Decoupling of massive right-handed neutrinos

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We investigate the effect of $(B+L)$-violating anomalous generation of massive right-handed neutrinos on their decoupling, when the right-handed neutrino mass is considerably greater than the right-handed gauge boson masses. Considering fermion-antifermion annihilation channels, the Lee-Weinberg type of calculation, in this case, gives an upper bound of about 700 GeV, which casts doubt on the existence of such a right-handed neutrino mass greater than right-handed gauge boson masses. We examine the possibility that a consideration of anomalous effects related to the $\mathrm{SU}(2)_R$ gauge group, together with the effect of *WW* channels, may turn this into a *lower* bound $\sim 10^2$ TeV.

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I. INTRODUCTION

The neutrino oscillation interpretation of recent observations of solar and atmospheric neutrino fluxes, although presenting some inconsistencies, may be taken to strengthen the idea of nonzero neutrino masses. In this situation, in addition to the standard model left-handed neutrinos, the existence and masses of right-handed neutrinos assume topical interest.

The contribution of massive neutrinos to the mass density of the universe allows the setting of a lower bound to such a neutrino mass from the usual cosmological constraints on the age and mass density of the universe [1–3], when the neutrino mass is less than gauge boson masses. For a neutrino mass greater than gauge boson masses, the lower bound has been questioned [4–8]. In the present paper, using for calculation a $L-R$ symmetric extension of the standard model [9–11], we investigate whether a lower bound may, indeed, exist even when the right-handed neutrino mass is greater than gauge boson masses.

In these $L-R$ symmetric models, the breaking of $\mathrm{SU}(2)_R$ gauge symmetry is associated with a critical temperature. This may, typically, be of the order of 1–10 TeV [12–14], and right-handed electron neutrino masses $\approx$10 TeV have been considered, yielding a left-handed electron neutrino mass $\approx 10^{-10}$ GeV, by a see-saw mechanism [14]. Now, $B+L$ is not conserved in standard electroweak theory due to an anomaly involving the SU(2) gauge group [15] and, at temperatures $\geqslant$1 TeV, $(B+L)$-violating transitions occur classically, via thermal fluctuations, at rates higher than the expansion rate of the universe [16]. So, we may expect that similar anomalous generation of right-handed neutrinos (in addition to the left-handed ones), via the $L-R$ symmetric gauge group, may become important near the $\mathrm{SU}(2)_R$-breaking critical temperature.

Although there is still a lot of fluidity in the matter, particle physics and cosmological bounds usually suggest right-handed Z_R and W_R boson masses with values $\geqslant$0.5 TeV and 1.6–3.2 TeV, respectively [17]. If, now, right-handed neutrinos of mass $\geqslant$10 TeV come under consideration in the literature, then it becomes necessary to investigate whether anomalous effects can, indeed, modify significantly the decoupling of right-handed neutrinos with mass greater than right-handed gauge boson masses.

The plan of the paper is as follows. Section I is the Introduction. In Sec. II, the $L-R$ symmetric model is used to evaluate the reduction rate of the right-handed neutrinos through $F\bar{F}$ channels in a standard Lee-Weinberg type of calculation, and to observe how the cosmological bound on their mass becomes an upper one, when this mass is greater than the right-handed gauge boson masses. The effect of the W^+W^- annihilation channel is then discussed. In Sec. III, the anomalous rate of reduction of right-handed neutrinos is related to the general anomalous rate of $B+L$-violating transitions, and the qualitative effect of the anomalous rate on the previously obtained mass bound is estimated, assuming a generic form for the $B+L$-violating rate arising from the anomaly involving the $\mathrm{SU}(2)_R$ gauge group. In Sec. IV, the influence of these anomalous effects on the mass bound is studied numerically, using numbers obtained by a simple extrapolation from the $\mathrm{SU}(2)_L$ result, and taking into account the effect of the W^+W^- channel.

II. DECOUPLING WITHOUT ANOMALOUS EFFECTS

A. Boltzmann equation for processes $N\bar{N}\to F\bar{F}$

We wish to set up a Boltzmann equation for the number density of right-handed neutrinos and, from a calculation of the asymptotic number density, estimate the contribution of these neutrinos to the mass density of the universe, and, hence, set bounds to the right-handed neutrino mass [1,3]. We will simplify matters by neglecting the decay of right-handed neutrinos. We first consider the reduction of right-handed neutrinos by the process $N\bar{N}\to F\bar{F}$, where F is a quark or a lepton, lighter than N. We are interested in investigating the situation when the right-handed neutrino mass is considerably greater than the right-handed gauge boson masses.

To calculate the rate of reduction of right-handed neutrinos we consider the $L-R$ symmetric model [10,11]. This model has pairs of fermion doublets f' belonging to different representations of $SU(2)_L\times SU(2)_R\times U(1)_{B-L}$, similar to

$$\begin{pmatrix}\nu_L\\ e_L\end{pmatrix}\left(\frac{1}{2},\quad 0,\quad -1\right),\quad \begin{pmatrix}\nu_R\\ e_R\end{pmatrix}\left(0,\quad \frac{1}{2},\quad -1\right),$$

$$\begin{pmatrix}u_L\\ d_L\end{pmatrix}\left(\frac{1}{2},\quad 0,\quad \frac{1}{3}\right),\quad \begin{pmatrix}u_R\\ d_R\end{pmatrix}\left(0,\quad \frac{1}{2},\quad \frac{1}{3}\right).$$

The numbers refer to the quantum numbers $T_{3L},T_{3R},B-L$, respectively. We will also write $\nu_R\equiv N$, $\nu_L\equiv\nu$.

The fermion gauge-boson interaction Lagrangian is

$$L_{\rm int}=g(\overline{f'}\gamma_\mu P_L\vec{T}_L f'\cdot\vec{W}^\mu_L+\overline{f'}\gamma_\mu P_R\vec{T}_R f'\cdot\vec{W}^\mu_R)+\frac{1}{2}g'\overline{f'}\gamma_\mu(B-L)f'B^\mu,$$

where $P_{R,L}=\frac{1}{2}(1\pm\gamma_5)$, $\vec{T}$ is the isospin operator and $\vec{W}^\mu,B^\mu$ are the gauge bosons. The neutral currents are set out in the basis

$$A^\mu=\sin\theta(W^\mu_{3L}+W^\mu_{3R})+\sqrt{\cos 2\theta}B^\mu,$$

$$Z^\mu=\cos\theta W^\mu_{3L}-\sin\theta\tan\theta W^\mu_{3R}-\tan\theta\sqrt{\cos 2\theta}B^\mu,$$

$$Z'^\mu=\frac{\sqrt{\cos 2\theta}}{\cos\theta}W^\mu_{3R}-\tan\theta B^\mu, \tag{1}$$

where θ is the Weinberg angle.

Neglecting $Z-Z'$ mixing, one gets the Z' neutral current Lagrangian [17]

$$L^{Z'}_{NC}=\frac{g}{\cos\theta\sqrt{\cos 2\theta}}\left(\sin^2\theta\sum_{f'}\overline{f'}\gamma^\mu[P_L T_{3L}-Q\sin^2\theta]f'+\cos^2\theta\sum_{f'}\overline{f'}\gamma^\mu[P_R T_{3R}-Q\sin^2\theta]f'\right)Z'_\mu. \tag{2}$$

Q is the charge operator.

The charged current Lagrangian consists of terms of the form

$$L_{CC\nu e}=\frac{g}{\sqrt{2}}(\bar{\nu}\gamma_\mu e_L W^\mu_L+\text{H.c.})+\frac{g}{\sqrt{2}}(\bar{N}\gamma_\mu e_R W^\mu_R+\text{H.c.}). \tag{3}$$

Assuming CP symmetry, and equilibrium conditions for all relevant particles except the N neutrinos, the rate of reduction of N neutrinos per unit volume can be obtained from the Boltzmann collision integral for the processes $N\bar{N}\to F\bar{F}$ [18,19]:

$$\Gamma_a=\sum_F\int d\pi_N d\pi_{\bar N} d\pi_F d\pi_{\bar F}(2\pi)^4\times\delta^4(p_N+p_{\bar N}-p_F-p_{\bar F})|\mathcal{M}_{\mathcal{F}}|^2(f_N f_{\bar N}-f_{N\rm eq}f_{\bar N\rm eq}). \tag{4}$$

Here, f is the phase space distribution function and $f_{\rm eq}$ is its equilibrium value. $|\mathcal{M}_{\mathcal{F}}|^2$ is the spin averaged matrix element squared, with proper symmetry factor, for the process $N\bar{N}\to F\bar{F}$, assumed, by $C\bar{P}$ symmetry, to be the same as that for the process $F\bar{F}\to N\bar{N}$. The measure $d\pi_i=g_i d^3p/[(2\pi)^3 2E]$, g_i being the degeneracy number. We assume that there is no significant Fermi degeneracy, so that $1-f\approx 1$.

Because CP symmetry has been assumed, we further assume that there is no N or $\bar{N}$ excess, and we can set $n=\bar{n}$, as well as $\mu_N=0=\mu_{\bar N}$, where n is the number density of the N neutrinos. We can, then, take

$$f_{N\rm eq}=e^{-E_N/T}.$$

The summation is over quarks and leptons lighter than N. Let us take ν and N to be electron neutrinos. We assume right-handed neutrinos of the other two generations to be much more massive than the N neutrinos, so that they are not relevant here.

It is usual to introduce the thermal average of the annihilation cross-section times relative velocity

$$\langle\sigma|v|\rangle=\frac{1}{n^2_{\rm eq}}\sum_F\int d\pi_N d\pi_{\bar N}d\pi_F d\pi_{\bar F}(2\pi)^4\times\delta^4(p_N+p_{\bar N}-p_F-p_{\bar F})|\mathcal{M}_{\mathcal{F}}|^2 e^{-E_N/T}e^{-E_{\bar N}/T},$$

and write Eq. (4) in the form [1]

$$\Gamma_a=\langle\sigma|v|\rangle(n^2-n^2_{\rm eq}),$$

where $n_{\rm eq}$ is the equilibrium value of n. Then the Boltzmann equation for the reduction of N neutrinos by these processes, in a universe expanding with $\dot{R}/R=H$, becomes [19]

$$\frac{dn}{dt}+3Hn=-\langle\sigma|v|\rangle(n^2-n^2_{\rm eq}). \tag{5}$$

B. Calculation of $\langle\sigma|v|\rangle$ from $L-R$ symmetric model

We consider the s-channel process $N(k)+\overline{N(\bar{k})}\xrightarrow{Z'(q)}F(p)+\overline{F(\bar{p})}$, and the t-channel process $N\bar{N}\xrightarrow{W}e\bar{e}$. We will work at temperatures $T<T_{\rm cr}$, the critical temperature corresponding to the breaking of $\mathrm{SU}(2)_L\times\mathrm{SU}(2)_R\times\mathrm{U}(1)_{B-L}$ to $\mathrm{SU}(2)_L\times\mathrm{U}(1)_Y$. We are going to consider N-type neutrinos with a mass M, which is at least an order or two of magnitude larger than $M_{Z'}$ ($M_{Z'}\geq 0.5$ TeV [17]). At this energy scale, we will approximate all quark and the e, μ, τ, ν masses to zero (mass of top≈175 GeV).

Next, we assume that ν and N have Majorana mass eigenstates [20]

$$\chi=\nu+\nu^c,\quad \omega=N+N^c,$$

where the superscript c refers to the charge conjugate field.

It is usual to consider a bidoublet and two triplet Higgs particles to generate Majorana states [11]. In this paper, however, we do not go into the details of any specific model of the Higgs sector. While evaluating the matrix element, we have considered N to be purely Majorana, i.e., we have neglected the contribution of the vector current and doubled that of the axial current by replacing $(1+\gamma_5)$ with $2\gamma_5$ [17].

The spin-averaged matrix element squared, with symmetry factor 1/2!, for the s-channel process gives, from Eq. (2),

$$|\mathcal{M}_{\mathcal{F}}|^2=\frac{1}{2}[g^4/(2\cos^2 2\theta)](C_{VF}^2+C_{AF}^2)[(p\cdot k)(\overline{p}\cdot\overline{k})+(\overline{p}\cdot k)(p\cdot\overline{k})-M^2 p\cdot\overline{p}]\frac{1}{(q^2-M_{Z'}^2)^2},$$

where

$$C_{VF}=T_3-2Q\sin^2\theta,\quad C_{AF}=T_3\cos 2\theta.$$

Now,

$$q^2=(k+\overline{k})^2=s=4E_{\text{c.m.}}^2=4(M^2+\mathbf{k}_{\text{c.m.}}^2)\gg M_{Z'}^2,$$

where $(E_{\text{c.m.}},\mathbf{k}_{\text{c.m.}})$ is the four-vector k in the c.m. frame. So, we approximate $1/(q^2-M_{Z'}^2)^2$ by $1/q^4$.

We calculate $\langle\sigma|v|\rangle$ in two steps. First, we calculate

$$I_F=\int d\pi_F d\pi_{\overline{F}}(2\pi)^4\delta^4(k+\overline{k}-p-\overline{p})|\mathcal{M}_{\mathcal{F}}|^2$$

in the c.m. frame. The result is

$$I_F=[g^4/(64\pi\cos^2 2\theta)](C_{VF}^2+C_{AF}^2)(2/3)\beta^2,$$

where β is the relative velocity$=2|\mathbf{k}_{\text{c.m.}}|/E_{\text{c.m.}}$. This p-wave term is a signature of Majorana neutrino annihilation.

In the Lee-Weinberg type of decoupling calculation, the N neutrinos may be considered to be nonrelativistic, as the relevant temperatures are of the order of M. Then, in the comoving ''lab'' frame, where $\overline{\mathbf{k}}$ makes an angle α with $\mathbf{k}$,

$$I_F=[g^4/(64\pi\cos^2 2\theta)](C_{VF}^2+C_{AF}^2)(2/3)\times(\mathbf{k}^2+\overline{\mathbf{k}}^2-2|\mathbf{k}|\,|\overline{\mathbf{k}}|\cos\alpha)/M^2. \tag{6}$$

In the second step we do the thermal averaging. Then,

$$\langle\sigma|v|\rangle_F=\frac{\int d\pi_N e^{-E_N/T}\int d\pi_{\overline{N}}e^{-E_{\overline{N}}/T}I_F}{\int[g_N d^3k/(2\pi)^3]e^{-E_N/T}\int[g_{\overline{N}}d^3\overline{k}/(2\pi)^3]e^{-E_{\overline{N}}/T}}.$$

Calculation, in the nonrelativistic approximation for the N neutrinos, gives

$$\langle\sigma|v|\rangle_F=\frac{g^4}{(64\pi\cos^2 2\theta)}(C_{VF}^2+C_{AF}^2)\frac{1}{M^2}\frac{T}{M}.$$

The effect of $N\overline{N}\to e\overline{e}$ can be taken into consideration by the usual Fierz rearrangement, which gives, in this case,

$$C_{Ve}/\cos 2\theta\to(C_{Ve}/\cos 2\theta)+1,$$

$$C_{Ae}/\cos 2\theta\to(C_{Ae}/\cos 2\theta)+1.$$

Finally, we get

$$\langle\sigma|v|\rangle=\frac{g^4}{(64\pi\cos^2 2\theta)}\frac{1}{M^2}\frac{T}{M}\sum_F(C_{VF}^2+C_{AF}^2). \tag{7}$$

So, effectively, $\langle\sigma|v|\rangle\sim 1/M^2$, as $T\sim M$.

C. Mass bound for right-handed neutrinos

Introducing $x=M/T$ and $Y=n/s$, where s is the entropy density, Eq. (5) becomes

$$(1.66g^{*(1/2)}/x^4)(M^5/M_{\text{Pl}})\frac{2\pi^2}{45}g_s^*\frac{dY}{dx}=-\left(\frac{2\pi^2}{45}g_s^*\right)^2\frac{M^6}{x^6}\langle\sigma|v|\rangle(Y^2-Y_{\text{eq}}^2) \tag{8}$$

or

$$\frac{dY}{dx}=-0.26g^{*(1/2)}\langle\sigma|v|\rangle(MM_{\text{Pl}}/x^2)(Y^2-Y_{\text{eq}}^2). \tag{9}$$

We take $M_{\text{Pl}}=1.22\times10^{19}$ GeV and $g^*\approx g_s^*\approx 100$ just below the critical temperature, considering N,W_R,Z' (and N_μ,N_τ) to be massive (we have not counted Higgs degrees of freedom).

Summing over all quarks and leptons, except the three right-handed neutrinos, we get, on calculation,

$$\sum_F(C_{VF}^2+C_{AF}^2)=3.28\,(\text{taking }\sin^2\theta=0.23).$$

Taking $g=0.65$, $\langle\sigma|v|\rangle=0.01/(M^2x)$.

For massive Majorana neutrinos, we get, in the nonrelativistic approximation [19]

$$Y_{\text{eq}}=2.89\times10^{-3}x^{3/2}e^{-x}. \tag{10}$$

From Eq. (9),

$$\frac{dY}{dx}=-[3.16\times10^{17}/(Mx^3)](Y^2-Y_{\text{eq}}^2). \tag{11}$$

We write $\Delta=Y-Y_{\text{eq}}$.

Then, before decoupling, $Y\approx Y_{\text{eq}}$, and $\Delta'\sim 0$, giving

$$\Delta\cong -Mx^3Y_{\text{eq}}'/[3.16\times10^{17}(2Y_{\text{eq}}+\Delta)].$$

Now, we put $\Delta=cY_{\text{eq}}$ at decoupling, where $c\sim 1$. According to the general numerical analysis of this type of decoupling [19], $c(c+2)=2$ when $\langle\sigma|v|\rangle\sim T$.

At decoupling, when $x=x_d\gg 1$, $Y_{\text{eq}}'\approx -Y_{\text{eq}}$ and

$$\Delta(x_d)\cong cY_{\rm eq}(x_d)=Mx_d^3/[3.16\times10^{17}(c+2)]. \quad (12)$$

This leads to

$$x_d\cong 35.14-\ln M-1.5\ln(35.14-\ln M). \quad (13)$$

After decoupling, $Y\gg Y_{\rm eq}$ and $\Delta\approx Y$.

From Eq. (11), we get

$$\Delta'=-3.16\times10^{17}\Delta^2/(Mx^3),$$

which gives, on integration, at $t\rightarrow\infty$,

$$\Delta_\infty=Y_\infty=2Mx_d^2/(3.16\times10^{17}),$$

assuming, $Y(x_d)\gg Y_\infty$. We will take, as our cosmological bound [19],

$$\Omega_N h^2<1, \quad \text{where } \Omega_N=\rho_N/\rho_c=Ms_0Y_\infty/\rho_c.$$

Here, it is assumed that $h>0.4$.

Taking $s_0=2970\,\mathrm{cm}^{-3}$ and $\rho_c=h^2 1.88\times10^{-29}\,\mathrm{g\,cm}^{-3}$, we get

$$\Omega_N h^2=2.80851\times10^8 MY_\infty \quad (14a)$$

$$=3.62\times10^{-9}M^2x_d^2, \quad (14b)$$

where M is to be taken in GeV. At the bound,

$$3.62\times10^{-9}M^2x_d^2=1.$$

Solving this equation and Eq. (13) simultaneously, using simple numerical methods, we get

$$x_d=23.55, \quad M=706\,\mathrm{GeV}.$$

Now, if we omit $\ln M$ in the third term on the right-hand side of Eq. (13), we get, approximately,

$$x_d=29.80-\ln M. \quad (15)$$

If we make this approximation, the error in x_d is less than 5 percent, even if M is as large as 10^6 GeV. Using (14b), we get

$$d(\Omega_N h^2)/dM=3.62\times10^{-9}\times2M(29.80-\ln M)$$
$$\times(28.80-\ln M),$$

which is positive for all practical purposes. This means that $\Omega_N h^2<1$ fixes an *upper* bound for M.

This can be seen transparently if we work in the approximation

$$Y_\infty\approx Y(x_d)\approx 2Y_{\rm eq}(x_d),$$

taking $c\approx1$ in Eq. (12). Then,

$$\Omega_N h^2\sim Mx_d^{3/2}e^{-x_d}.$$

Equation (15) shows that $\Omega_N h^2\sim M^2(29.80-\ln M)^{3/2}$, and, so, as M increases, $\Omega_N h^2$ increases, for all practical values of M.

TABLE I. Mass bound (no anomalous effects).

M(GeV)	$\Omega_N h^2$
10 000	160
5000	42.6
1000	1.95
750	1.12
706	1.00
500	0.516
250	0.136

This conclusion can be verified, numerically, by giving M different values in Eq. (13) and substituting the resulting x_d in Eq. (14b). The results are shown in Table I. In the usual Lee-Weinberg case, with $M\ll$gauge boson masses, one gets a *lower* bound because $\langle\sigma|v|\rangle\sim M^2$, which leads to $x_d\sim3\ln M+$const, and $\Omega_N h^2\sim1/M^2$. With $M\gg$gauge boson masses, $\langle\sigma|v|\rangle\sim1/M^2$, and this makes the difference.

$M<706$ GeV is, in effect, incompatible with our assumption of $M>$right-handed gauge boson masses, because, as we have remarked earlier, particle physics and cosmological bounds suggest right-handed gauge boson masses $\sim0.5-1$ TeV or more. We have to conclude that the assignment of any realistic mass, greater than right-handed gauge boson masses, to the right-handed neutrinos will violate the cosmological bound $\Omega_N h^2<1$.

D. Effect of $N\bar{N}\rightarrow W^+W^-$

However, we have considered only the processes $N\bar{N}\rightarrow F\bar{F}$. There are also $N\bar{N}\rightarrow W^+W^-$ processes with Z' or Higgs exchange in the s channel and charged lepton exchange in the t channel [6–8]. The arguments of Ref. [6] show that as $s\rightarrow4M^2$ (which is the interesting region for decoupling), Z' exchange dominates.

For $M\gg M_W$, using the results of Ref. [6], the authors of Ref. [8] find, for the $N\bar{N}\rightarrow W^+W^-$ cross section

$$\langle\sigma'|v|\rangle\cong\frac{g^4}{64\pi}\frac{M^2}{M_W^4}\frac{T}{M} \quad (16)$$

for one-handed Majorana neutrinos with standard model vertex factors. In the L-R model, Eq. (2) shows that the $N\bar{N}Z'$ vertex has an extra factor of $\cos^2\theta/\sqrt{\cos2\theta}$, while Eq. (1) shows that the $Z'W^+W^-$ vertex will have an extra factor of $\sqrt{\cos2\theta}/\cos^2\theta$. This means that we can use the $\langle\sigma'|v|\rangle$ of Eq. (16) directly in our calculations.

For a sufficiently large M/M_W ratio, $\langle\sigma'|v|\rangle$ of Eq. (16) will dominate over the $\langle\sigma|v|\rangle$ of Eq. (7). As $\langle\sigma'|v|\rangle\sim M^2$, we can expect, therefore, a *lower* bound for sufficiently large M/M_W.

Finally, we must consider the anomalous effects. We will consider, numerically, the detailed influence of these two additional effects on the bound after noting, in the next section, that anomalous effects, too, favor a lower bound.

III. INTRODUCTION OF ANOMALOUS EFFECTS

A. Anomalous generation of right-handed neutrinos

For the standard model, a classical, unstable, time-independent solution of the equations of motion has been identified [21,22]. This sphaleron solution corresponds to the barrier between vacua with different topological numbers. A sphaleron-mediated transition over the barrier leads to a fermion-number violating transition with $|\Delta L|=3$, $|\Delta B|=3$, of the type

$$|W_\mu^{cl}\alpha\rangle \rightarrow |W_\mu'^{cl}\alpha'\rangle,$$

where α, α' are fermion states, differing by $|\Delta L|=3$, $|\Delta B|=3$, and $W_\mu^{cl}, W_\mu'^{cl}$ are the initial and final SU(2) gauge boson configurations, which are essentially classical. [We are neglecting the small effect of the U(1) part [22].]

All colors and families of quarks and leptons will be generated equally, but, in any one transition, only one member per doublet will be found. For the rest of this paper, we will neglect family mixing and consider anomalous generation for a single (the lightest) family. In this case, $|\Delta L|=1$, $|\Delta B|=1$. α, α' will be restricted by the requirement that the sphaleron must be a color singlet, SU(2) singlet, and neutral mediator. There are then two relevant amplitudes, which we may write formally as $\langle W_\mu^{cl} uude\; W_\mu'^{cl}\rangle$ and $\langle W_\mu^{cl} udd\nu\; W_\mu'^{cl}\rangle$. All processes with these amplitudes can occur. For example, α may be the vacuum and α' may represent *uude* or *uddv*. In the *L-R* symmetric model, we expect, on general grounds [21,16,14], anomalous $B+L$ generation above or just below $T_{\rm cr}$, from both $SU(2)_L$ and $SU(2)_R$ gauge boson field configurations with nonvanishing topological charge.

However, the actual construction of the sphaleron solution depends on the details of the Higgs multiplet. The $SU(2)_L$ sphaleron [22] was worked out with a complex doublet. In the *L-R* symmetric case, the generation of Majorana masses at the higher energy scale results from spontaneous symmetry-breaking associated with a $SU(2)_R$ triplet scalar field [in addition to a $SU(2)_L$ triplet and a bidoublet which develop *VEV* at the lower energy scale] [11]. It has been shown [14] that the topological condition necessary for the existence of a sphaleron solution is fulfilled for a simplified model of $SU(2)_R$ symmetry breaking at the higher energy scale via a triplet complex scalar field. But, the construction of an explicit solution has proved very difficult.

In this situation, one has to assume [14,23] the occurrence of $B+L$ violation via sphalerons for the $SU(2)_R$ gauge group, in addition to $B+L$ violation for $SU(2)_L$, at the higher energy scale. Neglecting mixing parameters between left-handed and right-handed gauge bosons, we work with a highly simplified model in which the W_L^μ give rise to anomalous generation of leptons and baryons from left-handed doublets, and the W_R^μ from right-handed doublets. In particular, the W_R^μ will generate, anomalously, right-handed N neutrinos.

First, we want to relate the rate of production of the right-handed N neutrinos per unit volume to the total rate Γ of $\Delta B=1$, $\Delta L=1$ anomalous transitions per unit volume. We divide $\Delta L=+1$ processes into four types (assuming distinct flavor eigenstates for N and $\bar{N}$).

l: processes with an N in the final state, e.g.,

$$|W_{\mu R}^{cl}\text{vac}\rangle \rightarrow |W_{\mu R}'^{cl} uddN\rangle,$$

$\bar{l}$: processes with an $\bar{N}$ in the initial state, e.g.,

$$|W_{\mu R}^{cl}\bar{N}\bar{u}\bar{d}\bar{d}\rangle \rightarrow |W_{\mu R}'^{cl}\text{vac}\rangle,$$

m: processes with an e^- in the final state, e.g.,

$$|W_{\mu R}^{cl}\text{vac}\rangle \rightarrow |W_{\mu R}'^{cl} uude^-\rangle,$$

$\bar{m}$: processes with an e^+ in the initial state, e.g.,

$$|W_{\mu R}^{cl}\bar{u}\bar{u}\bar{d}e^+\rangle \rightarrow |W_{\mu R}'^{cl}\text{vac}\rangle.$$

Therefore,

$$\Gamma = \sum_l \Gamma_l + \sum_{\bar{l}} \Gamma_{\bar{l}} + \sum_m \Gamma_m + \sum_{\bar{m}} \Gamma_{\bar{m}},$$

where Σ_i is a sum over all processes of type i.

Each process has a rate which is determined in an essentially classical way: if the thermal fluctuation has sufficient energy to cross the barrier, the process will occur. If $i\omega^-$ is the frequency of the unstable (sphaleron) mode, a classical statistical mechanics calculation gives [24,25]

$$\Gamma_i = (\omega^-/\pi)(\text{Im}\,\mathcal{F}/T), \tag{17}$$

where $\mathcal{F}$ is the free energy. Also,

$$(\text{Im}\,\mathcal{F}/T) \sim e^{-(V_0/T)}, \tag{18}$$

where V_0 is the barrier height.

Because of this essentially classical nature, the barrier-crossing rate, at a given temperature, under equilibrium conditions, should be of the same order in different channels. In other words, we may expect that the rate of $\Delta L=1$, $\Delta B=1$ transitions, featuring one member of a lepton doublet, will be of the same order as the rate of such transitions, featuring the other member of the doublet. As a first approximation, we may take

$$\sum_l \Gamma_l + \sum_{\bar{l}} \Gamma_{\bar{l}} \approx \sum_m \Gamma_m + \sum_{\bar{m}} \Gamma_{\bar{m}} \approx \frac{1}{2}\Gamma. \tag{19}$$

The approximation will be bad when the N neutrinos are way out of equilibrium. In a decoupling study, however, one is interested in finding out when the species just falls out of equilibrium.

Let us now interpret $l,\bar{l}$ formally as Boltzmann collisional processes

$$l: i+j+\cdots \rightarrow N+a+b+\cdots.$$

$$\bar{l}: \bar{N}+\bar{a}+\bar{b}+\cdots \rightarrow \bar{i}+\bar{j}+\cdots.$$

CPT ensures that for every process of type l, there is a process of type $\overline{l}$ with the same matrix element $\mathcal{M}_l$. Then, we can write, formally [18,19],

$$\Gamma_l = \int d\pi_N d\pi_a d\pi_b \cdots d\pi_i d\pi_j \cdots |\mathcal{M}_l|^2 (2\pi)^4 \times \delta^4(p_N + p_a + p_b \cdots - p_i - p_j \cdots) f_N^{\mathrm{eq}} f_a f_b \cdots . \quad (20)$$

We have again assumed that all relevant species are in equilibrium except the right-handed neutrinos, and that there is no significant Fermi degeneracy or Bose condensation. Also,

$$\Gamma_{\overline{l}} = \int d\pi_{\overline{N}} d\pi_{\overline{a}} d\pi_{\overline{b}} \cdots d\pi_{\overline{i}} d\pi_{\overline{j}} \cdots |\mathcal{M}_l|^2 (2\pi)^4 \times \delta^4(p_{\overline{N}} + p_{\overline{a}} + p_{\overline{b}} \cdots - p_{\overline{i}} - p_{\overline{j}} \cdots) f_{\overline{N}} f_{\overline{a}} f_{\overline{b}} \cdots . \quad (21)$$

In these formal expressions, $|\mathcal{M}_l|^2$ is related to the classical probability and is not to be interpreted perturbatively.

As we are interested here in decoupling and not in baryogenesis, we will neglect small *CP*-asymmetric effects and assume *CP* symmetry. Then, we can assume, as in Sec. II A,

$$n = \overline{n}, \quad n_{\mathrm{eq}} = \overline{n}_{\mathrm{eq}};$$

also,

$$f_a = e^{-E_a/T}, \quad f_{\overline{a}} = e^{-E_{\overline{a}}/T}, \text{ etc.} \quad (22)$$

In this case, we can write, from Eqs. (20) and (21),

$$\Gamma_l = I_l n_{\mathrm{eq}}$$

and

$$\Gamma_{\overline{l}} = I_l n, \quad (23)$$

where I_l contains the result of the phase space integrations, apart from n_{eq} and n [19,26]. [$I_l = I_{\overline{l}}$ due to Eq. (22).] It can be interpreted as a thermally averaged width in mode l.

From Eqs. (19) and (23),

$$\sum_l I_l (n + n_{\mathrm{eq}}) = \frac{1}{2}\Gamma$$

and

$$\sum_l \Gamma_l = \frac{n_{\mathrm{eq}}}{2(n + n_{\mathrm{eq}})}\Gamma. \quad (24)$$

For $\Delta L = -1$ anomalous transitions, we will get, similarly, a process l' with a $\overline{N}$ in the final state, and a process $\overline{l'}$ with a N in the initial state, and a similar result

$$\sum_{\overline{l'}} \Gamma_{\overline{l'}} = \frac{n}{2(n + n_{\mathrm{eq}})}\Gamma'. \quad (25)$$

Now, we are neglecting baryon- and lepton-number excess or deficit. In this approximation, we can set $\mu_N = 0$. We can, then, take [25,27] $\Gamma = \Gamma'$, i.e., the rate of $\Delta L = 1$ transitions $\approx$the rate of $\Delta L = -1$ transitions.

Hence, the net rate of reduction of N neutrinos by anomalous processes, per unit volume, Γ_N = rate of such processes, per unit volume, with a N in the initial state—rate of such processes, per unit volume, with a N in the final state

$$= \sum_{\overline{l'}} \Gamma_{\overline{l'}} - \sum_l \Gamma_l .$$

We finally get

$$\Gamma_N = \frac{n - n_{\mathrm{eq}}}{2(n + n_{\mathrm{eq}})}\Gamma. \quad (26)$$

As expected, the anomalous rate vanishes if the N neutrinos are in equilibrium.

Assuming, therefore, *CP* symmetry, and equilibrium conditions for all relevant particles except right-handed neutrinos, the rate of reduction, per unit volume, of the N neutrinos can be written in the form of a Boltzmann equation

$$\frac{dn}{dt} + 3Hn = -\frac{(n - n_{\mathrm{eq}})}{2(n + n_{\mathrm{eq}})}\Gamma_R - [\langle \sigma |v| \rangle + \langle \sigma' |v| \rangle](n^2 - n_{\mathrm{eq}}^2). \quad (27)$$

This is our basic equation. We have put $\Gamma = \Gamma_R$ to indicate that we are considering the anomalous rate for right-handed neutrinos.

The assumption of $n = \overline{n}$ is a delicate one, in general. If there is an asymmetry, the post-annihilation relic neutrinos, with density $\rho_{N'}$, proportional to the asymmetry, will cluster with baryons inside the galaxies [28]. If ρ_G is the galactic density, there will be the bound $\rho_{N'} < \rho_G$. Bounds have been worked out, taking the asymmetry to be of the order of the baryon asymmetry [28,29,5].

Although we have maintained a technical distinction between flavor eigenstates of N and its Majorana mass eigenstates, the amplitude calculations have assumed pure Majorana particles. In this case of pure Majorana neutrinos, $N \equiv \overline{N}$ and, strictly speaking, there is no $(n - \overline{n})$ asymmetry. Yet, the argument that the decoupled neutrinos will finally end up in the galaxies retains its force, with $\rho_{N'} = \rho_N$. Then, the bound $\Omega_N h^2 < 1$ will be replaced by

$$\Omega_N h^2 < (\rho_G / 1.88 \times 10^{-29}), \quad (28)$$

where we have taken, as before, $\rho_c = 1.88 \times 10^{-29}\,\mathrm{h}^2\,\mathrm{g\,cm}^{-3}$.

Since we have Majorana mass states, lepton number violating processes with $|\Delta L| = 2$ can arise from L-violating terms in the Lagrangian, and produce a net L asymmetry (e.g., in the ν_L sector). A variety of such processes has been considered in the literature [14,23,30]. These theories are, broadly, of two types.

In one type of theory, a massive right-handed Majorana neutrino is allowed to decay. As we are neglecting decay, such theories have not been considered. In the other type of theory, L asymmetry has been generated by virtual N exchange. This L asymmetry, together with sphaleron-mediated $B+L$ violation, can destroy any pre-existing $B-L$ asymmetry, and send B to zero. Bounds on M have been proposed by demanding that such erasure of B should not take place. The temperature and mechanism of the $B-L$ production is, however, still uncertain, and we do not pursue these ideas further in this paper.

B. Anomalous effects in decoupling

For $T<T_{cl}$, the critical temperature for the spontaneous breakdown of $SU(2)_L\times U(1)_Y$, $\Gamma=\Gamma_L$ has been calculated [25,31]. For the right-handed case, with $T<T_{\rm cr}$, the complication of the Higgs sector has obstructed a calculation of Γ_R. However, the very general considerations mentioned in Sec. III A imply that $\Gamma\sim e^{-V_0/T}$.

V_0 can be estimated heuristically, as follows [32,33]. If we assume a sphaleron solution with energy $E_{\rm sp}$, we can put $V_0=E_{\rm sp}$. But $E_{\rm sp}$ arises from a classical solution, i.e., from a limit where many quanta are involved. We can take the energy per quantum $\sim M_W$, and the average number of quanta $\sim(1/\alpha_W)$, where $\alpha_W=g^2/4\pi$. Then, $E_{\rm sp}\sim M_W/\alpha_W$ is a very general estimate, which should hold for the right-handed case also, with $M_W=M_{WR}$. So, we can write, on general grounds,

$$\Gamma_R=\tilde{R}(M_{WR},T)e^{-BM_{WR}/(\alpha_W T)}, \tag{29}$$

where $\tilde{R},B$ depend on the precise form of the symmetry-breaking. However, we can say that $\tilde{R}$ will have dimension$\sim$(mass)^4, and B will be dimensionless and of order 1. Also, the whole idea of separating out the exponential is to isolate a prefactor $\tilde{R}$ which can be assumed to vary more slowly (in the left-handed case, the prefactor varies as powers of the arguments [25]).

Introducing x and Y, the Boltzmann equation (27) becomes, from Eqs. (8), (11), and (29) (considering only $\langle\sigma|v|\rangle$, at this stage, and not $\langle\sigma'|v|\rangle$),

$$\frac{dY}{dx}=-\frac{1}{(1.66g^{*1/2}/x^4)(M^5/M_{\rm Pl})(2\pi^2/45)g_s^*}\frac{Y-Y_{\rm eq}}{2(Y+Y_{\rm eq})}\times\tilde{R}'(M,a,x)e^{-Bx/(\alpha_W a)}-\frac{3.16\times10^{17}}{Mx^3}(Y^2-Y_{\rm eq}^2), \tag{30}$$

writing $a=(M/M_{WR})$. We are considering $a>1$.

Compactly, we can write

$$\frac{dY}{dx}=-R(M,a,x)e^{-Bx/(\alpha_W a)}\frac{Y-Y_{\rm eq}}{2(Y+Y_{\rm eq})}-\frac{3.16\times10^{17}}{Mx^3}(Y^2-Y_{\rm eq}^2), \tag{31}$$

where $\tilde{R}'$ and R are obtainable from $\tilde{R}$.

As a increases, the first term gains importance because of the exponential. Suppose a has a value for which the first term is predominant near decoupling. Let us characterize decoupling by the simple criterion

$$\Delta(x_d)=Y_{\rm eq}(x_d). \tag{32}$$

This is equivalent to taking $c=1$ in Eq. (12).

Before decoupling, $Y\approx Y_{\rm eq},\Delta'\sim0$, and

$$Y'_{\rm eq}=-Re^{-Bx/(\alpha_W a)}\frac{\Delta}{2(2Y_{\rm eq}+\Delta)}. \tag{33}$$

At decoupling, when $Y'_{\rm eq}=-Y_{\rm eq}$, Eq. (33) gives

$$[6Y_{\rm eq}(x_D)/R]e^{Bx_d/(\alpha_W a)}=1,$$

and, using the value of $Y_{\rm eq}$ from Eq. (10),

$$e^{[B/(\alpha_W a)-1]x_d}G(M,a,x_d)=1,$$

where the prefactor G can be assumed to have a slower variation with x_d and a than the exponential, because R is a prefactor for which this has been assumed. Then, assuming the exponential to dominate, we expect, approximately,

$$\left(\frac{B}{\alpha_W a}-1\right)x_d=\tilde{B}x_d\approx\text{const}. \tag{34}$$

Now, the sphaleron decay will produce an N neutrino only if the kinematic constraint $E_{\rm sp}>M$ is satisfied. As $E_{\rm sp}=BM_{WR}/\alpha_W$, so, $B/\alpha_W>a$ gives an upper limit a' on a for anomalous effects to occur. For $a<a'$, $\tilde{B}>0$, and, if a is increased, $\tilde{B}$ decreases, so that x_d increases.

We approximate Y_∞ by $Y(x_d)$, so that, from Eq. (32), $Y_\infty\approx2Y_{\rm eq}(x_d)$. Then, we get, from Eqs. (14) and (10),

$$\Omega_N h^2=1.62332\times10^6x_d^{3/2}e^{-x_d}aM_{WR}. \tag{35}$$

Since, from Eq. (34), $x_d\sim a/a'/(1-a/a')$, the exponential will dominate, and we can expect that, as a increases, $\Omega_N h^2$ will decrease. This means that $\Omega_N h^2<1$ will give a lower bound on a and, hence, on M, for a given M_{WR}, for $a<a'$.

If we can actually find values of the parameter $a=M/M_{WR}$, within the range $1<a<a'$, for which the anomalous term in Eq. (31) predominates, there will not be any hindrance from the Lee-Weinberg type of cosmological bound to right-handed neutrinos having masses greater than right-handed gauge boson masses. So, we find that anomalous reduction of right-handed neutrino number may have important effects on the decoupling of such neutrinos.

Whether these formal expectations will be borne out depends on the actual numbers in Γ_R. Extrapolating the known result for Γ_L to the right-handed case, keeping wide leeway, we will find that numerical results give cause for optimism.

IV. NUMERICAL RESULTS

We will take the Γ_L given in Ref. [25].

$$\Gamma_L=\frac{(1.4\times10^6)M_W^7}{g^6T^3}\exp\left[-\frac{16\pi M_W}{g^2T}\right]. \tag{36}$$

Here, the unstable mode ω^- is taken $\approx M_W$, and $E_{\rm sp} = 2(M_W/\alpha_W)\bar{E}$. $\bar{E}$ is a number which depends on (λ/g^2): $1.56 < \bar{E} < 2.72$ for $0 \le \lambda < \infty$ (λ is the four-Higgs self-coupling constant). We take $\bar{E} = 2$. M_W is temperature dependent.

$$M_W = M_W(0)[1-(T/T_C)^2]^{1/2}, \tag{37}$$

and $T_C = 3.8 M_W(0)$ [25]. There is an overall constant $\kappa \sim 1$ [25,31]. We take $\kappa = 1$.

This expression is valid for $2M_W \ll T \ll 2M_W/\alpha_W$. However, the range of T may be taken to be $M_W \ll T \ll M_W/\alpha_W$. [32]

We extrapolate this rate to get Γ_R, in a simple way, using the following prescription: (i) replace M_W by M_{WR}, (ii) write $T_{CR} = zM_{WR}(0)$, and (iii) include an overall factor b. z is not known reliably, because, as yet, there is not sufficient experimental data to evaluate the full L-R Lagrangian, including the Higgs sector [34]. For large z, Eq. (37) shows that $M_{WR} \approx M_{WR}(0)$. If z is too small, M_{WR} will become imaginary. We will vary z between 2 and 10. The numerical work will show that below $z=2$, the mass is not real, while there is little change above $z=10$.

Whereas the exponential part in Eq. (36) will almost certainly be right for Γ_R (apart from the order one quantity $\bar{E}$), the prefactor is bound to require considerable modification. Considering the prefactor to be a slowly varying quantity, whose main function is to set the numerical scale of the essentially exponential variation of Γ_R with $(1/T)$, we will allow b to vary from $10^{-3} - 10^3$, i.e., the decoupling will be investigated with anomalous rates for right-handed neutrino reduction varying over 6 orders of magnitude around the rate obtained by simple substitution of the right-handed W boson mass in the formula for the left-handed case.

We have, then,

$$\Gamma_R = \frac{(b1.4\times 10^6)M_{WR}(0)^7}{g^6 T^3}\left[1-\left(\frac{T}{zM_{WR}(0)}\right)^2\right]^{7/2} \times \exp\left[-\frac{16\pi M_{WR}(0)}{g^2 T}\left\{1-\left(\frac{T}{zM_{WR}(0)}\right)^2\right\}^{1/2}\right].$$

Introducing x and Y in the above expression, the Boltzmann equation (27) becomes

$$\frac{dY}{dx} = -f(x)(Y^2 - Y_{\rm eq}^2) - g(x)\left(\frac{Y-Y_{\rm eq}}{Y+Y_{\rm eq}}\right). \tag{38}$$

We will first consider the effect of taking the $F\bar{F}$ channel alone, i.e., we keep only $\langle\sigma|v|\rangle$ in Eq. (27). Then, from Eq. (11),

$$f(x) = \frac{3.16\times 10^{17}}{aM_{WR}(0)x^3},$$

Γ_R gives

TABLE II. Effect of uncertainty in $T_{\rm cr}$.

z	X	A
2	31.5	41
3	31.6	48
4	31.7	50
5	31.7	52
6	31.7	53
8	31.8	54
10	31.8	54
100	31.8	55

$$g(x) = \frac{b1.53\times 10^{23}x^7}{a^8 M_{WR}(0)}\left\{1-\left(\frac{a}{zx}\right)^2\right\}^{7/2} \times \exp\left[-\frac{118.98x}{a}\left\{1-\left(\frac{a}{zx}\right)^2\right\}^{1/2}\right],$$

where $a = M/M_{WR}(0)$.

For $x < x_d$, this equation simplifies, as in Sec. II C, to

$$\Delta = -\frac{Y'_{\rm eq}}{f(x)(2Y_{\rm eq}+\Delta) + g(x)/(2Y_{\rm eq}+\Delta)}. \tag{39}$$

We choose, again, as an approximate criterion for decoupling

$$\Delta(x_d) \approx Y_{\rm eq}(x_d) \Rightarrow Y(x_d) \approx 2Y_{\rm eq}(x_d).$$

At decoupling, $Y'_{\rm eq} = -Y_{\rm eq}$. Equation (39), then, leads to the decoupling condition

$$3f(x_d)Y_{\rm eq}(x_d) + \frac{g(x_d)}{3Y_{\rm eq}(x_d)} = 1. \tag{40}$$

Again, using the approximation

$$Y_\infty \approx Y(x_d) \approx 2Y_{\rm eq}(x_d),$$

the cosmological bound becomes, from Eq. (35),

$$1.62332\times 10^6 x_d^{3/2} e^{-x_d} a M_{WR}(0) < 1.$$

At the bound,

$$1.62332\times 10^6 x_d^{3/2} e^{-x_d} a M_{WR}(0) = 1. \tag{41}$$

First, we check the effect of z. Taking $M_{WR}(0) = 4000$ GeV and $b=1$, we solve Eqs. (40) and (41), numerically, to obtain values of $x_d = X$ and $a = A$, for which $\Omega_N h^2$ is just equal to 1. The results, displayed in Table II, show that, as z varies in the range $2 \le z \le 10$, X varies from 31.5 to 31.8, and A from 41 to 54. For $z=1$, M_{WR} is no longer real. Also, as expected, $z=100$ gives for X and A practically the same values as given by $z=10$.

Having seen that the effect of varying z is small, we set $z=4$ for subsequent numerical work.

We next check that the bound obtained is actually a *lower* bound. We do this by varying a around the value A. For each

TABLE III. Mass bound (with anomalous effects).

a	x_d	$\Omega_N h^2$
100	234.8	2.4×10^{-87}
75	72.5	9.6×10^{-18}
60	43.4	1.6×10^{-5}
50.46	31.7	1.0
40	22.1	6.6×10^{3}
25	19.6	4.4×10^{4}
10	20.4	8.1×10^{3}

TABLE V. Mass bound (W^+W^- and anomalous channels).

a	x_d	$\Omega_N h^2$
100	234.8	2.4×10^{-87}
75	72.5	9.2×10^{-18}
60	43.4	1.6×10^{-5}
49.55	31.7	1.0
40	30.8	1.9
25	29.4	4.3
10	26.8	20.5

assigned value of a, we solve Eq. (40) for x_d, and evaluate $\Omega_N h^2$ for this x_d from the left-hand side of (41). The results, displayed in Table III, show that as a increases through the value A, $\Omega_N h^2$ falls through 1, from higher to lower values.

Finally, we vary b from 10^{-3} to 10^3. The results are shown in Table IV. We find that X changes from 31.8 to 31.6, and A changes from 56 to 46. In every case, we have verified the nature of the bound, numerically. The results (not exhibited) parallel Table III. The bound remains a *lower* one.

It is necessary to verify that the restriction $M_{WR}<T<M_{WR}/\alpha_W$ is satisfied. For the lower limit, the worst case occurs when $M_{WR}\approx M_{WR}(0)=4000\,\text{GeV}$. Now, $T=4000A/X$, and the restriction is satisfied if $A>X$. A perusal of Tables III and IV will show that this is, indeed, so, for the parameter ranges considered by us. The stronger restriction, with M_{WR} replaced by $2M_{WR}$, is, however, not obeyed.

For the upper limit, the worst case occurs when z and, hence, M_{WR} is the least. Taking $z=2$, $X=31.5$, and $A=41$, from Table II, we find that $M_{WR}/\alpha_W\approx 90\,000$ Gev, while $T\approx 5200$ Gev. The restriction is obeyed.

We check the kinematical constraint $E_{\rm sp}>M$. As $E_{\rm sp}=2(M_{WR}/\alpha_W)\overline{E}$, we look only at the case when M_{WR} is the least, viz., $z=2$. $E_{\rm sp}$ comes out to be $>360\,000$ GeV, in this case, while, even for $A=55$, $M=220\,000$ GeV, less than $E_{\rm sp}$, as required.

Next, we consider the effect of adding the W^+W^- channel, i.e., we take $\langle\sigma'|v|\rangle$ in addition to $\langle\sigma|v|\rangle$ in the Boltzmann equation (27).

From Eqs. (9), (11), and (16), $f(x)$ of Eq. (38) becomes

$$f(x)=\frac{3.16\times 10^{17}}{aM_{WR}(0)x^3}+\frac{2.82\times 10^{16}a^3}{M_{WR}(0)x^3}.$$

We take $b=1$, $z=4$, and $M_{WR}(0)=4000\,\text{GeV}$. Repeating the numerical solution of Eqs. (40) and (41), we find that $\Omega_N h^2=1$ for $[M/M_{WR}(0)]=A=49.55$. The corresponding (M/T) at decoupling, viz., $x_d=31.68$. A check (Table V) shows that there is a *lower* bound. The corresponding mass bound is $M>198$ TeV.

Comparison with Table III shows that if the W^+W^- channel is not considered, $A=50.46$. Again, if we put $g(x)=0$ in Eq. (40), i.e., if the anomalous effects are neglected, but the W^+W^- channel is kept, then, simultaneous solution of Eqs. (40) and (41) gives $A=58.60$ and $x_d=31.86$. Again, a check (Table VI) shows that there is a lower bound.

So, if we take the W^+W^- and anomalous channels, in addition to the $F\overline{F}$ channels, we get a lower bound A for a and, hence, for M. This value of $A=49.55$, when compared with the value of $A=50.46$, obtained by neglecting the W^+W^- channel, and the value of $A=58.60$, obtained by neglecting the anomalous channel, shows that the anomalous channel is, at least, of comparable importance to the W^+W^- one near the bound. A comparison of Tables IV, V, and VI shows, further, that for values of $a\gg A$, the anomalous channel determines the value of $\Omega_N h^2$, and, for $a\ll A$, the W^+W^- channel determines $\Omega_N h^2$. This is as expected, because the deciding factor in the anomalous channel $\sim e^{-120x/a}$, while in the W^+W^- channel it is $\sim a^3/x^3$.

In addition, keeping all the channels, we have allowed b to change from 0.001 to 1000 as for Table IV. The value of A is found to change from 54 to 45. So, our check of varying the overall strength of the anomalous effects over six orders of magnitude preserves the importance of the anomalous processes in decoupling.

Finally, we consider the bound $\rho_N<$galactic density ρ_G, i.e.,

TABLE IV. Effect of overall uncertainty factor.

b	X	A
0.001	31.8	56
0.01	31.8	54
0.1	31.7	52
1	31.7	50
10	31.7	49
100	31.6	47
1000	31.6	46

TABLE VI. Mass bound (W^+W^-, but no anomalous effects).

a	x_d	$\Omega_N h^2$
100	33.4	0.4
75	32.6	0.7
58.60	31.9	1.0
50	31.4	1.3
40	30.8	1.9
25	29.4	4.3
10	26.8	20.5

$$\Omega_N h^2 < \frac{\rho_G}{1.88\times 10^{-29}}. \tag{42}$$

There is considerable uncertainty regarding the right-hand side of Eq. (42) [28,29,5,35]. We have taken it as 0.05, thereby getting $A=53.33$, keeping all channels, and putting $b=1$. The resulting lower bound on M becomes slightly stronger, becoming 213 TeV instead of 198 TeV. The slightness of the change is due to the high sensitivity of the factor $e^{-120x/a}$ to changes in a. If we take into account the maximum upper limit of 400 TeV set by unitarity [7,8], our results indicate a window of approximate size

$$200 \;\; \text{Tev} < M < 400\,\text{TeV}$$

for the N mass, with the M_{WR} mass$=4$ TeV.

V. CONCLUSIONS

Analyzing the decoupling of right-handed neutrinos with mass greater than right-handed gauge boson masses, using $F\bar{F}$ annihilation channels, we find that the cosmological bound $\Omega_N h^2 < 1$ leads to an *upper* bound on the right-handed neutrino mass M, of about 700 GeV. What this really means is that a right-handed neutrino mass greater than right-handed gauge boson masses is unlikely to be allowed cosmologically.

If we now introduce the W^+W^- annihilation channel and assume also that anomalous $(B+L)$-violating processes work at the right-handed symmetry-breaking scale by thermal diffusion over a barrier, separating states of different $B+L$, in the same way as this happens at the left-handed symmetry-breaking scale, then, we find that it is possible to have a *lower* bound for a right-handed neutrino mass greater than right-handed gauge boson masses.

A numerical extrapolation of the anomalous rate from the lower to the higher energy scale, allowing a leeway of six orders of magnitude, confirms that the anomalous channel may be as important as the W^+W^- channel in determining a bound for the N mass. Considering both the W^+W^- and anomalous channels, and taking $M_{WR}=4$ TeV, a lower bound appears for the right-handed neutrino mass at about 50 times the W_R boson mass. However, in the absence of an explicit calculation of the anomalous rate for the right-handed case, the numbers must only be considered as giving qualitative support to the idea that, at TeV energy scales, anomalous generation plays an important part in decoupling. To obtain reliable bounds, it is necessary to solve the problem of constructing explicitly the sphaleron solution for the right-handed case.

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