Constraints on mixing angles of Majorana neutrinos

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By combining the inputs from the neutrinoless double beta decay and the fits of cosmological models of dark matter with solar and atmospheric neutrino data, we obtain constraints on two of the mixing angles of Majorana neutrinos, which become stronger when coupled with the reactor neutrino data. These constraints are strong enough to rule out Majorana neutrinos if the small angle solution of solar neutrino puzzle is borne out.

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It is well-known [1,2] that unless neutrinos are very massive and nonrelativistic, or interact through both left- and right-handed currents, experimental data on neutrino-induced reactions cannot distinguish between Dirac and Majorana neutrinos. Neutrinoless double beta decay remains as the only feasible tool to probe this question. Although experiments [3–5] have so far provided only upper limits on the rate of this decay, recent limits [5] combined with other inputs on neutrino physics might already lead to important information on whether neutrinos are Dirac or Majorana particles. These other inputs are indications from the analysis of the cosmic microwave background for the presence of a hot dark matter component, which is presumably neutrinos of mass ≈ 1 eV [6,8,9], and the indications [10–12] from the analysis of solar and atmospheric neutrinos that neutrinos do oscillate and that the mass differences among the three neutrinos are much smaller than this scale of 1 eV.

We show in this paper that if neutrinos are Majorana particles, a combined study of all of the above pieces of data leads to rather stringent restrictions on two of the mixing angles that occur for three flavors of neutrinos. If these results are then confronted with the values of these mixing angles allowed by solar, atmospheric, and reactor neutrino data, the allowed regions are further narrowed and in fact, in a few cases, one is already close to contradiction, thus leading to the conclusion that neutrinos are not Majorana particles.

In the three flavor mixing scheme the neutrino flavor eigenstates $\nu_\alpha = \nu_{e,\mu,\tau}$ are related to the mass eigenstates $\nu_i = \nu_{1,2,3}$ by

$$\nu_\alpha = \sum_i U_{\alpha i} \nu_i, \tag{1}$$

where $U_{\alpha i}$ are the elements of the unitary mixing matrix U. We note that for the Majorana neutrino [2,13] there are three CP-violating phases in contrast with the case of the Dirac neutrino which has only one phase. We use the parametrization [13]

$$U = \begin{pmatrix} c_\omega c_\phi & s_\omega c_\phi e^{-i\delta_1} & s_\phi e^{-i\delta_2} \\ -s_\omega c_\psi e^{i\delta_1} - c_\omega s_\psi s_\phi e^{i(\delta_2+\delta_3)} & c_\omega c_\psi - s_\omega s_\psi s_\phi e^{i(\delta_3+\delta_2-\delta_1)} & s_\psi c_\phi e^{i\delta_3} \\ s_\omega s_\psi e^{i(\delta_1-\delta_3)} - c_\omega c_\psi s_\phi e^{i\delta_2} & -c_\omega s_\psi e^{-i\delta_3} - s_\omega c_\psi s_\phi e^{i(\delta_2-\delta_1)} & c_\psi c_\phi \end{pmatrix}, \tag{2}$$

where δ_1, δ_2, and δ_3 are the three CP-violating phases and c and s stand for sine and cosine of the associated mixing angle ω, ϕ, or ψ with the angle placed as subscript.

The rate for the neutrinoless double beta decay depends on the following combination of the neutrino parameters [2]:

$$m_{0\nu\beta\beta} = \left| \sum_{i=1,2,3} \eta_i U_{ei}^2 m_{\nu_i} \right|, \tag{3}$$

where m_{ν_i} are the Majorana neutrino masses, U_{ei} the elements of the first row of the mixing matrix given in Eq. (2),

$\eta_i = (1/i)\eta_i^{CP} = \pm 1$, and η_i^{CP} is the CP parity of the Majorana neutrino ν_i. Although neutrinoless double beta decay has not yet been seen experimentally, the experimental upper limits on this rate have recently improved to a significant extent. In particular one may refer to the results of the Tellurium [3] and Germanium experiments [4,5]. The strongest upper limit so far comes from the Germanium experiment [5,14] and it is

$$m_{0\nu\beta\beta} < 0.56 \text{ eV} \quad (99\% \text{ C.L.})$$

$$< 0.46 \text{ eV} \quad (90\% \text{ C.L.}). \tag{4}$$

These numbers have been obtained using the nuclear matrix elements calculated in [15]. We shall take into account the

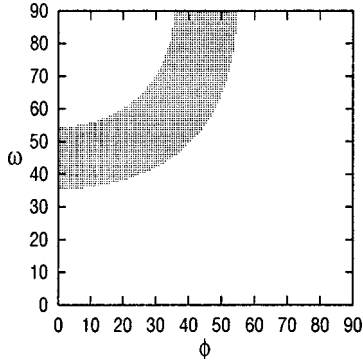*Email address: rathin@imsc.ernet.in
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FIG. 1. The allowed region in (ω, ϕ) is shown shaded for $m_\nu = 1.7$ eV, $\eta_1 = -\eta_2 = \eta_3 = \pm 1$, and $\delta_1 = \delta_2 = 0$.

uncertainties in this calculation (see below). Using the conservative number 0.56 eV we have

$$\left| \sum_{i=1,2,3} \eta_i U_{ei}^2 m_{\nu_i} \right| < 0.56 \text{ eV}. \tag{5}$$

Next we consider the fits to the recent data on the anisotropies of the cosmic microwave background radiation [16] and the large scale structure of the universe [17]. The best fit [9,8] requires a mixture of 10% ordinary baryonic matter, 70% cold, and 20% hot dark matter with $\Omega_m = 1$. If the hot component is identified with neutrinos, the model implies [6]

$$\sum_{i=1,2,3} m_{\nu_i} \approx 5 \text{ eV}. \tag{6}$$

The right-hand side of Eq. (6) is not expected to be less than 3 eV for $\Omega_m = 1$ [7] as otherwise there is too much small scale power [18,19]. On the other hand, solar and atmospheric neutrino data suggest that the two mass-squared differences among the three neutrinos are very small [10–12]; $m_2^2 - m_1^2 \approx 10^{-5}$ eV^2 or smaller and $m_3^2 - m_1^2 \approx 10^{-3} - 10^{-2}$ eV^2. Hence we take all three neutrinos as almost degenerate in mass and using Eq. (6) we obtain

$$m_{\nu_i} \approx m_\nu \approx 1.7 \text{ eV}. \tag{7}$$

We shall allow m_ν to vary over a range around 1.7 eV. This will take care of the uncertainties of cosmological models as well as those of the calculations of the nuclear matrix elements in double beta decay, since only the ratio $0.56/m_\nu$ enters into our analysis. Any possible improvement in the neutrinoless double beta decay limit can also be incorporated by scaling m_ν appropriately.

Combining all the inputs, we have the basic inequality

$$\left| (\eta_1 \cos^2 \omega + \eta_2 \sin^2 \omega e^{-i2\delta_1}) \cos^2 \phi + \eta_3 \sin^2 \phi e^{-i2\delta_2} \right|$$

$$< \frac{0.56}{m_\nu}, \tag{8}$$

where m_ν is expressed in eV. One can rewrite this inequality in terms of two effective phases by combining η_i and δ_i.
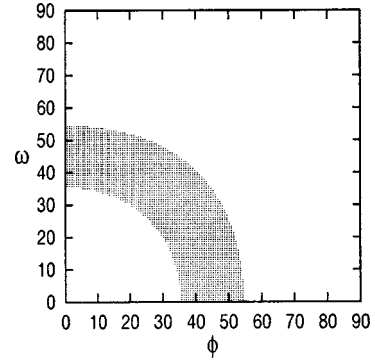

FIG. 2. Same as Fig. 1 but $-\eta_1 = \eta_2 = \eta_3 = \pm 1$.

However, to make our discussion on *CP* conserving and *CP*-violating cases more transparent we have kept both η_i and δ_i above. We proceed to extract the bounds on the mixing angles ω and ϕ implied by this inequality for various choices of η_i, δ_i, and typical values of m_ν favored by the cosmological models. It is to be noted that in contrast to the usual oscillation phenomena studied in neutrino physics, *CP* violation plays an important role in neutrinoless double beta decay.

We shall first consider $\delta_1 = \delta_2 = 0$ in Eq. (8). Out of eight possible combinations for different values of η_i in Eq. (8), four combinations are equivalent to the other four, as only the overall magnitude in the left-hand side of this inequality is constrained. So we shall analyze Eq. (8) on the basis of four cases: case I, $\eta_1 = \eta_2 = \eta_3 = \pm 1$; case II, $\eta_1 = -\eta_2 = \eta_3 = \pm 1$; case III, $-\eta_1 = \eta_2 = \eta_3 = \pm 1$; and case IV, $\eta_1 = \eta_2 = -\eta_3 = \pm 1$. Case I is the natural choice for η_i, if there exists a symmetry linking the three generations. But then the left-hand side of Eq. (8) is unity and so unless $m_\nu \lesssim 0.56$ eV, the inequality cannot be satisfied [20]. Since such low values of m_ν are not expected in the cosmological models, we conclude that the case of equal intrinsic *CP* parities for the three Majorana neutrinos is not favored. The allowed regions in ω and ϕ for cases II, III, and IV are plotted in Figs. 1, 2, and 3, respectively, for $m_\nu = 1.7$ eV. For both cases II and III, small ϕ ($\phi \approx 0$) requires ω to be in the region of 45°. Whereas small ω in case II is not allowed at all, small ω in case III requires large ϕ. Case IV leads to a constraint condition on ϕ that is independent of ω and requires ϕ to be in the region of 45°. Figures 4 and 5
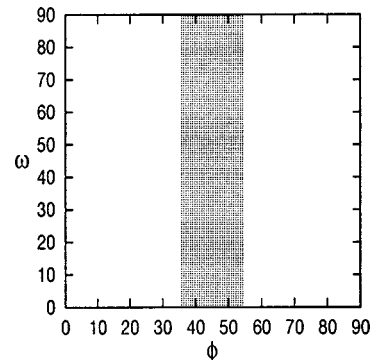

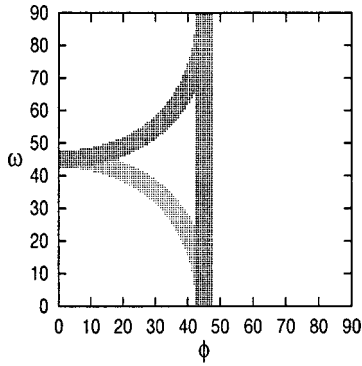FIG. 3. Same as Fig. 1 but $\eta_1 = \eta_2 = -\eta_3 = \pm 1$.

FIG. 4. Allowed region for all possible combinations of η_i, $m_\nu = 6$ eV and $\delta_1 = \delta_2 = 0$.

FIG. 6. Allowed region for all possible η_i, $m_\nu = 1.7$ eV and $\delta_1 = 0$, $\pi/8 \le \delta_2 \le 3\pi/8$.

show the total allowed regions for all possible combinations of η_i (i.e., all the cases I, II, III, and IV) for $m_\nu = 6$ eV and 0.64 eV, respectively. One may note that for $m_{0\nu\beta\beta}/m_\nu \to 0$, the allowed values of ω and ϕ are constrained to lie on the three curves $\tan^2\phi = \pm\cos 2\omega, 1$. Figure 4 (where $m_{0\nu\beta\beta}/m_\nu = 0.093$) gives a small width to these curves.

We next consider the *CP*-violating case. Now the choice $\eta_1 = \eta_2 = \eta_3 = \pm 1$ is *not* ruled out. Thus *CP* violation is capable of changing the conclusions dramatically. Figures 6–8 show the allowed regions in ω and ϕ for a few choices of the parameters δ_1 and δ_2.

We have not considered "maximal" *CP* violation $\delta_1 = \pi/2$ and/or $\delta_2 = \pi/2$ since these cases are subsumed by the choices of relative negative η_i, as far as inequality (8) is concerned. Therefore, it is important to keep in mind that Figs. 1–5 are relevant for such maximal *CP*-violating cases also. In particular, Fig. 1 and Fig. 2 can be considered to be the maximally *CP*-violating case of $\delta_1 = \pi/2$ and $\delta_2 = 0$.

Let us now compare the above bounds on the mixing angles with the results of the analysis of data from reactor, solar, and atmospheric neutrino experiments. Before we make this comparison, we must comment on the role of *CP* violation in these experiments. We note that even for Majorana neutrinos, the oscillation phenomena are controlled by a single *CP*-violating phase δ [21].

The CHOOZ reactor experiment [22] interpreted within a three flavor framework [23] leads to the constraint
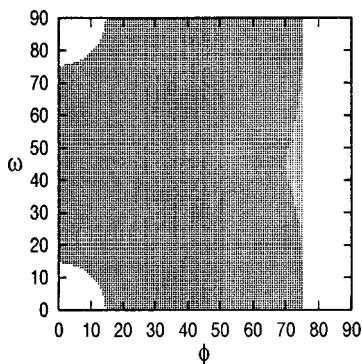
$$\phi \le 12.5°. \tag{9}$$

Although *CP* violation was neglected in reference [23], it is easy to see that Eq. (9) is valid even if *CP* is violated. Comparing Eq. (9) with Fig. 3, we see that case IV is ruled out, while comparisons with Figs. 1 and 2 rule out all values of ω except those in the region of 45°. These are for $m_\nu = 1.7$ eV. The results are weakened for $m_\nu = 0.64$ eV (Fig. 5) and are further strengthened for $m_\nu = 6$ eV (Fig. 4). If *CP* is violated, with $\delta_1 = 0$ and $\delta_2 \neq 0$ (Fig. 6), the above conclusion is still valid. But for $m_\nu = 1.7$ eV, $\pi/8 \gtrsim \delta_1 \lesssim 3\pi/8$, and $\delta_2 = 0$ (Fig. 7), as well as $\pi/8 \gtrsim \delta_1$, $\delta_2 \lesssim 3\pi/8$ (Fig. 8) there is no allowed region at all after using Eq. (9).

It is important to note that as a consequence of Eq. (9), the effect of *CP* violation in all the neutrino-oscillation phenomena is much reduced since the *CP*-violating phase factors $e^{\pm i\delta}$ always occur in combination with $\sin\phi$, as $\sin\phi e^{\pm i\delta}$. Hence it is legitimate to compare with the results of analyses of solar and atmospheric neutrinos even though *CP* violation is usually ignored in those analyses.

Analysis of solar neutrino data on the basis of the Mikheyev-Smirnov-Wolfenstein (MSW) effect [24] leads to the allowed ω values of either $2°–3°$ or $20°–40°$ for $\phi \le 12.5°$. So comparing with the above result, we conclude that the result of the present analysis contradicts the small ω MSW solution ($\omega \approx 2°-3°$), but there exists some overlap with the large ω MSW solution ($\omega \approx 20°-40°$). Also, there is no contradiction with the vaccum oscillation as a solution of the solar neutrino problem since this also requires large ω [27]. Finally there is no contradiction with the results of the
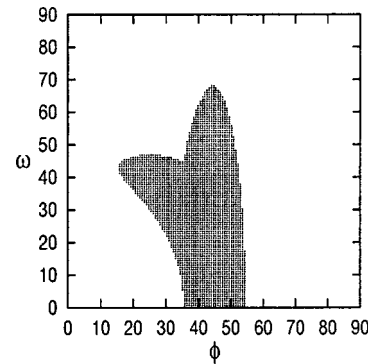

FIG. 5. Same as Fig. 4 but for $m_\nu = 0.64$ eV.

FIG. 7. Allowed region for all possible η_i, $m_\nu = 1.7$ eV and $\pi/8 \le \delta_1 \le 3\pi/8$, $\delta_2 = 0$.
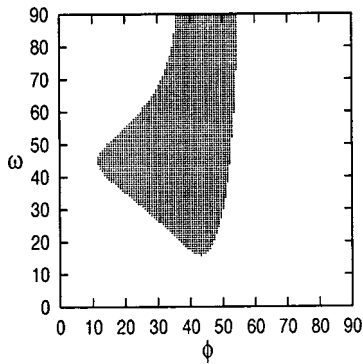
FIG. 8. Allowed region for all possible η_i, $m_\nu = 1.7$ eV and $\pi/8 \leqslant \delta_1$, $\delta_2 \leqslant 3\pi/8$.

atmospheric neutrino analysis [24] since this does not involve ω.

So far we have used the information on m_ν from cosmology to get results on the mixing angles which were then compared with the results of reactor, solar, and atmospheric neutrinos. In view of the uncertainties of cosmological models, one can ask what kind of information on the quasidegenerate mass for Majorana neutrinos can be obtained from our analysis, if we drop the cosmological input completely. It is clear from Fig. 5 that for the small ω MSW solution of the solar neutrino problem, we get an upper bound on m_ν of about 0.7 eV and we have checked that this upper bound becomes 4 eV for the large ω MSW solution.

The quantitative results of our analysis are contained in Figs. 1–8 in the form of restrictions in ω and ϕ. We may also state two qualitative conclusions that emerge from our analysis.

(1) If neutrinos are Majorana fermions and *CP* is conserved, all three neutrinos cannot have the same *CP* parities. This conclusion (which perhaps is well-known and is included here only for the sake of completeness) may be important for model building.

(2) If neutrinos are Majorana fermions, the mixing angle ω cannot be small. Hence, if the small ω solution turns out to be the only correct solution of the solar neutrino problem, then neutrinos cannot be Majorana fermions. This will have serious consequences for models intended to explain small neutrino masses.

The above results and conclusions are based on the present indications that the experiments on the neutrinoless double beta decay require $m_{0\nu\beta\beta}$ to be less than a fraction of an eV while models with neutrinos as the hot component of dark matter require m_ν to be higher than about 1 eV. The restrictions become more severe and the conclusions become stronger if the upper limit on $m_{0\nu\beta\beta}$ decreases [25,26] and/or m_ν increases.

Note added. After the first submission of our manuscript, we came across the papers of Georgi and Glashow [28] and of Branco, Rebelo, and Silva-Marcos [29], whose contents have partial overlap with our work. Our analysis is more general than both of these works in which ϕ is put as zero and in addition we have considered the *CP*-violating cases in detail.

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