{B_a}} \, \mathrm{Tr}[\bar{H}_a \Omega \Gamma H^{c\bar{b}}], \tag{2.4}$$

where Ω is the most general Dirac matrix proportional to the four-velocity v and to the residual momentum q. The calculation using Eqs. (2.2),(2.3) shows that the various matrix elements reduce to

$$\begin{aligned}
\langle B_a, v, q | V_\mu | B_c, v \rangle &= \sqrt{2M_{B_c} 2M_{B_a}} [\Omega_1^a v_\mu + a_0 \Omega_2^a q_\mu], \\
\langle B_a^*, v, q | V_\mu | B_c, v \rangle &= -i \sqrt{2M_{B_c} 2M_{B_a^*}} a_0 \Omega_2^a \\
&\quad \times \epsilon_{\mu\nu\alpha\beta} \epsilon^{*\nu} q^\alpha v^\beta,
\end{aligned} \tag{2.5}$$

[1]For a discussion of the heavy quark formalism applied to the quarkonium system see Ref. [24] and references therein.

$$\langle B_a^*, v, q|A_\mu|B_c, v\rangle = \sqrt{2M_{B_c}2M_{B_a^*}}[\Omega_1^a \epsilon_\mu^* + a_0\Omega_2^a \epsilon^*\cdot q v_\mu],$$

where V_μ and A_μ represent the weak flavor-changing ($c \to s,d$) vector and axial current, respectively, and ϵ is the B_a^* polarization vector. Therefore, as shown by Eq. (2.5), the six form factors parametrizing the B_c into B_a and B_a^* matrix elements can be expressed in terms of two invariant functions, Ω_1^a and Ω_2^a. The main difference with respect to the spin-flavor symmetry, holding in heavy-light mesons, is that the normalization of the form factors is not predicted at any point of the kinematic range and, in particular, it is not fixed at the nonrecoil point $q=0$.

Actually, the form factors Ω_2^a give rise to terms proportional to the lepton mass in the calculation of the semileptonic rates. Moreover, Ω_2^a do not contribute at zero recoil. The scale parameter a_0 is related to the size of the B_c meson, it can be assumed as proportional to the B_c Bohr radius and represents the typical range of variation of the form factors [19].

The relations (2.5) are valid near the zero-recoil point, where both B_c and the meson produced in the decay are nearly at rest. In the case of the transitions $B_c \to B_s^{(*)}, B_d^{(*)}$ the physical phase space is quite narrow (the maximum momentum transfer t to the lepton pair is $t_{max} \simeq 1$ GeV^2) and therefore one can assume that Eqs. (2.5) completely determine the semileptonic matrix elements (modulo a set of corrections mentioned below). The situation is different for processes induced, at the quark level, by the b-quark transitions. Let us consider the decays $B_c \to (D,D^*) l\nu$, induced by the $b \to u$ transition. In this case, the energy released to the final meson is small only near the zero-recoil point, where $q^2 \ll m_c^2$. At such kinematic point one can repeat the considerations for the transition $B_c \to B_s l \nu$, obtaining the relations

$$\langle D, v, q|V_\mu|B_c, v\rangle = \sqrt{2M_{B_c}2M_D}[\Sigma_1 v_\mu + a_0\Sigma_2 q_\mu],$$

$$\langle D^*, v, q|V_\mu|B_c, v\rangle = -i\sqrt{2M_{B_c}2M_{D^*}}a_0\Sigma_2 \times \epsilon_{\mu\nu\alpha\beta}\epsilon^{*\nu} q^\alpha v^\beta, \tag{2.6}$$

$$\langle D^*, v, q|A_\mu|B_c, v\rangle = \sqrt{2M_{B_c}2M_{D^*}}[\Sigma_1 \epsilon_\mu^* + a_0\Sigma_2 \epsilon^*\cdot q v_\mu].$$

Far from the nonrecoil point, the light recoiling quark keeps a large momentum, and therefore terms of the order of q/m_c cannot be neglected in the effective theory leading to Eq. (2.6).

Finally, we consider B_c decays into quarkonium states, such as η_c and J/ψ. The spin decoupling of both the beauty and charm quark allows us now to relate the six form factors to a single one:

$$\langle \eta_c, v, q|V_\mu|B_c, v\rangle = \sqrt{2M_{B_c}2M_{\eta_c}}\Delta v_\mu,$$

$$\langle J/\psi, v, q|A_\mu|B_c, v\rangle = \sqrt{2M_{B_c}2M_{J/\psi}}\Delta \epsilon_\mu^*. \tag{2.7}$$

Also in this case Eqs. (2.7) are only valid near the zero-recoil point. Nevertheless, in the following we use them, as well as Eqs. (2.6), for all physical values of the momentum transfer t, in order to compute semileptonic and nonleptonic B_c decay rates. This is admittedly a strong assumption, and the related uncertainty must be added to the uncertainties coming from finite mass and QCD corrections that in principle relate the invariant functions to the physical semileptonic matrix elements [19]. However, assuming Eqs. (2.7) and (2.6) in the whole kinematic range, a number of predictions can be collected; the experimental results will then provide us with indications on the numerical importance of the corrections.

III. B_c FORM FACTORS FROM A CONSTITUENT QUARK MODEL

In this section we compute the form factors Δ, Ω_1^a, and Σ_1 by using a relativistic potential model which allows to account for two QCD effects. The first one is confinement, which produces a suppression, at large distances, of the meson wave functions, due to the linearly increasing interquark potential. The second effect is represented by the deviation of the quark dynamics from the nonrelativistic limit. By taking such two effects into account, we are able to compute the form factor Δ in Eq. (2.7) as an overlap integral of B_c and J/ψ wave functions. Moreover, we can apply the formalism to the transitions $B_c \to B_s^{(*)}, B_d^{(*)}$ and $D_d^{(*)}$ at the nonrecoil point, and then extrapolate the result to the whole kinematic region spanned by the various semileptonic transitions.

Let us consider Δ in Eq. (2.7). In order to compute it, we consider the costituent quark model studied in [21], whose essential features can be easily summarized. First, we write down an expression for the B_c^+ meson state, in the B_c^+ rest frame, in terms of quark and antiquark creation operators, and of a meson wave function:

$$|B_c^+\rangle = i\frac{\delta_{\alpha\beta}}{\sqrt{3}}\frac{\delta_{rs}}{\sqrt{2}}\int d\vec{k}\ \ \psi_{B_c}(\vec{k}) b^\dagger(-\vec{k},r,\alpha) c^\dagger(\vec{k},s,\beta)|0\rangle, \tag{3.1}$$

where α and β are color indices, r and s spin indices. The operator $b^\dagger$ creates an anti-b quark with momentum $-\vec{k}$, while $c^\dagger$ creates a charm quark with momentum $\vec{k}$. A similar expression holds for the η_c ($\overline{c}c$) state, as well as for vector 1^- states, as described in [21]. In the meson state, as written in Eq. (3.1), the contribution of other Fock states, such as, e.g., states containing one or more gluons, is neglected.

The wave function $\psi_{B_c}(\vec{k})$ describes the momentum distribution of the quarks in the meson. It is obtained by solving the wave equation

$$\{\sqrt{\vec{k}^2+m_b^2} + \sqrt{\vec{k}^2+m_c^2} - M_{B_c}\}\psi_{B_c}(\vec{k}) + \int d\vec{k}'\, V(\vec{k},\vec{k}')\psi_{B_c}(\vec{k}') = 0 \tag{3.2}$$

stemming from the quark-antiquark Bethe-Salpeter equation, in the approximation of an instantaneous interaction represented by the potential V. Equation (3.2) partially takes into account the relativistic behavior of the quarks in the kinetic term; m_c and m_b represent the mass of the constituent charm and beauty quark, and M_{B_c} the mass of the bound state.

The QCD interaction is described assuming a static interquark potential having the form, in the coordinate space [25],

$$V(r)=\frac{8\pi}{33-2n_f}\Lambda\left[\Lambda r-\frac{f(\Lambda r)}{\Lambda r}\right], \tag{3.3}$$

with Λ a scale parameter, n_f the number of active flavors, and the function $f(t)$ given by

$$f(t)=\frac{4}{\pi}\int_0^\infty dq\,\frac{\sin(qt)}{q}\left[\frac{1}{\ln(1+q^2)}-\frac{1}{q^2}\right]. \tag{3.4}$$

The interest for this form of the potential is that it continuously interpolates the linearly confining behavior at large distances with the QCD Coulombic behavior at short distances, where the logarithmic reduction of the strong coupling constant, due to the asymptotic freedom property of QCD, is implemented. A further smoothing of the potential at short distances is adopted, according to quark-hadron duality arguments [21].

The wave equation (3.2), together with the form (3.3) of the potential and Eq. (3.1) of the meson state, completely determines the model, which has been extensively studied to describe static as well as dynamic properties of mesons containing heavy quarks [26–28]. Notice that the spin interaction effects are neglected since, in the case of heavy mesons, the chromomagnetic coupling is of the order of the inverse heavy quark masses. Therefore, both the pseudoscalar and the vector mesons, being degenerate in mass, are described by the same wave function.

An equation for the form factor $\Delta(\vec{q}=0)$ in Eq. (2.7) can be obtained expressing the $b\to c$ flavor-changing weak currents in terms of quark and antiquark operators; for the vector current, the expression is

$$\begin{aligned}V^\mu=&\frac{\delta_{\alpha\beta}}{(2\pi)^3}\int d\vec{q}\,d\vec{q}\,'\left[\frac{m_b m_c}{E_b(\vec{q})E_c(\vec{q}\,')}\right]^{1/2}:[\overline{u}_b(\vec{q},r)b_b^\dagger(\vec{q},r,\alpha)\\&+\overline{v}_b(\vec{q},r)d_b(\vec{q},r,\alpha)]\gamma^\mu[u_c(\vec{q}\,',s)b_c(\vec{q}\,',s,\beta)\\&+\overline{v}_c(\vec{q}\,',s)d_c^\dagger(\vec{q}\,',s,\beta)]:\end{aligned} \tag{3.5}$$

$[E_q(\vec{k})=\sqrt{k^2+m_q^2},\ k=|\vec{k}|]$; an analogous expression describes the axial current. Then, writing down the matrix elements (2.7) and applying canonical anticommutation relations [21,26], we obtain

$$\Delta(\vec{q}=0)=\frac{1}{2\sqrt{2M_{B_c}2M_{\eta_c}}}\int_0^\infty dk\,\frac{u_{B_c}(k)u_{\eta_c}(k)}{\sqrt{E_bE_c}}\times\frac{(E_b+m_b)(E_c+m_c)-k^2}{[(E_b+m_b)(E_c+m_c)]^{1/2}}, \tag{3.6}$$

where the reduced wave functions $u_M(k)$ are related to the $L=0$ wave functions ψ_M according to

$$u_M(k)=\frac{k\psi_M(|\vec{k}|)}{\sqrt{2}\pi}. \tag{3.7}$$

The covariant normalization is adopted: $\int_0^\infty dk|u_M(k)|^2=2M_M$.

The wave functions u_{B_c} and u_{η_c} can be obtained by solving Eq. (3.2) by numerical methods, choosing the values of the masses m_c and m_b of the constituent quarks, together with the scale parameter Λ, in such a way that the charmonium and bottomonium spectra are reproduced: $m_b=4.89$ GeV and $m_c=1.452$ GeV, with $\Lambda=397$ MeV [21]. A fit of the heavy-light meson masses also fixes the values of the constituent light-quark masses: $m_u=m_d=38$ MeV and $m_s=115$ MeV [21]. It is worth observing that, for the $\overline{b}c$ system, all the input parameters needed in Eq. (3.2) are fixed from the analysis of other channels, and the predictions do not depend on new external quantities.

The numerical solution of Eq. (3.2) produces the spectrum of the $\overline{b}c$ bound states; the predicted mass and the leptonic constant of the first S-wave resonance are [28] $M_{B_c}=6.28$ GeV (the value we use in our analysis) and $f_{B_c}=432$ MeV, in agreement with other theoretical determinations based on constituent quark models [29], QCD sum rules ($M_{B_c}=6.35$ GeV [6]) and lattice QCD ($M_{B_c}=6.388\pm9\pm98\pm15$ GeV [30]). Within the errors, the B_c mass agrees with the CDF result: $M_{B_c}=6.40\pm0.39$ (stat)±0.13 (syst) GeV [1].

The obtained B_c wave function $u_{B_c}(k)$ is depicted in Fig. 1. In the same figure we plot the wave functions of the other mesons involved in B_c semileptonic decays: B_s and B_d, the $\overline{c}c$ states η_c and J/ψ together with the first radial excitation η_c' and $\psi(2S)$, and the D meson.

Let us come back to Eq. (3.6) which provides the form factor Δ. For quark masses larger than the typical relative quark-antiquark momentum k, Eq. (3.6) becomes

$$\begin{aligned}\Delta(\vec{q}=0)&=\frac{1}{(2\pi)^3}\frac{1}{\sqrt{2M_{B_c}2M_{\eta_c}}}\int d\vec{k}\,\psi_{B_c}(\vec{k})\psi^*_{\eta_c}(\vec{k})\\&=\frac{1}{\sqrt{2M_{B_c}2M_{\eta_c}}}\int d\vec{x}\,\Psi_{B_c}(\vec{x})\Psi^*_{\eta_c}(\vec{x}),\end{aligned} \tag{3.8}$$

where $\Psi_M(\vec{x})$ is defined as

$$\Psi_M(\vec{x})=\frac{1}{(2\pi)^3}\int d\vec{k}\,e^{i\vec{k}\cdot\vec{x}}\psi_M(\vec{k}). \tag{3.9}$$

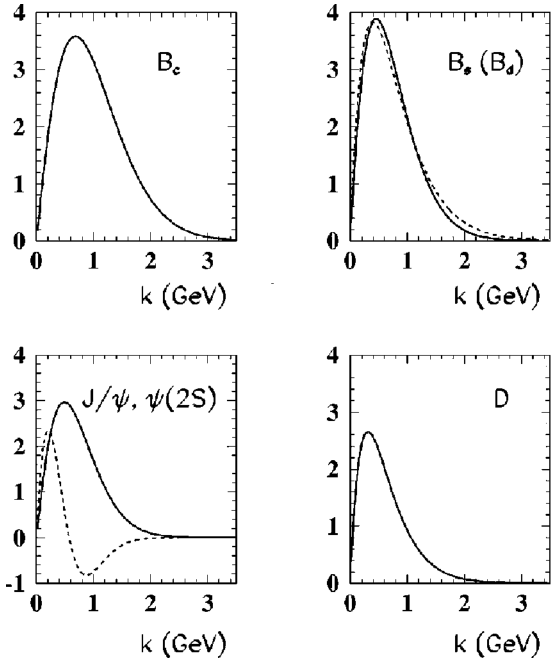

FIG. 1. Reduced $L=0$ wave functions $u_M(k)$ of heavy-heavy (B_c, J/Ψ, $\psi(2S)$) and heavy-light (B_s, B_d, D) mesons. The wave functions are obtained by solving the wave equation (3.2); they describe both the pseudoscalar 0^- and vector 1^- mesons.

Equation (3.8) shows that the form factor Δ, at the zero-recoil point, is simply given by the overlap integral of the B_c and η_c wave functions in the coordinate space. This result has already been obtained in [19], as it is typical of the calculation of form factors by quark models [26,31]. The interest in Eq. (3.8) is that no factors appear in the integral other than the wave functions; this implies that, in the limit where the B_c and η_c wave functions are equal (modulo the normalization condition), the form factor Δ is 1. Although such an overlap is not constrained by symmetry arguments, as in the case of the flavor symmetry in heavy-light mesons, from Eq. (3.8) it turns out that the deviation from unity of the invariant function at the zero-recoil point is due to the actual shapes of the meson wave functions. In our specific case, as reported in Table I, the deviation from unity is a 5% effect.

The calculation of Δ near the zero-recoil point, for a small momentum $\vec{q}$, can be performed by modifying Eq. (3.8), as discussed in [19]:

TABLE I. Parameters of the form factors [$\psi'=\psi(2S)$]. The functional dependence is in Eq. (3.15).

Channel	Form factor	$F(1)$	ρ^2	c
$B_c\to B_s(B_s^*)$	Ω_1^s	0.66	8	0
$B_c\to B_d(B_d^*)$	Ω_1^d	0.66	8	0
$B_c\to \eta_c(J/\psi)$	Δ	0.94	2.9	3
$B_c\to \eta_c'(\psi')$	Δ'	0.23	0	0
$B_c\to D(D^*)$	Σ_1	0.59	1.3	0.4

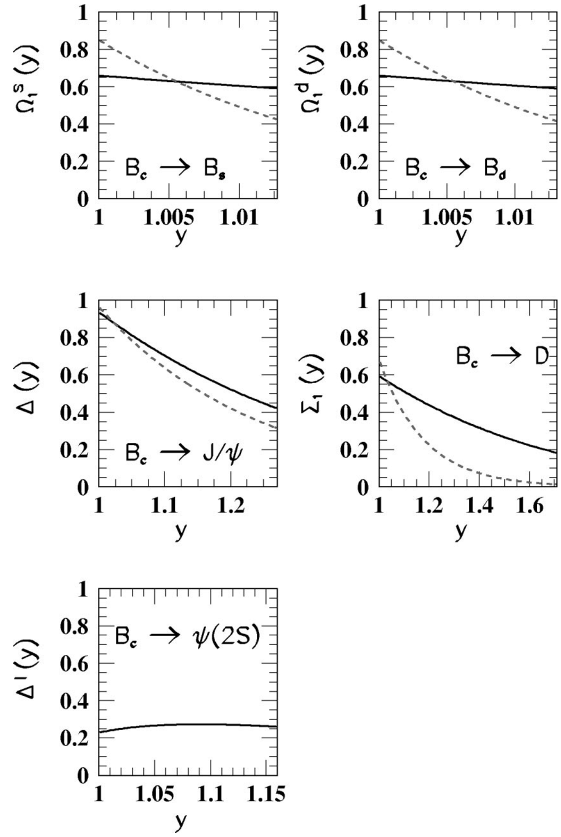

FIG. 2. Form factors of B_c semileptonic decays. The variable y is related to the squared momentum t, transferred to the lepton pair, by the relation $y=(M_{B_c}^2+M_M^2-t)/2M_{B_c}M_M$. The solid lines correspond to the form factors obtained by the model discussed in the paper; the dashed lines refer to the model in Ref. [15].

$$\Delta(\vec{q})=\frac{1}{\sqrt{2M_{B_c}2M_{\eta_c}}}\int d\vec{x}\, e^{i\vec{q}\cdot\vec{x}/2}\Psi_{B_c}(\vec{x})\Psi^*_{\eta_c}(\vec{x}), \tag{3.10}$$

and using the relation (valid near the zero-recoil point) $y=p_{B_c}p_{\eta_c}/M_{B_c}M_{\eta_c}=\sqrt{1+\vec{q}^2/M_{\eta_c}^2}$. We choose to perform an extrapolation of the result in the whole kinematic region, obtaining the form factor depicted in Fig. 2. The extrapolation provides a form factor having a nearly linear (with a small curvature term) y dependence in the kinematic range of the decays $B_c\to\eta_c l\nu$ and $B_c\to J/\psi l\nu$.

The same method and the same formulas can be used to calculate the form factor Δ' of $B_c\to\eta_c'$ and $B_c\to\psi(2S)$; the only new ingredient is the wave function of the $\psi(2S)$ radial excitation. Due to the oscillating behavior of $u_{\psi(2S)}$, the function Δ' is suppressed with respect to Δ; interestingly enough, it has a negligible y dependence, as one can observe in Fig. 2.

Before discussing the phenomenology of the decays $B_c\to\eta_c(J/\psi)l\nu$ and $B_c\to\eta_c'(\psi(2S))l\nu$, let us consider the matrix elements relevant for the transitions $B_c\to B_s(B_s^*)$. A feature of the model we are considering is that both heavy-heavy and heavy-light mesons are described by the same formalism. Therefore, Eq. (3.6) can be applied to calculate $\Omega_1^s(\vec{q}=0)$, substituting m_b with m_s and the wave function u_{η_c} with u_{B_s}. In the limit $m_s \to 0$ and for a large value of the b-quark mass, Eq. (3.6) becomes

$$\Omega_1^s(\vec{q}=0)=\frac{1}{\sqrt{2}}\frac{1}{\sqrt{2M_{B_c}2M_{B_s}}}\int d\vec{x}\,\Psi_{B_c}(\vec{x})\Psi^*_{B_s}(\vec{x}), \tag{3.11}$$

which differs by a factor $1/\sqrt{2}$ with respect to the analogous relation for Δ. This factor is a consequence of considering a heavy-light meson in the final state instead of a heavy-heavy meson, and produces a suppression of the corresponding form factor. Equation (3.11) suggests that, for similar (modulo the normalization condition) B_c and B_s wave functions, the form factor $\Omega_1^s(\vec{q}=0)$ is close to the value $\Omega_1^s(\vec{q}=0)=1/\sqrt{2}$. The actual value, reported in Table I, differs from this value by a 7% effect.

The two results $\Delta(\vec{q}=0)\simeq 1$ and $\Omega_1^s(\vec{q}=0)\simeq 1/\sqrt{2}$ are the main predictions of our analysis. They would deserve independent checks by different theoretical methods, namely by QCD sum rules in the heavy quark limit.

From Eq. (3.11) it is also possible to derive a relation, proposed in [19], between the form factor Ω_1^s and the leptonic constant of the B_s meson. As a matter of fact, in the framework of the constituent quark model, the B_s leptonic constant, defined by the matrix element $\langle 0|A_\mu|B_s(p)\rangle = i f_{B_s} p_\mu$, is given by [21]

$$f_{B_s}=\frac{\sqrt{3}}{2\pi M_{B_s}}\int_0^\infty dk\, k u_{B_s}(k)\left[\frac{(E_b+m_b)(E_s+m_s)}{E_b E_s}\right]^{1/2}\times\left[1-\frac{k^2}{(E_b+m_b)(E_s+m_s)}\right]. \tag{3.12}$$

For vanishing m_s and large m_b, f_{B_s} is simply related to the B_s wave function at the origin:

$$f_{B_s}=\frac{\sqrt{3}}{M_{B_s}}\Psi_{B_s}(0), \tag{3.13}$$

a relation analogous to the van Royen–Weisskopf formula for the quarkonium state. Expanding $\Psi_{B_s}(x)$ near the origin in Eq. (3.11), we obtain

$$\Omega_1^s(\vec{q}=0)\simeq\frac{1}{2\sqrt{3}}f_{B_s}\sqrt{M_{B_s}}\frac{1}{\sqrt{2M_{B_c}}}\int d\vec{x}\;\;\Psi_{B_c}(\vec{x}) + \text{corrections}. \tag{3.14}$$

The numerical comparison of Eq. (3.14) with Eq. (3.11), however, suggests that the next-to-leading corrections in Eq. (3.14) are sizable, and therefore the expansion (truncated at the first term) leading to Eq. (3.14) appears to be of limited usefulness.

The value of Ω_1^s at zero recoil is reported in Table I, and the plot of the form factor, extrapolated in the whole kinematic region, is depicted in Fig. 2; the form factor presents a soft y dependence in the narrow kinematic range spanned by the semileptonic $B_c \to B_s, B_s^*$ transitions.

The same procedure can be applied to compute Ω_1^d and Σ_1, and the results are also depicted in Fig. 2. The only new information is that, keeping finite values of the light quark masses, a $SU(3)_F$ breaking effect between Ω_1^d and Ω_1^s of less than 3% is predicted.

All the invariant functions can be represented by the three-parameter formula

$$F(y)=F(0)(1-\rho^2(y-1)+c(y-1)^2) \tag{3.15}$$

in terms of the value at zero recoil, the slope ρ^2 and the curvature c; the corresponding values are collected in Table I.

A remark concerns the invariant functions $\Omega_2^{s,d}$ and Σ_2. As mentioned in Sec. II, such form factors do not contribute at the zero-recoil point, since they appear in the term proportional to the small momentum q. In our approach, based on considering overlap integrals of wave functions of mesons at rest, we cannot provide an independent calculation of $\Omega_2^{s,d}$ and Σ_2, which therefore will be neglected in our analysis. Such an approximation, however, could have relevant consequences only in the case of the transitions $B_c \to D^{(*)} l\nu$; as already underlined, for the decays $B_c \to B_s^{(*)}$ and $B_c \to B^{(*)}$ the contribution from Ω_2 is always proportional to the momentum q, which remains small in these processes.

Let us conclude the section comparing our form factors Δ, Ω_1^a and Σ_1 with the outcome of the ISGW model [15], which has been widely applied to describe the heavy meson decays. In the ISGW approach, the form factors exponentially depend on the squared momentum transfer to the lepton pair, and at zero-recoil they are given by products of parameters relative to the mesons involved in the decays. We depict in Fig. 2 the various invariant functions obtained in this approach, observing some agreement with our results in the case of Δ; as for Ω_1^s, the result based on [15] deviates considerably from the value $1/\sqrt{2}$ suggested by our model.

IV. B_c SEMILEPTONIC DECAYS

The form factors Ω_1^s and Ω_1^d, Δ, Δ' and Σ_1 can be used to predict the semileptonic B_c decay rates, as well as various decay distributions. Before doing the calculation let us stress again that an extrapolation is performed for the relevant matrix elements far from the symmetry point (zero-recoil) where the form factors are originally computed. Such a procedure would require the calculation of the corrections, which could be sizable far from the symmetry point, an analysis beyond the aim of the present work. Considering the small range of momentum transfer t involved in $c \to (s,d)$ transitions, it is plausible that the extrapolation is quite under control for the decays $B_c \to B_s^{(*)} l\overline{\nu}$, $B_d^{(*)} l\overline{\nu}$. As for $B_c \to \eta_c$, $J/\psi l\overline{\nu}$, the extrapolation is done on a wider range of momentum transfer to the lepton pair. However, also in this

TABLE II. Semileptonic B_c^+ decay widths and branching fractions.

Channel	$\Gamma(10^{-15}$ GeV)	$\Gamma_L(10^{-15}$ GeV)	$\Gamma_T(10^{-15}$ GeV)	BR
$B_c^+ \to B_s e^+ \nu$	11.1(12.9)	-	-	$0.8(0.9)\times 10^{-2}$
$B_c^+ \to B_s^* e^+ \nu$	33.5(37.0)	19.1(21.4)	7.2(7.8)	$2.3(2.5)\times 10^{-2}$
$B_c^+ \to B_d e^+ \nu$	0.9(1.0)	-	-	$0.06(0.07)\times 10^{-2}$
$B_c^+ \to B_d^* e^+ \nu$	2.8(3.2)	1.6(1.8)	0.6(0.8)	$0.19(0.22)\times 10^{-2}$
$B_c^+ \to \eta_c e^+ \nu$	2.1(6.9)	-	-	$0.15(0.5)\times 10^{-2}$
$B_c^+ \to J/\psi e^+ \nu$	21.6(48.3)	13.2(33.2)	4.2(7.6)	$1.5(3.3)\times 10^{-2}$
$B_c^+ \to \eta_c' e^+ \nu$	0.3(0.3)	-	-	$0.02(0.02)\times 10^{-2}$
$B_c^+ \to \psi' e^+ \nu$	1.7(1.7)	1.1(1.1)	0.3(0.3)	$0.12(0.12)\times 10^{-2}$
$B_c^+ \to D^0 e^+ \nu$	0.005(0.03)	-	-	$0.0003(0.002)\times 10^{-2}$
$B_c^+ \to D^{*0} e^+ \nu$	0.12(0.5)	0.08(0.35)	0.02(0.05)	$0.008(0.03)\times 10^{-2}$

case it is interesting to make predictions and to compare them with the experimental results. Notice that we only consider massless charged leptons in the final state.

Concerning the parameters needed in the analysis, we use the experimental values of the masses of η_c, J/ψ, $\psi(2S)$, $D^{(*)}$, $B^{(*)}$, and B_s mesons; for the η_c' we use $M_{\eta_c'}=3.66$ GeV, and for $M_{B_s^*}$ we set $M_{B_s^*}=M_{B_s}+(M_{B_d^*}-M_{B_d})$. For the CKM matrix elements we use $V_{cb}=0.039$ and $V_{ub}=0.0032$; the values of V_{cs} and V_{cd} are fixed to $V_{cs}=0.975$ and $V_{cd}=0.22$. The results for the decay widths are reported in Table II where we also report the corresponding branching fractions, obtained assuming for τ_{B_c} the CDF central value: $\tau_{B_c}=0.46$ ps.

In order to understand the effect of the t dependence of the form factors, we also report in Table II the results obtained assuming t independent invariant functions, with the values fixed at the zero-recoil point. The results provide us with an upper bound for the various decay widths. As expected, the momentum transfer dependence is mild in the case of the $B_c \to B_s^{(*)}, B_d^{(*)}$ decays, where it only provides an effect of less than 10% in the decay rates. This is mainly due to the narrow t range spanned in such decay modes. In the case of $B_c \to \eta_c$ and J/ψ, there is a sizable effect due to the t dependence of the form factors. On the contrary, in the case of decays into radial excited states, η_c' and $\psi(2S)$, the t dependence is negligible. The t dependence is important for the Cabibbo suppressed B_c decays into D and D^*.

From Table II we conclude that the semileptonic modes are dominated by two channels, $B_c \to B_s l \nu$ and $B_c \to B_s^* l \nu$, in spite of the small phase space available for both the transitions; the two modes nearly represent the 60% of the semileptonic width, a result in agreement with the predictions available in the literature.

As for the $b \to c$ induced semileptonic B_c transitions, a peculiar role is played by the B_c decay into J/ψ, due to the clear signature represented by three charged leptons from the same decay vertex, two of them coming from J/ψ. This signature has been exploited to identify the B_c meson at Tevatron [1], and will be mainly employed at the future colliders [34]. Our prediction for the width of the decay $B_c \to J/\psi l \nu$ is $\Gamma(B_c \to J/\psi l \nu) \simeq 21.6 \times 10^{-15}$ GeV, with an upper bound of 48×10^{-15} GeV obtained using a t-independent form factor Δ. The agreement of this result with other calculations in the literature suggests that the finite mass corrections, responsible of subleading form factors in the matrix elements, should not be large. Tests on the size of such corrections can be performed by measuring the B_c decay rates into longitudinally and transversely polarized J/ψ: $\Gamma_{L,T}=\Gamma(B_c \to J/\psi_{L,T} l \nu)$, together with the corresponding decay distributions. Using the parametrization in Eq. (2.7) the decay widths are given by

$$\Gamma_L = \frac{G_F^2 V_{cb}^2 M_{J/\psi}^5}{12\pi^3} \int_1^{1+\delta} dy \;\; [\Delta(y)]^2 \sqrt{y^2-1} [ry-1]^2,$$

$$\Gamma_T = \frac{G_F^2 V_{cb}^2 M_{J/\psi}^5}{12\pi^3} \int_1^{1+\delta} dy \;\; [\Delta(y)]^2 \sqrt{y^2-1} \times [r^2+1-2 \;\; ry], \tag{4.1}$$

where $r=M_{B_c}/M_{J/\psi}$ and $\delta=(M_{B_c}-M_{J/\psi})^2/2M_{B_c}M_{J/\psi}$. The measurement of $d\Gamma_i/dy$ provides information on Δ and V_{cb}; in particular, if the curvature term in $\Delta(y)$ is neglected, the ratio Γ_T/Γ_L gives access to the slope ρ^2. The combination $V_{cb}\Delta(1)$ can be obtained from the measurement of Γ_L and from the total width, and therefore a measurement of V_{cb} is possible using this decay channel [34,32]. Such new determinations of the CKM element V_{cb}, even though not accurate as from B_d and B_u decays, would represent an important consistency check of the standard model.

Tests of the spin symmetry are provided by the measurement of the decay distributions in the y variable, whose deviations from the distributions related to a unique form factor Δ would imply the presence of spin symmetry-breaking terms.

Let us finally observe that our prediction for the rates of the decays into 0^- $(\overline{c}c)$ states, $B_c \to \eta_c l \nu$ and $B_c \to \eta_c' l \nu$, is smaller than the value reported by other analyses.

V. NONLEPTONIC B_c DECAYS

Estimates of the decay rates of several two-body nonleptonic B_c transitions can be obtained adopting the factorization approximation. Such an approximation finds theoretical support in few cases (large N_c limit; $m_b\to\infty$ limit in $b\to u$ transitions involving heavy-light meson systems [35]); nevertheless, it is widely used to estimate nonleptonic decay rates of mesons containing heavy quarks.

Let us first consider nonleptonic B_c decay modes induced, at the quark level, by the $b\to c$ and u transitions. The effective Hamiltonian governing the processes reads

$$H_{eff}=\frac{G_F}{\sqrt{2}}\{V_{cb}[c_1(\mu)Q_1^{cb}+c_2(\mu)Q_2^{cb}]+V_{ub}[c_1(\mu)Q_1^{ub}+c_2(\mu)Q_2^{ub}]+\text{H.c.}\}+\text{penguin operators}; \tag{5.1}$$

G_F is the Fermi constant, V_{ij} are CKM matrix elements and $c_i(\mu)$ scale-dependent Wilson coefficients. The four-quark operators Q_1^{cb} and Q_2^{cb} are given by

$$Q_1^{cb}=[V_{ud}^*(\bar{d}u)_{V-A}+V_{us}^*(\bar{s}u)_{V-A}+V_{cd}^*(\bar{d}c)_{V-A}+V_{cs}^*(\bar{s}c)_{V-A}](\bar{c}b)_{V-A},$$

$$Q_2^{cb}=[V_{ud}^*(\bar{c}u)_{V-A}(\bar{d}b)_{V-A}+V_{us}^*(\bar{c}u)_{V-A}(\bar{s}b)_{V-A}+V_{cd}^*(\bar{c}c)_{V-A}(\bar{d}b)+V_{cs}^*(\bar{c}c)_{V-A}(\bar{s}b)], \tag{5.2}$$

with $(\bar{q}_1q_2)_{V-A}=\bar{q}_1\gamma_\mu(1-\gamma_5)q_2$; analogous relations hold for Q_1^{ub} and Q_2^{ub}.

As well known, the factorization approximation amounts to evaluate the matrix elements of the four-quark operators in Eq. (5.2) between the initial B_c state and the final two-body hadronic states as the product of quark-current matrix elements. We adopt this approximation in the calculation of the rates, neglecting the contribution of penguin operators, since their Wilson coefficients are small with respect to c_1 and c_2 (interference effects of penguin diagrams are of prime importance in producing CP violating asymmetries in B_c decays). Moreover, we do not take into account the weak annihilation contribution represented by a B_c meson annihilating into a charged W; in this amplitude, the final hadronic state is entirely produced out of the vacuum, and therefore the contribution should be characterized by a sizable form factor suppression. Annihilation processes are presumably

TABLE III. Nonleptonic $(b\to c,u)$ B_c^+ decay widths and branching fractions.

Channel	$\Gamma(10^{-15}$ GeV)	BR	Channel	$\Gamma(10^{-15}$ GeV)	BR
$\eta_c\pi^+$	a_1^2 0.28	2.6×10^{-4}	η_cK^+	a_1^2 0.023	2×10^{-5}
$\eta_c\rho^+$	a_1^2 0.75	6.7×10^{-4}	η_cK^{*+}	a_1^2 0.041	3.6×10^{-5}
$\eta_ca_1^+$	a_1^2 0.96	8.6×10^{-4}	$\eta_cK_1^+$	a_1^2 0.05	4.4×10^{-5}
$\eta_c'\pi^+$	a_1^2 0.074	6.6×10^{-5}	$\eta_c'K^+$	a_1^2 0.0055	5×10^{-6}
$\eta_c'\rho^+$	a_1^2 0.16	1.5×10^{-4}	$\eta_c'K^{*+}$	a_1^2 0.008	7.4×10^{-6}
$\eta_c'a_1^+$	a_1^2 0.15	1.4×10^{-4}	$\eta_c'K_1^+$	a_1^2 0.0075	6.7×10^{-6}
$J/\psi\pi^+$	a_1^2 1.48	1.3×10^{-3}	$J/\psi K^+$	a_1^2 0.076	6.8×10^{-5}
$J/\psi\rho^+$	a_1^2 4.14	3.7×10^{-3}	$J/\psi K^{*+}$	a_1^2 0.23	2×10^{-4}
$J/\psi a_1^+$	a_1^2 5.78	5.2×10^{-3}	$J/\psi K_1^+$	a_1^2 0.3	2.7×10^{-4}
$\psi'\pi^+$	a_1^2 0.22	1.9×10^{-4}	$\psi'K^+$	a_1^2 0.01	9.3×10^{-6}
$\psi'\rho^+$	a_1^2 0.54	4.8×10^{-4}	$\psi'K^{*+}$	a_1^2 0.03	2.6×10^{-5}
$\psi'a_1^+$	a_1^2 0.65	5.8×10^{-4}	$\psi'K_1^+$	a_1^2 0.033	3×10^{-5}
$D^+\bar{D}^0$	a_2^2 0.15	8.4×10^{-6}	$D_s^+\bar{D}^0$	a_2^2 0.01	6×10^{-7}
$D^+\bar{D}^{*0}$	a_2^2 0.13	7.5×10^{-6}	$D_s^+\bar{D}^{*0}$	a_2^2 0.009	5.3×10^{-7}
$D^{*+}\bar{D}^0$	a_2^2 1.46	8.4×10^{-5}	$D_s^{*+}\bar{D}^0$	a_2^2 0.087	5×10^{-6}
$D^{*+}\bar{D}^{*0}$	a_2^2 2.4	1.4×10^{-4}	$D_s^{*+}\bar{D}^{*0}$	a_2^2 0.15	8.4×10^{-6}
η_cD_s	$(a_1\ 7.8+a_2\ 1.6)^2\times10^{-1}$	5×10^{-3}	η_cD^+	$(a_1\ 0.86+a_2\ 0.46)^2\times10^{-1}$	5×10^{-5}
$\eta_cD_s^*$	$(a_1\ 3.6+a_2\ 6.05)^2\times10^{-1}$	3.8×10^{-4}	η_cD^{*+}	$(a_1\ 0.7+a_2\ 0.9)^2\times10^{-1}$	2×10^{-5}
$\eta_c'D_s$	$(a_1\ 1.5+a_2\ 3.2)^2\times10^{-1}$	3.7×10^{-5}	$\eta_c'D^+$	$(a_1\ 0.28+a_2\ 0.7)^2\times10^{-1}$	1×10^{-6}
$\eta_c'D_s^*$	$(a_1\ 0.79+a_2\ 1.8)^2\times10^{-1}$	1×10^{-5}	$\eta_c'D^{*+}$	$(a_1\ 0.17+a_2\ 0.8)^2\times10^{-1}$	6×10^{-8}
$J/\psi D_s$	$(a_1\ 6.7+a_2\ 2.3)^2\times10^{-1}$	3.4×10^{-3}	$J/\psi D^+$	$(a_1\ 1.31+a_2\ 0.47)^2\times10^{-1}$	1.3×10^{-4}
$J/\psi D_s^*$	$(a_1\ 11+a_2\ 10.4)^2\times10^{-1}$	5.9×10^{-3}	$J/\psi D^{*+}$	$(a_1\ 2.02+a_2\ 2.3)^2\times10^{-1}$	1.9×10^{-4}
$\psi'D_s$	$(a_1\ 1.4+a_2\ 1.33)^2\times10^{-1}$	1×10^{-4}	$\psi'D^+$	$(a_1\ 0.35+a_2\ 0.36)^2\times10^{-1}$	5.8×10^{-6}
$\psi'D_s^*$	$(a_1\ 2.75+a_2\ 7.8)^2\times10^{-1}$	5.7×10^{-5}	$\psi'D^{*+}$	$(a_1\ 0.55+a_2\ 1.76)^2\times10^{-1}$	8.7×10^{-7}

TABLE IV. Nonleptonic ($c \to s,d$) B_c^+ decay widths and branching fractions.

Channel	$\Gamma(10^{-15}$ GeV)	BR	Channel	$\Gamma(10^{-15}$ GeV)	BR
$B_s\pi^+$	a_1^2 30.6	4×10^{-2}	B_sK^+	a_1^2 2.15	2.7×10^{-3}
$B_s\rho^+$	a_1^2 13.6	1.7×10^{-2}	B_sK^{*+}	a_1^2 0.043	5.4×10^{-5}
$B_s^*\pi^+$	a_1^2 35.6	4.5×10^{-2}	$B_s^*K^+$	a_1^2 1.6	2×10^{-3}
$B_s^*\rho^+$	a_1^2 110.1	1.4×10^{-1}			
$B_d\pi^+$	a_1^2 1.97	2.5×10^{-3}	B_dK^+	a_1^2 0.14	1.8×10^{-4}
$B_d\rho^+$	a_1^2 1.54	2×10^{-3}	B_dK^{*+}	a_1^2 0.032	4×10^{-5}
$B_d^*\pi^+$	a_1^2 2.4	3×10^{-3}	$B_d^*K^+$	a_1^2 0.12	1.6×10^{-4}
$B_d^*\rho^+$	a_1^2 8.6	1×10^{-2}	$B_d^*K^{*+}$	a_1^2 0.34	4.4×10^{-4}

relevant mainly for rare or suppressed B_c decays; in these cases they deserve a dedicated analysis.

A further remark concerns the Wilson coefficients $c_1(\mu)$ and $c_2(\mu)$. Writing the factorized amplitudes and taking into account the contribution of the Fierz reordered currents, it turns out that the relevant coefficients are the combinations: $a_1=c_1+\xi c_2$ and $a_2=c_2+\xi c_1$, with the QCD parameter ξ given by $\xi=1/N_c$. Several discussions concerning this parameter are available in the literature. We choose $a_1=c_1$ and $a_2=c_2$, i.e., $\xi=0$, in the spirit of the large N_c limit, and use c_1 and c_2 computed at an energy scale of the order of m_b. A detailed analysis of $1/N_c$ corrections to the coefficients a_1, a_2 as well as of the role of color-octet current operators in B decays can be found in [36]. Analogous considerations hold for the decays induced by the $c\to s(d)$ transitions; in this case we choose the coefficients c_1 and c_2 at the scale of the charm mass.

The factorized amplitudes can be expressed in terms of the form factors in Eqs. (2.5), (2.6), and (2.7), and of leptonic decay constants defined by the matrix elements $\langle 0|A_\mu|M(p)\rangle = i f_M p_\mu$ and $\langle 0|V_\mu|V(p,\epsilon)\rangle = f_V M_V \epsilon_\mu$. We use the following values: $f_{\pi^+}=0.131$ GeV, $f_{\rho^+}=0.208$ GeV, and $f_{a_1}=0.229$ GeV; $f_{K^+}=0.159$ GeV, $f_{K^{*+}}=0.214$ GeV, and $f_{K_1}=0.229$ GeV; $f_{\eta_c}=0.31$ GeV, $f_{\eta_c'}=0.23$ GeV, $f_\psi=0.38$ GeV, $f_{\psi'}=0.28$ GeV, and finally $f_D=0.2$ GeV, $f_{D_s}=0.24$ GeV and $f_{D^*}=0.23$ GeV, $f_{D_s^*}=0.275$ GeV. Such values correspond to experimental results or to average values from lattice QCD and QCD sum rules.[2]

The decay rates of several nonleptonic B_c transitions, obtained using $c_1(m_b)=1.132$, $c_2(m_b)=-0.286$ and $c_1(m_c)=1.351$, $c_2(m_c)=-0.631$, are collected in Tables III and IV. Also in this case we use the physical phase space together with the expression of the matrix elements in Eqs. (2.5)–(2.7).

A few comments are in order. We observe the dominance of the decay modes induced by the charm transition, and in particular of the channel $B_c^+\to B_s^*\rho^+$, which represents more than 10% of the total B_c width. It would be interesting to experimentally confirm this prediction, even though the final state presents severe reconstruction difficulties. From the experimental point of view, more promising are the decay modes having a J/ψ meson in the final state; among such modes, the decay channels $B_c^+\to J/\psi\pi^+$ and $B_c^+\to J/\psi\rho^+$ are particularly useful for the precise measurement of the B_c mass, by the complete reconstruction of the final state. Also the decay into a_1 is of particular interest, due to the large decay rate.

Several tests of factorization can be carried out, mainly using the decay channels having a J/ψ in the final state. For example, the assumption of the factorization approximation, together with the heavy quark spin symmetry, implies that the relation

$$\frac{\Gamma(B_c^+\to J/\psi\pi^+)}{\left.\dfrac{d\Gamma(B_c^+\to J/\psi l^+\nu)}{dy}\right|_{y=y_\pi}} = \frac{3\pi^2 V_{ud}^2 a_1^2 f_\pi^2}{M_{B_c} M_{J/\psi}} \tag{5.3}$$

holds in the limit $M_\pi\to 0$ [$y_\pi=(M_{B_c}^2+M_{J/\psi}^2)/2M_{B_c}M_{J/\psi}$]. An analogous relation holds for the B_c transition into the radial excited state $\psi(2S)$:

$$\frac{\Gamma(B_c^+\to \psi(2S)\pi^+)}{\left.\dfrac{d\Gamma(B_c^+\to \psi(2S) l^+\nu)}{dy}\right|_{y=y_\pi}} = \frac{3\pi^2 V_{ud}^2 a_1^2 f_\pi^2}{M_{B_c} M_{\psi(2S)}}. \tag{5.4}$$

In the case of a ρ meson in the final state one has

$$\frac{\Gamma(B_c^+\to J/\psi\rho^+)}{\left.\dfrac{d\Gamma(B_c^+\to J/\psi l^+\nu)}{dy}\right|_{y=y_\rho}} = \frac{3\pi^2 V_{ud}^2 a_1^2 f_\rho^2[8M_{J/\Psi}^2 M_\rho^2 + (M_{B_c}^2 - M_{J/\psi}^2 - M_\rho^2)^2]}{8 M_{B_c}^2 M_{J/\psi}^5} \times \frac{\lambda^{1/2}(M_{B_c}^2, M_{J/\psi}^2, M_\rho^2)}{\sqrt{y^2-1}[r^2 y_\rho^2 - 6 r y_\rho + 2r^2 + 3]}, \tag{5.5}$$

[2]A description of the current theoretical situation concerning the heavy meson leptonic decay constants is reported in Appendices C and D of Ref. [33].

λ being the triangular function, $r = M_{B_c}/M_{J/\psi}$, and $y_\rho = (M_{B_c}^2 + M_{J/\psi}^2 - M_\rho^2)/2M_{B_c}M_{J/\psi}$.

To test Eqs. (5.3)–(5.5) two-body decay rates and the differential $B_c^+ \to J/\psi l^+ \nu$ decay width are required; the measurement of such quantities, possible at the hadronic facilities, would provide us with important information on the heavy quark spin symmetry as well as on the factorization approximation in B_c decays.

VI. CONCLUSIONS

We have presented a determination of the invariant functions parametrizing the semileptonic B_c matrix elements in the infinite heavy quark mass limit. The form factors are obtained as overlap integrals of meson wave functions, obtained in the framework of a QCD relativistic potential model. An interesting result is that, although not constrained by symmetry arguments, the normalization of the form factor Δ describing the transition $B_c \to J/\psi l \nu$ is close to 1 at the zero-recoil point, as being the overlap of similar wave functions. On the contrary, the form factors relative to the transitions into heavy-light mesons, at zero-recoil point, are suppressed by a factor $\simeq 1/\sqrt{2}$ with respect to Δ. These results have several phenomenological consequences, in semileptonic and nonleptonic B_c decay processes, which can be experimentally tested. Moreover, they affect other important processes, such as radiative flavor-changing B_c decays [37] and CP violating B_c transitions [38,18]. In particular, the invariant functions computed in this paper can be useful to identify the B_c decay channels characterized by a clean experimental signature, a large branching fraction, and a visible CP asymmetry; the identification of this kind of decay mode is of paramount importance for the physics program of the experiments at the future accelerators.

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