Rare radiative $B_s \rightarrow \tau^+ \tau^- \gamma$ decay in the two Higgs doublet model

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The radiative $B_s \rightarrow \tau^+ \tau^- \gamma$ decay is investigated in the framework of the two Higgs doublet model. The dependence of the differential branching ratio on the photon energy and the branching ratio on the two Higgs doublet model parameters $m_{H^{\pm}}$ and $\tan \beta$ are studied. It is shown that there is an enhancement in the predictions of the two Higgs doublet model compared to the standard model case. We also observe that contributions of neutral Higgs bosons to the decay are sizable when $\tan \beta$ is large.

I. INTRODUCTION

Rare B-meson decays are one of the important research areas to test theoretical models and make estimates about their free parameters. In the standard model (SM) they are induced by flavor changing neutral currents (FCNC) at the loop level. This ensures a precise determination of the fundamental parameters of the SM, such as Cabibbo-Kabayashi-Maskawa (CKM) matrix elements, leptonic decay constants, etc. In addition, the studies on rare B-meson decays give powerful clues about the existence of models beyond the SM, such as two the Higgs doublet model (2HDM), minimal supersymmetric extension of the SM (MSSM) [1], etc. Among rare B decays, $B \rightarrow l^+ l^- \gamma$ decays are of special interest due to their cleanliness and sensitivity to new physics. They have been investigated in the framework of the SM in Refs. [2,3] for $l=e,\mu$ and in Ref. [4] for $l=\tau$. The theoretical results given in Refs. [3] and [4] are BR($B_s \rightarrow e^+ e^- \gamma$) $=2.35 \times 10^{-9}$, BR($B_s \rightarrow \mu^+ \mu^- \gamma$)$=1.9 \times 10^{-9}$, and BR($B_s \rightarrow \tau^+ \tau^- \gamma$)$=9.54 \times 10^{-9}$ without long distance (LD) contributions, respectively. When LD contributions are taken into account, BR($B_s \rightarrow \tau^+ \tau^- \gamma$) is calculated as 1.52×10^{-8} in Ref. [4]. These decays get negligible contributions from the diagrams, where photon is radiated from any charged internal line due to the fact that they will have a factor m_b^2 / M_W^2 in the Wilson coefficients. When photon is radiated from the final charged leptons, the contribution is proportional to the lepton mass m_l. Therefore, for $l=e,\mu$ case, it is negligible; however, for $l=\tau$ it gives a considerable contribution to the amplitude. In the 2HDM, there is a part coming from exchanging neutral Higgs bosons and in contrast to $B_s \rightarrow l^+ l^- \gamma$ ($l=e,\mu$) decays, we could expect that they significantly contribute for $B_s \rightarrow \tau^+ \tau^- \gamma$ decays. Therefore, in this work we study the $B_s \rightarrow \tau^+ \tau^- \gamma$ process in the framework of the 2HDM (models I and II).

2HDM is one of the simplest extensions of the SM, obtained by the addition of a second Higgs doublet. In this model, there are one physical charged Higgs scalar, two neutral Higgs scalars, and one neutral Higgs pseudoscalar. The Yukawa Lagrangian causes that the model possesses tree-level FC couplings of the neutral Higgs particles. To avoid such terms, it is proposed an *ad hoc* discrete symmetry [5] on the 2HDM potential and the Yukawa interaction. As a result, it appears two different choices for how to couple the

quarks to the two Higgs doublets: In the first choice (model I), the quarks do not couple to the first Higgs doublet, but couple to the second one. In the second choice (model II), the first Higgs doublet couples only to down-type quarks and the second one to only up-type quarks.

The paper is organized as follows. In Sec. II, we present the theoretical framework for the $B_s \rightarrow \tau^+ \tau^- \gamma$ decay and describe some details of its decay rate calculation . We give a numerical analysis and discussion of our results in Sec. III. Appendixes contain a list of the operators and the Wilson coefficients, as well as some relevant formulas about the long distance contributions.

II. $B_s \rightarrow \tau^+ \tau^- \gamma$ DECAY IN THE FRAMEWORK OF THE 2HDM

The exclusive decay $B_s \rightarrow \tau^+ \tau^- \gamma$ can be obtained from the inclusive one $b \rightarrow s \tau^+ \tau^- \gamma$. In order to calculate the relevant physical quantities for the decay $b \rightarrow s \tau^+ \tau^- \gamma$, we start with the QCD corrected amplitude for the process $b \rightarrow s \tau^+ \tau^-$. At this stage, the effective Hamiltonian is obtained by matching the full theory with the effective low energy one at the high scale μ. The Wilson coefficients are evaluated from μ down to the lower scale $\mu \sim O(m_b)$ using the renormalization group equation (RGE). The effective Hamiltonian in the 2HDM for the process $b \rightarrow s \tau^+ \tau^-$ is [6]

$$\mathcal{H} = \frac{-4 G_F}{\sqrt{2}} V_{tb} V_{ts}^* \left\{ \sum_{i=1}^{10} C_i(\mu) O_i(\mu) + \sum_{i=1}^{10} C_{Q_i}(\mu) Q_i(\mu) \right\}.$$

(1)

In this equation O_i are current-current ($i=1,2$), penguin ($i=3,..,6$), magnetic penguin ($i=7,8$), and semileptonic ($i=9,10$) operators . The additional operators Q_i,($i=1,\ldots,10$) are due to the neutral Higgs boson exchange diagrams, which give considerable contributions in the case that the lepton pair is $\tau^+ \tau^-$ [6]. $C_i(\mu)$ and $C_{Q_i}(\mu)$ are Wilson coefficients renormalized at the scale μ. All these operators and the Wilson coefficients, together with their initial values calculated at $\mu=m_W$ in the SM and also the additional coefficients coming from the new Higgs scalars are presented in Appendixes A and B. The QCD corrected amplitude for the inclusive $b \rightarrow s \tau^+ \tau^-$ decay in the 2HDM (model I or II) is

$$\mathcal{M}=\frac{\alpha G_F}{\sqrt{2}\,\pi}V_{tb}V_{ts}^*\left\{C_9^{\text{eff}}(\bar{s}\gamma_\mu P_L b)\,\bar{\tau}\gamma_\mu\tau\right.$$

$$+C_{10}(\bar{s}\gamma_\mu P_L b)\,\bar{\tau}\gamma_\mu\gamma_5\tau$$

$$-2C_7\frac{m_b}{p^2}(\bar{s}i\sigma_{\mu\nu}p_\nu P_R b)\bar{\tau}\gamma_\mu\tau+C_{Q_1}(\bar{s}\gamma_\mu P_R b)\bar{\tau}\tau$$

$$\left.+C_{Q_2}(\bar{s}\gamma_\mu P_R b)\bar{\tau}\gamma_5\tau\right\},\qquad(2)$$

where $P_{L,R}=(1\mp\gamma_5)/2$, p is the momentum transfer and V_{ij}'s are the corresponding elements of the CKM matrix.

In order to obtain the matrix element for $b\to s\tau^+\tau^-\gamma$ decay, a photon line should be attached to any charged internal or external line. The contributions coming from the attachement of photon to any internal line are suppressed and we neglect them in the following analysis. We now start with the case in which a photon is attached to the initial quark lines. The corresponding matrix element for the $B_s\to\tau^+\tau^-\gamma$ decay is

$$\mathcal{M}_1=\langle\gamma|\mathcal{M}|B_s\rangle=\frac{\alpha G_F}{2\sqrt{2}\,\pi}V_{tb}V_{ts}^*\left\{C_9^{\text{eff}}\bar{\tau}\gamma_\mu\tau\langle\gamma|\bar{s}\gamma_\mu\right.$$

$$\times(1-\gamma_5)b|B_s\rangle+C_{10}\bar{\tau}\gamma_\mu\gamma_5\tau\langle\gamma|\bar{s}\gamma_\mu(1-\gamma_5)b|B_s\rangle$$

$$-2C_7\frac{m_b}{p^2}\langle\gamma|\bar{s}i\sigma_{\mu\nu}p_\nu(1+\gamma_5)b|B_s\rangle\bar{\tau}\gamma_\mu\tau$$

$$+C_{Q_1}\bar{\tau}\tau\langle\gamma|\bar{s}(1+\gamma_5)b|B_s\rangle+C_{Q_2}\bar{\tau}\gamma_5\tau$$

$$\left.\times\langle\gamma|\bar{s}(1+\gamma_5)b|B_s\rangle\right\}.\qquad(3)$$

These matrix elements can be written in terms of the two independent, gauge invariant, parity conserving and parity violating form factors [3,7]:

$$\langle\gamma|\bar{s}\gamma_\mu(1\mp\gamma_5)b|B_s\rangle=\frac{e}{m_{B_s}^2}\{\epsilon_{\mu\alpha\beta\sigma}\epsilon_\alpha^*p_\beta q_\sigma g(p^2)$$

$$\pm i[\epsilon_\mu^*(pq)-(\epsilon^*p)q_\mu]f(p^2)\}\qquad(4)$$

and

$$\langle\gamma|\bar{s}i\sigma_{\mu\nu}p_\nu(1\mp\gamma_5)b|B_s\rangle=\frac{e}{m_{B_s}^2}\{\epsilon_{\mu\alpha\beta\sigma}\epsilon_\alpha^*p_\beta q_\sigma g_1(p^2)$$

$$\mp i[\epsilon_\mu^*(pq)$$

$$-(\epsilon^*p)q_\mu]f_1(p^2)\}.\qquad(5)$$

Here ϵ_μ and q_μ are the four vector polarization and four momentum of the photon, respectively. To calculate the ma-

trix elements $\langle\gamma|\bar{s}(1\pm\gamma_5)b|B_s\rangle$, we multiply both sides of Eq. (4) by p_μ and use the equations of motion. However, neglecting the mass of the strange quark they vanish,

$$\langle\gamma|\bar{s}(1\pm\gamma_5)b|B_s\rangle=0.\qquad(6)$$

Substituting Eqs. (4) and (5) in Eq. (3), for the matrix element $\mathcal{M}_1$ (structure dependent part) we get

$$\mathcal{M}_1=\frac{\alpha G_F}{2\sqrt{2}\,\pi}V_{tb}V_{ts}^*e\{\epsilon_{\mu\alpha\beta\sigma}\epsilon_\alpha^*p_\beta q_\sigma[A\,\bar{\tau}\gamma_\mu\tau+C\,\bar{\tau}\gamma_\mu\gamma_5\tau]$$

$$+i[\epsilon_\mu^*(pq)-(\epsilon^*p)q_\mu][B\bar{\tau}\gamma_\mu\tau+D\bar{\tau}\gamma_\mu\gamma_5\tau]\},\qquad(7)$$

where

$$A=\frac{1}{m_{B_s}^2}\left[C_9^{\text{eff}}g(p^2)-2C_7\frac{m_b}{p^2}g_1(p^2)\right],$$

$$B=\frac{1}{m_{B_s}^2}\left[C_9^{\text{eff}}f(p^2)-2C_7\frac{m_b}{p^2}f_1(p^2)\right],$$

$$C=\frac{C_{10}}{m_{B_s}^2}g(p^2),\quad D=\frac{C_{10}}{m_{B_s}^2}f(p^2).\qquad(8)$$

Note that the neutral Higgs exchange interactions do not give any contribution when photon is attached to the either one of the initial quark lines. However, when a photon is radiated from the final τ leptons the situation is different and the corresponding matrix element (Bremsstrahlung part) is

$$\mathcal{M}_2=\frac{\alpha G_F}{2\sqrt{2}\,\pi}V_{tb}V_{ts}^*eif_{B_s}\left\{\left(2m_\tau C_{10}+\frac{m_B^2}{m_b}C_{Q_2}\right)\right.$$

$$\times\left[\bar{\tau}\left(\frac{\not{\epsilon}\not{P}_{B_s}}{2p_1q}-\frac{\not{P}_{B_s}\not{\epsilon}}{2p_2q}\right)\gamma_5\tau\right]+\frac{m_{B_s}^2}{m_b}C_{Q_1}$$

$$\times\left[2m_\tau\left(\frac{1}{2p_1q}+\frac{1}{2p_2q}\right)\bar{\tau}\not{\epsilon}\tau+\bar{\tau}\left(\frac{\not{\epsilon}\not{P}_{B_s}}{2p_1q}-\frac{\not{P}_{B_s}\not{\epsilon}}{2p_2q}\right)\tau\right]\right\},$$

$$(9)$$

where we have used

$$\langle0|\bar{s}\gamma_\mu\gamma_5b|B_s\rangle=-if_{B_s}P_{B_s\mu},$$

$$\langle0|\bar{s}\sigma_{\mu\nu}(1\pm\gamma_5)b|B_s\rangle=0,$$

$$\langle0|\bar{s}\gamma_5b|B_s\rangle=if_{B_s}\frac{m_{B_s}^2}{m_b},$$

$$\langle0|\bar{s}b|B_s\rangle=0.\qquad(10)$$

and the conservation of the vector current. Here P_{B_s} is the momentum of the B_s meson.

Finally, we get the total matrix element for the $B_s \rightarrow \tau^+ \tau^- \gamma$ decay as

$$\mathcal{M} = \mathcal{M}_1 + \mathcal{M}_2. \qquad (11)$$

To calculate the decay rate, we need the square of this matrix element. By summing over the spins of the τ leptons and the

polarization of the photon, we obtain

$$|\mathcal{M}|^2 = |\mathcal{M}_1|^2 + |\mathcal{M}_2|^2 + 2\,\text{Re}(\mathcal{M}_1\mathcal{M}_2^*), \qquad (12)$$

where

$$|\mathcal{M}_1|^2 = \left| \frac{\alpha G_F}{2\sqrt{2}\,\pi} V_{tb}V_{ts}^* \right|^2 4\pi\alpha\, (8\,\text{Re}(B^*C + A^*D)p^2(p_1q - p_2q)(p_1q + p_2q) + 4[|C|^2 + |D|^2]\{(p^2 - 2m_\tau^2)[(p_1q)^2 + (p_2q)^2] - 4m_\tau^2(p_1q)(p_2q)\} + 4[|A|^2 + |B|^2]\{(p^2 + 2m_\tau^2)[(p_1q)^2 + (p_2q)^2] + 4m_\tau^2(p_1q)(p_2q)\}), \qquad (13)$$

$$2\,\text{Re}(\mathcal{M}_1\mathcal{M}_2^*) = \left| \frac{\alpha G_F}{2\sqrt{2}\,\pi} V_{tb}V_{ts}^* \right|^2 4\pi\alpha \left\{ 16\,C_{10}f_{B_s}m_\tau^2 \left[\text{Re}(A)\frac{(p_1q + p_2q)^3}{(p_1q)(p_2q)} + \text{Re}(D)\frac{(p_1q + p_2q)^2(p_1q - p_2q)}{(p_1q)(p_2q)} \right] \right.$$
$$- \frac{m_{B_s}^2}{m_b}C_{Q_1}\left[\text{Re}(B)\frac{(p_1q + p_2q)^3}{(p_1q)(p_2q)} - \text{Re}(C)\frac{(p_1q + p_2q)^2(p_1q - p_2q)}{(p_1q)(p_2q)} \right]$$
$$\left. + \frac{m_{B_s}^2}{m_b}\text{Re}(B)\left[\frac{(m_\tau^2 - 3p_2q)(p_1q)}{p_2q} + \frac{(2m_\tau^2 - p^2)(p_2q)}{p_1q} \right] \right\}, \qquad (14)$$

$$|\mathcal{M}_2|^2 = -\left| \frac{\alpha G_F}{2\sqrt{2}\,\pi} V_{tb}V_{ts}^* \right|^2 4\pi\alpha \left\{ -16\left[\left(2m_\tau C_{10} + \frac{m_{B_s}^2}{m_b}C_{Q_2} \right)^2 + \left(\frac{m_{B_s}^2 C_{Q_1}}{m_b} \right)^2 \right] \right.$$
$$+ \frac{2m_\tau^2}{(p_1q)^2}\left[\left(2m_\tau C_{10} + \frac{m_{B_s}^2}{m_b}C_{Q_2} \right)^2 (p^2 + 2p_2q) + \left(\frac{m_{B_s}^2 C_{Q_1}}{m_b} \right)^2 (p^2 + 2p_2q - 4m_\tau^2) \right] + \frac{4}{p_1q}\left[\left(2m_\tau C_{10} \right. \right.$$
$$+ \frac{m_{B_s}^2}{m_b}C_{Q_2} \right)^2 + \left(\frac{m_{B_s}^2 C_{Q_1}}{m_b} \right)^2 \right][3m_\tau^2 - p^2 - 2p_2q] + \frac{2m_\tau^2}{(p_2q)^2}\left[\left(2m_\tau C_{10} + \frac{m_{B_s}^2}{m_b}C_{Q_2} \right)^2 (p^2 + 2p_1q) \right.$$
$$\left. + \left(\frac{m_{B_s}^2 C_{Q_1}}{m_b} \right)^2 (p^2 + 2p_1q - 4m_\tau^2) \right] + \frac{4}{p_2q}\left[\left(2m_\tau C_{10} + \frac{m_{B_s}^2}{m_b}C_{Q_2} \right)^2 + \left(\frac{m_{B_s}^2 C_{Q_1}}{m_b} \right)^2 \right][3m_\tau^2 - p^2 - 2p_1q]$$
$$+ \frac{2}{(p_1q)(p_2q)}$$
$$\left. \times \left[\left(2m_\tau C_{10} + \frac{m_{B_s}^2}{m_b}C_{Q_2} \right)^2 p^2(2m_\tau^2 - p^2) - \left(\frac{m_{B_s}^2 C_{Q_1}}{m_b} \right)^2 (p^2 + 2p_2q - 4m_\tau^2) \right] \right\}. \qquad (15)$$

Here p_1, p_2 are momenta of the final τ leptons.

In the rest frame of the B_s meson, the photon energy E_γ and the lepton energy E_1 are restricted in the region given by

$$0 \leq E_\gamma \leq \frac{m_{B_s}^2 - 4m_\tau^2}{2m_{B_s}},$$

$$\frac{m_{B_s} - E_\gamma}{2} - \frac{E_\gamma}{2}\sqrt{1 - \frac{4m_\tau^2}{m_{B_s}^2 - 2m_{B_s}E_\gamma}} \leq E_1 \leq \frac{m_{B_s} - E_\gamma}{2} + \frac{E_\gamma}{2}\sqrt{1 - \frac{4m_\tau^2}{m_{B_s}^2 - 2m_{B_s}E_\gamma}}. \qquad (16)$$

In $|\mathcal{M}_2|^2$ it appears an infrared divergence, which originates in the Bremsstrahlung processes when photon is soft and in this limit, the $B_s \to \tau^+ \tau^- \gamma$ decay cannot be distinguished from $B_s \to \tau^+ \tau^-$. Therefore, in order to cancel the infrared divergences in the decay rate both processes must be considered together. In Ref. [4] it has been shown that infrared singular terms in $|\mathcal{M}_2|^2$ exactly cancel the $O(\alpha)$ virtual correction in $B_s \to \tau^+ \tau^-$ amplitude. However, in this work

we consider the photon in $B_s \to \tau^+ \tau^- \gamma$ as a hard photon, following the approach described in Ref. [4] and impose a cut on the photon energy. The lower limit of this cut is chosen so that the radiated photon can be detected in the experiments, namely, $E_\gamma \geqslant 50$ MeV ($\simeq a \; m_{B_s}$ with $a \geqslant 0.01$). After integrating over the phase space and taking into account the cut for the photon energy we get for the decay rate

$$
\Gamma = \left| \frac{\alpha G_F}{2\sqrt{2}\,\pi} V_{tb} V_{ts}^* \right|^2 \frac{\alpha}{(2\pi)^3} m_{B_s}^5 \pi \Biggl\{ \frac{m_{B_s}^2}{12} \int_\delta^{1-4r} x^3 \, dx \sqrt{1 - \frac{4r}{1-x}} [(|A|^2 + |B|^2)(1 - x + 2r) + (|C|^2 + |D|^2)(1 - x - 4r)]
$$

$$
- 4f_{B_s} r \Biggl\{ \left(C_{10} + \frac{m_{B_s}^2}{2m_b m_\tau} C_{Q_2} \right) \int_\delta^{1-4r} x^2 \, dx \, \mathrm{Re}(A) \ln \frac{1 + \sqrt{1-4r/(1-x)}}{1 - \sqrt{1-4r/(1-x)}} + \frac{m_{B_s}^2}{2m_b m_\tau} C_{Q_1} \int_\delta^{1-4r} x \, dx \, \mathrm{Re}(B)
$$

$$
\times \left[(1-x) \sqrt{1 - \frac{4r}{1-x}} + (x - 2r) \frac{1 + \sqrt{1-4r/(1-x)}}{1 - \sqrt{1-4r/(1-x)}} \right] \Biggr\} - 8f_{B_s}^2 r \frac{1}{m_{B_s}^2} \Biggl\{ \left(C_{10} + \frac{m_{B_s}^2}{2m_b m_\tau} C_{Q_2} \right)^2
$$

$$
\times \int_\delta^{1-4r} dx \left[\frac{(1-x)}{x} \sqrt{1 - \frac{4r}{1-x}} + \left(1 + \frac{2r}{x} - \frac{1}{x} - x \right) \ln \frac{1 + \sqrt{1-4r/(1-x)}}{1 - \sqrt{1-4r/(1-x)}} - \frac{1}{r} \left(\frac{m_{B_s} C_{Q_1}}{2m_b} \right)^2 \right.
$$

$$
\times \int_\delta^{1-4r} dx \left[(4r - 1) \frac{(1-x)}{x} \sqrt{1 - \frac{4r}{1-x}} + \left(-1 + \frac{8r^2}{x} + \frac{1}{x} + x + \frac{r}{x}(4x - 6) \right) \ln \frac{1 + \sqrt{1-4r/(1-x)}}{1 - \sqrt{1-4r/(1-x)}} \right] \Biggr\}, \tag{17}
$$

where $r = m_\tau^2 / m_{B_s}^2$, $\delta = 2a$ and $x = 2E_\gamma / m_{B_s}$ is the dimensionless photon energy satisfying

$$
\delta \leqslant x \leqslant 1 - \frac{4m_\tau^2}{m_{B_s}^2}.
$$

In our numerical calculations, we use the dipole forms of the form factors given by

$$
g(p^2) = \frac{1 \text{ GeV}}{(1 - p^2/5.6^2)^2}, \qquad f(p^2) = \frac{0.8 \text{ GeV}}{(1 - p^2/6.5^2)^2},
$$

$$
g_1(p^2) = \frac{3.74 \text{ GeV}^2}{(1 - p^2/40.5)^2}, \qquad f_1(p^2) = \frac{0.68 \text{ GeV}^2}{(1 - p^2/30)^2}, \tag{18}
$$

which were calculated in the framework of the light-cone QCD sum rules [7,8].

III. RESULTS AND DISCUSSION

In the 2HDM there are number of free parameters, namely masses of the charged and neutral Higgs bosons ($m_{H^\pm}$, m_{h^0}, m_{A^0}), the ratio of vacuum expectation values of Higgs bosons, $\tan\beta = v_2/v_1$, and the angle α due to the mixing of neutral Higgs bosons A^0 and h^0. The values of

these parameters are restricted by using the existing experimental data. The nonobservation of charged $H^\pm$ pair in Z decays [9] gives the model independent lower bound of the mass of the charged Higgs boson $H^\pm$, $m_{H^\pm} \geqslant 44$ GeV. However there is no experimental upper bound for $m_{H^\pm}$ except $m_{H^\pm} \leqslant 1$ TeV coming from the unitarity condition [10]. Further, top decays give $m_{H^\pm} \geqslant 147$ GeV for large $\tan\beta$ [11]. The other parameter of 2HDM, $\tan\beta$, is restricted as $\tan\beta > 0.7$ from $Z \to \bar{b} b$ decay [12]. The ratio $\tan\beta/m_{H^\pm}$ can also
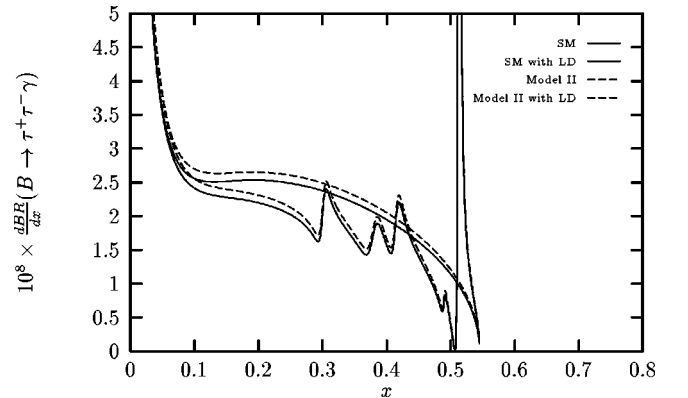

FIG. 1. Differential branching ratio as a function of $x = 2E_\gamma / m_{B_s}$ in the SM and model II for $m_{H^\pm} = 400$ GeV and $\tan\beta = 2$. In this figure, curves with sharp peaks represent the long distance contributions.

TABLE I. The values of the input parameters used in the numerical calculations.

Parameter	Value
m_c	1.4 (GeV)
m_b	4.8 (GeV)
α_{em}^{-1}	137
$|V_{tb}V_{ts}^*|$	0.045
m_{B_s}	5.28 (GeV)
$\tau(B_s)$	1.64×10^{-12} (s)
m_t	176 (GeV)
m_W	80 (GeV)
m_Z	91.19 (GeV)
m_τ	1.78 (GeV)
m_{h^0}	80 (GeV)
m_{H^0}	150 (GeV)
m_{A^0}	80 (GeV)
μ	m_b
Λ_{QCD}	0.225 (GeV)
$\alpha_s(m_Z)$	0.117
$\sin \theta_W$	0.2325

be restricted and it has been estimated as $\tan \beta / m_{H^\pm} \leqslant 0.38$ GeV^{-1} [13] and $\tan \beta / m_{H^\pm} \leqslant 0.46$ GeV^{-1} [14] from the experimental results of the branching ratios of the decays $B \rightarrow \tau \bar{\nu}$ and $B \rightarrow X \tau \bar{\nu}$. The upper bound has also been given for the same ratio as $\tan \beta / m_{H^\pm} = 0.06$ GeV^{-1} in the case that sufficient data could be taken and the theoretical uncertainties could be reduced for the exclusive decay $B \rightarrow D \tau \bar{\nu}$ [15]. Recently, the relation between $m_{H^\pm}$ and $\tan\beta$ has been estimated in Ref. [16], taking into account the CLEO measurement of the decay $B \rightarrow X_s \gamma$ [17]:

$$\text{Br}(B \rightarrow X_s \gamma) = (3.15 \pm 0.35 \pm 0.32) \times 10^{-4}. \quad (19)$$

In our calculations, we take the masses m_{h^0} and m_{A^0} equal and not too heavy since the b-quark dipole moment is strongly sensitive to the difference between these masses in the 2HDM [18]. Further, we choose the value of the angle α as being zero since the mixing between h^0 and A^0 is weak. For completeness, we have also checked the dependence of the branching ratio on α for the fixed values the other 2HDM parameters and seen that this dependence is negligible.

In the present work, we study the 2HDM parameters dependence of the BR and dimensionless photon energy dependence of the differential branching ratio (dBR/dx) in models I and II. Doing this, we have used the input parameters given in Table I.

In Fig. 1, we present dBR($B \rightarrow \tau^+ \tau^- \gamma$)/$dx$ as a function of $x = 2E_\gamma / m_{B_s}$ in the SM and in model II for $m_{H^\pm} = 400$ GeV and $\tan \beta = 2$. We do not display the predictions of model I there, since they are very close to those of model II. In this figure, curves with sharp peaks represent the long distance contributions. From Fig. 1, we see that there is an enhancement in the 2HDM compared to the SM case.
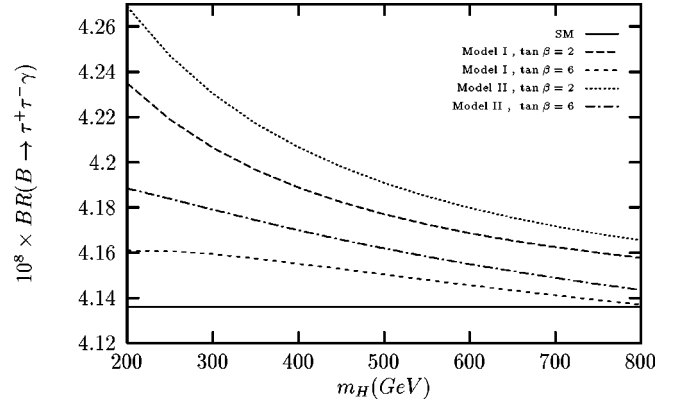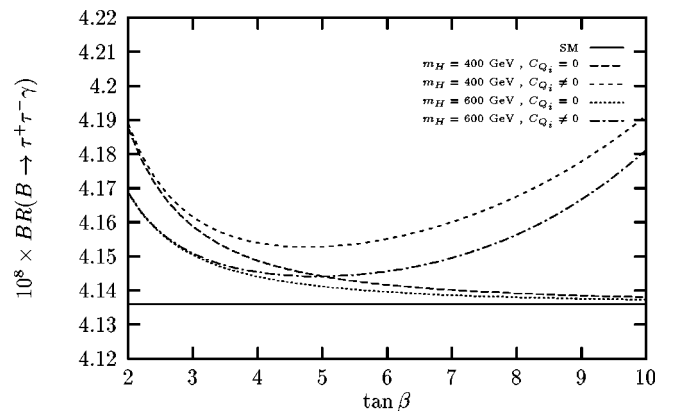

FIG. 2. Branching ratio as a function of $m_{H^\pm}$ in the SM, model I, and II for different values of $\tan \beta$.

Figure 2 shows the dependence of the BR on the Higgs boson mass $m_{H^\pm}$ for different values of the parameter $\tan \beta$ for models I and II, as well as for the SM. We again observe an enhancement for the BR in 2HDM compared to the SM case, especially for the small values of $m_{H^\pm}$. For example, for $m_{H^\pm} = 400$ GeV and $\tan \beta = 2$, BR($B_s \rightarrow \tau^+ \tau^- \gamma$) = 4.18 $\times 10^{-8}$ in model I, and BR($B_s \rightarrow \tau^+ \tau^- \gamma$) = 4.20×10^{-8} in model II. These values are slightly greater than the SM predictions, which is BR($B_s \rightarrow \tau^+ \tau^- \gamma$) = 4.13×10^{-8}. In addition, the $m_{H^\pm}$ dependence of the BR becomes weaker with increasing values of $\tan \beta$ for both models. Note that these results are sensitive to the choice of form factors and there can exist considerable differences in the results. Therefore, it is difficult to predict the new physics effects for large $m_H^\pm$ and small $\tan \beta$ values lying in the restriction region.

We present the BR as a function of $\tan \beta$ for different values of $m_{H^\pm}$ in models I and II in Fig. 3 and Fig. 4, respectively. It is seen that additional contributions coming from neutral Higgs exchange diagrams (i.e., contributions with $C_{Q_i} \neq 0$) causes the BR to increase with the increasing values of $\tan \beta$ in contrast to the case that neutral Higgs bosons do not contribute ($C_{Q_i} = 0$). The reason for these two different behaviors can easily be understood by comparing

FIG. 3. Branching ratio as a function of $\tan \beta$ in the SM and model I for different values of $m_{H^\pm}$. Curves with $C_{Q_i} \neq 0$ ($C_{Q_i} = 0$) represent the contributions including (not including) the neutral Higgs boson interactions.
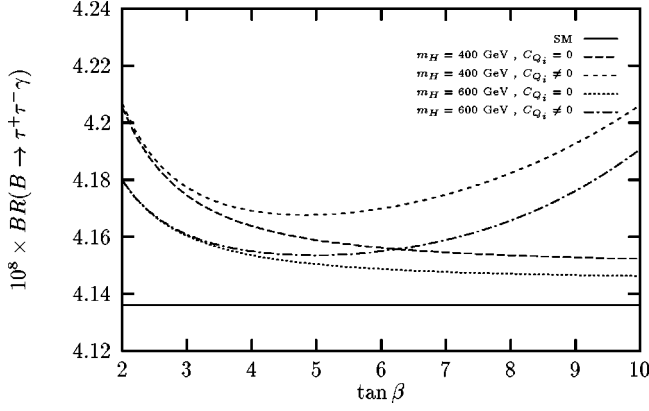
FIG. 4. The same as Fig. 3, but for model II.

Eqs. (B3) and (B2), which represent the neutral Higgs bosons and the remaining contributions, respectively, namely, the first one is proportional to $\tan^2\beta$, while the second is $1/\tan^2\beta$ so that for the larger values of $\tan\beta$, neutral Higgs boson contributions dominate in the BR.

In conclusion, we observe an enhancement in the differential branching ratio and the branching ratio of the exclusive process $B_s\to\tau^+\tau^-\gamma$ in the framework of the 2HDM as compared to the SM. Further, this enhancement becomes more detectable for large $\tan\beta$ values lying in experimentally restricted regions. Therefore, the measurement of this exclusive decay can give important clues about the new physics beyond the SM, corresponding model parameters and also the effects of neutral Higgs contributions.

APPENDIX A: THE OPERATOR BASIS

The operator basis in the 2HDM (models I and II) for the process under consideration is [19,20]

$$O_1=(\bar{s}_{L\alpha}\gamma_\mu c_{L\beta})(\bar{c}_{L\beta}\gamma^\mu b_{L\alpha}),$$

$$O_2=(\bar{s}_{L\alpha}\gamma_\mu c_{L\alpha})(\bar{c}_{L\beta}\gamma^\mu b_{L\beta}),$$

$$O_3=(\bar{s}_{L\alpha}\gamma_\mu b_{L\alpha})\sum_{q=u,d,s,c,b}(\bar{q}_{L\beta}\gamma^\mu q_{L\beta}),$$

$$O_4=(\bar{s}_{L\alpha}\gamma_\mu b_{L\beta})\sum_{q=u,d,s,c,b}(\bar{q}_{L\beta}\gamma^\mu q_{L\alpha}),$$

$$O_5=(\bar{s}_{L\alpha}\gamma_\mu b_{L\alpha})\sum_{q=u,d,s,c,b}(\bar{q}_{R\beta}\gamma^\mu q_{R\beta}),$$

$$O_6=(\bar{s}_{L\alpha}\gamma_\mu b_{L\beta})\sum_{q=u,d,s,c,b}(\bar{q}_{R\beta}\gamma^\mu q_{R\alpha}),$$

$$O_7=\frac{e}{16\pi^2}\bar{s}_\alpha\sigma_{\mu\nu}(m_bR+m_sL)b_\alpha\mathcal{F}^{\mu\nu},$$

$$O_8=\frac{g}{16\pi^2}\bar{s}_\alpha T^a_{\alpha\beta}\sigma_{\mu\nu}(m_bR+m_sL)b_\beta\mathcal{G}^{a\mu\nu},$$

$$O_9=\frac{e}{16\pi^2}(\bar{s}_{L\alpha}\gamma_\mu b_{L\alpha})(\bar{l}\gamma^\mu l),$$

$$O_{10}=\frac{e}{16\pi^2}(\bar{s}_{L\alpha}\gamma_\mu b_{L\alpha})(\bar{l}\gamma^\mu\gamma_5 l),$$

$$Q_1=\frac{e^2}{16\pi^2}(\bar{s}^\alpha_L b^\alpha_R)(\bar{\tau}\tau),$$

$$Q_2=\frac{e^2}{16\pi^2}(\bar{s}^\alpha_L b^\alpha_R)(\bar{\tau}\gamma_5\tau),$$

$$Q_3=\frac{g^2}{16\pi^2}(\bar{s}^\alpha_L b^\alpha_R)\sum_{q=u,d,s,c,b}(\bar{q}^\beta_L q^\beta_R),$$

$$Q_4=\frac{g^2}{16\pi^2}(\bar{s}^\alpha_L b^\alpha_R)\sum_{q=u,d,s,c,b}(\bar{q}^\beta_R q^\beta_L),$$

$$Q_5=\frac{g^2}{16\pi^2}(\bar{s}^\alpha_L b^\beta_R)\sum_{q=u,d,s,c,b}(\bar{q}^\beta_L q^\alpha_R),$$

$$Q_6=\frac{g^2}{16\pi^2}(\bar{s}^\alpha_L b^\beta_R)\sum_{q=u,d,s,c,b}(\bar{q}^\beta_R q^\alpha_L),$$

$$Q_7=\frac{g^2}{16\pi^2}(\bar{s}^\alpha_L\sigma^{\mu\nu} b^\alpha_R)\sum_{q=u,d,s,c,b}(\bar{q}^\beta_L\sigma_{\mu\nu}q^\beta_R),$$

$$Q_8=\frac{g^2}{16\pi^2}(\bar{s}^\alpha_L\sigma^{\mu\nu} b^\alpha_R)\sum_{q=u,d,s,c,b}(\bar{q}^\beta_R\sigma_{\mu\nu}q^\beta_L),$$

$$Q_9=\frac{g^2}{16\pi^2}(\bar{s}^\alpha_L\sigma^{\mu\nu} b^\beta_R)\sum_{q=u,d,s,c,b}(\bar{q}^\beta_L\sigma_{\mu\nu}q^\alpha_R),$$

$$Q_{10}=\frac{g^2}{16\pi^2}(\bar{s}^\alpha_L\sigma^{\mu\nu} b^\beta_R)\sum_{q=u,d,s,c,b}(\bar{q}^\beta_R\sigma_{\mu\nu}q^\alpha_L),$$

$$(A1)$$

where α and β are SU(3) color indices and $\mathcal{F}^{\mu\nu}$ and $\mathcal{G}^{\mu\nu}$ are the field strength tensors of the electromagnetic and strong interactions, respectively.

APPENDIX B: THE INITIAL VALUES OF THE WILSON COEFFICIENTS

The initial values of the Wilson coefficients for the relevant process in the SM are [21]

$$C^{SM}_{1,3,\ldots6,11,12}(m_W)=0,$$

$$C^{SM}_2(m_W)=1,$$

$$C^{SM}_7(m_W)=\frac{3x^3-2x^2}{4(x-1)^4}\ln x+\frac{-8x^3-5x^2+7x}{24(x-1)^3},$$

$$C_8^{SM}(m_W) = -\frac{3x^2}{4(x-1)^4} \ln x + \frac{-x^3+5x^2+2x}{8(x-1)^3},$$

$$C_9^{SM}(m_W) = -\frac{1}{\sin^2\theta_W} B(x) + \frac{1-4\sin^2\theta_W}{\sin^2\theta_W} C(x)$$

$$-D(x) + \frac{4}{9},$$

$$C_{10}^{SM}(m_W) = \frac{1}{\sin^2\theta_W}[B(x) - C(x)],$$

$$C_{Q_i}^{SM}(m_W) = 0 \quad i = 1, \dots, 10. \tag{B1}$$

The initial values for the additional part due to charged Higgs bosons are

$$C_{1,\dots 6}^H(m_W) = 0,$$

$$C_7^H(m_W) = X F_1(y) + Y F_2(y),$$

$$C_8^H(m_W) = X G_1(y) + Y G_2(y),$$

$$C_9^H(m_W) = X H_1(y),$$

$$C_{10}^H(m_W) = X L_1(y), \tag{B2}$$

and due to the neutral Higgs bosons are [6]

$$C_{Q_1}^H(m_W) = \frac{m_b m_\tau}{m_{h^0}^2} \frac{1}{\sin^2\theta_W} \frac{x}{4} X^{-1} \left\{ \frac{\sin^2 2\alpha}{2m_{H^\pm}^2} \left(m_{h^0}^2 - \frac{(m_{h^0}^2 - m_{H^0}^2)^2}{2m_{H^0}^2} \right) f_3(y) + (\sin^2\alpha + h\cos^2\alpha) f_1(x,y) \right.$$

$$\left. + [m_{h^0}^2/m_W^2 + (\sin^2\alpha + h\cos^2\alpha)(1-z)] f_2(x,y) \right\},$$

$$C_{Q_2}^H(m_W) = -\frac{m_b m_\tau}{m_{A^0}^2} X^{-1} \left\{ f_1(x,y) + \left(1 + \frac{(m_{H^\pm}^2 - m_{A^0}^2)}{2m_W^2} \right) f_2(x,y) \right\},$$

$$C_{Q_3}^H(m_W) = \frac{m_b e^2}{m_\tau g^2}[C_{Q_1}(m_W) + C_{Q_2}(m_W)],$$

$$C_{Q_4}^H(m_W) = \frac{m_b e^2}{m_\tau g^2}[C_{Q_1}(m_W) - C_{Q_2}(m_W)],$$

$$C_{Q_i}(m_W) = 0, i = 5, \dots, 10, \tag{B3}$$

where

$$x = \frac{m_t^2}{m_W^2}, \quad y = \frac{m_t^2}{m_H^\pm}, \quad z = \frac{x}{y}, \quad h = \frac{m_{h^0}^2}{m_{H^0}^2}, \quad f_1(x,y) = \frac{x\ln x}{x-1} - \frac{y\ln y}{y-1},$$

$$f_2(x,y) = \frac{x\ln y}{(z-x)(x-1)} + \frac{\ln z}{(z-1)(x-1)}, \quad f_3(y) = \frac{1-y+y\ln y}{(y-1)^2}. \tag{B4}$$

and

$$X = \frac{1}{\tan^2\beta} \left(\frac{1}{\tan^2\beta} \right), \quad Y = \frac{-1}{\tan^2\beta} \quad (1) \text{ in model I (II)}. \tag{B5}$$

The explicit forms of the functions $F_{1(2)}(y)$, $G_{1(2)}(y)$, $H_1(y)$, and $L_1(y)$ are given as

$$F_1(y) = \frac{y(7-5y-8y^2)}{72(y-1)^3} + \frac{y^2(3y-2)}{12(y-1)^4} \ln y,$$

$$F_2(y) = \frac{y(5y-3)}{12(y-1)^2} + \frac{y(-3y+2)}{6(y-1)^3} \ln y,$$

$$G_1(y) = \frac{y(-y^2+5y+2)}{24(y-1)^3} + \frac{-y^2}{4(y-1)^4} \ln y,$$

$$G_2(y) = \frac{y(y-3)}{4(y-1)^2} + \frac{y}{2(y-1)^3} \ln y,$$

$$H_1(y) = \frac{1-4\sin^2\theta_W}{\sin^2\theta_W} \frac{xy}{8} \left[\frac{1}{y-1} - \frac{1}{(y-1)^2} \ln y \right] - y \left[\frac{47y^2-79y+38}{108(y-1)^3} \right.$$

$$\left. - \frac{3y^3-6y+4}{18(y-1)^4} \ln y \right],$$

$$L_1(y) = \frac{1}{\sin^2\theta_W} \frac{xy}{8} \left[-\frac{1}{y-1} + \frac{1}{(y-1)^2} \ln y \right]. \tag{B6}$$

Finally, the initial values of the coefficients in the 2HDM are

$$C_i^{2\text{HDM}}(m_W) = C_i^{\text{SM}}(m_W) + C_i^{\text{H}}(m_W). \tag{B7}$$

Using these initial values, we can calculate the coefficients $C_i^{2\text{HDM}}(\mu)$ and $C_{Q_i}^{2\text{HDM}}(\mu)$ at any lower scale in the effective theory with five quarks, namely u,c,d,s,b similar to the SM case. Wilson coefficients $C_7^{2\text{HDM}}(\mu)$, $C_9^{2\text{HDM}}(\mu)$, $C_{10}^{2\text{HDM}}(\mu)$, $C_{Q_1}^{2\text{HDM}}(\mu)$ and $C_{Q_2}^{2\text{HDM}}(\mu)$ play the essential role in this process and the others enter into expressions due to operator mixing. For completeness we would like to give the explicit expressions of the cofficients essential in this process. The effective coefficient $C_7^{\text{eff}}(\mu)$ is defined as [22]

$$C_7^{\text{eff}}(\mu) = C_7^{2\text{HDM}}(\mu) + Q_d [C_5^{2\text{HDM}}(\mu) + N_c C_6^{2\text{HDM}}(\mu)] + Q_u \left(\frac{m_c}{m_b} C_{12}^{2\text{HDM}}(\mu) + N_c \frac{m_c}{m_b} C_{11}^{2\text{HDM}}(\mu) \right), \tag{B8}$$

where the leading order QCD corrected Wilson coefficients $C_7^{\text{LO,2HDM}}(\mu)$ are given by [19,20,23]:

$$C_7^{\text{LO,2HDM}}(\mu) = \eta^{16/23} C_7^{2\text{HDM}}(m_W) + (8/3)(\eta^{14/23} - \eta^{16/23}) C_8^{2\text{HDM}}(m_W) + C_2^{2\text{HDM}}(m_W) \sum_{i=1}^{8} h_i \eta^{a_i}, \tag{B9}$$

and $\eta = \alpha_s(m_W)/\alpha_s(\mu)$, h_i and a_i are the numbers which appear during the evaluation [23]. The perturbative part of the Wilson coefficient $C_9^{\text{eff}}(\mu)$ can be defined as [20,23]

$$C_9^{\text{pert}}(\mu) = C_9^{2HDM}(\mu) \tilde{\eta}(\hat{s}) + h(z,\hat{s})[3C_1(\mu) + C_2(\mu) + 3C_3(\mu) + C_4(\mu) + 3C_5(\mu) + C_6(\mu)] - \frac{1}{2} h(1,\hat{s})[4C_3(\mu)$$

$$+ 4C_4(\mu) + 3C_5(\mu) + C_6(\mu)] - \frac{1}{2} h(0,\hat{s})[C_3(\mu) + 3C_4(\mu)] + \frac{2}{9}[3C_3(\mu) + C_4(\mu) + 3C_5(\mu) + C_6(\mu)]. \tag{B10}$$

Here the contributions of the coefficients $C_1(\mu), \ldots, C_6(\mu)$ are due to the operator mixing. In Eq. (B10) $\tilde{\eta}(\hat{s})$ represents the one gluon correction to the matrix element O_9 with $m_s = 0$ [20] and the function $h(z,\hat{s})$ arises from the one loop contributions of the four quark operators $O_1, \ldots, O_6$. Their explicit expressions are

$$\tilde{\eta}(\hat{s}) = 1 + \frac{\alpha_s(\mu)}{\pi} \omega(\hat{s}), \tag{B11}$$

where

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$$\omega(\hat{s}) = -\frac{2}{9}\pi^2 - \frac{4}{3}\mathrm{Li}_2(\hat{s}) - \frac{2}{3}\ln\hat{s}\ln(1-\hat{s}) - \frac{5+4\hat{s}}{3(1+2\hat{s})}\ln(1-\hat{s}) - \frac{2\hat{s}(1+\hat{s})(1-2\hat{s})}{3(1-\hat{s})^2(1+2\hat{s})}\ln\hat{s} + \frac{5+9\hat{s}-6\hat{s}^2}{6(1-\hat{s})(1+2\hat{s})} \quad \text{(B12)}$$

and

$$h(z,\hat{s}) = -\frac{8}{9}\ln\frac{m_b}{\mu} - \frac{8}{9}\ln z + \frac{8}{27} + \frac{4}{9}x - \frac{2}{9}(2+x)|1-x|^{1/2}\begin{cases}\left(\ln\left|\frac{\sqrt{1-x}+1}{\sqrt{1-x}-1}\right| - i\pi\right) & \text{for} \quad x\equiv\frac{4z^2}{\hat{s}}<1,\\[3mm] 2\arctan\frac{1}{\sqrt{x-1}} & \text{for} \quad x\equiv\frac{4z^2}{\hat{s}}>1,\end{cases} \quad \text{(B13)}$$

$$h(0,\hat{s}) = \frac{8}{27} - \frac{8}{9}\ln\frac{m_b}{\mu} - \frac{4}{9}\ln\hat{s} + \frac{4}{9}i\pi, \quad \text{(B14)}$$

where $z = m_c/m_b$ and $\hat{s} = p^2/m_b^2$. In addition to the perturbative part, there exist also the long distance (LD) one due to the conversion of the real $\bar{c}c$ into the lepton pair $\tau^+\tau^-$, described by the reaction $B \rightarrow \gamma\psi_i \rightarrow \gamma\tau^+\tau^-$, where $i = 1, \ldots, 6$. Adding this contribution to the perturbative one coming from the $c\bar{c}$ loop, the NLO QCD corrected $C_9^{\mathrm{eff}}(\mu)$ can be written as

$$C_9^{\mathrm{eff}}(\mu) = C_9^{\mathrm{pert}}(\mu) + Y_{\mathrm{reson}}(\hat{s}), \quad \text{(B15)}$$

where $Y_{\mathrm{reson}}(\hat{s})$ in NDR scheme is defined as

$$Y_{\mathrm{reson}}(\hat{s}) = -\frac{3}{\alpha_{em}^2}\kappa\sum_{V_i=\psi_i}\frac{\pi\Gamma(V_i\rightarrow\tau^+\tau^-)m_{V_i}}{q^2 - m_{V_i} + im_{V_i}\Gamma_{V_i}}[3C_1(\mu) + C_2(\mu) + 3C_3(\mu) + C_4(\mu) + 3C_5(\mu) + C_6(\mu)]. \quad \text{(B16)}$$

The phenomenological parameter κ in Eq. (B16) is taken as 2.3 [24].

Finally, the Wilson coefficients $C_{Q_1}(\mu)$ and $C_{Q_2}(\mu)$ are given by [6]

$$C_{Q_i}(\mu) = \eta^{-12/23}C_{Q_i}(m_W), \quad i = 1,2. \quad \text{(B17)}$$

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