Gauge invariance and finite width effects in radiative two-pion τ lepton decay

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The contribution of the $\rho^{\pm}$ vector meson to the $\tau\rightarrow\pi\pi\nu\gamma$ decay is considered as a potential source for the determination of the magnetic dipole moment of this light vector meson. In order to keep the gauge invariance of the whole decay amplitude, a procedure similar to the fermion-loop scheme for charged gauge bosons is implemented to incorporate the finite width effects of the $\rho^{\pm}$ vector meson. The absorptive pieces of the one-loop corrections to the propagators and electromagnetic vertices of the $\rho^{\pm}$ meson and $W^{\pm}$ gauge boson have identical forms in the limit of massless particles in the loops, suggesting this to be a universal feature of spin-one unstable particles. Model-dependent contributions to the $\tau\rightarrow\pi\pi\nu\gamma$ decay are suppressed by fixing the two-pion invariant mass distribution at the rho meson mass value. The resulting photon energy and angular distribution is relatively sensitive to the effects of the ρ magnetic dipole moment.

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I. INTRODUCTION

The two-pion mode is by far the dominant hadronic channel of semileptonic τ decays. According to Ref. [1] the measured resonant and non-resonant pieces of the $\tau\rightarrow\pi\pi\nu_{\tau}$ branching ratios are given by $(25.32\pm0.15)\%$ and $(0.30\pm0.32)\%$, respectively. The two-pion invariant mass distribution has also been measured in a wide region around the ρ^{+} mass [2], and it reveals a rich resonant structure dominated by charged vector mesons $\rho(770)$, ρ'.

The impressive accuracy attained in the measurement of these properties has been used for several purposes. For instance, it provides a precise test of the CVC (conserved vector current) hypothesis (which relates the two-pion tau decays to the I=1 contribution in $e^{+}e^{-}\rightarrow\pi^{+}\pi^{-}$), it reduces the hadronic uncertainties in the evaluation of $(g-2)_{\mu}$ and in the running of the QED fine structure constant at the m_Z scale [3], and it has been suggested even as a good place to determine the τ lepton charged weak dipole moments [4]. On the other hand, since multi-pion (multi-kaon) semileptonic channels have been found to be dominated by intermediate light hadronic resonances, these τ decays can be used to measure the intrinsic properties of these resonances [2]. Therefore, in this paper we explore the potential of the radiative two-pion τ decays in order to determine the magnetic dipole moment of the charged $\rho(770)$ vector meson.

In recent papers [5,6], we have considered the possibility to measure the magnetic dipole moment of light charged vector resonances (ρ and K^{*}) in their production [5] and decay [6] processes. These works have the limitation of considering vector mesons as stable particles. Since vector mesons are highly unstable particles (the width-mass ratio are 0.2 and 0.06 for the ρ and K^{*}, respectively), their properties (mass, width, magnetic dipole moment) would depend on the specific model used to describe its production and decay mechanisms [7]. A model independent measurement of its mass and width can only be obtained by identifying the pole position of the S-matrix amplitude [7,8].

In the present paper we consider the full S-matrix amplitude for the production *and* decay of the $\rho^{\pm}(770)$ vector meson in $\tau\rightarrow\pi\pi\nu\gamma$ decay in order to explore the sensitivity of this decay to the effects of the magnetic dipole moment of the $\rho^{\pm}$ vector meson. Since the evaluation of the relevant contributions to the τ lepton decay amplitude involve the propagator and the electromagnetic vertex of the charged ρ meson [9], some care must be taken in order to preserve the electromagnetic gauge invariance of the S-matrix amplitude in the presence of the finite width of the vector meson. To maintain gauge-invariance, in this paper we introduce a procedure similar to the so-called *fermion loop-scheme* proposed recently to keep gauge-invariance in processes involving the $WW\gamma$ vertex [10]. As discussed in the first two references of [10], violation of gauge invariance in the processes $q\overline{q}\rightarrow l\nu_{l}\gamma$ and $e^{+}e^{-}\rightarrow u\overline{d}e^{-}\overline{\nu}_{e}$ (that involve the $WW\gamma$ vertex) can have catastrophic effects for certain kinematical configurations in those reactions.

We have organized this paper as follows. In Sec. II we compute the absorptive parts of one loop corrections to the propagator and electromagnetic vertex of the charged vector meson. Since the leading contributions to the absorptive corrections arise from loops with two pseudoscalar mesons, we call it the *boson loop-scheme*. As discussed in Ref. [10] for the W boson case, the addition of these corrections provides a convenient and consistent way to preserve the electromagnetic Ward identity in the presence of a finite width of the unstable particle. Our results for the absorptive corrections with massless mesons in the loops are identical to those obtained in the fermion loop scheme for the $W^{\pm}$ gauge boson in the limit of massless fermions. In Sec. III we compute the full S-matrix gauge-invariant amplitude for the process $\tau^{-}\rightarrow\pi^{-}\pi^{0}\nu_{\tau}\gamma$, using the gauge-invariant Green functions derived in Sec. II. In Sec. IV we study the effects of the ρ^{-} magnetic dipole moment in the two-pion invariant mass and double-differential photon distribution of the $\tau\rightarrow\pi\pi\nu\gamma$ decay. In Sec. V we summarize and discuss our results. Two appendices are deserved to compute the corrections to the electromagnetic vertex (Appendix A) and to provide (Appendix B) the relevant scalar, vector and tensor integrals required to evaluate explicitly the absorptive parts of the propagator and vertex corrections.

Before we start our discussion, let us mention that our results can be straightforwardly extended to the $K^{*\pm}(892)$ resonance contribution in $\tau \to K\pi\nu_\tau\gamma$ decays with proper inclusion of the two isospin channels ($K^+\pi^0$ and $K^0\pi^+$) in the absorptive corrections.

II. GAUGE INVARIANCE AND BOSON LOOP-SCHEME FOR VECTOR MESONS

The electromagnetic gauge-invariance of amplitudes involving intermediate spin-one charged resonances in radiative processes can be broken if one naively incorporates the finite width of these resonances in their propagators [10]. This problem can be cured by different, but rather arbitrary, procedures (see Argyres in Ref. [10]). In the case of the unstable $W^\pm$ gauge boson, one of the recently proposed methods is the so-called *fermion loop-scheme* [10]. It consists in the addition of the absorptive parts of the fermionic one-loop corrections to the electromagnetic vertex and the propagator of the W gauge boson. In this way, the electromagnetic Ward identity between these two- and three-point functions is satisfied at the one-loop level and the gauge-invariance of the amplitude with intermediate unstable gauge-bosons is guaranteed.

Following this idea, in this section we compute the absorptive corrections to the propagator and electromagnetic vertex of the $\rho^\pm$ vector meson that arise from the one-loop diagrams with two-pseudoscalar mesons (see Figs. 1 and 2). Despite the fact that the interaction Lagrangian of pions and vector mesons is not renormalizable, we will not be concerned with these technical points as far as we focus only on the one-loop absorptive corrections which is free of infinities. This procedure serves our purposes to cure gauge-invariance in amplitudes of radiative processes involving intermediate unstable vector mesons.

The reader may wonder if a perturbative analysis of these Green functions makes sense given the strong interactions of the ρ vector meson. As it was shown long ago [11] in general, the approach to strong interactions based on dispersion theory and the one based on (perturbative) field theory, give equivalent and complementary results in the calculation of transition amplitudes. As a particular example, let us consider the $\pi^\pm$ electromagnetic form factor $F_\pi(s)$ in the timelike region $s>0$ which is dominated by the ρ^0 vector meson. In this case, dispersion theory techniques used to relate $F_\pi(s)$ to the $l=1$ phase shift of $\pi\pi$ scattering [13] and a perturbative analysis [based on the interaction Lagrangian given below in Eq. (6)] of the ρ meson propagator [9,12] give identical results for the pion electromagnetic form factor. As is well known [14], the Gounaris-Sakurai parametrization [13] gives a very good description of the experimental data for $|F_\pi(s)|$ extracted from $e^+e^- \to \pi^+\pi^-$ in a wide kinematical region of the center of mass energy $\sqrt{s}$. This equivalence of dispersion relation and field theory approaches for the ρ vector meson propagator gives us good confidence to compute the $\rho\rho\gamma$ vertex in a perturbative framework.

Let us start our discussion with the lowest order propagator $[D_0^{\mu\nu}(q)]$ and electromagnetic vertex ($\Gamma_0^{\mu\nu\lambda}$) of the $\rho^\pm$ vector meson. Using the conventions given in Fig. 1(a) we have

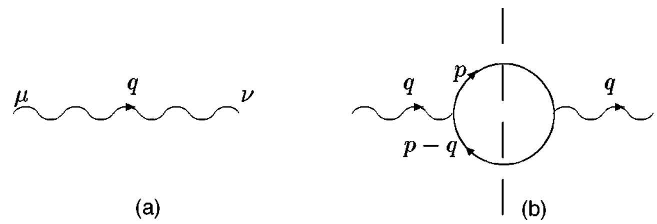

FIG. 1. Propagator of the ρ^+ meson: (a) tree level; (b) one-loop $\pi^+\pi^0$ absorptive correction.

$$D_0^{\mu\nu}(q) = i\left(\frac{-g^{\mu\nu} + \dfrac{q^\mu q^\nu}{m^2}}{q^2 - m^2}\right) = -\frac{iT^{\mu\nu}(q)}{q^2-m^2} + \frac{iL^{\mu\nu}(q)}{m^2}, \tag{1}$$

where $T^{\mu\nu}(q) \equiv g^{\mu\nu} - q^\mu q^\nu/q^2$ and $L^{\mu\nu}(q) \equiv q^\mu q^\nu/q^2$ are the transverse and longitudinal projectors, respectively. On the other hand, Lorentz covariance and CP-invariance impose the electromagnetic vertex to be given by [15] [vector mesons are taken as virtual and the photon is real; the momenta flow as shown in Fig. 2(a)]

$$e\Gamma_0^{\mu\nu\lambda} = e((q_1+q_2)^\mu g^{\nu\lambda} + (k^\nu g^{\mu\lambda} - k^\lambda g^{\mu\nu})\beta(0) - q_1^\nu g^{\mu\lambda} - q_2^\lambda g^{\mu\nu}). \tag{2}$$

In the previous equation e denotes the positron charge, and $\beta(0)$ is the magnetic dipole moment of the vector meson in units of $e/2m_V$, with m_V the mass of the vector meson.

The special value $\beta(0)=2$, which is considered as a criterion of elementarity [16], would correspond to the *canonical* value of the giromagnetic ratio. Also, it has been shown [17] that this is the natural value of the magnetic dipole moment of a composite spin-one system that consists of two spin-1/2 elementary components moving collinearly, with equal charge-mass ratios ($e_1/m_1 = e_2/m_2$). Therefore, deviations from this canonical value would reflect the dynam-

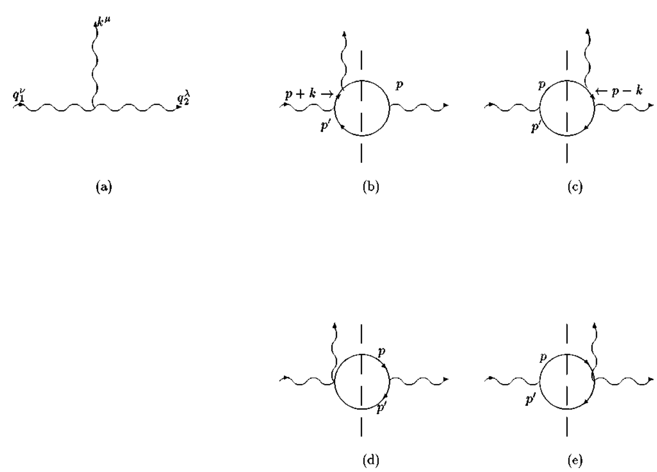

FIG. 2. Electromagnetic vertex of the ρ^+ meson: (a) tree level; (b)–(e) one-loop $\pi^+\pi^0$ absorptive correction.

ics of the internal structure of the meson. For example, as obtained from different phenomenological quark models, the magnetic dipole moment of the $\rho(770)$ and $K^*(892)$ vector mesons are predicted to be [18]: $\beta_\rho(0)\approx 2.2\sim 3.0$ and $\beta_{K^*}(0)\approx 2.37$ in the corresponding units of $e/2m_V$.

We can easily check that Eqs. (1) and (2) satisfy the lowest order electromagnetic Ward identity given by

$$k_\mu \Gamma_0^{\mu\nu\lambda} = [iD_0^{\nu\lambda}(q_1)]^{-1} - [iD_0^{\nu\rho}(q_2)]^{-1}. \tag{3}$$

In order to satisfy the Ward identity in the presence of a finite width of the vector meson, let us follow a method similar to Refs. [10]. Following the usual procedure [10], we add the absorptive correction shown in Fig. 1(b) to the lowest order propagator and we perform the Dyson summation of these graphs to end with the next form of the dressed propagator:

$$D_{\mu\nu}(q) = -\frac{iT^{\mu\nu}(q)}{q^2 - m^2 + i\,\mathrm{Im}\,\Pi^T(q^2)} + \frac{iL^{\mu\nu}(q)}{m^2 - i\,\mathrm{Im}\,\Pi^L(q^2)}, \tag{4}$$

where $\mathrm{Im}\,\Pi^T(q^2)$ and $\mathrm{Im}\,\Pi^L(q^2)$ are the transverse and longitudinal pieces of the absorptive part of the self-energy correction:

$$\mathrm{Im}\,\Pi^{\mu\nu}(q) = \mathrm{Im}\,\Pi^T(q^2) T^{\mu\nu}(q) + \mathrm{Im}\,\Pi^L(q^2) L^{\mu\nu}(q). \tag{5}$$

The Feynman rules needed to evaluate the absorptive corrections can be obtained from the gauged version of the VPP interaction Lagrangian:

$$\mathcal{L} = \frac{ig}{\sqrt{2}} \mathrm{Tr}(V_\mu P D^\mu P - V_\mu D^\mu P P) \tag{6}$$

where $V_\mu = \lambda_a V_\mu^a/\sqrt{2}$ and $P = \lambda_a P^a/\sqrt{2}$ (λ_a the Gell-Mann matrices) stand for the SU(3) octet of vector and pseudoscalar mesons and, $g\approx 6.0$ is the $\rho\pi\pi$ coupling constant obtained from $\rho\to\pi\pi$. The matrix form of the photonic covariant derivative is $D^\mu P = \partial^\mu P + ie[Q,P]A^\mu$ where $Q = \mathrm{diag}(2/3,-1/3,-1/3)$ is the quark-charge matrix and A_μ is the electromagnetic four-potential.

Using cutting techniques and the Feynman rules obtained from Eq. (6), the absorptive part of the self-energy correction becomes [see Fig. 1(b)]

$$\mathrm{Im}\,\Pi^{\mu\nu}(q) = -\frac{g^2 \lambda^{1/2}(q^2, m_\pi^2, m_{\pi'}^2)}{64\pi^2 q^2} \times \int d\Omega (2p-q)^\mu (2p-q)^\nu \theta \times (q^2 - (m_\pi + m_{\pi'})^2). \tag{7}$$

Using the results given in Appendix B for the one-point integrals and the decomposition given in Eq. (5) we get

$$\mathrm{Im}\,\Pi^T(q^2) = \sqrt{q^2}\,\Gamma_\rho(q^2), \tag{8}$$

$$\mathrm{Im}\,\Pi^L(q^2) = -\frac{g^2 \lambda^2(q^2, m_\pi^2, m_{\pi'}^2)}{16\pi} \left(\frac{m_\pi^2 - m_{\pi'}^2}{q^2}\right)^2, \tag{9}$$

where $\lambda(x,y,z) \equiv x^2+y^2+z^2-2xy-2xz-2yz$, m_π ($m_{\pi'}$) denotes the charged (neutral) pion mass, and

$$\Gamma_\rho(q^2) = \frac{g^2}{48\pi q^2}\left(\frac{\lambda(q^2, m_\pi^2, m_{\pi'}^2)}{q^2}\right)^{3/2} \tag{10}$$

denotes the energy-dependent (or off-shell) decay width of the ρ meson. Therefore, the denominator in Eq. (4) gets the Breit-Wigner shape used to describe the energy distribution typical of a resonance.

The absorptive corrections to the electromagnetic vertex can be computed from the cut diagrams shown in Figs. 2(b)–2(e). The relevant Feynman rules describing the $\rho\pi\pi$ and $\rho\pi\pi\gamma$ vertices are obtained from the Lagrangian density given above. A lengthy but straightforward evaluation of the four Feynman graphs in Fig. 2 leads to the following form for the full electromagnetic vertex (see Appendices A and B for details):

$$e\Gamma^{\mu\nu\lambda} = e(\Gamma_0^{\mu\nu\lambda} + \Gamma_1^{\mu\nu\lambda}), \tag{11}$$

where the absorptive correction is given by

$$e\Gamma_1^{\mu\nu\lambda} = \sum_{i=a}^{d} I^{\mu\nu\lambda}(i), \tag{12}$$

with the $I^{\mu\nu\lambda}(i)$ terms as given in Appendix A. In the right hand side of Eq. (12) we will drop terms proportional to k^μ because we are considering the electromagnetic vertex with a real photon and two virtual vector mesons, and k^μ terms do not contribute to the Ward identity or the $\tau\to\pi\pi\nu\gamma$ decay amplitude.

It is straightforward to check that the explicit results obtained for the electromagnetic vertex [Eq. (11)] and the propagator [Eq. (4)] of the $\rho^\pm$ vector meson satisfy the electromagnetic Ward identity:

$$k_\mu \Gamma^{\mu\nu\lambda} = [iD^{\nu\lambda}(q_1)]^{-1} - [iD^{\nu\lambda}(q_2)]^{-1} \tag{13}$$

or, in terms of the absorptive one-loop corrections, it reads

$$k_\mu \Gamma_1^{\mu\nu\lambda} = i\,\mathrm{Im}\,\Pi^{\nu\lambda}(q_1) - i\,\mathrm{Im}\,\Pi^{\nu\lambda}(q_2). \tag{14}$$

After we have proved that electromagnetic gauge-invariance is satisfied with the two- and three-point Green functions given in Eqs. (4) and (11), it is interesting to consider two special cases. The first is to realize that in the limit of isospin symmetry, namely $m_\pi = m_{\pi'}$, the longitudinal piece of the absorptive self-energy correction [see Eq. (9)] vanishes and we obtain the explicit expressions:

$$D_I^{\mu\nu}(q) = i\left(\frac{-g^{\mu\nu} + \dfrac{q^\mu q^\nu}{m^2}(1 + i\gamma_I(q^2))}{q^2 - m^2 + i\gamma_I(q^2) q^2}\right), \tag{15}$$

$$\Gamma_1^{\mu\nu\lambda}(I)=\frac{g^2}{16\pi(q_1^2-q_2^2)}\{B_1F_1^{\mu\nu}k^\lambda+B_2F_2^{\mu\lambda}k^\nu+A_1q_1^\mu T_1^{\nu\lambda}-A_2q_2^\mu T_2^{\nu\lambda}+((A_1+B_1)[F_1^{\mu\lambda}F_1^{\alpha\nu}+F_1^{\mu\nu}F_1^{\alpha\lambda}]\times(q_1)_\alpha-(1\rightarrow 2))\}. \quad (16)$$

where the tensors $F_i^{\alpha\beta}$ and $T_i^{\alpha\beta}$ are defined in Appendix A, and $\gamma_I(q^2)\equiv\Gamma_I(q^2)/\sqrt{q^2}$, with $\Gamma_I(q^2)$ the width of the vector meson Eq. (10) taken in the isospin symmetry limit. The coefficients A_i, B_i $(i=1,2)$ that appear in Eq. (16) are functions of q_i^2 defined by

$$A_i=\frac{2(q_i^4-4q_i^2m_\pi^2)^{3/2}}{3q_i^4}, \quad (17)$$

$$B_i=2m_\pi^2\ln\left(\frac{q_i^2+\sqrt{q_i^4-4q_i^2m_\pi^2}}{q_i^2-\sqrt{q_i^4-4q_i^2m_\pi^2}}\right)-\sqrt{q_i^4-4q_i^2m_\pi^2}. \quad (18)$$

A second interesting case is the limit of massless pseudoscalar mesons $(m_\pi\rightarrow 0)$ appearing in the loop corrections (we will attach a label ch, for chiral, to the corresponding results). In this case $A_i\rightarrow 2q_i^2/3$, $B_i\rightarrow -q_i^2$, hence the propagator and the electromagnetic vertex of Eqs. (15) and (16) get the simple forms:

$$D_{ch}^{\mu\nu}(q)=i\left(\frac{-g^{\mu\nu}+\dfrac{q^\mu q^\nu}{m^2}(1+i\gamma)}{q^2-m^2+i\gamma q^2}\right), \quad (19)$$

$$\Gamma_1^{\mu\nu\lambda}(ch)=i\gamma\Gamma_0^{\mu\nu\lambda}, \quad (20)$$

where $\gamma=\Gamma/m$ with $\Gamma=\Gamma_\rho(q^2=m^2)$ as given by Eq. (10) in the chiral limit.

Observe that Eq. (19) can be rewritten as

$$D_{ch}^{\mu\nu}(q)=i\left(\frac{-g^{\mu\nu}+\dfrac{q^\mu q^\nu}{m_\rho^2-im_\rho\Gamma_\rho}}{(1+i\gamma_\rho)(q^2-m_\rho^2+im_\rho\Gamma_\rho)}\right), \quad (21)$$

if we redefine the mass and width of the unstable meson according to [19]:

$$m_\rho=\frac{m}{\sqrt{1+\gamma^2}},\quad \gamma_\rho=\frac{\Gamma_\rho}{m_\rho}=\frac{\Gamma}{m}=\gamma. \quad (22)$$

The form of propagator for the unstable charged spin-one particle given in Eq. (21), was derived in Ref. [20] in general terms. This form has the advantage to maintain gauge-invariance of an amplitude that involves this resonance as an intermediate state.

Equations (19) and (20) are identical to the results obtained for the absorptive corrections to the propagator and the electromagnetic vertex of the W gauge boson in the fermion loop-scheme, when fermions running in the loops are massless [10]. This is an interesting result because gauge-invariance restricts the form of the two- and three-point Green functions for the W and ρ particles to be the same, despite the fact the origin of these corrections (loops with fermions and bosons, respectively) is very different in each case. Based on this observation, we might conclude that the absorptive parts of the one-loop corrections (with massless particles in the loop) to the propagator and electromagnetic vertex of charged spin-one unstable particles have the universal forms given in Eqs. (19) and (20).

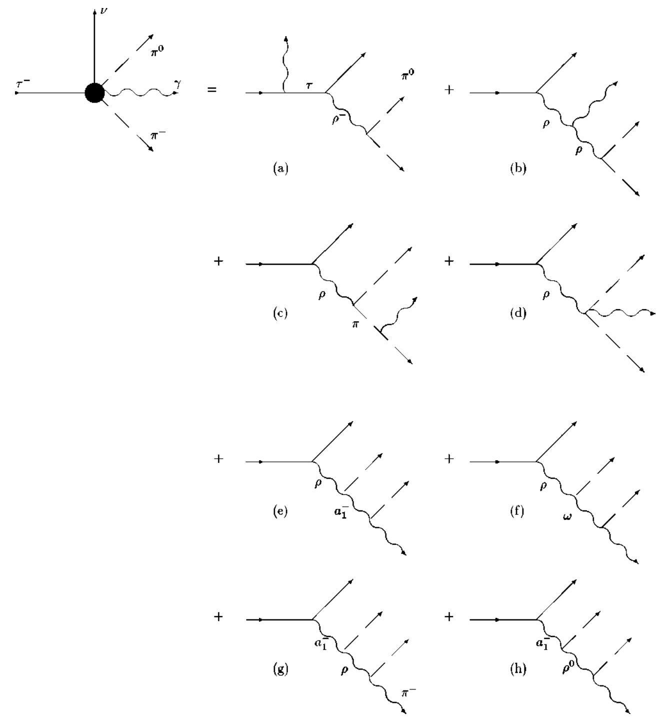

FIG. 3. Feynman diagrams for the $\tau\rightarrow\pi\pi\nu_\tau\gamma$ decay: (a)–(d) pure ρ^+ contributions, and (e)–(h) model-dependent contributions.

III. CONTRIBUTIONS TO THE $\tau\rightarrow\pi\pi\nu\gamma$ AMPLITUDE

In this section we compute the gauge-invariant amplitude for the $\tau^-\rightarrow\pi^-\pi^0\nu_\tau\gamma$ decay. This amplitude can be computed in a simple way following Low's theorem procedure of attaching the photon to the external charged particles of the non-radiative process and fixing the contributions from internal lines emission by requiring gauge invariance (see for example [9]). This method however does not allow to fix the contribution of the $\rho^\pm$ magnetic dipole moment because this term is gauge-invariant by itself. Therefore, we use a dynamical model that incorporates the electromagnetic vertex and the propagator of the intermediate $\rho^\pm$ unstable vector meson given in the previous section.

Our convention for the four-momenta and polarization four-vectors of the particles are indicated in Fig. 3. We find convenient to introduce the following four-vectors: $Q=p-p'=q+q'+k$ and $Q'=q+q'$ (Q'^2 is the squared invariant mass of the two-pion system), such that the energy-momentum conservation is expressed through $Q=Q'+k$.

Since the photon is real, we have the conditions

$$k\cdot\epsilon=k^2=0$$

$$Q\cdot k=Q'\cdot k=(Q^2-Q'^2)/2.$$

We can split the amplitude according to the two types of contributions:

$$\mathcal{M}=\mathcal{M}(\rho)+\mathcal{M}(MD), \tag{23}$$

where $\mathcal{M}(\rho)$ denotes the contributions involving only the ρ meson as intermediate states [Figs. 3(a)–3(d)], and $\mathcal{M}(MD)$, which we call model-dependent terms, denotes the remaining [Figs. 3(e)–3(h)] contributions. We will focus our attention on the first term on Eq. (23), since we expect the pure ρ contributions to dominate the process for values of the two-pion invariant mass distribution in the vicinity of the ρ meson mass.

In the evaluation of the different contributions to $\mathcal{M}(\rho)$ we use the electromagnetic vertex and the propagator of the ρ vector meson given in Eqs. (20) and (21), i.e. with the absorptive corrections for massless pseudoscalars. As we can check, the simple form given in Eq. (21) does not exactly account for the measured Breit-Wigner shape of the two-pion invariant mass distribution of the $\tau\to\pi\pi\nu_\tau$ decay [2]. However, we can use this simple form for the purposes to estimate the effects of the $\rho^\pm$ magnetic dipole moment in the two-pion invariant mass distribution *close* to the ρ resonance region.

After some simplifications, the pure ρ contributions to the amplitude can be written as follows:

$$\mathcal{M}(\rho)=\frac{G_F V_{ud}}{\sqrt{2}}egg_\rho\{l^\alpha H_\alpha+l'^\alpha H'_\alpha\}\times\frac{1}{1+i\gamma_\rho}, \tag{24}$$

where the factor $[1+i\gamma_\rho]^{-1}$ arises from the denominator in Eq. (21). The other factors in Eq. (24) denote the following: G_F is the Fermi constant, V_{ud} is the ud Cabibbo-Kobayashi-Maskawa matrix element, and $g_\rho\approx 0.166$ GeV^2 (see for example Ref. [22]) denote the strength of the $W-\rho^+$ coupling. The four-vectors l^α, l'^α, H_α, H'_α are leptonic and hadronic currents defined as follows:

$$l^\alpha=\overline{u}(p')\gamma^\alpha(1-\gamma_5)u(p), \tag{25}$$

$$l'^\alpha=\overline{u}(p')\gamma^\alpha(1-\gamma_5)\left(\frac{\not k\not\epsilon}{-2p\cdot k}\right)u(p), \tag{26}$$

$$\begin{aligned}H_\alpha=\frac{1}{Q^2-\tilde{m}_\rho^2}\Bigg\{&-a(q-q')_\alpha-\left(1+\frac{2q\cdot k}{Q'^2-\tilde{m}_\rho^2}\beta(0)\right)c_\alpha\\&-4a\left(1-\frac{\beta(0)}{2}\right)\left(\frac{Q_\alpha}{\tilde{m}_\rho^2}\right)\frac{q\cdot kQ\cdot k}{Q'^2-\tilde{m}_\rho^2}+\frac{Q\cdot k}{Q'^2-\tilde{m}_\rho^2}\beta(0)d_\alpha\Bigg\}\\&+\frac{b}{Q'^2-\tilde{m}_\rho^2}(q-q')_\alpha,\end{aligned} \tag{27}$$

$$H'_\alpha=-\frac{(q-q')_\alpha}{Q'^2-\tilde{m}_\rho^2}, \tag{28}$$

where $\tilde{m}_\rho^2\equiv m_\rho^2-im_\rho\Gamma_\rho$ is the ρ pole position.

The quantities a, b, c_α and d_α in Eq. (27) denote gauge invariant factors defined as follows:

$$a=\frac{q\cdot\epsilon}{q\cdot k}-\frac{Q\cdot\epsilon}{Q\cdot k}, \tag{29}$$

$$b=\frac{p\cdot\epsilon}{p\cdot k}-\frac{Q\cdot\epsilon}{Q\cdot k}, \tag{30}$$

$$c_\alpha=\frac{q\cdot\epsilon}{q\cdot k}k_\alpha-\epsilon_\alpha, \tag{31}$$

$$d_\alpha=\frac{Q\cdot\epsilon}{Q\cdot k}k_\alpha-\epsilon_\alpha. \tag{32}$$

The four-vectors c_α, d_α satisfy the following conditions:

$$k\cdot c=k\cdot d=0, \tag{33}$$

$$\epsilon\cdot c=\epsilon\cdot d=1, \tag{34}$$

$$c\cdot c=d\cdot d=c\cdot d=-1, \tag{35}$$

$$Q\cdot c=(Q\cdot k)a,\quad Q\cdot d=0, \tag{36}$$

$$q\cdot c=0,\quad q\cdot d=-(q\cdot k)a, \tag{37}$$

$$p\cdot c=p\cdot k(a-b),\quad p\cdot d=-(p\cdot k)b. \tag{38}$$

Since Eqs. (29)–(32) vanish identically when $\epsilon\to k$, electromagnetic gauge-invariance of the decay amplitude is guaranteed.

IV. EFFECTS OF THE MAGNETIC DIPOLE MOMENT IN THE DIFFERENTIAL DISTRIBUTION OF PHOTONS

In this section we discuss the effects of the magnetic dipole moment of the ρ meson in the differential decay distribution of photons in the $\tau\to\pi\pi\nu\gamma$ decay. We shall start our analysis by fixing the kinematics.

A four body decay can be described in terms of five independent kinematical variables. In the rest frame of the τ lepton we choose them as follows: the charged pion energy (E), the total energy of the two pion system (Q'_0), the photon energy (ω), the angle between the photon and charged pion three-momenta (θ) and the squared invariant mass of the two-pion system (Q'^2). Based on our experience with the analysis of the magnetic dipole moment effects in the radiative decay [5] and production [6] processes of the ρ^+ meson, we will focus on the distribution of radiated photons at small values of θ. Furthermore, in order to suppress the contributions due to diagrams in Figs. 3(e)–3(h), we will include also the distribution in the two-pion invariant mass and set $Q'^2=m_\rho^2$.

In terms of these variables, the differential decay rate in

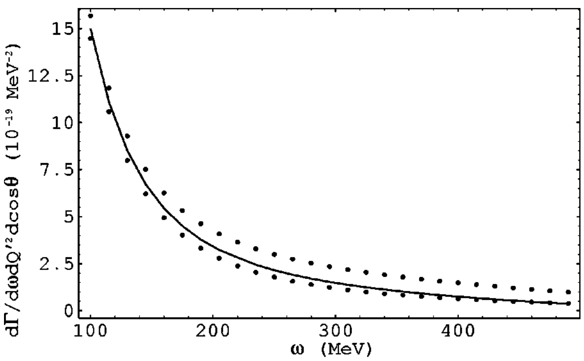

FIG. 4. Two-pion invariant mass and angular-energy photon distributions in $\tau\rightarrow\pi\pi\nu_\tau\gamma$ decay for $\theta=15°$ and $Q'^2=m_\rho^2$: solid line $\beta(0)=2$, upper dotted line $\beta(0)=3$ and lower dotted line $\beta(0)=1$.

the rest frame of the τ lepton can be written as follows:

$$\frac{d\Gamma}{dQ'^2 d\omega d\cos\theta}=\frac{1}{32m_\tau(2\pi)^5}\times\int_{Q'^{min}_0}^{Q'^{max}_0}\overline{|\mathcal{M}(\rho)|^2}\sqrt{1-\frac{4m_\pi^2}{Q'^2}}dQ'_0, \tag{39}$$

where $\overline{|\mathcal{M}(\rho)|^2}$ is the unpolarized decay probability averaged over the τ lepton polarizations.

The limits of the integration region in Eq. (39) are given by

$$Q'^{min}_0=\frac{(m_\tau-2\omega)^2+Q'^2}{2(m_\tau-2\omega)}$$

$$Q'^{max}_0=\frac{m_\tau^2+Q'^2}{2m_\tau}.$$

In Fig. 4 we plot the differential distribution given in Eq. (39) as a function of the photon energy for a fixed value of the small angle $\theta(=15°)$ and by setting $Q'^2=m_\rho^2$. The different curves correspond to three different values of the magnetic dipole moment of the vector meson: $\beta(0)=1$ (lower dotted curve), $\beta(0)=2$ (solid line) and $\beta(0)=3$ (upper dotted curve). These plots are not as sensitive to the effects of the ρ^+ magnetic dipole moment as their corresponding counterparts in radiative ρ^+ decay [5] and production [6]. However, the observable Eq. (39) increases by more than 30% in a wide range of photon energies when $\beta(0)$ increases by one unit of $e/2m_\rho$ with respect to its canonical value. This is interesting because the quark model predictions [18] for the ρ^+ magnetic dipole moment lie systematically above $\beta(0)=2$.

One of the reasons for the lost sensitivity of the angular and energy distributions of photons in $\tau\rightarrow\pi\pi\nu_\tau\gamma$ with respect to $\rho^+\rightarrow\pi^+\pi^0\gamma$ (or $\tau^-\rightarrow\rho^-\nu_\tau\gamma$) decays is that the effects of the magnetic dipole moment in the former enter at order ω while in the second it does at order ω^0.

V. SUMMARY AND CONCLUSIONS

In this paper we have analyzed the contribution of the $\rho^+(770)$ vector meson to the $\tau\rightarrow\pi\pi\nu_\tau\gamma$ decay with the aim to study the effects of its magnetic dipole moment in this reaction. We have introduced a dynamical model to incorporate the finite width effects (which we call *boson loop-scheme*) of the vector meson that is consistent with electromagnetic gauge invariance. In this boson loop-scheme, the finite width is naturally incorporated by adding the absorptive parts of the one loop corrections to the propagator and electromagnetic vertex of the $\rho^\pm$ vector meson. The results obtained for these Green functions in the chiral limit (loops with massless pseudoscalars) are identical to those obtained in the fermion loop-scheme [10] (with massless fermions) for the $W^\pm$ gauge boson.

As is known [21], the decay amplitude for the $\tau\rightarrow\pi\pi\nu_\tau\gamma$ can be rendered gauge-invariant in a simple form (with an arbitrary form of the $\rho^\pm$ vector meson propagator) by computing the photon emission from external lines and fixing internal contributions by imposing gauge invariance on the total amplitude. This method, however, does not permit to fix the contribution of the $\rho^\pm$ magnetic dipole moment because this term is gauge-invariant by itself. This is the reason to require the use of a dynamical model to incorporate the $\rho\rho\gamma$ vertex and the finite width effects in a consistent way.

In the present analysis of the τ lepton radiative decay we have suppressed the model-dependent contributions by setting the invariant mass of the two pions to the rho meson mass value. As is known [21] these contributions would enter the decay amplitude at the same order in the photon energy as it does the magnetic moment of the $\rho^\pm$ vector meson. The photon energy spectrum in the τ decay considered, taken at small angles of photon emission (with respect to the final charged pion), is not as sensitive to the effects of the magnetic dipole moment as in the case of decay [5] or production [6] processes of the vector meson. However, the observable given in Eq. (39) shows an appreciable sensitivity when $\beta(0)$ increases its value with respect to its canonical value.

To conclude, let us emphasize the pertinence of the present work. In our previous papers [5,6] the vector meson was assumed to be stable. That would be a good approximation if the production (decay) process could be separated experimentally from the decay (production) mechanism in a model-independent way and/or if some appropriate kinematical cuts allowed the isolation of these partial processes to be done. On the other hand, from a conceptual point of view, the representation of a vector meson as an asymptotic state in the evaluation of the S-matrix amplitude (as done in Refs. [5,6]) is not completely justified because this vector meson is a broad resonance. Those difficulties justify the relevance of the present work.

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APPENDIX A: ABSORPTIVE CORRECTIONS TO THE ELECTROMAGNETIC VERTEX

The flow of particle momenta in the absorptive corrections to the electromagnetic vertex of the $\rho^{\pm}$ vector meson is indicated in Figs. 2(b)–2(e). The absorptive amplitudes corresponding to the different contributions in Fig. 3 are given, respectively, by

$$I^{\mu\nu\lambda}(a)=\frac{1eg^2}{8\pi^2}\int\frac{d^3p}{2E}\frac{d^3p'}{2E'}\delta^4(q_2-p+p')\times\frac{2p^{\mu}(k+p+p')^{\nu}(p+p')^{\lambda}}{2p\cdot k},\tag{A1}$$

$$I^{\mu\nu\lambda}(b)=\frac{eg^2}{8\pi^2}\int\frac{d^3p}{2E}\frac{d^3p'}{2E'}\delta^4(q_1-p+p')\times\frac{2p^{\mu}(p+p')^{\nu}(p+p'-k)^{\lambda}}{-2p\cdot k},\tag{A2}$$

$$I^{\mu\nu\lambda}(c)=-\frac{eg^2}{8\pi^2}\int\frac{d^3p}{2E}\frac{d^3p'}{2E'}\delta^4\times(q_2-p+p')g^{\mu\nu}(p+p')^{\lambda},\tag{A3}$$

$$I^{\mu\nu\lambda}(d)=-\frac{eg^2}{8\pi^2}\int\frac{d^3p}{2E}\frac{d^3p'}{2E'}\delta^4\times(q_1-p+p')g^{\mu\lambda}(p+p')^{\nu}.\tag{A4}$$

Let us define the factors $U_i\equiv eg^2\lambda^{1/2}(q_i^2,m_{\pi}^2,m_{\pi'})/(32\pi^2q_i^2)$ $(i=1,\,2)$.

Since the photon is real, we can drop the terms proportional to k^{μ} in the evaluation of the above integrals. The integration of Eqs. (A1)–(A4) with the help of the results from Appendix B gives

$$\frac{2I^{\mu\nu\lambda}(a)}{U_2}=2b_1(q_2^2)((k-q_2)^{\nu}F_2^{\mu\lambda}-q_2^{\lambda}F_2^{\mu\nu})+4f_2(q_2^2)\times(k^{\lambda}F_2^{\mu\nu}+k^{\nu}F_2^{\mu\lambda})+4f_1(q_2^2)(q_2^{\lambda}F_2^{\mu\nu}+q_2^{\nu}F_2^{\mu\lambda}+q_2^{\mu}T_2^{\nu\lambda})+\frac{4\pi\Delta^2}{q_2\cdot k}\frac{q_2^{\mu}q_2^{\lambda}}{q_2^2}\left(k^{\nu}+\frac{\Delta^2}{q_2^2}q_2^{\nu}\right)\tag{A5}$$

$$\frac{2I^{\mu\nu\lambda}(b)}{U_1}=2b_1(q_1^2)((q_1+k)^{\lambda}F_1^{\mu\nu}+q_1^{\nu}F_1^{\mu\lambda})-4f_2(q_1^2)\times(k^{\lambda}F_1^{\mu\nu}+k^{\nu}F_1^{\mu\lambda})-4f_1(q_1^2)(q_1^{\lambda}F_1^{\mu\nu}+q_1^{\nu}F_1^{\mu\lambda}+q_1^{\mu}T_1^{\nu\lambda})+\frac{4\pi\Delta}{q_1\cdot k}\frac{q_1^{\mu}q_1^{\nu}}{q_1^2}\left(\frac{\Delta^2}{q_1^2}q_1^{\lambda}-k^{\lambda}\right)\tag{A6}$$

$$\frac{2I^{\mu\nu\lambda}(c)}{U_2}=-\frac{4\pi\Delta^2}{q_2^2}q_2^{\lambda}g^{\mu\nu},\tag{A7}$$

$$\frac{2I^{\mu\nu\lambda}(d)}{U_1}=-\frac{4\pi\Delta^2}{q_1^2}q_1^{\nu}g^{\mu\lambda},\tag{A8}$$

where $\Delta^2=m_{\pi}^2-m_{\pi'}^2$. The f_i, a_i and b_i coefficients are functions of q_k^2 $(k=1,2)$, and their explicit forms are given in Appendix B. The second rank tensors are defined as $F_i^{\mu\nu}\equiv g^{\mu\nu}-q_i^{\mu}k^{\nu}/(q_i\cdot k)$ and $T_i^{\mu\nu}\equiv g^{\mu\nu}-q_i^{\mu}q_i^{\nu}/q_i^2$. Note that $F_1^{\mu\nu}=F_2^{\mu\nu}$, because $q_1\cdot k=q_2\cdot k$ and we can drop terms proportional to k^{μ}. Note also that $F_i^{\mu\nu}$ are not symmetric tensors.

Adding up the four contributions we obtain the absorptive part of the electromagnetic vertex corrections as follows:

$$e\Gamma_1^{\mu\nu\lambda}=\sum_{i=a}^{d}I^{\mu\nu\lambda}(i).\tag{A9}$$

APPENDIX B: TENSOR INTEGRALS

In the evaluation of the absorptive corrections we need to compute different tensor integrals that appear in the two- and three-point Green functions. The explicit form of these integrals are as follows.

1. Two-point integrals

In this case we have only one available external momenta (q) and the metric tensor to express the integrals

$$\int d\Omega p^{\alpha}=\frac{2\pi}{q^2}(q^2+\Delta^2)q^{\alpha}\tag{B1}$$

$$\int d\Omega p^{\alpha}p^{\beta}=\frac{\pi}{3q^2}\left\{4(\lambda(q^2,m_{\pi}^2,m_{\pi'}^2)+3q^2m_{\pi}^2)\times\frac{q^{\alpha}q^{\beta}}{q^2}-\lambda(q^2,m_{\pi}^2,m_{\pi'}^2)g^{\alpha\beta}\right\}\tag{B2}$$

$$\int d\Omega p^{\alpha}p^{\beta}p^{\gamma}=-\frac{\pi(q^2+\Delta^2)}{6q^4}\left\{\lambda(q^2,m_{\pi}^2,m_{\pi'}^2)(g^{\alpha\beta}q^{\gamma}+g^{\alpha\gamma}q^{\beta}+g^{\beta\gamma}q^{\alpha})-2\left(\frac{\lambda(q^2,m_{\pi}^2,m_{\pi'}^2)}{q^2}+2m_{\pi}^2\right)q^{\alpha}q^{\beta}q^{\gamma}\right\}.\tag{B3}$$

2. Three-point integrals

Here, there are two independent external momenta at our disposal (q and k) and the metric tensor to express the integrals

$$\int\frac{d\Omega}{p\cdot k}=N\tag{B4}$$

$$\int d\Omega p^{\alpha}=2\pi\left(1+\frac{\Delta^2}{q^2}\right)q^{\alpha} \tag{B5}$$

$$\int d\Omega \frac{p^{\alpha}}{p\cdot k}=a_1 q^{\alpha}+a_2 k^{\alpha} \tag{B6}$$

$$\int d\Omega \frac{p^{\alpha}p^{\beta}}{p\cdot k}=b_1\left(g^{\alpha\beta}-\frac{k^{\alpha}q^{\beta}+k^{\alpha}q^{\beta}}{q\cdot k}\right)+b_2 q^{\alpha}q^{\beta}+b_3 k^{\alpha}k^{\beta} \tag{B7}$$

$$\int d\Omega \frac{p^{\alpha}p^{\beta}p^{\gamma}}{p\cdot k}=f_1(F^{\alpha\beta}q^{\gamma}+F^{\beta\gamma}q^{\alpha}+F^{\gamma\alpha}q^{\beta})+f_2(g^{\alpha\beta}k^{\gamma}+g^{\beta\gamma}k^{\alpha}+g^{\alpha\gamma}k^{\beta})+f_3 q^{\alpha}q^{\beta}q^{\gamma}+f_4 k^{\alpha}k^{\beta}k^{\gamma}-\frac{2f_2}{q\cdot k}(k^{\alpha}k^{\beta}q^{\gamma}+k^{\alpha}q^{\beta}k^{\gamma}+q^{\alpha}k^{\beta}k^{\gamma}), \tag{B8}$$

where $F^{\alpha\beta}\equiv g^{\alpha\beta}-q^{\alpha}q^{\beta}/q^2$.

The factors N, a_i, b_i and f_i are functions of the scalar q^2:

$$N(q^2)=\frac{2\pi\sqrt{q^2}}{q\cdot kEv}\ln\left(\frac{1+v}{1-v}\right) \tag{B9}$$

$$a_1(q^2)=\frac{4\pi}{q\cdot k} \tag{B10}$$

$$a_2(q^2)=\frac{2\pi q^2}{(q\cdot k)^2}\left[\frac{1}{v}\ln\left(\frac{1+v}{1-v}\right)-2\right] \tag{B11}$$

$$b_1(q^2)=\frac{\pi E\sqrt{q^2}}{q\cdot k}\left[\frac{(1-v^2)}{v}\ln\left(\frac{1+v}{1-v}\right)-2\right] \tag{B12}$$

$$b_2(q^2)=\frac{4\pi E}{q\cdot k\sqrt{q^2}} \tag{B13}$$

$$b_3(q^2)=\frac{\pi E q^4}{(q\cdot k)^3\sqrt{q^2}}\left[\frac{(3-v^2)}{v}\ln\left(\frac{1+v}{1-v}\right)-6\right] \tag{B14}$$

$$f_1(q^2)=-\frac{4\pi E^2 v^2}{3(q\cdot k)} \tag{B15}$$

$$f_2(q^2)=\frac{\pi E^2 q^2}{(q\cdot k)^2}\left[\frac{4v^2}{3}+\frac{(1-v^2)}{v}\ln\left(\frac{1+v}{1-v}\right)-2\right] \tag{B16}$$

$$f_3(q^2)=\frac{4\pi E^2}{3(q\cdot k)q^2}(3+v^2) \tag{B17}$$

$$f_4(q^2)=\frac{\pi E^2 q^4}{(q\cdot k)^4}\left[\frac{(5-3v^2)}{v}\ln\left(\frac{1+v}{1-v}\right)+\frac{8v^2}{3}-10\right], \tag{B18}$$

where $E=(q^2+\Delta^2)/(2\sqrt{q^2})$ and $v=\lambda^{1/2}(q^2,m_{\pi}^2,m_{\pi'}^2)/(q^2+\Delta^2)$.

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