Scattering from an AdS_3 bubble and an exact AdS_3 space

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(Received 9 March 1999; published 27 December 1999)

We investigate the close relationship between the potential and absorption cross section for test fields in an anti–de Sitter $(\mathrm{AdS})_3$ bubble (a five-dimensional black hole) and an exact AdS_3 space. There are two solutions in type-IIB string theory: an AdS_3 bubble corresponds to the dilatonic solution, while an exact AdS_3 space is the nondilatonic solution. In order to obtain the cross section for an AdS_3 bubble, we introduce the {out}-state scattering picture with the AdS_3-asymptotically flat space matching procedure. For an exact AdS_3 space, one considers the {in}-state scattering picture with the AdS_3-AdS_3 matching. Here the non-normalizable modes are crucially taken into account for the matching procedure. It turns out that the cross sections for the test fields in an AdS_3 bubble take the same forms as those in an exact AdS_3 space. This suggests that in the dilute gas and low-energy limits, the *S* matrix for an AdS_3 bubble can be derived from an exact AdS_3 space.

PACS number(s): 04.70.Bw, 04.50.+h, 04.70.Dy

I. INTRODUCTION

There has been great progress in string theory of the D1-D5 brane system with momentum modes along the string direction (S^1). This gives us a five-dimensional (5D) black hole (M_5) with three charges (Q_1,Q_5,Q_n). The first progress was achieved in the Bekenstein-Hawking entropy [1]. Apart from the success of counting the microstates of a 5D black hole through D-brane physics, dynamical considerations become an important issue [2–5]. This is so because the semiclassical absorption cross section (gray-body factor) for a test field arises as a consequence of its potential barrier surrounding the horizon. That is, this is an effect of the space-time curvature. More precisely, it is worth noting that a semiclassical absorption cross section can be derived from a solution to the differential equation of a test field (most often, $\bar{\nabla}^2\phi=0$) on the supergravity side.

The anti–de Sitter (AdS) conformal field theory (CFT) correspondence states that string theory in the AdS space is dual to a conformal field theory defined on its remote boundary of AdS space [6]. The semiclassical limit of spacetime physics is related to the large-*N* limit of the dual CFT. A 5D black hole $(M_5\times S^1\times T^4)$ becomes $\mathrm{AdS}_3\times S^3\times T^4$ near the horizon but with an asymptotically flat space (AFS) [7]. Recently, this has been called an AdS_3 bubble in AFS and corresponds to the dilatonic solution [8]. In this case the matching procedure is crucial for obtaining an absorption coefficient and here we need to match an AdS_3 bubble to AFS.

On the other hand, one obtains $\mathrm{AdS}_3\times S^3\times T^4$ as the other solution to the type-IIB string theory. This is an exact AdS_3 space with asymptotic AdS_3 and corresponds to the nondilatonic solution [9]. Further it includes the Banados-Teitelboim-Zanelli (BTZ) black hole [10]. We point out that the near horizon limit of a 5D black hole yields the BTZ black hole, whereas this solution accomodates the BTZ black-hole spacetime as a whole.

It was understood that the gray-body factor calculation makes sense when one finds an asymptotically flat region as in Sec. II [6]. Hence, it may not be possible in an AdS_3 space because there is no asymptotic state corresponding to particle at infinity of AdS_3. However, the authors in [9] calculated the gray-body factor for a free scalar and the dilaton both in $M_5\times S^1\times T^4$ (an AdS_3 bubble) and $\mathrm{AdS}_3\times S^3\times T^4$ (an exact AdS_3 space) within the type-IIB supergravity. Here in the exact AdS_3 calculation we choose the non-normalizable modes to obtain the gray-body factor. This corresponds to the AdS_3-AdS_3 matching procedure. This expression denotes shorthand for a certain choice of boundary conditions where non-normalizable modes inject flux into AdS_3. It turns out that two results of a free scalar are exactly the same. And the results for the dilaton are the same up to a factor of 3. These show a close relationship between two approaches. More recently, vacuum correlators of the dual CFT_4 were expressed as truncated *n*-point functions for the non-normalizable modes in AdS_5. One can interpret this result as an *S* matrix of an exact AdS_5 space arising from a limit of scattering from an AdS_5 bubble [8]. This supports that our calculation based on the non-normalizable modes is correct.

In this paper we will show that the *S* matrix of an AdS_3 bubble can be derived partly from an exact AdS_3 space. This is one of the current issues in the AdS-CFT correspondence. For this purpose, we investigate the close relationship between the potential and absorption cross section in an AdS_3 bubble and an exact AdS_3 space. Comparing an AdS_3 bubble with an exact AdS_3 space leads to an assumption that the potential of an exact AdS_3 space is the left-hand side of an AdS_3 bubble. For this study, we introduce the {in} and {out}-state pictures for an AdS_3 bubble. For the exact AdS_3 study, one needs the {in}-state as well as non-normalizable modes. Further, we introduce the test fields for scattering analysis. These are in an AdS_3 bubble: a free scalar (ϕ) which, in the decoupling limit, relates to a (1,1) operator $\mathcal{O}$ in the holographically dual theory; two fixed scalars (ν,λ) to (2,2), (3,1), and (1,3) operators; two intermediate scalars (η,ξ) to (1,2) and (2,1) operators. On the exact AdS_3 side, the test fields are a free scalar (ψ), the dilaton (Φ), an intermediate scalar (η), and the tachyon (T).

The organization of this work is as follows. Section II is devoted to analyzing the scattering from an AdS_3 bubble within a 5D black hole. This corresponds to a conventional scattering study. We study the scattering of the test fields in an exact AdS_3 space in Sec. III. In this case we are careful for obtaining the absorption coefficient because we cannot define the asymptotically flat space. Finally, we discuss our results in Sec. IV.

II. SCATTERING FROM AN AdS_3 BUBBLE IN A 5D BLACK HOLE

Initially we introduce all perturbing modes in a 5D black-hole background. It is pointed out that in s-wave calculation the fixed scalars are physically propagating modes and other fields belong to be redundant modes [5]. Hence we choose two fixed scalars (ν,λ) and a free scalar (ϕ) as the relevant modes. We begin with the 5D black hole with three charges:

$$ds^2_{5\mathrm{D}}=-hf^{-2/3}dt^2+f^{1/3}(h^{-1}dr^2+r^2d\Omega_3^2), \tag{1}$$

where

$$f=f_1f_2f_n=\left(1+\frac{r_1^2}{r^2}\right)\left(1+\frac{r_5^2}{r^2}\right)\left(1+\frac{r_n^2}{r^2}\right),\quad h=\left(1-\frac{r_0^2}{r^2}\right). \tag{2}$$

Here the radii are related to the boost parameters (α_i) and the charges (Q_i) as

$$r_i^2=r_0^2\sinh^2\alpha_i=\sqrt{Q_i^2+\frac{r_0^4}{4}}-\frac{r_0^2}{2},i=1,5,n. \tag{3}$$

Hence the D-brane black hole depends on the four parameters (r_1,r_5,r_n,r_0). The event horizon (outer horizon) is clearly at $r=r_0$. When all three charges are nonzero, the surface $r=0$ becomes a smooth inner horizon (Cauchy horizon). When at least one of the charges is zero, the surface $r=0$ becomes singular. The extremal case corresponds to the limit of $r_0\to 0$ with the boost parameters $\alpha_i\to\pm\infty$, keeping the charges (Q_i) fixed. We are interested in the limit of $r_0,r_n\ll r_1,r_5$, which is called the dilute gas limit. This is so because this limit corresponds to the near-outer horizon. Here we choose $Q_1=r_1^2,Q_5=r_5^2$, and $r_n=r_0\sinh\alpha_n$ with a finite α_n. This corresponds to the near-extremal black hole and its thermodynamic quantities (energy, entropy, Hawking temperature) are given by

$$E_{\mathrm{next}}=\frac{2\pi^2}{\kappa_5^2}\left[r_1^2+r_5^2+\frac{1}{2}r_0^2\cosh 2\alpha_n\right], \tag{4}$$

$$S_{\mathrm{next}}=\frac{4\pi^3r_0}{\kappa_5^2}r_1r_5\cosh\alpha_n, \tag{5}$$

$$\frac{1}{T_{\mathrm{H,next}}}=\frac{2\pi}{r_0}r_1r_5\cosh\alpha_n, \tag{6}$$

where κ_5^2 is the 5D gravitational constant. The above energy and entropy are those of a gas of massless 1D particles. In this case the temperatures for left and right moving string modes are given by

$$T_L=\frac{1}{2\pi}\left(\frac{r_0}{r_1r_5}\right)e^{\alpha_n},\quad T_R=\frac{1}{2\pi}\left(\frac{r_0}{r_1r_5}\right)e^{-\alpha_n}. \tag{7}$$

This implies that the (left and right moving) momentum modes along the string direction are excited, while the excitations of D1-anti D1 and D5-anti D5 branes are suppressed. The Hawking temperature is given by their harmonic average

$$\frac{2}{T_H}=\frac{1}{T_L}+\frac{1}{T_R}. \tag{8}$$

A. Potential analysis

For a free scalar $[\phi=\phi(r)e^{i\omega t}Y_l(\theta_1,\theta_2,\theta_3)]$, the linearized equation $\bar{\nabla}^2\phi=0$ in the background of Eq. (1) leads to [2,3,11]

$$\left[(hr^3\partial_r)^2+\omega^2r^6f-\frac{l(l+2)h}{r^2}\right]\phi=0. \tag{9}$$

The s-wave $(l=0)$ linearized equation for the fixed scalars takes the form [4,5]

$$\left[(hr^3\partial_r)^2+\omega^2r^6f-\frac{8hr^4r_\pm^4}{(r^2+r_\pm^2)^2}\left(1+\frac{r_0^2}{r_\pm^2}\right)\right]\phi_\pm=0, \tag{10}$$

where one gets ν for ϕ_+ and λ for ϕ_-. Here $r_\pm^2=[r_1^2+r_5^2+r_n^2\pm\sqrt{r_1^4+r_5^4+r_n^4-r_1^2r_5^2-r_1^2r_n^2-r_5^2r_n^2}]/3$. Considering $N=r^{-3/2}\tilde{N}$, for $N=\nu,\lambda,\phi$ and introducing a tortoise coordinate $r^*=\int(dr/h)=r+(r_0/2)\ln|(r-r_0)/(r+r_0)|$ [2], then the equation takes the form

$$\frac{d^2\tilde{N}}{dr^{*2}}+(\omega^2-\tilde{V}_N)\tilde{N}=0. \tag{11}$$

Here we take $r_1=r_5=R$ and $r_0=r_n$ for simplicity. In the dilute gas limit $(R\gg r_0),\tilde{V}_N(r)$ is given by

$$\tilde{V}_\nu(r)=-\omega^2(f-1)+h\left[\frac{3}{4r^2}\left(1+\frac{3r_0^2}{r^2}\right)+\frac{8R^4}{r^2(r^2+R^2)^2}\right], \tag{12}$$

$$\tilde{V}_\lambda(r)=-\omega^2(f-1)+h\left[\frac{3}{4r^2}\left(1+\frac{3r_0^2}{r^2}\right)+\frac{8R^4}{r^2(3r^2+R^2)^2}\right], \tag{13}$$

$$\tilde{V}_\phi(r)=-\omega^2(f-1)+h\left[\frac{3}{4r^2}\left(1+\frac{3r_0^2}{r^2}\right)+\frac{l(l+2)}{r^2}\right], \tag{14}$$

where

$$f-1=\frac{r_0^2+2R^2}{r^2}+\frac{(2r_0^2+R^2)R^2}{r^4}+\frac{r_0^2R^4}{r^6}. \tag{15}$$

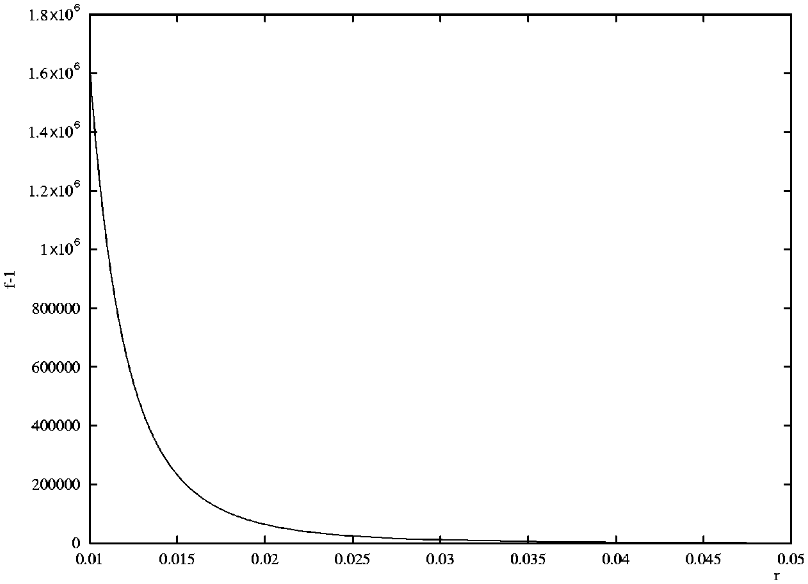

FIG. 1. The graph of $(f-1)$ in a 5D black hole with $r_0=0.01, R=0.3$. A peak appears at the outer horizon $(r=r_0)$.

We note that $\tilde{V}_N$ depends on two parameters (r_0, R) as well as the energy (ω). As Eq. (11) stands, it is far from the Schrödinger-type equation. The ω dependence is a matter of peculiar interest to us compared with the Schwarzschild black-hole potentials (V_{RW}, V_Z, V_ψ) [12]. This makes the interpretation of $\tilde{V}_N$ as a potential difficult. As is shown in Fig. 1, this arises because $(f-1)$ is very large, as large as 10^6 for $r_0=0.01, R=0.3$ in the near horizon. In order for $\tilde{V}_N$ to be a potential, it is necessary to take the low-energy limit of $\omega \to 0$. It is suitable to be 10^{-3}. And $\omega^2(f-1)$ is of the order $\mathcal{O}(1)$ and thus it can be ignored in comparison to the remaing ones. Now we can define a potential $V_N = \tilde{V}_N + \omega^2(f-1)$. Hence, in the low-energy limit $(\omega \to 0)$, Eq. (11) becomes similar to the Schrödinger-type equation. Further the last terms in Eqs. (12),(13) are important to compare each other. After the partial fraction, these lead to

$$\frac{8R^4}{r^2(r^2+R^2)^2} = \frac{8}{r^2} - \frac{8}{r^2+R^2} - \frac{8R^2}{(r^2+R^2)^2}, \tag{16}$$

$$\frac{8R^4}{r^2(3r^2+R^2)^2} = \frac{8}{r^2} - \frac{24}{3r^2+R^2} - \frac{24R^2}{(3r^2+R^2)^2}. \tag{17}$$

The last term of a free scalar in Eq. (14) with $l=2$ keeps the first terms in Eqs. (16),(17) only. One finds immediately the sequence

$$V_{\phi_0} \leqslant V_\lambda \leqslant V_\nu \leqslant V_{\phi_2}. \tag{18}$$

Here ϕ_0 denotes the s-wave $(l=0)$ free scalar and ϕ_2 the free one with $l=2$. This is also observed from the graphs of potential in Fig. 2 with $r_0=0.01, R=0.3$. Because the shape of their potentials takes nearly the same form $(V_\lambda \simeq V_\nu \simeq V_\phi^{l=2})$, these give us nearly the same reflection coefficient $\mathcal{R}=|R|^2$ and absorption one $\mathcal{A}=|A|^2$. For example, in the low-energy limit of $\omega \to 0$, λ, ν, ϕ_2 take nearly the zero-absorption cross section. Furthermore, all potentials go to zero, as r approaches infinity. This implies the existence of

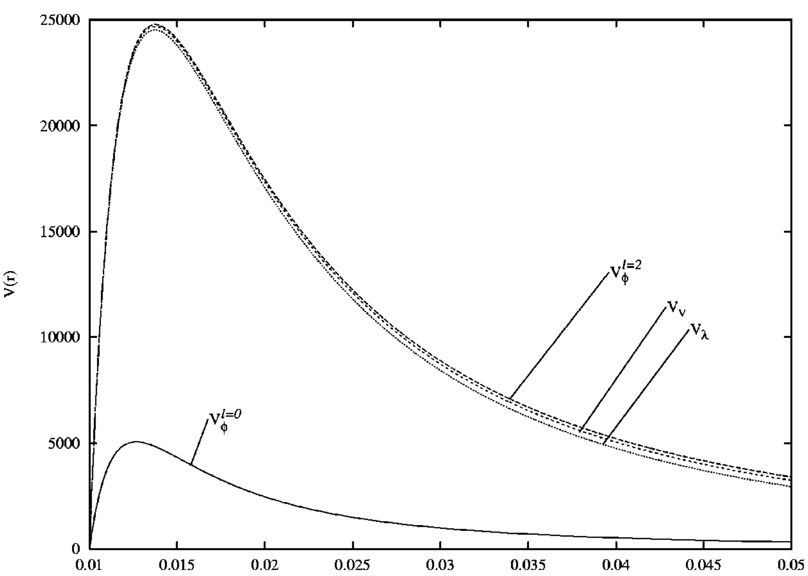

FIG. 2. Four potential graphs $(V_\phi^{l=0}, V_\nu, V_\lambda, V_\phi^{l=2})$ for an AdS_3 bubble in a 5D black hole with $r_0=0.01, R=0.3$.

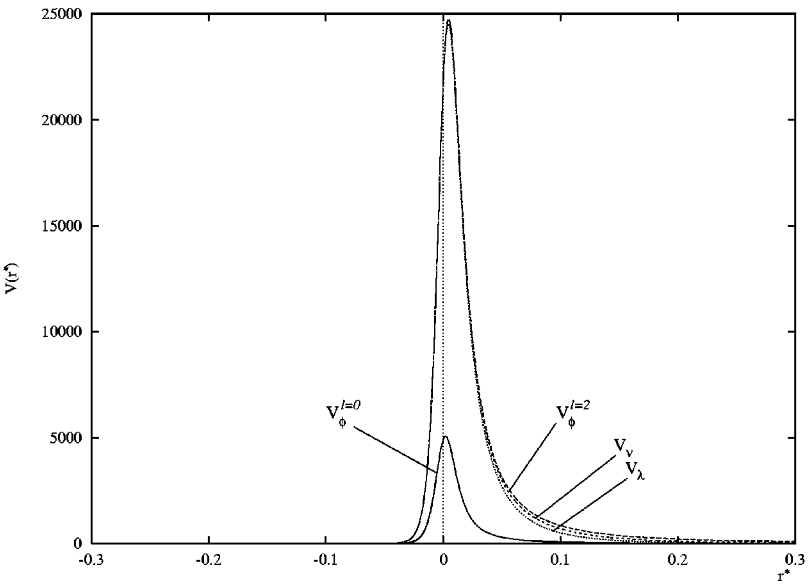

FIG. 3. Four potential graphs ($V_\phi^{l=0}, V_\nu, V_\lambda, V_\phi^{l=2}$) as functions of r^* for an AdS_3 bubble in a 5D black hole with $r_0 = 0.01, R = 0.3$.

the asymptotic states outside an AdS_3 bubble. The size of an AdS_3 bubble is from $r_0 = 0.01$ to $R = 0.3$.

B. Scattering from an AdS_3 bubble

We are interested in the scattering of the test fields off $V_N(r^*)$. It is well known that the scattering analysis is usually done by choosing a coordinate such as $-\infty \leqslant r^* \leqslant \infty$. It is always possible to visualize the black hole as presenting an effective potential barrier (or well) to an incoming test wave. One expects that some of the incident wave will be irreversibly absorbed by the black hole, while the remaining fraction will be scattered back to the infinity. In this scattering we can calculate the reflection and transmission (absorption) coefficients [13]. As is shown in Fig. 3, we note that all potential barriers $V_N(r^*)$ take nearly the symmetric forms around $r^* = 0$ when they are rewritten by a tortoise coordinate r^* [14]. Also they are localized at $r^* = 0$. Since $V_N(r^*) \to 0$ as $r^* \to \pm\infty$, one finds

$$\frac{d^2 \tilde{N}_{\pm\infty}}{dr^{*2}} + \omega^2 \tilde{N}_{\pm\infty} = 0. \tag{19}$$

Asymptotically ($r^* \to \infty$), the solution is given by

$$N_{+\infty}^{\rm out} = e^{i\omega r^*} + R_N^{\rm out}(\omega) e^{-i\omega r^*}. \tag{20}$$

Considering the time part of $e^{i\omega t}$, the first is an incoming wave ($\leftarrow$) and the last is an outgoing wave ($\rightarrow$). Near the horizon it is purely incoming ($\leftarrow$) as

$$N_{-\infty}^{\rm out} = A_N^{\rm out}(\omega) e^{i\omega r^*}. \tag{21}$$

We call this type of solution $\{\rm out\}_N$ and the corresponding vacuum state is defined as $b_i|0\rangle_{\rm out} = 0$.

In order to study the Hawking radiation, we introduce another boundary condition. Asymptotically the wave is purely outgoing ($\rightarrow$),

$$N_{+\infty}^{\rm in} = A_N^{\rm in}(\omega) e^{-i\omega r^*}, \tag{22}$$

but near the horizon it has both outgoing ($\rightarrow$) and incoming ($\leftarrow$) parts:

$$N_{-\infty}^{\rm in} = e^{-i\omega r^*} + R_N^{\rm in}(\omega) e^{i\omega r^*}. \tag{23}$$

We call this type of solution $\{\rm in\}_N$ and its vacuum state is defined as $a_i|0\rangle_{\rm in} = 0$. The vacuum states $|0\rangle_{\rm out}$ and $|0\rangle_{\rm in}$ form two different bases of which any state can be expanded in terms of the other. These are two different Fock space vacuum states and $\{\rm out\}_N$ and $\{\rm in\}_N$ are related to each other by the Bogoliubov transformation,

$$b_i = \sum_j (\alpha_{ij}^* a_j - \beta_{ij}^* a_j^\dagger), \tag{24}$$

$$b_i^\dagger = \sum_j (\alpha_{ij} a_j^\dagger - \beta_{ij} a_j). \tag{25}$$

The computation of Hawking shows in a semiclassical approximation that the thermal radiation from the black hole with temperature T_H is given by [15]

$$\langle N_\omega \rangle = {}_{\rm in}\langle 0|b_i^\dagger b_i|0\rangle_{\rm in} = \sum_k |\beta_{ik}|^2 = \frac{\sigma_{\rm 5D}^N}{e^{\omega/T_H} - 1}, \tag{26}$$

with an absorption cross section $\sigma_{\rm 5D}^N = |A_N^{\rm out}|^2 \times 4\pi/\omega^3 = \mathcal{A}_N^{\rm out} 4\pi/\omega^3$. Note that if $\sigma_{\rm 5D}^N$ is a constant, $\langle N_\omega \rangle$ is the same as that of a blackbody. Typically, $\sigma_{\rm 5D}^N$ is not constant but varies. The deviations from the black-body spectrum have earned it the name ''gray-body factor.'' Here we define the *S*-matrix from $\{\rm in\}_N$ and $\{\rm out\}_N$ as [14]

$$S(\omega) = \begin{pmatrix} A_N^{\rm out}(\omega) & A_N^{\rm in}(\omega) \\ R_N^{\rm out}(\omega) & R_N^{\rm in}(\omega) \end{pmatrix}. \tag{27}$$

Here an incident wave ($e^{i\omega r^*}$) of unit amplitude from $r^*=+\infty$ gives rise to $A_N^{\rm out}(\omega)$ and $R_N^{\rm out}(\omega)$. On the other hand, an incident wave ($e^{-i\omega r^*}$) of unit amplitude from $r^*=-\infty$ gives rise to $A_N^{\rm in}(\omega)$ and $R_N^{\rm in}(\omega)$. The relation between these is given by

$$A_N^{\rm out}(\omega)=A_N^{\rm in}(\omega)=A_N(\omega), \tag{28}$$

$$\frac{R_N^{\rm out}(\omega)}{A_N(\omega)}=-\frac{R_N^{\rm in}(-\omega)}{A_N(-\omega)},\quad \frac{R_N^{\rm out}(-\omega)}{A_N(-\omega)}=-\frac{R_N^{\rm in}(\omega)}{A_N(\omega)}, \tag{29}$$

$$A_N^*(\omega)=A_N(-\omega),\quad R_N^{\rm out}{}^*(\omega)=R_N^{\rm out}(-\omega),$$

$$R_N^{\rm in}{}^*(-\omega)=R_N^{\rm in}(\omega). \tag{30}$$

The above relations establish the symmetry and unitarity of the S matrix in the AdS_3 bubble scattering. Actually $A_N(\omega)$ can be calculated from the backscattering of an incident wave N off the potential $V_N(r^*)$. It is not easy to find out the absorption amplitude A_N directly in the complicated potentials such as $V_N(r^*)$. In this case one uses the flux $\mathcal{F}$ of the incoming wave to obtain the absorption coefficient

$$\mathcal{A}_N^{\rm out}=\frac{\mathcal{F}(-\infty)}{\mathcal{F}(\infty)}, \tag{31}$$

where $\mathcal{F}(-\infty)$ $[\mathcal{F}(\infty)]$ are the fluxes at the horizon (infinity). In this way we can calculate the semiclassical absoprtion cross section.

C. Exact analysis of near-horizon (AdS_3 bubble)

First let us consider a free scalar. Using $z=h$, the wave equation (9) can be rewritten as

$$z(1-z)\frac{d^2\phi}{dz^2}+(1-z)\frac{d\phi}{dz}+\left\{-C+\frac{Q}{z}+\frac{E}{1-z}\right\}\phi=0, \tag{32}$$

where

$$C=\left(\frac{\omega r_1 r_5 r_n}{2r_0^2}\right)^2=\frac{\omega^2}{64\pi^2}\left(\frac{1}{T_L}-\frac{1}{T_R}\right)^2, \tag{33}$$

$$E=-\frac{l(l+2)}{4}+\frac{\omega^2(r_1^2+r_5^2+r_n^2)}{4}\simeq-\frac{l(l+2)}{4}, \tag{34}$$

$$Q=\left(\frac{\omega}{4\pi T_H}\right)^2\left[\left(1+\frac{r_0^2}{r_1^2}+\frac{r_0^2}{r_5^2}\right)+4\pi^2 r_n^2 T_H^2\right]\simeq\left(\frac{\omega}{4\pi T_H}\right)^2. \tag{35}$$

Here $\simeq$ means both the dilute gas limit ($r_0,r_n\ll r_1,r_5$) and low-energy limit ($\omega\to 0$). Then one has $\omega r_0,\omega r_n \ll\omega r_1,\omega r_5<1$. In order to compare Eq. (32) with the hypergeometric equation, one has to transform it into the pole-free equation. With an unknown constant A, we find the ingoing mode at the horizon

$$\phi^n=Az^{-i\sqrt{Q}}(1-z)^{(1-\nu)/2}F(a,b,c;z), \tag{36}$$

where

$$\nu=\sqrt{(l+1)^2-\omega^2(r_1^2+r_5^2+r_n^2)}\simeq l+1, \tag{37}$$

$$a=\frac{1-\nu}{2}-i\sqrt{Q}+i\sqrt{C}\simeq-\frac{l}{2}-i\frac{\omega}{4\pi T_R}, \tag{38}$$

$$b=\frac{1-\nu}{2}-i\sqrt{Q}-i\sqrt{C}\simeq-\frac{l}{2}-i\frac{\omega}{4\pi T_L}, \tag{39}$$

$$c=1-2i\sqrt{Q}\simeq 1-i\frac{\omega}{2\pi T_H}. \tag{40}$$

The large-r behavior ($z\to 1$) of ϕ^n can be obtained from the ($z\to 1-z$) transformation rule for the hypergeometric functions as

$$\phi^{n\to f}=\frac{A\Gamma(c)\Gamma(c-a-b)}{\Gamma(c-a)\Gamma(c-b)}u^{\nu-1}+\frac{A\Gamma(c)\Gamma(a+b-c)}{\Gamma(a)\Gamma(b)}u^{-(\nu+1)}. \tag{41}$$

For the fixed scalars (ν,λ), considering both Eqs. (16), (17) and the near-horizon condition of $r\simeq r_0\ll r_1,r_5$, one finds that Eq. (10) leads to Eq. (9) with $l=2$. Thus one can obtain their near-horizon behaviors from ϕ_2.

D. Asymptotic states

Let us first consider a free scalar. In the far region, we introduce $\phi=\check{\phi}/r$ and $u=\omega r$ and then Eq. (9) leads to

$$\frac{d^2\check{\phi}}{du^2}+\frac{1}{u}\frac{d\check{\phi}}{du}+\left[1-\frac{\nu^2}{u^2}\right]\check{\phi}=0. \tag{42}$$

The solution is given by the Bessel function when ν is not an integer

$$\phi^f=\left[\alpha\frac{J_\nu(u)}{u}+\beta\frac{J_{-\nu}(u)}{u}\right], \tag{43}$$

where α,β are unknown constants. From the large u-behavior ($r\to\infty,\omega r\gg 1$), one finds the asymptotic states

$$\phi_\infty^f=\sqrt{\frac{1}{2\pi}}\frac{e^{-iu}}{u^{3/2}}\{\alpha e^{i(\nu+1/2)\pi/2}+\beta e^{-i(-\nu+1/2)\pi/2}\} \tag{44}$$

and its incoming flux

$$\mathcal{F}(\infty)=-\frac{2}{\omega^2}|\alpha e^{i(\nu+1/2)\pi/2}+\beta e^{i(-\nu+1/2)\pi/2}|^2. \tag{45}$$

The small u-behavior ($\omega r<1$) of ϕ^f is

$$\phi^{\mathrm{f}\to\mathrm{inter}}=\frac{1}{u}\left[\alpha\left(\frac{u}{2}\right)^{\nu}\frac{1}{\Gamma(\nu+1)}+\beta\left(\frac{u}{2}\right)^{-\nu}\frac{1}{\Gamma(-\nu+1)}\right]. \tag{46}$$

On the other hand, the asymptotic behavior of the *s*-wave fixed scalars ($\phi_{\pm}=\breve{\phi}_{\pm}/r$) is governed by

$$\frac{d^2\breve{\phi}_{\pm}}{du^2}+\frac{1}{u}\frac{d\breve{\phi}_{\pm}}{du}+\left[1-\frac{\nu^2}{u^2}\right]\breve{\phi}_{\pm}=0 \tag{47}$$

with $\nu\simeq1$. Its solution is given by

$$\phi^{f}_{\pm}\simeq\left[\alpha_{\pm}\frac{J_{\nu}(u)}{u}+\beta_{\pm}\frac{J_{-\nu}(u)}{u}\right]. \tag{48}$$

Here $\alpha_{\pm},\beta_{\pm}$ are unknown constants.

E. AdS_3-AFS matching procedure and absorption cross section

Here we use the matching procedure between an AdS_3 bubble (near-horizon of a 5D black hole) and asymptotically flat space to obtain an absorption coefficient. First consider the matching of a free scalar. Here the matching point resides on $0<r^*<\infty$ in Fig. 3. In the intermediate zone ($u<1$), from Eqs. (41) and (46) one finds

$$\alpha=Au_0\frac{2^{\nu}\Gamma(\nu+1)\Gamma(\nu)\Gamma(c)u_0^{-\nu}}{\Gamma(c-a)\Gamma(c-b)},$$

$$\beta=Au_0\frac{\Gamma(-\nu+1)\Gamma(-\nu)\Gamma(c)u_0^{\nu}}{2^{\nu}\Gamma(a)\Gamma(b)}. \tag{49}$$

Since $u_0=\omega r_0\ll1$, one finds $\alpha\gg\beta$. In this case we take an incoming flux effectively. Furthermore, the incoming flux at the horizon is found as

$$\mathcal{F}(0)=-8\pi r_0^2\sqrt{Q}|A|^2. \tag{50}$$

The absportion coefficient is given by

$$\mathcal{A}_{\phi}^{\mathrm{out}}=\frac{\mathcal{F}(0)}{\mathcal{F}(\infty)}\simeq4\pi u_0^2\sqrt{Q}\left|\frac{A}{\alpha}\right|^2. \tag{51}$$

The absorption cross section takes the form [11,16]

$$\sigma_{\mathrm{5D}}^{\phi}=(l+1)^2\frac{4\pi}{\omega^3}\mathcal{A}_{\phi}^{\mathrm{out}}\simeq\frac{\mathcal{A}_H^{\mathrm{5D}}}{[l!(l+1)!]^2}(l+1)^2\left(\frac{\omega r_0}{2}\right)^{2l}\times\left|\frac{\Gamma\left(\frac{l+2}{2}-i\frac{\omega}{4\pi T_L}\right)\Gamma\left(\frac{l+2}{2}-i\frac{\omega}{4\pi T_R}\right)}{\Gamma\left(1-i\frac{\omega}{2\pi T_H}\right)}\right|^2 \tag{52}$$

with the area of horizon $\mathcal{A}_H^{\mathrm{5D}}=2\pi r_1r_5r_n$. We have, for even l

$$\sigma_{\mathrm{5D}}^{\phi_l}=(l+1)^2\frac{\pi^3}{2^{4l}}\frac{(r_1r_5)^{2l+2}\omega^{2l+1}}{[l!(l+1)!]^2}\times[\omega^2+(2\pi T_L)^2 2^2]\cdots[\omega^2+(2\pi T_L)^2l^2]\times[\omega^2+(2\pi T_R)^2 2^2]\cdots[\omega^2+(2\pi T_R)^2l^2]\times\frac{e^{\omega/T_H}-1}{(e^{\omega/2T_L}-1)(e^{\omega/2T_L}-1)}. \tag{53}$$

The matching procedure for the s wave $\phi_{\pm}$ is nearly the same as in a free scalar [5]. It leads to

$$\sigma_{\mathrm{5D}}^{\phi_{\pm}}=\frac{\pi^3r_1^6r_5^6}{64r_{\pm}^4}\omega(\omega^2+16\pi^2T_L^2)(\omega^2+16\pi^2T_R^2)\times\frac{e^{\omega/T_H}-1}{(e^{\omega/2T_L}-1)(e^{\omega/2T_L}-1)}. \tag{54}$$

In the limit of $\omega\ll T_L,T_R,T_H$, the low-energy absorption cross sections are calculated as

$$\sigma_{\mathrm{5D}}^{\phi_0}=\mathcal{A}_H^{\mathrm{5D}}, \tag{55}$$

$$\sigma_{\mathrm{5D}}^{\phi_2}=\frac{3}{16}\mathcal{A}_H^{\mathrm{5D}}(\omega r_0)^4=\frac{3}{4}(\omega R)^4\left\{\frac{\mathcal{A}_H^{\mathrm{5D}}}{4}\left(\frac{r_0}{R}\right)^4\right\}, \tag{56}$$

$$\sigma_{\mathrm{5D}}^{\nu}=\frac{\mathcal{A}_H^{\mathrm{5D}}}{4}\left(\frac{r_0}{R}\right)^4, \tag{57}$$

$$\sigma_{\mathrm{5D}}^{\lambda}=9\frac{\mathcal{A}_H^{\mathrm{5D}}}{4}\left(\frac{r_0}{R}\right)^4, \tag{58}$$

where we impose the relation $r_1=r_5=R, r_0=r_n$. Here we find a sequence of the cross section

$$\sigma_{\mathrm{5D}}^{\phi_0}\geqslant\sigma_{\mathrm{5D}}^{\lambda}\geqslant\sigma_{\mathrm{5D}}^{\nu}\geqslant\sigma_{\mathrm{5D}}^{\phi_2}. \tag{59}$$

This originates from the potential sequence in Eq. (18). It is consistent with our naive expectation that the absorption cross section increases, as the height of potential decreases. Here we wish to point out the difference between a free scalar and the fixed scalars. In the dilute gas limit ($R\gg r_0$) and the low-energy limit ($\omega\to0$), the s-wave cross section for a free scalar ($\sigma_{\mathrm{5D}}^{\phi_0}$) goes to $\mathcal{A}_H^{\mathrm{5D}}$ [3], while the *s*-wave cross sections for fixed scalars (ν,λ) including ϕ_2 approach zero [4]. Also this can be confirmed from Fig. 2.

III. SCATTERING FROM AN EXACT AdS_3 SPACE IN AdS_3 (BTZ BLACK HOLE) $\times S^3\times T^4$

A. Potential analysis

Here we consider the geometry of an exact AdS_3 space ($AdS_3\times S^3\times T^4$) as the other solution to the type-IIB string theory [9]. This corresponds to the nondilatonic solution. This geometry can lead the Bañados-Teitelboim-Zanelli

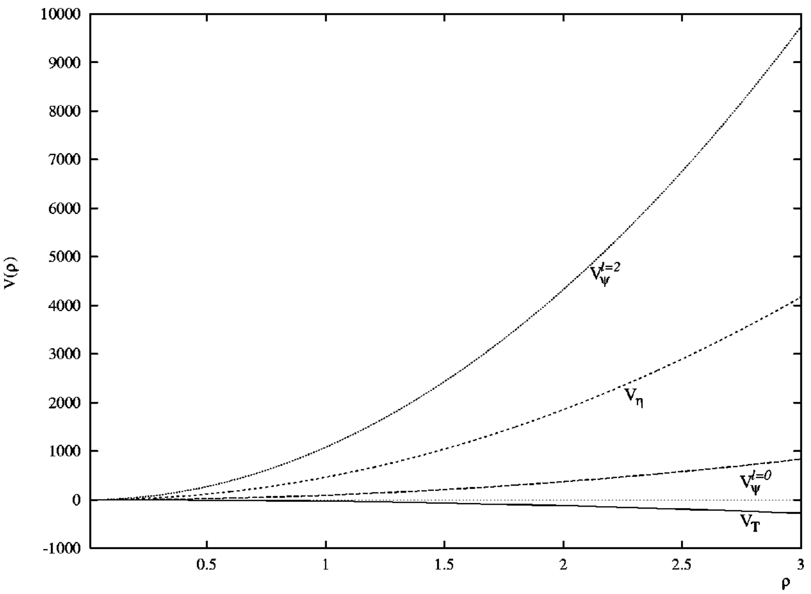

FIG. 4. Four potential graphs ($V_{\rm T},V_{\psi}^{l=0},V_{\eta},V_{\psi}^{l=2}$) as functions of ρ for an exact AdS_3 space.

(BTZ) black hole space time as a whole by the periodic identification. A ten-dimensional minimally coupled scalar satisfies

$$\Box_{10}\Psi=0. \qquad (60)$$

Ψ can be decomposed into

$$\Psi=e^{-i\omega t}e^{im\varphi}e^{iK_ix^i}Y_l(\theta_1,\theta_2,\theta_3)\psi(\rho). \qquad (61)$$

Then Eq. (60) leads to

$$\nabla^2_{\rm BTZ}\psi(\rho)+\frac{\mu}{R^2}\psi(\rho)=0 \qquad (62)$$

with $\mu=-l(l+2)-K^2r_5^2$. The $\mu=-8(l=2)$ case contains both the dilaton (Φ) and a free scalar(ψ) with $l=2$. The $\mu=-3(l=1)$ case corresponds to an intermediate scalar (η) and $\mu=1$ leads to tachyon (T). Here the BTZ black-hole spacetime is given by [10]

$$ds^2_{\rm BTZ}=-f^2dt^2+\rho^2\left(d\varphi-\frac{J}{2\rho^2}dt\right)^2+f^{-2}d\rho^2 \qquad (63)$$

with $f^2=\rho^2/R^2-M+J^2/4\rho^2 \;=(\rho^2-\rho_+^2)(\rho^2-\rho_-^2)/\rho^2R^2$. The mass, angular momentum, angular velocity at the horizon, and area of horizon are

$$M=(\rho_+^2+\rho_-^2)/R^2,\quad J=2\rho_+\rho_-/R,\quad \Omega_H=\frac{J}{2\rho_+^2},$$

$$\mathcal{A}_H^{\rm BTZ}=2\pi\rho_+. \qquad (64)$$

Further, one finds the relation between the BTZ and a 5D black hole as

$$T_H^{\rm BTZ}=(\rho_+^2-\rho_-^2)/2\pi R^2\rho_-=T_H,$$

$$\frac{1}{T_{L/R}^{\rm BTZ}}=\frac{1}{T_H^{\rm BTZ}}\left(1\pm\frac{\rho_+}{\rho_-}\right)=\frac{1}{T_{L/R}}. \qquad (65)$$

Its s-wave equation with $m=0$ takes the form

$$\left[f^2\partial_\rho^2+\left\{\frac{1}{\rho}\partial_\rho(\rho f^2)\right\}\partial_\rho+\frac{\omega^2}{f^2}+\frac{\mu}{R^2}\right]\psi(\rho)=0. \qquad (66)$$

Defining $\psi(\rho)=\tilde{\psi}/\sqrt{\rho}$ and then Eq. (66) takes the form

$$f^2\tilde{\psi}''+(f^2)'\tilde{\psi}'+\left[\frac{f^2}{4\rho^2}-\frac{(f^2)'}{2\rho}+\frac{\omega^2}{f^2}+\frac{\mu}{R^2}\right]\tilde{\psi}=0, \qquad (67)$$

where the prime ($'$) denotes the differentiation with respect to ρ. In order to obtain the Schrödinger-type equation, we introduce the tortoise coordinate ρ^* as [17]

$$\rho^*=\int\frac{d\rho}{f^2}=\frac{R^2}{2(\rho_+^2-\rho_-^2)}\left[\rho_+\ln\left(\frac{\rho-\rho_+}{\rho+\rho_-}\right)-\rho_-\ln\left(\frac{\rho-\rho_-}{\rho+\rho_-}\right)\right]. \qquad (68)$$

We note that $\rho_+\leqslant\rho\leqslant\infty$, while $-\infty\leqslant\rho^*\leqslant 0$ for the outside horizon. On the other hand, in a 5D black hole one finds that $r_0\leqslant r\leqslant\infty$ is mapped into $-\infty\leqslant r^*\leqslant\infty$. Then Eq. (67) leads to

$$\frac{d^2\psi}{d\rho^{*2}}+(\omega^2-V_\mu)\tilde{\psi}=0, \qquad (69)$$

where the potential is given by

$$V_\mu(\rho)=f^2\left[-\frac{f^2}{4\rho^2}+\frac{(f^2)'}{2\rho}-\frac{\mu}{R^2}\right]. \qquad (70)$$

Four potential graphs ($V_{\rm T},V_{\psi}^{l=0},V_{\eta},V_{\psi}^{l=2}$) in an exact AdS_3 background are shown in Fig. 4. The parameters are chosen as $\rho_+=0.01,\rho_-=0.001,R=0.3$. These are all monotonically increasing functions with ρ, in contrast to V_N for an AdS_3

bubble. This shows a peculiar property of AdS_3, which in this spacetime the asymptotic states cannot be defined. But the tachyon potential (V_T) is a monotonically decreasing function. This field couples to the minimal weight primary operator with (1/2,1/2) [18]. Also this satisfies both the stability condition for the AdS_3 space and the Dirichlet boundary condition [19], which are clearly related to the shape of its potential.

B. Asymptotically AdS_3 behavior and non-normalizable modes

In the near horizon ($\rho \to \rho_+, V_\mu \to 0$), Eq. (69) reduces to

$$\frac{d^2\tilde{\psi}_{\rm NH}}{d\rho^{*2}} + \omega^2 \tilde{\psi}_{\rm NH} = 0, \tag{71}$$

which leads to the plane-wave solution

$$\tilde{\psi}_{\rm EH} = e^{-i\omega\rho^*} + R_\psi^{\rm in}(\omega) e^{i\omega\rho^*}. \tag{72}$$

Here the first term is an outgoing mode ($\rightarrow$) and the second is an incoming mode ($\leftarrow$). Now let us discuss the asymptotically AdS_3 behavior. Near the timelike boundary ($\rho \to \infty, \rho^* \to 0$), one finds

$$\frac{d^2\tilde{\psi}_\infty}{d\rho^{*2}} - \left[\frac{3}{4} - \mu\right]\frac{\rho^2}{R^4}\tilde{\psi}_\infty = 0. \tag{73}$$

Here we introduce the relation between ρ and ρ^*

$$\frac{\rho_+}{\rho(\rho^*)} = \frac{Y}{1-\sigma^2}\sum_{n=0}^{\infty} a_n(\sigma) Y^{2n}, \quad Y = \tanh \lambda \rho^*,$$

$$\sigma = \frac{\rho_-}{\rho_+}, \quad \lambda = \frac{\rho_+}{R^2}(1-\sigma^2), \quad a_0 = 1. \tag{74}$$

If σ^2 is very small and $\rho^* \to 0$, one finds

$$\rho = \frac{\rho_+}{Y}(1-\sigma^2) = \rho_+(1-\sigma^2)\coth \lambda \rho^* \simeq \frac{R^2}{\rho^*}. \tag{75}$$

Using Eqs. (75), (73) leads to

$$\frac{d^2\tilde{\psi}_\infty}{d\rho^{*2}} - \left[\frac{3}{4} - \mu\right]\frac{\tilde{\psi}_\infty}{\rho^{*2}} = 0. \tag{76}$$

Its solution takes the form

$$\tilde{\psi}_\infty = \rho^{*\frac{1 \pm 2\sqrt{1-\mu}}{2}}. \tag{77}$$

Finally, we have

$$\psi_\infty(\rho^*) = \frac{\tilde{\psi}_\infty}{\sqrt{\rho}} \propto \rho^{*(1 \pm \sqrt{1-\mu})}. \tag{78}$$

For the s-wave free scalar ($\mu = 0$), ψ_∞ takes the form (ρ^{*2}, const) and for the dilaton field ($\mu = -8$), one finds ($\rho^{*4}, 1/\rho^{*2}$). We find ($\rho^{*3/2}, \rho^{*1/2}$) for a conformally coupled scalar ($\mu = 3/4$) [17]. For the tachyon ($\mu = 1$), one has (ρ^*, ρ^*) and for an intermediate scalar ($\mu = -3$), one finds ($\rho^{*3}, 1/\rho^*$). Instead of the plane-wave form, here one finds the power-law behaviors of ρ^* [20]. The non-normalizable modes are found to be $1/\rho^{*2}$ for the dilaton and $1/\rho^*$ for an intermediate scalar, which diverge as ρ^* approaches the timelike boundary ($\rho^* \to 0$). The positive powers of ρ^* all belong to be the normalizable modes, which converge at spatial infinity. We note that the tachyon takes only the normalizable modes [19]. This can be easily conjectured from its shape of potential V_T. The s-wave free scalar takes a constant behavior at infinity. This makes it difficult to divide ψ_∞ into ingoing and outgoing modes at spatial infinity [21]. Actually, it is impossible to define an ingoing wave and an outgoing wave at the spatial infinity of an exact AdS_3 space. Instead in the asymptotically AdS_3 space it contains the normalizable as well as the non-normalizable modes. The latter will play an important role in calculation of the absorption coefficient $\mathcal{A}_\psi^{\rm in}$.

C. Scattering from an exact AdS_3 space

First we note that ρ^* covers only the left-hand side ($-\infty \leqslant \rho^* \leqslant 0$) of the whole space. In the case of where the black-hole geometry is asymptotically flat as a 5D black hole, the tortoise coordinate r^* goes from $-\infty$ to ∞. Hence, this is similar to the infinite string problem in which the initial data propagates towards left and right indefinitely [17]. The initial data no longer enjoys this privilege when the background is asymptotically AdS_3 because the tortoise coordinate ρ^* goes from $-\infty$ to 0 only. One may consider this as the semi-infinite string problem (or a finite cavity with reflecting walls in AdS_5 space [22]). Then the Dirichlet or Neumann boundary condition at spatial infinity ($\rho^* = 0$) is required to formulate the problem appropriately. However, we take a different point of view to attack an asymptotically AdS_3 problem. This is based on the observation of the shape of the potential and the global structure of an exact AdS_3 space [10]. We first construct the potential $V_\mu(\rho^*)$ by replacing ρ in Eq. (70) with $\rho \simeq \rho_+ \coth(\rho_+\rho^*/R^2)$. The potentials in Fig. 5 look like one half of the AdS_3-bubble potentials. Especially, we observe that the free-field potentials $V_\psi^{l=0}(\rho^*)$ and $V_\psi^{l=2}(\rho^*)$ in Fig. 5 take nearly the same form as in $V_\phi^{l=0}(r^*)$ and $[V_\phi^{l=2}(r^*), V_\nu(r^*), V_\lambda(r^*)]$ in the region of $-\infty \leqslant r^* \leqslant 0$ in Fig. 3. Furthermore, the Penrose diagram of an exact $AdS_3(-\infty \leqslant \rho^* \leqslant 0)$ is one half of the would-be whole diagram in $-\infty \leqslant \rho^* \leqslant \infty$ [10].

In this work we assume that the potential of an exact AdS_3 space is the left-hand side of an AdS_3 bubble. The important thing is to calculate the absorption cross section in the background of an exact AdS_3 space. Considering the $\{\rm in\}_\psi$-state picture, it is not hard to calculate the absorption coefficient $\mathcal{A}_\psi^{\rm in}$. Although the $\{\rm out\}_\psi$ state cannot be defined, we can derive $\mathcal{A}_\psi^{\rm in}$ from the backscattering of a test field ψ off $V_\psi(\rho^*)$. If we choose the boundary condition appropriately, the potential of Fig. 5 is enough to calculate the absorption coefficient, regardless of the would-be right-hand side ($0 \leqslant \rho^* \leqslant \infty$). In this backscattering process, the key point is to use an appropriate matching procedure between a

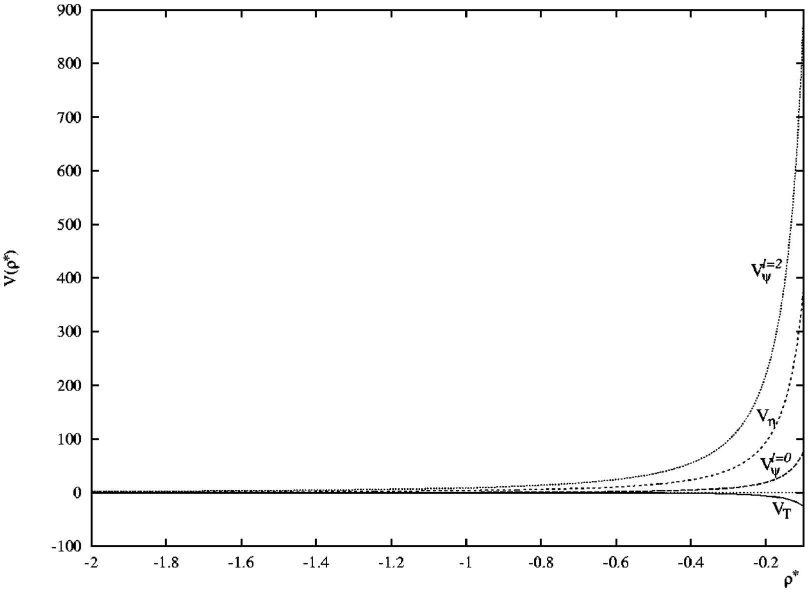

FIG. 5. Four potential graphs $(V_T, V_\psi^{l=0}, V_\eta, V_\psi^{l=2})$ as functions of ρ^* for an exact AdS_3 space.

near AdS_3 and an asymptotic AdS_3. We remind the reader that $\rho^*=0$ is a timelike bounadry and thus information enters or exits from it. This is an exact middle point if one assumes a whole space of $-\infty \leqslant \rho^* \leqslant \infty$. Thus requiring the conventional boundary condition may lead to a wrong result in deriving the absorption coefficient. Instead of the Dirichlet condition of $\psi|_{\rho^*=0}=0$, one may use the non-normalizable modes. The non-normalizable mode is a divergent quantity at $\rho^*=0$ but its flux is finite at $\rho^*=0$. Also it corresponds to specifying another boundary condition at spatial infinity. On the other hand, if one use the normalizable modes which satisfy the Dirichlet boundary condition, one may not succeed in obtaining the absorption coefficient in an exact AdS_3.

D. AdS_3-AdS_3 matching procedure and absorption cross section

In order to calculate the semiclassical absorption cross section, we have to solve the exact differential equation (62) with an appropriate boundary condition. Since it is difficult to solve Eq. (62) directly, one has to use the matching procedure between the near-horizon ($\rho \sim \rho_+$) AdS_3 and the far-region ($\rho \to \infty$) AdS_3. Here the matching point resides on $-\infty < \rho^* < 0$ in Fig. 5.

In the far region Eq.(62) becomes

$$\psi_\infty'' + \frac{3}{x}\psi_\infty' + \frac{\mu}{x^2}\psi_\infty = 0 \tag{79}$$

with a dimensionless variable $x=\rho/R$. One easily finds the far-region solution

$$\psi_\infty(x) = [\tilde{\alpha} x^{-1+\sqrt{1-\mu}} + \tilde{\beta} x^{-1-\sqrt{1-\mu}}] \tag{80}$$

with two unknown constants $\tilde{\alpha}, \tilde{\beta}$. The first term is a divergent quantity at $\rho=\infty$ but behaves well in the interior region. This corresponds to the non-normalizable modes and is coupled to the boundary operator $\mathcal{O}$ at infinity. The second one is the normalizable mode and propagates in the bulk. This can be used to construct two-, three-, and four-point functions. Although one cannot define an ingoing flux at infinity of AdS_3, one can calculate the total flux. The flux at spatial infinity is given by

$$\mathcal{F}(\infty) = -2\pi\sqrt{1-\mu}\,|\tilde{\alpha} - i\tilde{\beta}|^2. \tag{81}$$

We note here that for the tachyon with $\mu=1$, $\mathcal{F}^T(\infty)=0$. This is because the tachyon takes only the normalizable modes which are to be zero at infinity. Thus we exclude it from our analysis.

In order to obtain the near-horizon behavior, we introduce the new variable $z=(\rho^2-\rho_+^2)/(\rho^2-\rho_-^2) = (x^2-x_+^2)/(x^2-x_-^2)$. Then Eq. (62) leads to

$$z(1-z)\frac{d^2\psi}{dz^2} + (1-z)\frac{d\psi}{dz} + \left(\frac{A_1}{z} + \frac{\mu/4}{1-z} - B_1\right)\psi = 0, \tag{82}$$

where

$$A_1 = \left(\frac{\omega - m\Omega_H}{4\pi T_H}\right)^2, \quad B_1 = -\frac{\rho_-^2}{\rho_+^2}\left(\frac{\omega - m\Omega_H \rho_+^2/\rho_-^2}{4\pi T_H^{\mathrm{BTZ}}}\right)^2. \tag{83}$$

In the case of the s-wave propagation with $m=0$, the near-horizon AdS_3 equation (82) leads exactly to the near-horizon equation (32) of a 5D black hole. Explicitly, the relationship between these is given by

$$A_1 \to Q, \quad \frac{\mu}{4} \to E, \quad B_1 \to C. \tag{84}$$

In this sense, Eq. (32) is called an AdS_3 bubble. The ingoing wave is given by the hypergeometric function

$$\psi(z) = C_1 z^{-i\sqrt{A_1}}(1-z)^{(1-\sqrt{1-\mu})}F(a,b,c;z), \tag{85}$$

where

$$\sqrt{1-\mu}\simeq l+1, \tag{86}$$

$$a=\frac{1-\sqrt{1-\mu}}{2}-i\sqrt{A_1}+i\sqrt{B_1}\simeq-\frac{l}{2}-i\frac{\omega}{4\pi T_R^{\rm BTZ}}, \tag{87}$$

$$b=\frac{1-\sqrt{1-\mu}}{2}-i\sqrt{A_1}-i\sqrt{B_1}\simeq-\frac{l}{2}-i\frac{\omega}{4\pi T_L^{\rm BTZ}}, \tag{88}$$

$$c=1-2i\sqrt{A_1}=1-i\frac{\omega}{2\pi T_H^{\rm BTZ}}. \tag{89}$$

Here $\simeq$ means $K^2r_5^2\simeq 0$. The corresponding flux is

$$\mathcal{F}(0)=-8\pi\sqrt{A_1}(x_+^2-x_-^2)|C_1|^2 \tag{90}$$

with $x_+^2-x_-^2=(r_0/R)^2\ll 1$. The absorption coefficient will be taken as

$$\mathcal{A}_\psi^{\rm in}=\frac{\mathcal{F}(0)}{\mathcal{F}(\infty)}=\frac{4\sqrt{A_1}(x_+^2-x_-^2)}{\sqrt{1-\mu}}\frac{|C_1|^2}{|\tilde{\alpha}-i\tilde{\beta}|^2}. \tag{91}$$

In order to obtain $\tilde{\alpha}$ and $\tilde{\beta}$, we use the matching procedure. It is important to remember that the present spacetime is an exact AdS_3. Thus we have to match the near-AdS_3 with the asymptotic AdS_3 to find an absorption coefficient. We know the far-region behavior of Eq. (85). This can be found from the $z\to 1-z$ for the hypegeometric function

$$\psi_{n\to f}(x)=[C_1E_1(x_+^2-x_-^2)^{(1-\sqrt{1-\mu})/2}x^{-1+\sqrt{1-\mu}}+C_1E_2(x_+^2-x_-^2)^{(1+\sqrt{1-\mu})/2}x^{-1-\sqrt{1-\mu}}], \tag{92}$$

where

$$E_1=\frac{\Gamma(c)\Gamma(c-a-b)}{\Gamma(c-a)\Gamma(c-b)},\quad E_2=\frac{\Gamma(c)\Gamma(-c+a+b)}{\Gamma(a)\Gamma(b)}. \tag{93}$$

Matching Eq. (80) with Eq. (92) in the far-region ($x\gg 1$) leads to

$$\tilde{\alpha}=C_1E_1\left(\frac{r_0}{R}\right)^{1-\sqrt{1-\mu}},\quad \tilde{\beta}=C_1E_2\left(\frac{r_0}{R}\right)^{1+\sqrt{1-\mu}}. \tag{94}$$

Considering $R\gg r_0$, one finds $\tilde{\alpha}\gg\tilde{\beta}$ even for the $l=0$ case. Hence, we can neglect $\tilde{\beta}$ in favor of $\tilde{\alpha}$. This amounts to taking the flux of the non-normalizable modes. The key point is an AdS_3-AdS_3 matching in this backscattering process. Then the absorption coefficient is approximately given by

$$\mathcal{A}_\psi\simeq\frac{4\sqrt{A_1}(x_+^2-x_-^2)}{\sqrt{1-\mu}}\left(\frac{r_0}{R}\right)^{2(\sqrt{1-\mu}-1)}\frac{1}{|E_1|^2}. \tag{95}$$

The absorption cross section for $AdS_3\times S^3$ with $m=0$ leads to

$$\sigma_{\rm AdS}^{\psi}=\frac{\mathcal{A}_\psi}{\omega}\simeq\frac{\tilde{\mathcal{A}}_H^{\rm 6D}}{l!(l+1)!}\left(\frac{r_0}{R}\right)^{2l}\left|\frac{\Gamma\left(\frac{l+2}{2}-i\frac{\omega}{4\pi T_L^{\rm BTZ}}\right)\Gamma\left(\frac{l+2}{2}-i\frac{\omega}{4\pi T_R^{\rm BTZ}}\right)}{\Gamma\left(l-i\frac{\omega}{2\pi T_H^{\rm BTZ}}\right)}\right|^2 \tag{96}$$

with $\tilde{\mathcal{A}}_h^{\rm 6D}=\mathcal{A}_H^{\rm BTZ}\times 2\pi^2R^3$.

In the low-energy limit $\omega\to 0$, it turns out that the 6D cross sections for an exact AdS theory take the same form as Eqs. (55) and (57):

$$\sigma_{\rm AdS}^{\psi_0}=\tilde{\mathcal{A}}_H^{\rm 6D}, \tag{97}$$

$$\sigma_{\rm AdS}^{\psi_2}=\sigma_{\rm AdS}^{\Phi}=\frac{1}{3}\frac{\tilde{\mathcal{A}}_H^{\rm 6D}}{4}\left(\frac{r_0}{R}\right)^4. \tag{98}$$

This 6D result is derived from $AdS_3\times S^3$. In order for this to match with the cross section of a 5D black hole, it needs to introduce a compactified circle (S^1) in $M_5\times S^1\times T^4$. In this case one finds $\tilde{\mathcal{A}}_H^{\rm 6D}=\mathcal{A}_H^{\rm 5D}\times 2\pi R$ with a radius of $S^1(R)$. We note that ϕ_2, $\nu(=\Phi)$ and λ give us slightly different cross sections in an AdS_3 bubble, whereas these (ψ_2,Φ) do not make any distinction in an exact AdS theory.

IV. DISCUSSIONS

It seems that the S-matrix cannot be extracted from the anti–de Sitter space even in a limit [22,23]. This is based on the fact that in an exact AdS_3 the asymptotic states cannot be defined, due to the timelike boundary and the periodicity of geodesics. However, the authors in [8] showed that the correlation functions of the dual CFT_4 to AdS_5 are considered as the bulk S matrices. The vacuum correlators $\langle\mathcal{O}(x_1)\mathcal{O}(x_2)\cdots\mathcal{O}(x_n)\rangle_{{\rm CFT}_4}$ of the CFT_4 are expressed as truncated n-point functions convolved against the non-normalizable modes. These can be interpreted as an S matrix for an exact AdS_5 space arising from a limit of scattering from an AdS_5 bubble in asymptotically flat space.

In this work, we show that the S matrix of an AdS_3 bubble can be derived from an exact AdS_3 in the dilute gas and low-energy limits. We confirm this from the calculation of the absorption cross section. This originates from the fact that the near-horizon equations for an AdS_3 bubble (32) and an exact AdS_3 space, Eq. (82) are the same form, but they have different boundary conditions at infinity. In the AdS_3 bubble calculation, one uses the AdS_3-AFS matching to obtain the absorption coefficient $\mathcal{A}_N^{\rm out}$. On the other hand, in the exact AdS_3 calculation we use both the {in}-state picture and the non-normalizable mode to obtain the absorption coefficient $\mathcal{A}_\psi^{\rm in}$. This amounts to taking the AdS_3-AdS_3 matching. The *s*-wave gray-body factor of a free scalar of an AdS_3 bubble has exactly the same form as that of an exact AdS_3 space. For the dilaton we find the same form of cross section $\sigma = c\tilde{\mathcal{A}}_H^{6D}(r_0/R)^4$ but with $c=1/4$ for an AdS_3 bubble and $c=1/12$ for an exact AdS_3 space.

Let us compare our results with the others. The general formula for the gray-body factor is derived from the vacuum two-point function $\langle \mathcal{O}(x)\mathcal{O}(0)\rangle_{\rm CFT_2}$ of a boundary operator $\mathcal{O}$ in the effective string [24] and the boundary CFT_2 approaches [25]. These give us the same result for a free scalar but for the dilaton, $c=1/4$ as in an AdS_3 bubble. Consequently, the two-point correlator provides us the gray-body factor in the dilute gas and low energy limits. This quantity takes exactly the same form in the CFT_2 and AdS_3 bubble approaches. Further, in the exact AdS_3 approach one finds the same form of the gray-body factor. This means that the *S* matrix can be derived from an exact AdS_3 space. It is obvious that the conformal limit of the gauge theory (CFT_2) corresponds with scattering from an exact AdS_3 space. Finally, we present here a scattering picture in an exact AdS_3 space and compare it with the scattering of an AdS_3 bubble in AFS.

ACKNOWLEDGMENTS

This work was supported in part by the Basic Science Research Institute Program, Ministry of Education, Projects No. BSRI-98-2413 and BSRI-98-2441.

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