Parity conservation in supersymmetric vectorlike theories

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We give the reconfirmation that parity is conserved in vectorlike supersymmetric theories, such as supersymmetric QCD with massive quarks with no cubic couplings among chiral multiplets, based on fermionic path integrals, originally developed by Vafa and Witten. We also look into the effect of supersymmetric breaking through gluino masses, and see that parity conservation is in tact also in this case. Our conclusion is valid, when only bosonic parity-breaking observable terms are considered in path integrals such as the original Vafa-Witten formulation.

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I. INTRODUCTION

Nonperturbative chiral symmetry breaking [1] is an important aspect for phenomenological model building based on vectorlike theories, such as in composite particle models [2,3]. It has been proven that parity symmetry is conserved in nonsupersymmetric vectorlike QCD theories, and is not broken spontaneously even nonperturbatively [4,5]. This proof [4] is based on the evaluation of fermionic path integrals, which always gives a non-negative vacuum energy after adding parity-breaking terms. However, the question has risen whether the Vafa-Witten constraint [4,5] for the nonsupersymmetric case can be avoided in supersymmetric vectorlike theories because of the new interactions among gluino-quark-squarks [6,7], and whether parity is broken like other global symmetries [8]. These particular interaction terms with scalar dependence seem to be the main obstruction for the proof for the positive definiteness of the determinant in the fermionic path integrals [5–7].

On the other hand, gauge symmetry breaking is known to occur for massless supersymmetric QCD, when the number of flavor N_f is smaller than the number of colors: $N_f < N$ [9]. This seems to suggest the parity breaking in supersymmetric vectorlike theories may well occur, depending on the numbers N_f or N. However, at the same time, it contradicts the other universal wisdom about supersymmetry that supersymmetric vacuum is stable when the Witten index $\mathrm{Tr}(-1)^F$ [10] is nonzero, e.g., $\mathrm{Tr}(-1)^F = N$ for the gauge group $SU(N)$, and therefore the vacuum energy most probably stays zero with no parity breaking. It has been also recently pointed out [11] that supersymmetric QCD has condensate-free phase with no gluino condensate, based on the important papers by Seiberg [12] clarifying the nonperturbative conservation of parity. Considering all of these developments, it seems worthwhile to look into this question of nonperturbative parity conservation in vectorlike supersymmetric theories, from the viewpoint of the Vafa-Witten constraint [4], as an independent and different formulation.

Motivated by this observation, in our present paper, we reconfirm that parity is conserved nonperturbatively in exactly supersymmetric vectorlike theories, as well as in broken supersymmetric cases with massive gluini. Our method is to evaluate the explicit fermionic path integrals, originally developed by Vafa and Witten [4], which can be applied easily also to broken supersymmetric theories. We show that we can overcome the above-mentioned difficulty with the gluino-quarks-squark mixing, by using two-component spinor notation.

The model we deal with in this paper is a globally supersymmetric vectorlike theory with massive chiral multiplets coupled to a non-Abelian vector multiplet with no cubic coupling among chiral multiplets. Our proof is based on three major assumptions: the first one for the massiveness of all the quark chiral multiplets, the second one about the absence of the Yukawa-couplings among chiral multiplets, and the third one that we rely on the method in [4] for purely bosonic parity-breaking observables. Therefore our method does not cover the fermionic parity-breaking observables like Wilson fermions [13] treated in lattice QCD.[1] The massiveness of quarks are also important for nonperturbative conservation of supersymmetry, due to the well-defined nonzero Witten index $\mathrm{Tr}(-1)^F$ in such cases [10]. This is because supersymmetry is conserved, only if the vacuum energy is zero. Therefore the nonperturbative breaking of supersymmetry would cause the shift of vacuum energy, causing the breaking of parity symmetry [4,5]. Interestingly, we will find that parity is conserved also for a vectorlike supersymmetric theory, like nonsupersymmetric vectorlike theory. We also look into the effect of gluino masses, which will not disturb the main body of the proof for the supersymmetric case, and therefore parity is also conserved in broken supersymmetric vectorlike theories.

II. REVIEW FOR NONSUPERSYMMETRIC VECTORLIKE THEORY

We start with reviewing the parity conservation in the nonsupersymmetric case [4,5] first, in order also to elucidate our notation. Suppose the total Lagrangian $\mathcal{L}(\lambda) \equiv \mathcal{L} - \lambda X$ with a parameter λ is a generalization of the parity-conserving Lagrangian $\mathcal{L}$, such as that of QCD, with a parity-non-conserving observable X, such as the $F\tilde{F}$ term, with a real constant λ. If parity is broken in the vacuum and

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[1]For reviews for supersymmetric QCD on lattice, see, e.g., [14].

$\langle X \rangle \neq 0$, then the theory can choose a vacuum state in which $\lambda \langle X \rangle < 0$ due to the signature ambiguity of $\langle X \rangle$. Hence the vacuum energy $E(\lambda)$ can be lower than $E(0)$ of the parity-conserving vacuum energy: $E(\lambda \neq 0) < E(0)$.

However, an explicit evaluation of path integrals reveals that this would not happen; i.e., there is no such vacuum whose energy is lower than that of the parity-conserving one [4]. Consider the path integral in Euclidean space for the vacuum energy $E(\lambda)$:

$$e^{-VE(\lambda)} = \int [dA_a{}^I][d\chi][d\bar{\chi}][d\xi][d\bar{\xi}]$$

$$\times \exp\left[-\int d^4x (\mathcal{L} + i\lambda X)\right], \quad (2.1)$$

where V is the Euclidean volume and $A_a{}^I$ is the gluon field. The indices $I, J \ldots$ are for adjoint representations of the gauge group G. For example for $G = SU(N)$, we have $I, J, \ldots = 1, 2, \ldots, N^2 - 1$. In order to clarify basic constituents of our system, we use 2-component spinors in this paper. Since we are dealing with a vectorlike theory, our two-component Weyl spinors χ_i and ξ^i with the flavor indices $i, j, \ldots = 1, 2, \ldots, N$ for quarks are in the conjugate representations to each other. The factor of i in the λX term is due to the usual Wick rotation. We specify the Lagrangian as

$$\mathcal{L} = -\frac{1}{4}(F_{ab}{}^I)^2 + \mathcal{L}_F,$$

$$\mathcal{L}_F = +i(\bar{\chi}^{\dot{\alpha}i} \slashed{D}^\beta{}_{\dot{\alpha}} \chi_{\beta i}) + i(\xi^{\alpha i} \slashed{D}_\alpha{}^{\dot{\beta}} \bar{\xi}_{\dot{\beta}i}) + m_i{}^j (\bar{\chi}^{\dot{\alpha}i} \bar{\xi}_{\dot{\alpha}j})$$

$$+ m_j{}^i (\xi^{\alpha j} \chi_{\alpha i}), \quad (2.2)$$

where $m \equiv (m_i{}^j)$ is an $N \times N$ Hermitian mass matrix:

$$(m_i{}^j)^* = m_j{}^i,$$

which can be arranged to have only positive eigenvalues. Since we are dealing in this paper only with a vectorlike theory, the quark fermions χ_i and ξ^i are in the representations conjugate to each other, e.g., **N** and **N*** representations of $SU(N)$, respectively. Accordingly, our covariant derivative D_a contains the minimal coupling of the gauge field to these fermions. We are using the notations similar to that in [15], e.g., we use the Minkowskian four-dimensional (4D) vector indices $a, b, \ldots = 0, 1, 2, 3$, with the signature ($\eta_{ab}$) $= \text{diag}(+, -, -, -)$, while $\alpha, \beta, \ldots = 1, 2$ and $\dot{\alpha}, \dot{\beta}, \ldots = \dot{1}, \dot{2}$ for the 2-component spinors. Other relevant relations are such as

$$\slashed{D}_{\alpha\dot{\beta}} \equiv (\sigma^c)_{\alpha\dot{\beta}} D_c, \quad [(\sigma^c)_{\alpha\dot{\beta}}]^* = (\sigma^c)_{\beta\dot{\alpha}}, \quad (2.3)$$

$$\psi^\alpha = C^{\alpha\beta} \psi_\beta, \quad \bar{\psi}_{\dot{\alpha}} = \bar{\psi}^{\dot{\beta}} C_{\dot{\beta}\dot{\alpha}},$$

$$(\psi^\alpha)^\dagger = +\bar{\psi}^{\dot{\alpha}}, \quad (\psi_\alpha)^\dagger = -\bar{\psi}_{\dot{\alpha}},$$

$$(\psi_1^{\alpha_1} \cdots \psi_j^{\alpha_j} \bar{\chi}_1^{\dot{\beta}_1} \cdots \bar{\chi}_k^{\dot{\beta}_k})^\dagger = \chi_k^{\beta_k} \cdots \chi_1^{\beta_1} \bar{\psi}_j^{\dot{\alpha}_j} \cdots \bar{\psi}_1^{\dot{\alpha}_1},$$

$$\partial_a^\dagger = -\partial_a, \quad (C_{\alpha\beta}) = (C_{\dot{\alpha}\dot{\beta}}) = \begin{pmatrix} 0 & -i \\ +i & 0 \end{pmatrix}.$$

Based on these, it is easy to confirm the reality of each term in our Lagrangian.

As was shown by Vafa and Witten in a vectorlike theory [5], the fermionic space for the path integral (2.1) can be a direct sum of the positive and negative eigenstates of the Dirac operator in a finite volume V. Let $\chi_{\alpha i \mu}$ and $\bar{\xi}_{\dot{\alpha}i\mu}$ correspond to such eigenstates [16]:

$$\slashed{D}^{\beta\dot{\alpha}} \chi_{\beta i \mu} = +\mu \bar{\xi}^{\dot{\alpha}}{}_{i\mu}, \quad \slashed{D}^{\alpha\dot{\beta}} \bar{\xi}_{\dot{\beta}i\mu} = +\mu \chi_{i\mu}{}^\alpha. \quad (2.4)$$

We then easily see that the set of $\chi_{\alpha(-\mu)} \equiv \chi_{\alpha(+\mu)}$, $\bar{\xi}_{\dot{\alpha}i(-\mu)} \equiv -\bar{\xi}_{\dot{\alpha}i(+\mu)}$ corresponds to the eigenvalue $-\mu$:

$$\slashed{D}^{\beta\dot{\alpha}} \chi_{\beta i(-\mu)} = -\mu \bar{\xi}^{\dot{\alpha}}{}_{i(-\mu)}, \quad \slashed{D}^{\alpha\dot{\beta}} \bar{\xi}_{\dot{\beta}i(-\mu)} = -\mu \chi_{i(-\mu)}{}^\alpha. \quad (2.5)$$

Therefore the whole fermionic space is not only a direct sum of positive and negative eigenstates, but also they are always paired up between $+\mu > 0$ and $-\mu < 0$.

Or equivalently, in terms of a four-component Dirac spinor

$$\psi \equiv (\psi_{\alpha i}) \equiv \begin{pmatrix} \chi_{\alpha i} \\ \bar{\xi}_{\dot{\alpha}i} \end{pmatrix}, \quad \bar{\psi} \equiv (\bar{\psi}^{\alpha i}) \equiv \begin{pmatrix} \bar{\chi}^{\dot{\alpha}i} \\ \xi^{\alpha i} \end{pmatrix}, \quad (2.6)$$

with the four-component spinorial indices $\alpha \equiv (\alpha, \dot{\alpha})$, $\beta \equiv (\beta, \dot{\beta}), \ldots$, we have

$$\tilde{\slashed{D}} \psi = \begin{pmatrix} O & \slashed{D}_\alpha{}^{\dot{\beta}} \\ \slashed{D}_{\dot{\alpha}}{}^\beta & O \end{pmatrix} \begin{pmatrix} \chi_{\beta i} \\ \bar{\xi}_{\dot{\beta}i} \end{pmatrix}, \quad \tilde{\slashed{D}} \equiv \begin{pmatrix} 0 & \slashed{D}_\alpha{}^{\dot{\beta}} \\ \slashed{D}_{\dot{\alpha}}{}^\beta & 0 \end{pmatrix}. \quad (2.7)$$

We can also introduce the $2N \times 2N$ mass matrix $\tilde{m}$ and the usual 4×4 γ_5 matrix for the four-component notation by

$$\tilde{m} \equiv \begin{pmatrix} m_i{}^j & 0 \\ 0 & m_i{}^j \end{pmatrix}, \quad \gamma_5 \equiv \begin{pmatrix} \delta_\alpha{}^\beta & 0 \\ 0 & -\delta_{\dot{\alpha}}{}^{\dot{\beta}} \end{pmatrix}. \quad (2.8)$$

Note also that $\tilde{m}$ is Hermitian: $\tilde{m}^\dagger = \tilde{m}$. The eigenstates in Eq. (2.6) are much more transparent now for $\tilde{\slashed{D}}$ as

$$\tilde{\slashed{D}} \psi_\mu = +\mu \psi_\mu, \quad \tilde{\slashed{D}} \psi_{(-\mu)} = -\mu \psi_{(-\mu)}, \quad (2.9)$$

The eigenstates $\psi_{(-\mu)}$ correspond to $\psi_{(-\mu)} \equiv \gamma_5 \psi_\mu$, because γ_5 satisfies $\{\gamma_5, \tilde{\slashed{D}}\} = 0$, and therefore

$$\tilde{\slashed{D}} \psi_{(-\mu)} = \tilde{\slashed{D}}(\gamma_5 \psi_\mu) = -\gamma_5 \tilde{\slashed{D}} \psi_\mu = -\mu \gamma_5 \psi_\mu = (-\mu) \psi_{(-\mu)}. \quad (2.10)$$

Therefore the pairing between μ and $-\mu$ eigenstates in Eq. (2.6) is clear. Accordingly, the Lagrangian $\mathcal{L}_F$ is simply

$$\mathcal{L}_F = \bar{\psi}(i\tilde{\slashed{D}} + \tilde{m})\psi. \quad (2.11)$$

Our path integral (2.1) is now

$$e^{-VE(\lambda)} = \int [dA_a{}^I] \int [d\psi][d\bar{\psi}] e^{-\int d^4 x(\mathcal{L}_F + \mathcal{L}_B + i\lambda X)}$$

$$= \int [dA_a{}^I] \int [d\psi][d\bar{\psi}] e^{-\int d^4 x(\mathcal{L}_B + i\lambda X)} \exp\left[-\int d^4 x \bar{\psi}(i\slashed{D} + \tilde{m})\psi \right]$$

$$= \int [dA_a^I] I_F \exp\left[-\int d^4 x(\mathcal{L}_B + i\lambda X) \right]. \tag{2.12}$$

Here I_F is the fermionic determinant from the fermionic path integral:

$$I_F \equiv \int [d\psi][d\bar{\psi}] \exp\left[-\int d^4 x \bar{\psi}(i\slashed{D} + \tilde{m})\psi \right]$$

$$= \mathrm{Det}(i\slashed{D} + \tilde{m})$$

$$= \prod_\mu \det(i\mu I_{2N} + \tilde{m})$$

$$= (\det \tilde{m})^{n_{L0} + n_{R0}} \left[\prod_{\mu>0} \det(i\mu I_{2N} + \tilde{m}) \right] \left[\prod_{\mu<0} \det(i\mu I_{2N} + \tilde{m}) \right]$$

$$= (\det \tilde{m})^{2n_0} \left[\prod_{\mu>0} \det(i\mu I_{2N} + \tilde{m}) \right] \left[\prod_{\mu>0} \det(-i\mu I_{2N} + \tilde{m}) \right] \tag{2.13a}$$

$$= (\det \tilde{m})^{2n_0} \prod_{\mu>0} \det(i\mu I_{2N} + \tilde{m}) \det(-i\mu I_{2N} + \tilde{m})$$

$$= (\det \tilde{m})^{2n_0} \prod_{\mu>0} \det(i\mu I_{2N} + \tilde{m}) \det(+i\mu I_{2N} + \tilde{m})^\dagger \tag{2.13b}$$

$$= (\det \tilde{m})^{2n_0} \prod_{\mu>0} [\det(i\mu I_{2N} + \tilde{m})][\det(+i\mu I_{2N} + \tilde{m})]^*$$

$$= (\det \tilde{m})^{2n_0} \left| \prod_{\mu>0} \det(i\mu I_{2N} + \tilde{m}) \right|^2 > 0. \tag{2.13c}$$

Here I_{2N} is an $2N \times 2N$ unit matrix, and in Eq. (2.13a), n_0 is the number of $\mu = 0$ modes, satisfying $n_{L0} = n_{R0} \equiv n_0$. This is because we have to consider only the instanton number zero background $n_{L0} - n_{R0} = 0$ that is connected with the original vacuum with energy $E(\lambda = 0)$ [17]. The previously-mentioned parings $\mu \leftrightarrow -\mu$ are used also in Eq. (2.13a). The determinants in Eq. (2.13a) are taken for the $2N \times 2N$ matrix for flavor indices, distinguished from the symbol ''Det'' for the fermionic path integral. In Eq. (2.13b) we have also used the Hermiticity of $\tilde{m}$. Now the positive definiteness of I_F is clear from Eqs. (2.13a)–(2.13c). For the case of $N = 1$, Eq. (2.13c) is in agreement with [4].

Once the fermionic determinant (2.13a)–(2.13c) is positive, we see that the path integral (2.1) is positive, except for the phase factor $\exp(i\lambda \int d^4 x X)$, which does not lower the ground state energy. This is why $E(\lambda)$ must have a minimum only at $\lambda = 0$ [4].

Before ending this section, we give the following lemma which will be of importance in the next section. Note that the Lagrangian $\mathcal{L}_F$ is rewritten as

$$\mathcal{L}_F = \sum_\mu \mathcal{L}_\mu \equiv \sum_\mu [+i\mu(\bar{\chi}^{\dot{\alpha} i}{}_\mu \bar{\xi}_{\dot{\alpha} i \mu}) + i\mu(\xi^{\alpha i}{}_\mu \chi_{\alpha i \mu}) + m_i{}^j(\bar{\chi}^{\dot{\alpha} i}{}_\mu \bar{\xi}_{\dot{\alpha} j \mu}) + m_i{}^j(\xi^{\alpha i}{}_\mu \chi_{\alpha j \mu})]. \tag{2.14}$$

Accordingly, the path integral (2.13a)–(2.13c) is also equivalent to

$$I_F = \left(\prod_\mu \int [d\chi_\mu][d\bar{\chi}_\mu][d\xi_\mu][d\bar{\xi}_\mu] \right) \exp\left[\int d^4 x \sum_\mu \{\xi_{\alpha\mu} C^{\alpha\beta}(m + i\mu I_N)\chi_{\beta\mu} + \bar{\chi}_{\dot{\alpha}\mu} C^{\dot{\alpha}\dot{\beta}}(m + i\mu I_N)\bar{\xi}_{\dot{\beta}\mu}\} \right]$$

$$= (\det \tilde{m})^{2n_0} \left| \prod_{\mu>0} \det(i\mu I_{2N} + \tilde{m}) \right|^2 > 0. \tag{2.15}$$

In other words, I_F in Eqs. (2.13a)–(2.13c) can be computed in terms of 2-component spinors in Eq. (2.6). This relationship will be helpful when we consider complicated mixed Lagrangians between the gaugini and the quark or lepton fields in the next section.

III. VECTORLIKE THEORY WITH SUPERSYMMETRY

We now generalize the above method to supersymmetric theories with no Yukawa couplings among quarks. Suppose we have the massive quark fermions χ_i, $\bar{\chi}^i$, ξ^i, $\bar{\xi}_i$ together with the massless gluino Majorana fields $\lambda_\alpha{}^I$, $\bar{\lambda}_{\dot\alpha}{}^I$. As before, the indices $I, J, \ldots = 1, 2, \ldots, g = \dim G$ are for the adjoint representations of the gauge group G. All the fermion-dependent terms in our Lagrangian are

$$
\begin{aligned}
\mathcal{L}_F = {}&+ i(\bar{\chi}^{\dot\alpha i} D\!\!\!/_{\dot\alpha}{}^\beta \chi_{\beta i}) + i(\xi^{\alpha i} D\!\!\!/_\alpha{}^{\dot\beta}\bar{\xi}_{\dot\beta i}) + i(\bar{\lambda}^{\dot\alpha I} D\!\!\!/_{\dot\alpha}{}^\beta \lambda_\beta{}^I) \\
&+ m_i{}^j(\bar{\chi}^{\dot\alpha i}\bar{\xi}_{\dot\alpha j}) \\
&+ m_j{}^i(\xi^\alpha \chi_{\alpha i}) + i(T^I)_i{}^j[z^{*i}(\lambda^{\alpha I}\chi_{\alpha j}) - z_j(\bar{\lambda}^{\dot\alpha I}\bar{\chi}_{\dot\alpha}{}^j)] \\
&- i(T^I)_i{}^j[u^i(\bar{\lambda}^{\dot\alpha I}\bar{\xi}_{\dot\alpha j}) - u_j^*(\lambda^{\alpha I}\xi_\alpha{}^i)].
\end{aligned} \tag{3.1}
$$

The z_i and u^i are the spin 0 fields (squarks) in the chiral multiplets (z_i, χ_i) and (u^i, ξ^i) with $i, j, \ldots = 1, 2, \ldots, N$, in the representations $\mathbf{N}$ and $\mathbf{N}^*$. The $(T^I)_i{}^j$ are Hermitian generators of the gauge group. As in Eq. (2.2), we can assume that $m_i{}^j$ is Hermitian only with positive eigenvalues. The presence of these mixing terms with (pseudo)scalar dependence have been considered to be the main obstruction for the parity conservation in supersymmetric theories in the past [5–7], because they seem to prevent us from proving the positive definiteness of the fermionic determinant. However, we will see that this is not the obstruction. After the above prescription, there is no fermion-dependent term in the supersymmetric Lagrangian $\mathcal{L}$ left over other than $\mathcal{L}_F$: $\mathcal{L} = \mathcal{L}_F + \mathcal{L}_B$ with a purely bosonic Lagrangian $\mathcal{L}_B$.

We now consider the eigenstates for χ and ξ as in Eq. (2.4), and rewrite all the χ and ξ-dependent terms in $\mathcal{L}_F$, as

$$
\begin{aligned}
\mathcal{L}_{\chi,\xi} = \sum_\mu \mathcal{L}_{\chi,\xi,\mu} \equiv \sum_\mu \Big[&- \xi_\alpha{}^i{}_\mu C^{\alpha\beta}(m_i{}^j + i\mu\delta_i{}^j)\chi_{\beta j\mu} \\
&- \bar{\chi}_{\dot\alpha}{}^i{}_\mu C^{\dot\alpha\dot\beta}(m_i{}^j + i\mu\delta_i{}^j)\bar{\xi}_{\dot\beta j\mu} \\
&- \xi_\alpha{}^i{}_\mu C^{\alpha\beta}\rho_{\beta i} - \bar{\chi}_{\dot\alpha}{}^i{}_\mu C^{\dot\alpha\dot\beta}\bar{\omega}_{\dot\beta i} - \bar{\xi}_{\dot\alpha i\mu}C^{\dot\alpha\dot\beta}\bar{\rho}_{\dot\beta}{}^i \\
&- \chi_{\alpha i\mu}C^{\alpha\beta}\omega_\beta{}^i \Big],
\end{aligned} \tag{3.2}
$$

where

$$
\rho_{\alpha i} \equiv +i(T^I u^*)_i \lambda_\alpha{}^I, \quad \omega_\alpha{}^i \equiv +i(z^* T^I)^i \lambda_\alpha{}^I,
$$

$$
\bar{\rho}_{\dot\alpha}{}^i \equiv -i(u T^I)^i \bar{\lambda}_{\dot\alpha}{}^I, \quad \bar{\omega}_{\dot\alpha i} \equiv -i(T^I z)_i \bar{\lambda}_{\dot\alpha}{}^I. \tag{3.3}
$$

As usual in path integral, we can redefine the fields in such a way that the linear terms in χ or ξ disappear. In our case, this can be done by the field redefinitions[2]

$$
\begin{aligned}
\tilde{\xi}_\alpha{}^i{}_\mu &\equiv \xi_\alpha{}^i{}_\mu + \omega_\alpha{}^j(m + i\mu I_N)^{-1}{}_j{}^i, \\
\tilde{\chi}_{\alpha i\mu} &\equiv \chi_{\alpha i\mu} + (m + i\mu I_N)^{-1}{}_i{}^j \rho_{\alpha j},
\end{aligned} \tag{3.4a}
$$

$$
\begin{aligned}
\tilde{\bar{\xi}}_{\dot\alpha i\mu} &\equiv \bar{\xi}_{\dot\alpha i\mu} + (m + i\mu I_N)^{-1}{}_i{}^j \bar{\omega}_{\dot\alpha j}, \\
\tilde{\bar{\chi}}_{\dot\alpha}{}^i{}_\mu &\equiv \bar{\chi}_{\dot\alpha}{}^i{}_\mu + \bar{\rho}_{\dot\alpha}{}^j(m + i\mu I_N)^{-1}{}_j{}^i,
\end{aligned} \tag{3.4b}
$$

to have

$$
\begin{aligned}
\mathcal{L}_{\xi,\chi,\mu} = {}&- \tilde{\xi}_\alpha{}^i{}_\mu C^{\alpha\beta}(m + i\mu I_N)_i{}^j \tilde{\chi}_{\beta j\mu} - \tilde{\bar{\chi}}_{\dot\alpha}{}^i{}_\mu C^{\dot\alpha\dot\beta}(m + i\mu I_N)_i{}^j \\
&\times \tilde{\bar{\xi}}_{\dot\beta j\mu} + \omega_\alpha{}^i C^{\alpha\beta}(m + i\mu I_N)^{-1}{}_i{}^j \rho_{\beta j} \\
&+ \bar{\rho}_{\dot\alpha}{}^i C^{\dot\alpha\dot\beta}(m + i\mu I_N)^{-1}{}_i{}^j \bar{\omega}_{\dot\beta j}.
\end{aligned} \tag{3.5}
$$

Since m is Hermitian only with positive eigenvalues, $m + i\mu I_N$ is also diagonalizable only with nonzero eigenvalues, and there is no problem for defining the inverse $(m + i\mu I_N)^{-1}$. After this, $\mathcal{L}_F$ is now

$$
\mathcal{L}_F = \sum_\mu \mathcal{L}'_{\chi,\xi,\mu} + \sum_\mu \mathcal{L}_{\lambda^2,\mu} + \mathcal{L}_{\lambda D\!\!\!/\lambda}, \tag{3.6}
$$

where $\mathcal{L}'_{\chi,\xi,\mu}$ is the first line of Eq. (3.5) which coincides with the nonsupersymmetric case (2.7), $\mathcal{L}_{\lambda D\!\!\!/\lambda}$ is the gluino kinetic term, while $\mathcal{L}_{\lambda^2,\mu}$ is the λ^2 and $\bar{\lambda}^2$ terms after the field redefinition (3.4). Let us collect all of these λ-dependent terms into $\mathcal{L}_\lambda$:

$$
\begin{aligned}
\mathcal{L}_\lambda \equiv {}&- \bar{\lambda}_{\dot\alpha}{}^I D\!\!\!/^{\beta\dot\alpha}\lambda_\beta{}^I - \sum_\mu \lambda_\alpha{}^I C^{\alpha\beta}M_\mu{}^{IJ}\lambda_\beta{}^J \\
&- \sum_\mu \bar{\lambda}_{\dot\alpha}{}^I C^{\dot\alpha\dot\beta}\bar{M}_\mu{}^{IJ}\bar{\lambda}_{\dot\beta}{}^J \\
\equiv {}&- i\bar{\lambda}_{\dot\alpha}{}^I D\!\!\!/^{\beta\dot\alpha}\lambda_\beta{}^I - \lambda_\alpha{}^I C^{\alpha\beta}M^{IJ}\lambda_\beta{}^J \\
&- \bar{\lambda}_{\dot\alpha}{}^I C^{\dot\alpha\dot\beta}\bar{M}^{IJ}\bar{\lambda}_{\dot\beta}{}^J,
\end{aligned} \tag{3.7}
$$

where the matrices $M_\mu \equiv (M_\mu{}^{IJ})$, $\bar{M}_\mu \equiv (M_\mu{}^{IJ})$, $M \equiv (M^{IJ})$ and $\bar{M} \equiv (\bar{M}^{IJ})$ are defined by

$$
M_\mu{}^{IJ} \equiv -(z^* T^{(I|} \tilde{m}_\mu^{-1} T^{|J)} u^*),
$$

$$
\bar{M}_\mu{}^{IJ} \equiv -(u T^{(I|} \tilde{m}_\mu^{-1} T^{|J)} z),
$$

$$
M^{IJ} \equiv \sum_\mu M_\mu{}^{IJ}, \quad \bar{M}^{IJ} \equiv \sum_\mu \bar{M}_\mu{}^{IJ}, \quad \tilde{m}_\mu \equiv m + i\mu I_N. \tag{3.8}
$$

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[2]Note that Eq. (3.4b) is not necessarily the Hermitian conjugate of Eq. (3.4a). This is related to the Hermiticity only by the combination of $+\mu$ and $-\mu$.

Due to the antisymmetry of $C^{\alpha\beta}$ and $C^{\dot\alpha\dot\beta}$, the matrices M_μ, $\bar M_\mu$, M and $\bar M$ are all symmetric in $I\leftrightarrow J$. Note also that

$$M_\mu{}^\dagger = M_\mu{}^* = \bar M_{-\mu}, \quad M^\dagger = M^* = \bar M, \qquad (3.9)$$

the latter of which is confirmed by the former under Σ_μ, which is symmetric between $+\mu\leftrightarrow-\mu$. The M is not necessarily Hermitian, and it has both real and imaginary parts. Now $\mathcal{L}_\lambda$ is rewritten as

$$\mathcal{L}_\lambda = -(\lambda^{\alpha I}, \bar\lambda^{\dot\alpha I})\begin{pmatrix} \delta_\alpha{}^\beta M^{IJ} & i\delta^{IJ}\slashed{D}_\alpha{}^{\dot\beta} \\ i\delta^{IJ}\slashed{D}^\beta{}_{\dot\alpha} & \delta_{\dot\alpha}{}^{\dot\beta}\bar M^{IJ} \end{pmatrix}\begin{pmatrix} \lambda_\beta{}^J \\ \bar\lambda_{\dot\beta}{}^J \end{pmatrix}$$

$$= \bar\Lambda(i\mathcal{D}+\mathcal{M})\Lambda, \qquad (3.10a)$$

$$\mathcal{D}\equiv\begin{pmatrix} O & \slashed{D}_\alpha{}^{\dot\beta} \\ \slashed{D}^\beta{}_{\dot\alpha} & O \end{pmatrix}\otimes I_g, \quad \mathcal{M}\equiv I_2\otimes\begin{pmatrix} M & O \\ O & \bar M \end{pmatrix},$$

$$(3.10b)$$

$$\bar\Lambda\equiv-(\lambda^\alpha,\bar\lambda^{\dot\alpha})=(\lambda_\beta,\bar\lambda_{\dot\beta})\begin{pmatrix} C^{\beta\alpha} & O \\ O & C^{\dot\beta\dot\alpha} \end{pmatrix}=\Lambda^T\mathcal{C},$$

$$\mathcal{C}\equiv\begin{pmatrix} C^{\alpha\beta} & O \\ O & C^{\dot\alpha\dot\beta} \end{pmatrix}. \qquad (3.10c)$$

The original fermionic Lagrangian now is $\mathcal{L}_F=\Sigma_\mu\mathcal{L}'_{\chi,\xi,\mu}+\mathcal{L}_\lambda$, and the total path integral to be considered in the Euclidian space is

$$e^{-VE(\lambda)}=\int [dA_a{}^I][dz][dz^*][du][du^*]I_F e^{-\int d^4x(\mathcal{L}_B+i\lambda X)}, \qquad (3.11)$$

where the fermionic path integral I_F is from Eqs. (3.5) and (3.10) with $\mathcal{M}$ replaced by $\mathcal{M}$:

$$I_F=\int [d\chi][d\bar\chi][d\xi][d\bar\xi][d\lambda][d\bar\lambda]\exp\left[-\int d^4x\left(\sum_\mu \mathcal{L}'_{\chi,\xi,\mu}+\mathcal{L}_\lambda\right)\right]$$

$$=\left(\prod_\mu \int [d\tilde\chi_\mu][d\tilde{\bar\chi}_\mu][d\tilde\xi_\mu][d\tilde{\bar\xi}_\mu]e^{-\int d^4x\mathcal{L}'_{\chi,\xi,\mu}}\right)\int [d\lambda][d\bar\lambda]e^{-\int d^4x\mathcal{L}_\lambda}$$

$$=(\det m)^{2n_0}\left|\prod_{\mu>0} \det(m+i\mu I_N)\right|^2 [\mathrm{Det}(i\mathcal{D}+\mathcal{M})]^{1/2}. \qquad (3.12)$$

The first two factors are from the $\int[d\chi][d\bar\chi][d\xi][d\bar\chi]$ integral as in the nonsupersymmetric case (2.15) now with the shifted variables $\tilde\chi,\tilde{\bar\chi},\tilde\xi,\tilde{\bar\xi}$, and the remaining factor is from the $\int[d\lambda][d\bar\lambda]$ integral. There is potential phase ambiguity [18,19] for taking the square root in the last factor in Eq. (3.12). However, we will shortly show that there is no problem with this ambiguity in vectorlike theories.

Even though we cannot diagonalize $\mathcal{D}$ and $\mathcal{M}$ simultaneously, we still can use the eigenstate $|\nu\rangle$ for the eigenvalue $\nu\in\mathbb{R}$ of the operator $\mathcal{D}$:

$$\mathcal{D}|\nu\rangle=+\nu|\nu\rangle, \quad \text{i.e.,} \quad \mathcal{D}\Lambda_\nu=\mathcal{D}\begin{pmatrix} \lambda_\nu \\ \bar\lambda_\nu \end{pmatrix}=+\nu\begin{pmatrix} \lambda_\nu \\ \bar\lambda_\nu \end{pmatrix}\equiv\nu\Lambda_\nu, \qquad (3.13)$$

where the subscript $_\nu$ on Λ_ν, etc. denotes the eigenvalue for the four-component spinor Λ, with the adjoint index ^{I} omitted. As usual, we can define

$$\Gamma_5\equiv I_2\otimes\begin{pmatrix} I_g & O \\ O & -I_g \end{pmatrix}, \qquad (3.14)$$

satisfying $\{\Gamma_5,\mathcal{D}\}=0$, so that an eigenstate $|-\nu\rangle$ of $\mathcal{D}$ can be constructed by

$$\mathcal{D}[\Gamma_5|\nu\rangle]=-\Gamma_5\mathcal{D}|\nu\rangle=(-\nu)[\Gamma_5|\nu\rangle]\Rightarrow\Gamma_5|\nu\rangle=|-\nu\rangle. \qquad (3.15)$$

Therefore any eigenstate for $\forall\nu>0$ is always paired up with an eigenstate $-\nu<0$. We now see that a conjugate state $\langle\nu|$ is related to $|\nu\rangle$ as follows. Consider

$$\Lambda^\dagger=\begin{pmatrix} \lambda_\alpha \\ \bar\lambda_{\dot\alpha} \end{pmatrix}^\dagger=(-\bar\lambda_{\dot\alpha},-\lambda_\alpha)=(\lambda,\bar\lambda)\begin{pmatrix} O & -I_g \\ -I_g & O \end{pmatrix}=\Lambda^T\mathcal{F}$$

$$= \bar\Lambda\mathcal{C}^{-1}\mathcal{F}, \qquad (3.16a)$$

$$\mathcal{F}\equiv I_2\otimes\begin{pmatrix} O & -I_g \\ -I_g & O \end{pmatrix}, \quad \mathcal{F}^{-1}=\mathcal{F},$$

$$[\mathcal{F},\mathcal{C}]=0, \quad [\mathcal{F},\mathcal{M}]=0. \qquad (3.16b)$$

Here $\bar\Lambda$ is the usual Dirac conjugate of Λ, and $\mathcal{F}$ is needed for complex conjugation. Therefore

$$|\nu\rangle^\dagger=\langle\nu|\mathcal{C}^{-1}\mathcal{F}^{-1}, \quad \langle\nu|^\dagger=\mathcal{F}\mathcal{C}|\nu\rangle. \qquad (3.17)$$

Accordingly, as in Eq. (3.15) we can confirm that

$$\langle-\nu|=\langle\nu|\Gamma_5. \qquad (3.18)$$

Other important relations needed are

$$\mathcal{F}\mathcal{M}\mathcal{F}^{-1}=\mathcal{M}^{\dagger}, \quad \mathcal{C}\mathcal{M}\mathcal{C}^{-1}=\mathcal{M}, \quad [\Gamma_5,\mathcal{M}]=0, \tag{3.19}$$

$$\mathcal{F}\mathcal{D}\mathcal{F}^{-1}=+\hat{\mathcal{D}}\equiv\begin{pmatrix} 0 & \mathcal{D}^{\beta}{}_{\dot{\alpha}} \\ \mathcal{D}_{\alpha}{}^{\dot{\beta}} & 0 \end{pmatrix},$$

$$\mathcal{C}\mathcal{D}\mathcal{C}^{-1}=-\hat{\mathcal{D}}, \quad \mathcal{D}^{\dagger}=-\hat{\mathcal{D}}, \tag{3.20}$$

as easily confirmed. Using these as well as Eq. (3.16), we get

$$\Gamma_5(i\mathcal{D}+\mathcal{M})\Gamma_5=-i\mathcal{D}+\mathcal{M}=+\mathcal{C}^{-1}\mathcal{F}^{-1}(i\hat{\mathcal{D}}+\mathcal{M}^{\dagger})\mathcal{F}\mathcal{C}$$
$$=\mathcal{C}^{-1}\mathcal{F}^{-1}(-i\mathcal{D}^{\dagger}+\mathcal{M}^{\dagger})\mathcal{F}\mathcal{C}$$
$$=+\mathcal{C}^{-1}\mathcal{F}^{-1}(i\mathcal{D}+\mathcal{M})^{\dagger}\mathcal{F}\mathcal{C}. \tag{3.21}$$

The determinant in the square root in the last factor in Eq. (3.12) can be reexpressed as the usual definition of the determinant in terms of exponential, trace, and logarithmic functions:

$$\text{Det}(i\mathcal{D}+\mathcal{M})=(\det M)^{\tilde{n}_0}(\det \bar{M})^{\tilde{n}_0}$$

$$\times \prod_{\nu\neq 0} \exp[\langle\nu|\ln(i\mathcal{D}+\mathcal{M})|\nu\rangle]. \tag{3.22}$$

As in the case of quarks, we consider only the instanton number zero background, so that the number of left- and right-handed zero modes are the same: $\tilde{n}_{L0}=\tilde{n}_{R0}\equiv\tilde{n}_0$, whose contributions in Eq. (3.22) can be computed separately, as

$$(\det M)^{\tilde{n}_0}(\det \bar{M})^{\tilde{n}_0}=(\det M)^{\tilde{n}_0}(\det M)^{*\tilde{n}_0}=|\det M|^{2\tilde{n}_0}. \tag{3.23}$$

As for the $\nu\neq 0$ contributions, due to the pairing between the $|\nu\rangle$ and $|-\nu\rangle$, Eq. (3.22) is rewritten as

$$\text{Det}(i\mathcal{D}+\mathcal{M})=|\det M|^{2\tilde{n}_0}\left[\prod_{\nu>0} \exp[\langle\nu|\ln(i\mathcal{D}+\mathcal{M})|\nu\rangle]\right]\left[\prod_{\nu<0} \exp[\langle\nu|\ln(i\mathcal{D}+\mathcal{M})|\nu\rangle]\right]$$

$$=|\det M|^{2\tilde{n}_0}\prod_{\nu>0} \exp[\langle\nu|\ln(i\mathcal{D}+\mathcal{M})|\nu\rangle]\exp[\langle-\nu|\ln(i\mathcal{D}+\mathcal{M})|-\nu\rangle], \tag{3.24}$$

where the exponent in the last factor is simplified by the aid of Eqs. (3.17) and (3.21) as

$$\langle-\nu|\ln(i\mathcal{D}+\mathcal{M})|-\nu\rangle=\langle\nu|\Gamma_5\ln(i\mathcal{D}+\mathcal{M})\Gamma_5|\nu\rangle$$
$$=\langle\nu|\mathcal{C}^{-1}\mathcal{F}^{-1}\{\ln(i\mathcal{D}+\mathcal{M})\}^{\dagger}\mathcal{F}\mathcal{C}|\nu\rangle=[\langle\nu|\ln(i\mathcal{D}+\mathcal{M})|\nu\rangle]^{\dagger}$$
$$=[\langle\nu|\ln(i\mathcal{D}+\mathcal{M})|\nu\rangle]^{*}. \tag{3.25}$$

Therefore Eq. (3.24) is semipositive definite:

$$\text{Det}(i\mathcal{D}+\mathcal{M})=|\det M|^{2\tilde{n}_0}\prod_{\nu>0} |\exp(\nu|\ln(i\mathcal{D}+\mathcal{M})|\nu)|^2\geq 0. \tag{3.26}$$

Combining this with Eq. (3.12), we get the semipositive definiteness of the fermionic determinant:

$$I_F=|\det m|^{2n_0}|\det M|^{\tilde{n}_0}\left|\prod_{\mu>0} \det(m+i\mu I_N)\right|^2\prod_{\nu>0} |\exp\langle\nu|\ln(i\mathcal{D}+\mathcal{M})|\nu\rangle|\geq 0. \tag{3.27}$$

Notice that the usual phase ambiguity when taking the square root [18,19] does not arise here, because of the semipositive definite expression of Eq. (3.26), as contributions from $|\nu\rangle$ and $|-\nu\rangle$ always in pairs. The main ingredient in this proof is the usage of the eigenstate $|\nu\rangle$ with the properties of Γ_5 and complex conjugations, which do not require the diagonalization of $\mathcal{M}$, or even its commutator with $\mathcal{D}$. The crucial procedure we have relied on is the expression of the determinant in terms of exponential, trace, and logarith-

mic functions, which is to be the universal definition for a determinant.

Note that Eq. (3.27) implies only non-negativity of I_F which can still be zero. This is because the matrix $\mathcal{M}$ can depend on the scalar coordinates $Z\equiv(z,z^{*},u,u^{*})$. However, we can further show that there exists a measurable support (a connected domain with nonzero measure) in the Z space, on which $I_F(Z)>0$ and nonzero. In fact, consider the particular point $Z_0=0$ on which $\mathcal{M}=O$ due to Eqs. (3.8), (3.9), and

(3.10). It follows that

$$I_F(0) = |\det m|^{2n_0} |\det M|^{\tilde{n}_0}$$

$$\times \left| \prod_{\mu>0} \det(m + i\mu I_N) \right|^2 \prod_{\nu>0} |\exp\langle \nu | \ln(i\nu) | \nu \rangle| > 0.$$

$$(3.28)$$

Once we get $I_F(0) > 0$, then relying on the smoothness of $I_F(Z)$ as a function of Z, we can conclude that $I_F(Z) > 0$ on a measurable support including $Z_0 = 0$. The existence of a measurable support for $I_F > 0$ leads us to the positivity of the path-integral measure $\int [dz][dz^*][du][du^*] I_F > 0$, and we conclude that parity is conserved in supersymmetric vector-like theories.

In the above analysis, we have performed the most usual Wick rotation from the Minkowskian metric $(+,-,-,-)$ into the Euclidian one $(-,-,-,-)$ by replacing formally the co-ordinate $x^0 \to ix^4$. However, this may need more care, when it comes to the complex conjugation of spinors. Motivated by this, we have reconfirmed our result above by an alternative Wick rotation into the metric $(+,+,+,+)$. Additionally, the spinors in these Euclidean spaces are only ''formally'' defined, in such a way that their complex conjugation rule is essentially the parallel to the Minkowskian case, like the simple replacement $x^0 \to ix^4$, and this is the very reason why the Feynman rules in the usual Euclidian path integral are essentially the same as those in the Minkowskian. Rigorously speaking, spinors in the Euclidian spaces can exist only as $USp(2)$ spinors [20], and moreover the dotted and undotted spinors as eigenvectors of the γ_5 matrix are no longer related by complex conjugations [20].[3] For this precaution, we have also reformulated the Wick rotation, such that the dotted λ_α and undotted $\bar{\lambda}_{\dot{\alpha}}$ spinors in the final Eu-clidian space are not related to each other under complex conjugation, as they should be [20]. Interestingly, we have reached the same conclusion for the semipositive definite-ness of the determinant (3.27), even though the meaning of the bra and cket vectors are slightly modified, and all the pseudoscalar Yukawa couplings with γ_5 acquire an extra factor of i like the λX term in Eq. (2.1). One additional feature in this case we seem to rely on is that the gluino zero modes are unstable and disappear from the physical spec-trum, based on the analysis in Ref. [22]. Since the details of this formulation are rather technical leading essentially to the same conclusion, we skip them in this paper.

In this section, when dealing with the path integral in Euclidean space, we have ''implicitly'' assumed the exis-tence of a supersymmetric regularization [23], such as the dimensional reduction scheme originally developed by Sie-gel [24]. In dealing with a supersymmetric regularization for nonperturbative computations such as path integrals, we may have to be careful about a possible anomaly for supersym-metry, and the validity of regularization to higher orders. As

far as we understand, however, there has been no rigorous proof of the validity of such a regularization to all orders. Since this subtlety is outside of the scope of this paper, we do not address ourselves to this issue here.

IV. VECTORLIKE THEORY WITH BROKEN SUPERSYMMETRY

We mention the possibility of adding some gluino mass terms, which may be caused by some spontaneous, explicit, or nonperturbative breaking of supersymmetry. This is easily considered, by adding the gluino mass terms

$$\mathcal{L}_{m'\lambda^2} \equiv m'^{IJ}(\lambda^{\alpha I} \lambda_\alpha^J) + m'^{IJ}(\bar{\lambda}^{\dot{\alpha} I} \bar{\lambda}_{\dot{\alpha}}^J), \qquad (4.1)$$

to our original Lagrangian (3.1). Here $m' \equiv (m'^{IJ})$ is real and symmetric. Accordingly, Eq. (3.10a) is now

$$\mathcal{L}'_\lambda \equiv -(\lambda^\alpha, \bar{\lambda}^{\dot{\alpha}}) \begin{pmatrix} I_2 \otimes (M+m') & i\slashed{D} \otimes I_g \\ i\slashed{D}^T \otimes I_g & I_2 \otimes (\bar{M}+m') \end{pmatrix} \begin{pmatrix} \lambda_\beta \\ \bar{\lambda}_{\dot{\beta}} \end{pmatrix}.$$

$$(4.2)$$

This implies that the matrix M is replaced by $M + m'$ and $\bar{M}$ by $\bar{M} + m'$. Under this shift, the relations such as $M^\dagger = \bar{M}$ are in tact. Eventually Eq. (3.27) is now replaced by

$$I'_F = |\det m|^{2n_0} |\det M|^{\tilde{n}_0} \left| \prod_{\mu>0} \det(m + i\mu I_N) \right|^2$$

$$\times \left| \prod_{\nu>0} \exp\langle \nu | \ln(i\slashed{D} + \mathcal{M}') | \nu \rangle \right| \geq 0, \qquad (4.3)$$

where $\mathcal{M}'$ is a $4g \times 4g$ matrix similar to $\mathcal{M}$ defined by

$$\mathcal{M}' \equiv \mathcal{M} + I_2 \otimes \begin{pmatrix} m' & O \\ O & m' \end{pmatrix}, \quad \mathcal{M}'^\dagger = \mathcal{M}'^*. \quad (4.4)$$

Hence the presence of m' does not affect the semipositive definiteness of the fermionic determinant. Accordingly, we can also show that $I'_F > 0$ on a measurable support in the Z space, and therefore we conclude that parity is conserved also in broken supersymmetric vectorlike theories with non-zero gluino masses.

V. CONCLUDING REMARKS

In this paper we have reconfirmed the conservation of parity in supersymmetric vectorlike theories. Even though this result has been known for a long time now, we have reconfirmed the parity conservation, based on an indepen-dent and different methodology. The methodology of our proof is to perform fermionic path integrals, which were originally developed by Vafa and Witten [4]. We have con-firmed that the determinant as the fermionic path integral I_F is positive and nonzero on a measurable support under the bosonic integral $\int [dz][dz^*][du][du^*]$. We have also seen that the quark-gluino-squark mixing terms in the supersym-metric theory pose no problem. The supersymmetric vector-like theory seems to avoid the problem with these mixing

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[3]This situation is similar to what is called Atiyah-Ward space-time with the signature $(+,+,-,-)$ studied in [21].

terms thanks to pairings between the eigenstates of the Dirac operator, despite the complication caused by the mixing with gluini. Our result relies on the original method in [4], namely we deal only with bosonic parity-breaking observables in path integrals. Therefore our result does not cover the parity breaking *via* fermionic observables [13] which do not acquire the imaginary unit i under the Wick rotation.

In our analysis of the fermionic path integral, we first integrated over the quark fields χ, $\bar{\chi}$, ξ, $\bar{\xi}$, making the computation more organized, instead of integrating over the gluino field first as in [6]. By so doing, we have seen that the final gluino path integral is less involved and more controllable, in particular when we need to consider the Majorana gluino determinant which used to have subtlety with γ_5-pseudo-scalar couplings. The nice and simple feature of our result is that the fermionic path integral can be cast into the form of Eq. (3.12), and that the phase ambiguity becomes unimportant. In other words, the usual phase ambiguity in the square root of the determinants for a 2-component spinor [18,19] does not arise in a vectorlike theory, due to the pairing between the integrals over dotted and undotted spinors, combined with the pairing between the positive and negative eigenstates of the Dirac operator, including the zero modes.

We have also studied the effect of the gluino masses, as a result of either spontaneous, explicit, or nonperturbative breaking of supersymmetry, and reached the conclusion that parity is also conserved in these cases with broken supersymmetry. Due to the path-integral formulation we have adopted here, this analysis is easier than other methods in Refs. [12, 11. In principle, we can generalize our result to the case with the squark masses caused by the supersymmetry breaking, but since these terms are purely bosonic affecting only $\mathcal{L}_B$, they are not expected to change our analysis or result in this paper.

At first glance, our result here seems to contradict with Ref. [9], which shows that the vacuum structure is disturbed, when there are more colors than flavors: $N_f < N$. However, there is actually no conflict, because we interpret this as the result of the masslessness of quarks treated in Ref. [9]. In our system, due to the massive quarks from the outset, the Witten index $\mathrm{Tr}(-1)^F$ is well-defined and nonzero [10], e.g., $\mathrm{Tr}(-1)^F = N$ for the $SU(N)$ gauge group. Hence we expect no breaking of supersymmetry even at the nonperturbative level, which would have ruined the foundation of our proof. Since the topological stability due to the well-defined $\mathrm{Tr}(-1)^F$ is reliable for massive quarks, it is quite natural that chiral symmetry or parity symmetry is also conserved. To put it differently, the parity conservation is natural from the fact that the index is saturated by the breaking of the intrinsic Z_N symmetry acting on quarks. Additionally, our parity conservation is also consistent with the result of [11] about the phase with unbroken discrete axial symmetry. Some subtlety arises, when the masses of the quarks become zero, because in such a case the Witten index $\mathrm{Tr}(-1)^F$ is no longer well-defined [10,25], and therefore the vacuum loses its stability against chiral or parity breakings as in [9]. From this viewpoint, we see no contradiction of our result with Refs. [9, 25], in which the masses of quarks are zero from the outset.

We have also seen that the supersymmetry breaking, if its only effect on fermions is the gluino masses, does not alter the conservation of parity. Even though this statement seems contradictory with the previous paragraph, we understand that the breaking of supersymmetry lifts the vacuum energy higher than the original supersymmetric and parity-conserving vacuum with $E(0) = 0$. Actually, the viewpoint that parity operates on gluini rather trivially also supports the absence of phase transitions even for massive gluini.

We can try to apply our method to other arbitrary global or discrete symmetries, such as baryon number, in a supersymmetric vectorlike theory, using the prescription using upper bounds for fermionic propagators in Ref. [26]. However, there seems to be an obstruction caused by the zeroness of mass eigenvalues, e.g., our matrix $\mathcal{M}$ in Eq. (3.10) hitting zeros, that upsets the upper bound for fermionic propagators, undermining the foundation for the stability of fermion-antifermions Greens functions against symmetry-breaking parameters [5,26]. In other words, our method using the semipositive definite fermionic determinant is powerful only for parity symmetry, or other symmetries based only on the vacuum to vacuum amplitude.

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