Fully localized brane intersections: The plot thickens

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We study fully localized Bogomol'nyi-Prasad-Sommerfield brane solutions in classical supergravity using a perturbative approach to the coupled Born-Infeld or bulk supergravity system. We derive first order bulk supergravity fields for world-volume solitons corresponding to intersecting $M2$-branes and to a fundamental string ending on a $D3$-brane. One interesting feature is the appearance of certain off-diagonal metric components and the corresponding components of the gauge potentials. Making use of a supersymmetric ansatz for the exact fields, we formulate a perturbative expansion which applies to $M2\perp M2(0)$, $M5\perp M5(3)$ and $Dp\perp Dp$ $(p-2)$ intersections. We find that perturbation theory qualitatively distinguishes between certain cases: perturbation theory breaks down at second order for intersecting $M2$-branes and Dp-branes with $p\leqslant 3$ while it is well behaved, at least to this order, for the remaining cases. This indicates that the behavior of the full nonlinear intersecting Dp-brane solutions may be qualitatively different for $p\leqslant 3$ than for $p\geqslant 4$, and that fully localized asymptotically flat solutions for $p\leqslant 3$ may not exist. We discuss the consistency of these results with world-volume field theory properties.

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I. INTRODUCTION

The fact that supersymmetric gauge theories describe the low energy dynamics of branes in string theory has yielded many important insights, some of which are reviewed in [1,2]. The most interesting physics arises when supersymmetry is further broken by brane intersections. For example, Witten [3] (following earlier work in [4]) has shown how to recover the results of Seiberg and Witten [5] on $\mathcal{N}=2$ gauge theories in four dimensions via intersecting $M5$-branes in eleven dimensions. The smooth complex curve describing the $M5$-brane intersection in this construction provides a geometric realization of the Seiberg-Witten curve, describing the renormalization group flow of the Yang-Mills coupling.

The constructions referred to above involve branes embedded in flat space. One expects that useful complementary information could be obtained from the curved space descriptions of these same systems. Delocalized solutions for intersecting branes, in which the harmonic function associated with each brane is smeared out over the directions parallel to the other branes, have been known for some time (see e.g., [6–25], and especially [26–28] for reviews). These solutions are useful for many purposes, such as black hole entropy counting constructions, in which the delocalized directions are compactified on a torus. However, the smearing wipes out much of the interesting physics.

Despite a good deal of effort, spacetimes describing fully localized Bogomol'nyi-Prasad-Sommerfield (BPS) brane intersections have proved quite difficult to find. In fact, it has been shown [29,30] that certain intersections are necessarily delocalized. These results, however, were limited to either intersections in which one brane is fully contained within another or to ''partially'' localized intersections in which one of the branes is smeared over the directions parallel to the other brane. In this paper we will consider the existence of supergravity solutions for a class of fully localized intersections. By fully localized, we mean that the spacetime fields have nontrivial dependence on the coordinates ''along'' either brane, and that the sources are appropriate delta functions. Unlike the cases considered in [29,30], we are now faced with a nonlinear set of field equations to solve. Consequently, our analysis will be restricted to a weak field perturbative expansion. We start with $M2$-branes, and solve perturbatively for the spacetime fields due to a nonplanar $M2$-brane source; namely, one for which the $M2$-brane surface is a certain holomorphic curve associated with orthogonal intersecting branes. However, we find that the second order term in the perturbation series diverges. Intersections of $D2$-branes $(D2\perp D2(0))$ and $D3$-branes $(D3\perp D3(1))$ behave similarly. On the other hand, in the case of $M5$-branes or Dp branes with $p\geqslant 4$, the second order term is finite. One can interpret the second order results as supporting the gauge theory arguments of [30] about the existence and/or nonexistence of fully localized BPS intersecting D-brane solutions. Possibly the divergence in the perturbative expansion indicates a more general result that there are no fully localized, nonplanar, static, gravitating BPS $M2$-branes or Dp-branes for $p\leqslant 3$.

The question of existence probes an interesting aspect of the gauge theory description of brane dynamics. A version of the AdS conformal field theory (CFT) limit [31,32] implies [30] that the near-horizon properties of intersecting D-brane spacetimes have a dual gauge theory description. The scale over which brane intersections are delocalized in classical supergravity turns out to be dual to the quantum fluctuations of a massless modulus field in the gauge theory. Complete delocalization occurs when these fluctuations become large due to infrared effects. This in turn is determined by the dimensionality of the intersection. The gauge theory analysis in [30] accounts for the supergravity results that, for example, Dp-branes cannot be localized within $D(p+4)$-branes for $p=0$ or $p=1$, while they may be localized for $p\geqslant 2$.

As a starting point for the present analysis, consider the delocalized solution for a pair of $M2$-branes, one in the $(t,1,2)$ plane and one in the $(t,3,4)$ plane, intersecting at the origin. The spacetime fields are

$$ds^2=-(f_1f_2)^{-2/3}dt^2+f_1^{-2/3}f_2^{1/3}(dx_1^2+dx_2^2)+f_1^{1/3}f_2^{-2/3}(dx_3^2+dx_4^2)+(f_1f_2)^{1/3}(dx_5^2+\cdots+dx_{10}^2),$$

$$A_{t12}=f_1^{-1},\quad A_{t34}=f_2^{-1},\quad f_i=f_i(x_5,\ldots,x_{10}),\quad \nabla_\perp^2 f_i=0, \tag{1.1}$$

where $\nabla_\perp^2=\partial_5^2+\cdots+\partial_{10}^2$. One can try to find localized intersections by starting with an ansatz of the form (1.1) and allowing the functions f_1 and f_2 to depend on all the spatial coordinates, i.e., not only the directions $x_5,\ldots,x_{10}$ transverse to both branes, but also x_1, x_2, x_3, and x_4. However, the equations of motion turn out to require that at least one of the branes remains delocalized [33–36]. If, for example, the $M2$-brane in the $(t34)$ plane is to be localized, we must have translation invariance in the x^3,x^4 directions, i.e., the $(t12)$ brane cannot be localized. Furthermore the supergravity equations then reduce to

$$\nabla_\perp^2 f_1=0,\quad \nabla_\perp^2 f_2+f_1(\partial_1^2+\partial_2^2)f_2=0. \tag{1.2}$$

Given a solution to the first equation for f_1, the second equation for f_2 is then linear.[1]

It seems clear that in order to find fully localized intersections, one must consider a wider class of spacetimes. Some physical input is then necessary to determine an appropriate generalization of the diagonal ansatz. Our strategy is quite simple. The key ingredients in determining the weak field limit of a given brane intersection are appropriate source terms for the field equations. These are provided by coupling the bulk supergravity fields to brane sources via the Born-Infeld effective action. Now, since $M2$-branes have no world-volume gauge fields, the term ''Born-Infeld dynamics'' may seem inappropriate. However, under dimensional reduction what we refer to here as the Born-Infeld action of $M2$-branes reduces to the familiar Born-Infeld action for D-branes and it is convenient to use the same term for both systems.

Smooth world-volume solitons, sometimes called ''BI solitons,'' describing certain nonplanar branes in a background flat spacetime have been studied, beginning with the work of [3,37–39]. Following the literature, we will refer to these nonplanar branes as ''intersecting.'' These solitonic configurations are appropriate sources for the bulk field equations linearized around flat space. In Sec. II, we work out the linearized fields for the BI soliton source describing intersecting $M2$-branes. We will see from the linearized analysis that certain off-diagonal terms in the metric, as well as certain additional components of the gauge field, are generated by BI soliton sources.

Any BI soliton generates a solution to the bulk field equations linearized about flat space. Distinctions between different intersecting brane configurations, however, arise when we look at higher orders in the weak field expansion. Carrying out a perturbative expansion based directly on the field equations would be a tedious task. Fortunately, the supersymmetry of the BI soliton sources leads to considerable simplifications. Recently, Fayyazuddin and Smith [36] have presented an ansatz for the spacetime fields of fully localized intersecting $M5$-branes, which preserves half of the 32 supersymmetries of $D=11$ supergravity.[2] The remaining undetermined function in the ansatz satisfies a nonlinear set of equations.

It is straightforward to alter the form of the ansatz in [36] to obtain an appropriate ansatz for $M2$-brane intersections and this is done in Sec. III. The altered ansatz does in fact guarantee the existence of Killing spinors appropriate to intersecting $M2$-branes. A useful consistency check is that the ansatz matches the linearized solutions of Sec. II. By dimensional reduction and T duality, the same is true for solutions of Dp-branes intersecting Dp-branes on $p-2$ dimensional spatial manifolds in $D=10$ type II supergravities. We use this structure to investigate the second order perturbations to the bulk supergravity fields. Although they are finite for larger branes, for intersecting $M2$-, $D2$-, and $D3$-branes, we find that the second order perturbations diverge at every point in spacetime as we take a delta function limit of smooth sources to represent fully localized branes. Section III shows that this holds for a simple ''crossed-brane'' configuration, while the more complicated calculations associated with holomorphic curve brane configurations are presented in Appendix A. As will be discussed in Sec. IV, this meshes well with arguments of [30] based on the low-energy field theory on the D-branes and is likely to be connected with interesting properties of full nonlinear solutions.

Lastly, Appendix B considers the weak coupling limit of a fundamental string ending on a $D3$-brane. In this case, we do not yet have an ansatz for the full nonlinear spacetime fields, but we hope that our first order results will help to motivate such an ansatz. The weak field results do show that the Born-Infeld spike soliton of [37–39] generates the appropriate Neveu-Schwarz (NS) antisymmetric tensor field to be identified with a fundamental string.

II. $M2$-BRANE INTERSECTIONS

We begin by studying the weak field limit of a pair of $M2$-branes intersecting at a point. We take the action to be

[1]The partially localized intersections studied in [29,30] are solutions to equations having this form, but with different numbers of relative transverse and overall transverse directions. The relative transverse directions are taken to be compact and f_2 is expanded in Fourier modes. Solutions for which f_2 is localized can always be found when the two branes are separated in the transverse directions. However, as the transverse separation is taken to zero, the Fourier modes with nonzero wave number are driven to zero unless the number d of overall transverse directions satisfies $d\leqslant 3$.

[2]However, the actual intersecting solutions displayed in [36] are diagonal and describe one localized and one delocalized brane.

given by $S=S_{\rm bulk}+S_{\rm brane}$, where $S_{\rm bulk}$ is the $D=11$ supergravity action and $S_{\rm brane}$ is the Born-Infeld action for an $M2$-brane embedded in curved spacetime. The bosonic parts of these are given by

$$S_{\rm bulk}=\frac{1}{l_{pl}^9}\int d^{11}x\left\{\sqrt{-g}\left(R-\frac{1}{12}F^2\right)+\frac{2}{(72)^2}\epsilon^{\mu_1\cdots\mu_{11}}F_{\mu_1\mu_2\mu_3\mu_4}F_{\mu_5\mu_6\mu_7\mu_8}A_{\mu_9\mu_{10}\mu_{11}}\right\} \tag{2.1}$$

$$S_{\rm brane}=-T\int d^3\xi\left\{\sqrt{-\det G}-\frac{1}{6}\epsilon^{abc}(\partial_a X^\mu)(\partial_b X^\nu)\times(\partial_c X^\rho)A_{\mu\nu\rho}\right\}. \tag{2.2}$$

Here $G_{ab}=(\partial_a X^\mu)(\partial_b X^\nu)g_{\mu\nu}$ is the induced metric on the $M2$-brane world volume, $g_{\mu\nu}$ and $A_{\mu\nu\rho}$ are the spacetime metric and gauge field, respectively, and the $M2$-brane tension T is related to the $D=11$ Planck length by $T=1/l_{lp}^3$. An $M2$-brane configuration $X^\mu(\xi)$ carries stress-energy $T^{\mu\nu}_{\rm brane}$ given by

$$T^{\mu\nu}_{\rm brane}(x)\equiv\frac{1}{\sqrt{-g}}\frac{\delta S_{\rm brane}}{\delta g_{\mu\nu}(x)}=-\frac{T}{2\sqrt{-g}}\int d^3\xi\sqrt{-\det G}G^{ab}(\partial_a X^\mu)\times(\partial_b X^\nu)\delta^{11}(x-X(\xi)), \tag{2.3}$$

and current density for the antisymmetric tensor gauge field $J^{\mu\nu\rho}_{\rm brane}$ given by

$$J^{\mu\nu\rho}_{\rm brane}(x)\equiv\frac{1}{\sqrt{-g}}\frac{\delta S_{\rm brane}}{\delta A_{\mu\nu\rho}(x)}=\frac{T}{\sqrt{-g}}\int d^3\xi\epsilon^{abc}(\partial_a X^\mu)(\partial_b X^\nu)(\partial_c X^\rho)\times\delta^{11}(x-X(\xi)). \tag{2.4}$$

These are conserved if the $M2$-brane equations of motion are satisfied.

In a flat and empty background, intersecting $M2$-branes can be described by a BPS soliton solution to the world-volume equations of motion [3,37] with two separate asymptotic regions

$$(X^1+iX^2)(X^3+iX^4)=\alpha_0^2,\quad X^5=\cdots=X^{10}=0. \tag{2.5}$$

For $\alpha_0^2=0$ this describes a pair of orthogonally intersecting planes in the (1,2) and (3,4) planes. For $\alpha_0^2\neq 0$ the intersection region is smoothed out this scale and $|\alpha_0|$ is the size of the ''neck'' where the two branes join. The Born-Infeld equations of motion in the flat background are satisfied for any α_0 in the complex plane. Now, the Born-Infeld effective action is an approximation to the low-energy brane dynamics valid when certain derivatives are small. For large α_0, all curvatures of the brane are small and the Born-Infeld description will therefore be accurate. Furthermore, in [40] a related intersection of $D3$-branes and fundamental strings was studied in which, due to supersymmetry, the Born-Infeld description could be shown to be exact. In the present case, we again expect that our Born-Infeld description of the brane dynamics is exact for all values of α_0.

Note that the parameter α_0 has nothing to do with the charges of the branes; the symmetry of the holomorphic curve guarantees that we have the same number of branes in each of the two planes. A similar parameter occurs in D-brane intersections of the form $Dp\perp Dp(p-2)$ and, in that case, corresponds to a modulus in the low-energy field theory on the branes.

Let us choose coordinates $\xi^{0,1,2}=X^{0,1,2}$ on the $M2$-brane world volume. Note that this choice introduces an asymmetry between the two asymptotic regions. The brane stress tensor $T^{\mu\nu}_{\rm brane}$ evaluated in a flat background then has diagonal components

$$\begin{aligned}T^{00}(x)&=\frac{T}{2}\left(1+\frac{\alpha_0^4}{R^4}\right)\delta^8(x-x_0),\\ T^{11}(x)&=T^{22}(x)=-\frac{T}{2}\delta^8(x-x_0),\\ T^{33}(x)&=T^{44}(x)=-\frac{\alpha_0^4 T}{2R^4}\delta^8(x-x_0),\end{aligned} \tag{2.6}$$

and also nonzero off-diagonal components

$$\begin{aligned}T^{13}(x)&=T^{24}(x)=\frac{\alpha_0^2T(x_1^2-x_2^2)}{2R^4}\delta^8(x-x_0),\\ T^{23}(x)&=-T^{14}(x)=\frac{2\alpha_0^2Tx_1x_2}{2R^4}\delta^8(x-x_0),\end{aligned} \tag{2.7}$$

where $R^2=x_1^2+x_2^2$ and

$$\delta^8(x-x_0)\equiv\delta\left(x_3-\frac{\alpha_0^2x_1}{x_1^2+x_2^2}\right)\delta\left(x_4+\frac{\alpha_0^2x_2}{x_1^2+x_2^2}\right)\delta(x_5)\cdots\delta(x_{10}).$$

The nonzero gauge current density components are

$$\begin{aligned}J^{012}&=T\delta^8(x-x_0),\quad J^{034}=\frac{\alpha_0^4T}{R^4}\delta^8(x-x_0),\\ J^{013}&=J^{024}=-\frac{2\alpha_0^2Tx_1x_2}{R^4}\delta^8(x-x_0),\\ J^{014}&=-J^{023}=-\frac{\alpha_0^2T(x_1^2-x_2^2)}{R^4}\delta^8(x-x_0).\end{aligned} \tag{2.8}$$

We solve the linearized Einstein equation in the standard way. Let $g_{\mu\nu}=\eta_{\mu\nu}+h_{\mu\nu}$ define $\gamma_{\mu\nu}=h_{\mu\nu}-\frac{1}{2}\eta_{\mu\nu}h$, and choose Lorentz gauge $\partial^\rho \gamma_{\rho\mu}=0$. The linearized Einstein equation then reduces to $l_{pl}^{-9}\partial^\rho\partial_\rho\gamma_{\mu\nu}=-2T_{\mu\nu}^{\rm brane}$. We find that the solution is

$$\begin{aligned}ds^2=&-(1-\tfrac{2}{3}(f_1+f_2))dt^2+(1-\tfrac{2}{3}f_1+\tfrac{1}{3}f_2)(dx_1^2+dx_2^2)\\&+(1+\tfrac{1}{3}f_1-\tfrac{2}{3}f_2)(dx_3^2+dx_4^2)\\&+(1+\tfrac{1}{3}(f_1+f_2))\delta^{kl}dx_kdx_l+2\phi(dx_1dx_3+dx_2dx_4)\\&+2\psi(dx_2dx_3-dx_1dx_4),\end{aligned} \tag{2.9}$$

where the four functions $f_1(x)$, $f_2(x)$, $\phi(x)$, and $\psi(x)$ satisfy the flat, spatial, ten-dimensional Laplace equation with different source terms,

$$\begin{aligned}&\nabla^2 f_1=-l_{pl}^6\delta^8(x-x_0),\quad \nabla^2 f_2=-\frac{\alpha^4 l_{pl}^6}{R^4}\delta^8(x-x_0),\\&\nabla^2\phi=-\frac{\alpha^2 l_{pl}^6(x_1^2-x_2^2)}{R^4}\delta^8(x-x_0),\\&\nabla^2\psi=-\frac{2\alpha^2 l_{pl}^6 x_1x_2}{R^4}\delta^8(x-x_0).\end{aligned} \tag{2.10}$$

Similarly, the linearized gauge field is given by

$$\begin{aligned}A=&-f_1dx^0\wedge dx^1\wedge dx^2-f_2dx^0\wedge dx^3\wedge dx^4\\&+\psi(dx^0\wedge dx^1\wedge dx^3+dx^0\wedge dx^2\wedge dx^4)\\&+\phi(dx^0\wedge dx^1\wedge dx^4-dx^0\wedge dx^2\wedge dx^3).\end{aligned} \tag{2.11}$$

Integral expressions for the functions $f_1(x)$, $f_2(x)$, $\phi(x)$, and $\psi(x)$ are then easily obtained using the ten-dimensional Green's function. The resulting expression, for e.g., $f_1(x)$ is given by

$$\begin{aligned}f_1(x)=\lambda l_{pl}^6\int d^2y\{&(x_1-y_1)^2+(x_2-y_2)^2\\&+(x_3-\alpha^2y_1/(y_1^2+y_2^2))^2\\&+(x_4+\alpha^2y_2/(y_1^2+y_2^2))^2+x_kx^k\}^{-4},\end{aligned} \tag{2.12}$$

where $\lambda=1/8\omega_9$ and ω_9 is the volume of the unit 9 sphere. This integral, like the similar ones for f_2, ϕ, and ψ, cannot be simply evaluated analytically. However, the integral expressions can easily be manipulated to show that the following relations, necessary for the Lorentz gauge conditions to hold, are satisfied

$$\begin{aligned}&\partial_1 f_1=\partial_3\phi-\partial_4\psi,\quad \partial_2 f_1=\partial_4\phi+\partial_3\psi,\\&\partial_3 f_2=\partial_1\phi+\partial_2\psi,\quad \partial_4 f_2=\partial_2\phi-\partial_1\psi.\end{aligned} \tag{2.13}$$

III. NONLINEAR INTERSECTING *M*2-BRANES

In Sec. II, we saw that it was straightforward to derive the first order fields for a localized intersection of $M2$-branes. This raises the question of whether our analysis can be extended to a full perturbation scheme. Here, we study the second order perturbations, which already show surprising results.

To make the analysis tractable, we first find a compact formulation of the full nonlinear intersecting $M2$-brane problem. Our approach is based on the recent work of Fayyazuddin and Smith [36] on localized $M5$-brane intersections, in which a supersymmetric ansatz is given for the spacetime fields. We start by adapting the results of [36] to the $M2$-brane case. The ansatz in this case, as in [36], depends on a single unknown function which must satisfy a nonlinear partial differential equation. The spacetime fields found in the previous section, because they are determined by a BPS source, give a leading order solution to these nonlinear equations in weak field perturbation theory.

A. The non-perturbative ansatz

Fayyazudin and Smith [36] have given an ansatz for the spacetime fields of a pair of $M5$-branes intersecting on a 3-brane. In this case, as with $M2$-branes intersecting at a point, each brane has two spatial dimensions not shared by the other. The ansatz in [36] is built around a Kähler metric on this four-dimensional relative transverse space, with the unknown function being the Kähler potential. Define complex coordinates $s=x^1+ix^2$ and $v=x^3+ix^4$ on the relative transverse space. The ansatz for intersecting $M2$-branes, analogous to that in [36], is then given by

$$ds^2=-H^{-2/3}dt^2+2H^{-2/3}g_{m\bar{n}}dz^mdz^{\bar{n}}+H^{1/3}\delta_{\gamma\sigma}dx^\gamma dx^\sigma, \tag{3.1}$$

where γ, $\sigma=5,\dots,10$, z^m ranges over s, v, and $g_{m\bar{n}}$ is a Kähler metric on the associated four-space; i.e., we may introduce the potential K such that $g_{m\bar{n}}=\partial_m\partial_{\bar{n}}K$. The function H is related to the ''determinant'' g of $g_{m\bar{n}}$:

$$H=4g=4(g_{v\bar{v}}g_{s\bar{s}}-g_{v\bar{s}}g_{s\bar{v}}). \tag{3.2}$$

Taking the three-form gauge potential A to be related to the metric through

$$A_{0m\bar{n}}=\frac{1}{2}iH^{-1}g_{m\bar{n}}, \tag{3.3}$$

and calculating the supersymmetry variations of the fields shows that this ansatz guarantees the existence of Killing spinors η satisfying the projection condition

$$\Gamma_{0m\bar{n}}\eta=iH^{-1}g_{m\bar{n}}\eta. \tag{3.4}$$

It therefore yields a supersymmetric solution of 11-dimensional supergravity when the equations of motion for the gauge field are satisfied. One can show that these reduce to the same nonlinear equation for the Kähler potential K found for $M5$-brane intersections in [36],

$$\frac{1}{2}\partial_m\partial_{\bar{n}}(8g(K)+\partial_\gamma\partial_\gamma K)=J_{m\bar{n}}, \tag{3.5}$$

where in this case the source $J_{m\bar{n}}$ is related to the 3-form current $J^{\mu\nu\rho}_{\rm brane}$ defined in Eq. (2.4) by

$$J_{m\bar{n}}=\frac{i}{2}\epsilon_{mm_1}\epsilon_{\bar{n}\bar{n}_1}J^{0m_1\bar{n}_1}_{\text{brane}}\sqrt{-\det g_{11}}, \tag{3.6}$$

where $\det g_{11}$ is the determinant of the full 11-dimensional metric. The i in Eq. (3.6) guarantees that $J_{m\bar{n}}$ is Hermitian. Note that because the ''determinant'' $g=H/4$ is a density, rather than a scalar, Eq. (3.5) is not a tensor equation. Consequently, the form of Eq. (3.5) is invariant only under holomorphic changes of the coordinates z^m for which the Jacobian is the identity.

Following our general strategy, the source $J_{m\bar{n}}$ should be consistent with the coupled bulk and/or Born-Infeld dynamics. Introducing a complex spatial coordinate $\xi=\xi^1+i\xi^2$ on the $M2$-brane world volume, one can check that the BI equations of motion are satisfied for any static holomorphic configuration $X^\gamma=0$, $X^0=\xi^0$, $X^m=X^m(\xi)$, $X^{\bar{n}}=X^{\bar{n}}(\bar{\xi})$, whenever the bulk fields have the form described by Eqs. (3.1) and (3.3). One technical difficulty is that for a single brane with a delta function source, as in Sec. II, the metric and three-form potential diverge at the source and, consequently, the Born-Infeld equations of motion are not well defined. In order to deal with this, one may introduce a ''fluid'' or ''dust'' of $M2$-branes which provides a smooth source at which the bulk fields need not diverge. Given the nonlinearities, the existence of smooth solutions is nontrivial even for smooth sources. However, if we assume that they do in fact exist for arbitrary smooth sources, one may consider a limit in which the dust density approximates a δ function. In this sense, any holomorphic embedding of the $M2$-brane is a consistent source for the full coupled nonlinear problem. In particular, since an arbitrary smooth source does lead to smooth bulk fields at first order in perturbation theory, any holomorphic embedding of the $M2$-brane is a consistent source at second order in perturbation theory.

Before studying the weak field perturbation expansion of Eq. (3.5) for the Kähler potential, we introduce a new set of holomorphic coordinates for the relative transverse space (s,v), which will be useful in keeping the calculations compact. In complex coordinates the world volume of the $M2$-brane source is given by the holomorphic curve $sv=\alpha_0^2$. This is most easily described by making a holomorphic change of coordinates with unit Jacobian:

$$\alpha=\sqrt{sv},\quad \beta=\sqrt{sv}\ln s/v. \tag{3.7}$$

Translated into the present notation, the intersecting $M2$-brane holomorphic curve of Sec. II yields a source of the form

$$J_{\alpha\bar{\alpha}}=\frac{q}{2}\delta^{(2)}(\alpha-\alpha_0)\delta^{(6)}(x_\perp) \tag{3.8}$$

with $J_{\alpha\bar{\beta}}$, $J_{\beta\bar{\alpha}}$, and $J_{\beta\bar{\beta}}$ vanishing and $x_\perp$ representing x^γ for $\gamma=1$ to 6. Here q is a charge describing the number of branes that are present and α_0 is the parameter describing the ''neck'' of the holomorphic curve. Note that such a holomorphic source satisfies the obvious integrability condition for the existence of a solution to Eq. (3.5): there is a potential $J=(q/4\pi)\ln|\alpha-\alpha_0|\delta^{(6)}(x_\perp)$ such that $J_{m\bar{n}}=\partial_m\partial_{\bar{n}}J$. In terms of the ($\alpha$, β) coordinates, the field equations and source are just as for a flat brane, which will simplify the calculations below. Note however, that what makes the problem nontrivial in the coordinates (α, β) are the boundary conditions. These are determined by the fact that the asymptotic metric takes the standard Cartesian form in terms of the original s, v coordinates, and that the s, v coordinates are to range over (exactly) the complex plane. The result is that the coordinate β ranges only over a strip, and that the asymptotic form of the metric is complicated in terms of α and β. Thus, it is nontrivial to construct the exact solution.

However, for the purposes of this paragraph only, let us make the assumption that the boundary conditions at infinity are not important near the source. In this case, the standard flat-brane solution holds (approximately) in this region. One obtains a solution in which, as usual, the ''source'' at $\alpha=\alpha_0$ is replaced by a horizon through which the solution may be smoothly continued. Thus, if the boundary conditions are indeed unimportant near the source, we should in the end obtain a solution of the sourceless 11-dimensional supergravity equations.

B. Perturbation expansion

As noted in [36], the nonlinear equation (3.5) for the Kahler potential K can be solved using a weak field expansion. Expand the Kähler potential as $K=\Sigma_{n\geqslant 0}K^{(n)}$, where $K^{(n)}$ is proportional to q^n, and also introduce $g^{(n)}_{n\bar{m}}=\partial_n\partial_{\bar{m}}K^{(n)}$. We want to perturb around flat spacetime, so the zeroth order Kähler metric is $g^{(0)}_{m\bar{n}}=\delta_{m\bar{n}}$ (with $\delta_{s\bar{s}}=\frac{1}{2}$), which follows from the zeroth order Kähler potential $K^{(0)}=(s\bar{s}+v\bar{v})/2$. Since we perturb around flat spacetime, the asymptotic boundary conditions will play a central role.

The nonlinear equation for the Kähler potential (3.5) is the same for both the $M2$-brane intersections considered here and the $M5$-brane intersections studied in [36]. Solutions for intersecting $D2$-branes can be constructed by considering the setup for $M2$-branes, taking the source to be independent of x_{10} (i.e., smearing the branes along this direction), and using dimensional reduction. Further smearing of the source can create additional symmetry directions, and we can then use classical T duality of the supergravity and Born-Infeld theories to construct a fully localized $D_p\perp D_p(p-2)$ solution in type-II supergravity coupled to an appropriate brane source. Thus, by letting the index γ range over an appropriate number ($d=7-p$) of transverse directions, Eq. (3.5) in fact describes intersecting solutions of the form $D_p\perp D_p(p-2)$. However, as we perturb around flat space and impose asymptotically flat boundary conditions in the d dimensional transverse space, we will only analyze the cases with $d\geqslant 3$ in detail below [i.e., $D_p\perp D_p(p-2)$ with $p\leqslant 4$ or intersecting $M2$- or $M5$-branes].

Given the form of the zeroth order fields, the first order terms in Eq. (3.5) combine to give

$$\frac{1}{2}\nabla^2(g^{(1)}_{m\bar{n}})=J_{m\bar{n}}, \tag{3.9}$$

where ∇^2 here denotes the $(d+4)$-dimensional flat Laplacian in the four relative transverse coordinates $s, \bar{s}, v, \bar{v}$, and the d overall transverse coordinates x^{γ}, $\nabla^2 = 4\partial_s\partial_{\bar{s}} + 4\partial_v\partial_{\bar{v}} + \partial_\gamma\partial_\gamma$. Let us introduce the notation $[s] = -1 = [\bar{s}]$, $[v] = +1 = [\bar{v}]$. Then, with $m(\bar{n})$ ranging over s, v $(\bar{s}, \bar{v})$, all the source components of Eq. (2.8) assemble using Eq. (3.6) into the compact form

$$J_{m\bar{n}} = e^{[m]\beta/2\alpha} e^{[\bar{n}]\bar{\beta}/2\bar{\alpha}} \frac{q}{2} \delta^{(2)}(\alpha - \alpha_0)\delta^{(d)}(x_\perp), \quad (3.10)$$

and the components of the first order Kähler metric are given by

$$g^{(1)}_{m\bar{n}} = \frac{-g}{(d+2)\omega_{d+3}} \int d^2\beta' \frac{e^{[m]\beta'/2\alpha_0} e^{[\bar{n}]\bar{\beta}'/2\bar{\alpha}_0}}{\left(\sum_{\gamma=1}^{d} x^\gamma x^\gamma + |s - \alpha_0 e^{-\beta'/2\alpha_0}|^2 + |v - \alpha_0 e^{\beta'/2\alpha_0}|^2\right)^{(d+2)/2}}, \quad (3.11)$$

where ω_{d+3} is again the volume of the unit $(d+3)$-sphere. These results are a more compact form of those given in Eq. (2.9) in Sec. II for $d=6$.

The sources $J_{m\bar{n}}$ do not depend on the background metric. Therefore, the right-hand side of Eq. (3.5) only has contributions at first order. Continuing to the expansion of Eq. (3.5) we find that the terms of order j satisfy

$$\partial_m\partial_{\bar{n}}\left(\nabla^2 K^{(j)} + 8\sum_{1\leq k\leq j-1}(g^{(k)}_{s\bar{s}} g^{(j-k)}_{v\bar{v}} - g^{(k)}_{v\bar{s}} g^{(j-k)}_{s\bar{v}})\right) = 0, \quad (3.12)$$

for $j>1$. Boundary conditions at infinity for localized branes imply that the quantity in parenthesis must vanish. Hence, the higher order terms in K satisfy a flat ten-dimensional Laplace equation with sources given by products of lower order terms and are given formally by the integrals

$$K^{(j)}(x_0) = \frac{4}{(d+2)\omega_{d+3}} \int d^{10}x \frac{(g^{(k)}_{s\bar{s}} g^{(j-k)}_{v\bar{v}} - g^{(k)}_{v\bar{s}} g^{(j-k)}_{s\bar{v}})}{|x_0 - x|^{d+2}}, \quad (3.13)$$

where the notation x_0, x includes the complex coordinates s, v, as well as the transverse coordinates x^γ. When the integral (3.13) converges, it gives the unique solution to Eq. (3.12) satisfying the appropriate boundary conditions.

C. To converge, or not to converge?

The important question which needs to be addressed is whether the integrals (3.13) for K^j do in fact converge, starting with the second order term $j=2$. We consider here the limit $\alpha_0 \to 0$ in which the smooth intersection degenerates into the singular intersection of two perpendicular planes. Although, due to the large curvature at the intersection, the Born-Infeld description of the dynamics is not *a priori* justified in this limit, the considerably more complicated calculations for $\alpha_0 \neq 0$ lead to the same conclusions. These calculations are presented in Appendix A. In the $\alpha_0 \to 0$ limit, the nonzero source terms in Eq. (3.8) are given simply by

$$J_{s\bar{s}} = \frac{q}{2}\delta^2(s)\delta(x), \quad J_{v\bar{v}} = \frac{q}{2}\delta^2(v)\delta(x). \quad (3.14)$$

The fact that $J_{s\bar{v}}, J_{v\bar{s}} \to 0$ even at $s=v=0$ can be verified by integrating $J_{s\bar{v}}$ (at finite α_0) over any region invariant under $s \to e^{i\theta}s$, $v \to e^{-i\theta}v$. The first order metric for the crossed-plane source is just the superposition of the results for flat branes at $s=0$ and $v=0$. For example, the component $g^{(1)}_{s\bar{s}}$ is

$$g^{(1)}_{s\bar{s}} = \frac{-g}{(d+2)\omega_{1+3}(x^\gamma x^\gamma + |s|^2)^{d/2}}, \quad (3.15)$$

with an analogous expression for $g^{(1)}_{v\bar{v}}$. The off-diagonal terms $g_{s\bar{v}}$ and $g_{v\bar{s}}$ both vanish.

The integral in Eq. (3.13) for $K^{(2)}$ then has the form

$$K^{(2)}(x_0) = \frac{4}{(d+2)^3\omega^3_{d+3}} \int \frac{q^2}{|x_0 - x|^{d+2}} \times \frac{d^dx\, d^2s\, d^2v}{(x^\gamma x^\gamma + |s|^2)^{d/2}(x^\sigma x^\sigma + |v|^2)^{d/2}}. \quad (3.16)$$

Let us analyze this integral in a small region near $x^\gamma = s = v = 0$. In this region, we may approximate $|x_0 - x|$ by a constant. Introducing $p^2 = x^\gamma x^\gamma + |s|^2 + |v|^2$, the integral over this small region factors into an integral over angles and an integral over ρ of the form $\int p^{3-d}dp$. The integral over angles does not vanish as the integrand is strictly positive. Thus, when $d \geq 4$, the integral diverges for any x_0. However, for $d=3$, the integral converges and the second order perturbation is well defined. Although we have not explicitly considered the cases with $d<3$, it is clear that the second order perturbation will have no short distance divergences in those cases.

This calculation suggests that higher order perturbation theory breaks down when the number d of overall transverse dimensions satisfies $d \geq 4$, which includes the $M2$-brane intersection $(d=6)$. On the other hand, perturbation theory is potentially well defined for $d=3$, which includes $M5$-brane intersections. As will be discussed further in Sec. IV, these results fit well with both the supergravity results of [30] in similar, but slightly different situations and with the predictions of that work, based on arguments in the D-brane field theory, for supergravity solutions of the present form.

Since the divergence of second order perturbations may be unexpected, the reader may wonder if some subtlety has been passed over through the use of singular sources. To show that such subtleties are under control, we consider below the same calculations for smooth sources and study the limit in which the smooth sources approximate the delta-functions above.

D. Smooth sources

Still keeping $\alpha_0=0$, we smooth the sources according to

$$J_{s\bar{s}}=\frac{q}{2}f_L(|s|^2+x^\gamma x^\gamma),\quad J_{v\bar{v}}=\frac{q}{2}f_L(|v|^2+x^\gamma x^\gamma), \tag{3.17}$$

where f_L is a smooth, non-negative function which vanishes for $r>L$ and has unit normalization; i.e., satisfying $\omega_d\int_0^L f(r)r^{d+1}dr=1$. Note that this smoothing is simple to carry out because of the ''no force'' condition between BPS objects. With sources smoothed over any scale L, solutions exist at each order of perturbation theory. We want to study the behavior of solutions as we take $L\rightarrow 0$. Typically there are many ways to take such a limit in general relativity (see, e.g., [41]). However, the present BPS system is highly constrained. Fixing the volume integral of the current components $J_{m\bar{n}}$ determines the total charge via Gauss' law, $\nabla_\mu F^{\mu\rho\sigma\tau}=J^{\rho\sigma\tau}$. Therefore for each L the solution has the same charge in the above prescription. Expanding in multipole moments, we see that to leading order in r^{-1}, $K^{(1)}$ stays the same for all L.

Symmetry considerations guarantee that the first order fields evaluated outside the dust distribution are identical to those from the delta-function source, and that $g_{s\bar{v}}$ and $g_{v\bar{s}}$ remain identically zero. However, the first order fields are now smooth everywhere, so the integral defining $K^{(2)}$ converges. It therefore gives the correct second order perturbation for the smooth source.

Now, consider the limit in which $L\rightarrow 0$ and f_L becomes the appropriate delta function. For any smooth f_L approximating the singular source, we may divide the integral for $K^{(2)}$ into an integral over a region outside the support of f_L, and one over a region inside. Since the integrand in the outside region is just the same as in the delta-function case, we have already seen that, for $d\geqslant 4$, it grows without bound in the limit. Now note that since f is non-negative, $g^{(1)}_{ss}$ and $g^{(1)}_{vv}$ are positive and the source for $K^{(2)}$ is of a definite sign. Thus, the integral over the region containing the source contributes to $K^{(2)}$ with the same sign as in the exterior region. Thus, we conclude that for $d\geqslant 4$, in the limit in which the smooth source becomes a delta function, $K^{(2)}$ grows without bound at each x_0.

The effect of this divergence on a physical quantity is somewhat subtle. For example, although the divergence occurs at the same order in r^{-1} as the term from $K^{(1)}$ that encodes the total charge, it cannot in fact effect the total charge computed at infinity. This is fixed by charge conservation, and the divergence can only appear in $F^{\mu\rho\sigma\tau}$ at higher order in r^{-1}. It is useful to note that arguments of the above form apply directly to the second order metric perturbation $g^{(2)}_{m\bar{n}}$, and to the norm $\|\partial_t\|^2\sim H^{-2/3}$ of the timelike Killing field. The latter is a scalar under coordinate transformations, so that its divergence shows that the result is not an artifact of our particular choice of gauge. Thus, we conclude that perturbation theory breaks down at second order for localized solutions of intersecting $M2$-, $D2$-, and $D3$-branes. However, the second order perturbations do exist for localized intersecting solutions of larger branes for which $d\leqslant 3$.

Now, on the one hand, it is no surprise that perturbation theory cannot construct a full nonlinear solution corresponding to a delta-function source. We expect a full solution to have a horizon, which is a strong field effect. Sources may be characterized by a ''charge radius'' $r_c\sim q^{1/(d-1)}$, and by a length scale L associated with the support of f. One expects perturbation theory to be useful for weak sources with $r_c/L\ll 1$, but not for strong sources with $r_c/L\sim 1$ or greater. Of course, the difference between weak and strong sources is usually apparent only when one attempts to sum the perturbation series. What is interesting about our case is the explicit divergence of the second order term and the fact that the behavior is very different for $d\leqslant 3$ than for $d\geqslant 4$. Although we can say nothing definite about the full nonlinear solutions, this strongly suggests that their behavior is qualitatively different for $d\geqslant 4$ than for $d\leqslant 3$. In particular, it is consistent with the prediction of [30] that fully localized asymptotically flat solutions should exist only for $d\leqslant 3$. In Appendix A, we show that the same behavior holds for $\alpha_0\neq 0$.

IV. DISCUSSION

In this work we have explicitly constructed the first order perturbative bulk supergravity fields corresponding to intersecting $M2$-branes. The corresponding results for a fundamental string ending on a $D3$-brane appear in Appendix B. We also showed that, as one would expect, any solution of the coupled bulk supergravity/Born-Infeld system for intersections of the form $M2\perp M2(0)$ or $Dp\perp Dp(p-2)$ is controlled by equations of the form presented in [36] for $M5\perp M5(3)$. We used this structure to analyze the second order perturbations of the bulk fields. While these perturbations are finite and small, far from the branes for intersecting $M5$-branes and intersecting Dp-branes with $p\geqslant 4$, the second order perturbations diverge everywhere in the spacetime for intersecting $M2$-branes and for intersecting Dp-branes with $p=2,3$.

This result appears to fit well with the predictions of [30] based on field theory considerations. That work started from the observation [29] that there are no fully localized solutions for one-branes inside five-branes. Solutions do exist when the branes are separated in the transverse direction, but the one-branes necessarily delocalize as the transverse separation is removed. The limit of zero separation gives one-branes ''smeared'' over the five-branes. It was shown in [30] that similar results hold in a number of other contexts, such as $D(p-4)$-branes parallel to Dp-branes for $p=3,4$, or Dp-branes intersecting smeared Dp-branes on a $p-2$ surface for $p\leqslant 3$. This behavior is in contrast with the situation for larger branes in which the solutions remain localized as the transverse separation is removed.[3]

Similar effects are found in certain "near-horizon" spacetimes. Therefore, one expects to have a field theory description of this effect through an analogue of the dualities described in [31,32]. Understanding the field theory origin of delocalization was the main goal of [30]. Consider first the case of $D(p-4)$-branes parallel to Dp-branes. Since both are associated with a "width" of the $D(p-4)$ branes in the directions along the Dp-branes, a natural idea is that the delocalization in classical supergravity is somehow related to the scale size of the instantons that describe the smaller branes in the Higgs phase of the Dp-brane field theory. In dualities in general, strong field classical effects on one side are related to strongly quantum mechanical effects on the other. It turns out that the supergravity delocalization is related to the quantum fluctuations of the scale size in the field theory. Fluctuations which would be large due to ultraviolet effects are suppressed by a string-scale cutoff, but the fluctuations can still be large due to infrared effects. The relevant field theory lives on the intersection of the two branes and delocalization occurs in exactly those cases where this is $(0+1)$ or $(1+1)$-dimensional, for which the infrared effects do indeed make the fluctuations large. This "fluctuation-delocalization duality" correctly predicts both cases in which the supergravity should delocalize and the rate at which it does so as the transverse separation is removed. Now consider Dp-branes intersecting Dp-branes on a surface with $(p-2)$ spatial dimensions. Such intersecting branes are associated with holomorphic curves $Z_1Z_2=\alpha_0^2$ in C^2, where C denotes the complex numbers. It turns out that α_0 is a modulus and is related to the scale size modulus associated with $D(p-4)$-branes inside Dp-branes through T duality. Thus, one expects similar behavior in this case, with delocalization related to the quantum fluctuations of α_0. As little information was available regarding the classical supergravity solutions for fully localized intersecting branes, [30] could compare the field theory only with the classical supergravity solutions in which one brane was smeared over the world volume of the other. For such cases, agreement was once again found with regard both to which cases should delocalize and how fast this should happen as the transverse separation is removed. The natural prediction is of course that a fully localized solution in which two branes are separated in a transverse direction should also delocalize when this separation is removed and, therefore, that fully localized intersecting brane solutions with $Dp\perp Dp(p-2)$ should not exist for $p\leqslant 3$. As a result, one expects that M-theory solutions with $M2\perp M2(0)$ also should not exist. These are just the cases for which we found a divergence of the second order perturbations of the bulk fields. Note that since first order perturbation theory is linear, the lack of a well-defined second order perturbation is the natural signature of the nonexistence of fully localized asymptotically flat solutions. A small subtlety is that one should remember that the field theory is dual to the supergravity physics only in the near-horizon region. As a result, it is not clear just what the field theory arguments have to say about the existence of asymptotically flat (as opposed to near-horizon) supergravity solutions for which the neck size α_0 of the supergravity solution is comparable to or larger than the charge radius r_c of the branes. For this reason, [30] could conclude that such solutions fail to exist only for small α_0. It is interesting that our perturbative results were qualitatively the same for all values of α_0, but it is not clear to what extent the existence of full nonlinear solutions for large α_0 should be reflected in perturbation theory.

Having found that the second order perturbations fail to exist for $d\geqslant 4$, it is natural to ask about the higher order perturbations for the case $d\leqslant 3$. Do they in fact exist? This is far from clear. The source terms for the higher order perturbations are more complicated, and there is the potential for subtle cancellations even in the case $\alpha_0=0$. We leave this question for future work.

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APPENDIX A: SOURCES WITH $\alpha_0\neq 0$

We now wish to consider the second order perturbation $K^{(2)}$ for the case $\alpha_0\neq 0$. Again, we will find that $K^{(2)}$ exists only for $d\leqslant 3$. Let us consider the value of $K^{(2)}$ at some point $x_0=(x_0^\gamma, s_0, \bar{s}_0, v_0, \bar{v}_0)$. From Eq. (3.13), this is

$$K^{(2)}(x_0)=\frac{4}{(d+2)\omega_{d+3}}\int d^dx\, d^2s\, d^2v \times \frac{g^{(1)}_{,s'\bar{s}'}(x)\,g^{(1)}_{,v'\bar{v}'}(x)-g^{(1)}_{,s'\bar{v}'}\,g^{(1)}_{,v'\bar{s}'}}{|x_0-x|^{(d+2)}}, \quad \text{(A1)}$$

with $g^{(1)}_{m\bar{n}}$ given by Eq. (3.11). As before, a divergence can only result from integrating over the singularities in the first order fields that arise at the location of the source. Note, in particular, that adding to $g^{(1)}_{m\bar{n}}$ any smooth function of x with the same large x behavior will not alter the convergence of the above integral. This is the strategy we will invoke below.

If, instead of integrating over the entire β strip in Eq. (3.11), we restrict the integration to be over only the region $|\beta'-\beta|<2\epsilon_0$, then this changes $g^{(1)}_{m\bar{n}}$ only by a smooth function of the sort mentioned above. In the remaining (small $\beta'-\beta$) region, it is useful to expand $e^{[m]\beta'/2\alpha_0}e^{[\bar{m}]\beta'/2\bar{\alpha}_0}$ in powers of $\epsilon/\alpha_0:=(\beta'-\beta)/2\alpha_0$. We

[3]The first such localized solutions were found in [42] in the near-core limit.

write the resulting infinite series as $e^{[m]\beta/2\alpha_0}e^{[\bar{m}]\bar{\beta}/2\bar{\alpha}_0}P^{(1)}_{m\bar{n}}$. The expression $P^{(1)}_{m\bar{n}}$ is a series in ϵ, $\bar{\epsilon}$ with constant coefficients.

We also expand terms in the denominator in powers of $\delta/\alpha_0 := \alpha/\alpha_0 - 1$. Note that we have

$$|s-s'|^2 = e^{-\beta/2\alpha_0}e^{-\bar{\beta}/2\bar{\alpha}_0}|\delta(1+\beta/2\alpha) - \epsilon + O(\delta^2,\delta_\epsilon,\epsilon^2)|^2 \tag{A2}$$

and similarly for $|v-v'|^2$. The singularity of $|x-x'|$ will be controlled by

$$\Delta = x^\gamma x^\gamma + 2\cosh(\beta/2\alpha_0 + \bar{\beta}/2\bar{\alpha}_0)\epsilon\bar{\epsilon} + \left[\left(1+\frac{\beta}{2\alpha_0}\right)\left(1+\frac{\bar{\beta}}{2\bar{\alpha}_0}\right)e^{-\beta/2\alpha_0}e^{-\bar{\beta}/2\bar{\alpha}_0} + \left(1-\frac{\beta}{2\alpha_0}\right)\left(1-\frac{\bar{\beta}}{2\bar{\alpha}_0}\right)e^{+\beta/2\alpha_0}e^{+\bar{\beta}/2\bar{\alpha}_0}\right]\delta\bar{\delta},$$
$$+\left[\left(1+\frac{\beta}{2\alpha_0}\right)e^{-\beta/2\alpha_0}e^{-\bar{\beta}/2\bar{\alpha}_0} - \left(1+\frac{\beta}{2\alpha_0}\right)e^{+\beta/2\alpha_0}e^{+\bar{\beta}/2\bar{\alpha}_0}\right]\bar{\epsilon}\delta,$$
$$+\left[\left(1+\frac{\bar{\beta}}{2\bar{\alpha}_0}\right)e^{-\beta/2\alpha_0}e^{-\bar{\beta}/2\bar{\alpha}_0} - \left(1+\frac{\bar{\beta}}{2\bar{\alpha}_0}\right)e^{+\beta/2\alpha_0}e^{+\bar{\beta}/2\bar{\alpha}_0}\right]\epsilon\bar{\delta} \tag{A3}$$

since we may write

$$|x-x'|^2 = \Delta + O(\delta^3,\delta^2\epsilon,\delta\epsilon^2,\epsilon^3)$$
$$= \Delta\left(1+\frac{\delta^2}{\Delta}O(\delta,\epsilon) + \frac{\epsilon^2}{\Delta}O(\delta,\epsilon)\right). \tag{A4}$$

Since, for any $\beta,\bar{\beta}$ the object Δ is a positive definite quadratic form in x^γ,δ,ϵ, the functions δ^2/Δ, ϵ^2/Δ are bounded by functions of $\beta,\bar{\beta}$. Thus, we may write

$$|x-x'|^{-(d+2)} = \Delta^{-(d+2)/2}P^{(2)}, \tag{A5}$$

where P^2 is a series in both $|\epsilon/\alpha_0|$ and $|\delta/\alpha_0|$ whose coefficients involve functions of the form ϵ^2/Δ and δ^2/Δ. The most important property of $P^{(2)}$ is that it does not depend on $m,\bar{n}$.

Collecting these observations together, we have

$$(g^{(1)}_{m\bar{n}})_{\text{sing}} = -\frac{e^{[m]\beta/2\alpha}e^{[\bar{n}]\bar{\beta}/2\bar{\alpha}}}{(d+2)\omega_{d+3}}\int_{|\epsilon<\epsilon_0|}\frac{d^2\epsilon\, P^{(1)}_{m\bar{n}}P^{(2)}}{\Delta^{(d+2)/2}}. \tag{A6}$$

Note that when considering sufficiently high order terms that arise in the product $P^{(1)}_{m\bar{n}}P^{(2)}$, the integral over $d^2\beta'$ is nonsingular, even for $x^\gamma=0$, $\alpha=\alpha'$. Thus, dropping these terms again changes $g^{(1)}_{m\bar{n}}$ only by another smooth function of appropriate decrease at infinity.

Having dropped the terms in $P^{(1)}_{m\bar{n}}P^{(2)}$ that are not singular at $\epsilon = x^\gamma = \delta = 0$, let us consider taking $\epsilon_0 \to \infty$ to remove the restriction on the region of integration. The highest remaining terms lead to logarithmic diverges at large ϵ, but the other terms remain finite. Thus, if we add appropriate counterterms to regulate the logarithmic divergence, taking $\epsilon_0 \to \infty$ changes $g^{(1)}_{m\bar{n}}$ only by a bounded function and does not effect the convergence of the second order perturbations (A1) to the Kähler potential. The details of treating the large ϵ logarithms are not important, as we will see that the convergence of Eq. (A1) at small ϵ is controlled by lower order terms in $P^{(1)}_{m\bar{n}}P^{(2)}$.

Extending the integration region in this way over the entire complex ϵ-plane, the integral (A6) may be evaluated exactly (see, for example, [43]). The result has the form

$$[g^{(1)}_{m\bar{n}}]_{\text{sing}} = -\frac{e^{[m]\beta/2\alpha}e^{[\bar{n}]\bar{\beta}/2\bar{\alpha}}}{(d+2)\omega_{d+3}}\frac{Q_{m\bar{n}}}{(x^\gamma x^\gamma + 2\Omega^2\delta\bar{\delta})^{d/2}}, \tag{A7}$$

where $Q_{m\bar{n}}$ is a polynomial in $|\delta|$ whose coefficients are determined by those of $P^{(1)}_{nm}P^{(2)}$ and

$$\Omega^2(\beta,\bar{\beta}) = (1-\beta/2\alpha_0)(1-\bar{\beta}/2\bar{\alpha}_0)e^{\beta/2\alpha_0}e^{\bar{\beta}/2\bar{\alpha}_0} + (1+\beta/2\alpha_0)(1+\bar{\beta}/2\bar{\alpha}_0)e^{-\beta/2\alpha_0}e^{-\bar{\beta}/2\bar{\alpha}_0}$$
$$-\frac{|(1-\beta/2\alpha_0)e^{\beta/2\alpha_0}e^{\bar{\beta}/2\bar{\alpha}_0} - (1+\beta/2\alpha_0)e^{-\beta/2\alpha_0}e^{-\bar{\beta}/2\bar{\alpha}_0}|^2}{2\cosh(\beta/2\alpha_0 + \bar{\beta}/2\bar{\alpha}_0)}. \tag{A8}$$

We must now see how the various terms in Eq. (A7) effect the second order Kähler potential (A1). Note that the first order fields enter quadratically, through the combination $4H^{(1)}=g^{(1)}_{ss}g^{(1)}_{\bar{v}\bar{v}}-g^{(1)}_{s\bar{v}}g^{(1)}_{\bar{v}s}$. The singular part of this expression may be written

$$(H^{(1)})_{\text{sing}}=\frac{4}{(d+2)^2\omega_{d+3}^2}\frac{Q_{s\bar{s}}Q_{v\bar{v}}-Q_{s\bar{v}}Q_{v\bar{s}}}{(x^\gamma x^\gamma+\Omega^2\delta\bar{\delta})^{d/2}}. \tag{A9}$$

The effects of a term in $H^{(1)}$ of given order in $|\delta|$ on the second order perturbation $K^{(2)}$ are straightforward to analyze. After rescaling δ by Ω, the $\beta,\bar{\beta}$ dependence factors out. The integral over $\beta,\bar{\beta}$ converges, and the only integrals remaining to be done are of the form

$$\int \frac{d^dx\, d^2\delta|\delta|^k}{(x^2+|\delta|^2)^d}. \tag{A10}$$

The convergence of such integrals can be studied by introducing the radial coordinate $\rho=\sqrt{x^\gamma x^\gamma+|\delta|^2}$. The expression (A10) factors into a convergent angular integral and a radial integral that converges for $k+1\geqslant d$.

Clearly, the relevant issue is which values of k actually contribute. This is just the question of determining the smallest power of $|\delta|$ that appears in the numerator of $(H^{(1)})_{\text{sing}}$, which in turn can be found by studying how the first order fields (3.11) enter into $(H^{(1)})_{\text{sing}}$. Let us first consider terms of the form (A10) that arise from the constant term in $P^{(2)}$; i.e., for the moment take $P^{(2)}=1$.

Note that the first few terms in $P^{(1)}_{m\bar{n}}$ are

$$P^{(1)}_{m\bar{n}}=1+[m]\epsilon/\alpha_0+[\bar{n}]\bar{\epsilon}/\bar{\alpha}_0+\frac{[m]^2}{2}\epsilon^2/\alpha_0^2+\frac{[\bar{n}]^2}{2}\bar{\epsilon}^2/\bar{\alpha}_0^2$$
$$+[m][\bar{n}]\epsilon\bar{\epsilon}/\alpha_0\bar{\alpha}_0+O(\epsilon^3/\alpha_0). \tag{A11}$$

To this same order, taking $P^{(2)}=1$, the singular part of $H^{(1)}$ is therefore

$$(H^{(1)})^{P^{(2)}=1}_{\text{sing}}=\frac{4}{(d+2)^2\omega_{d+3}^2}\left(\int\frac{P^{(1)}_{s\bar{s}}d^2\epsilon}{\Delta^{(d+2)/2}}\int\frac{P^{(1)}_{v\bar{v}}d^2\epsilon}{\Delta^{(d+2)/2}}-\int\frac{P^{(1)}_{s\bar{v}}d^2\epsilon}{\Delta^{(d+2)/2}}\int\frac{P^{(1)}_{v\bar{s}}d^2\epsilon}{\Delta^{(d+2)/2}}\right)$$
$$=\frac{16}{(d+2)^2\omega_{d+3}^2\alpha_0\bar{\alpha}_0}\left(\int\frac{\epsilon\bar{\epsilon}d^2\epsilon}{\Delta^{(d+2)/2}}\int\frac{d^2\epsilon}{\Delta^{(d+2)/2}}-\int\frac{\epsilon\, d^2\epsilon}{\Delta^{(d+2)/2}}\int\frac{\bar{\epsilon}\, d^2\epsilon}{\Delta^{(d+2)/2}}\right)+\cdots, \tag{A12}$$

as all terms of less than second order cancel out. The ellipses above denote terms of higher order. The important question is whether the second order terms above also cancel. It turns out that this is not the case. To see this, write Δ as $A\bar{A}\epsilon\bar{\epsilon}+B\epsilon+\bar{B}\bar{\epsilon}+C\bar{C}=|(A\epsilon+B/\bar{A})|^2+C\bar{C}-|B/A|^2$ and change integration variables to $\omega=A\epsilon+B/\bar{A}$. Since Δ is even in ω, integrals of the form

$$\int\frac{\omega\, d^2\omega}{\Delta^{(d+2)/2}}$$

vanish. As a result, we may write

$$(H^{(1)})^{P^{(2)}=1}_{\text{sing}}=\frac{16}{(d+2)^2\omega_{d+3}^2\alpha_0\bar{\alpha}_0|A|^6}$$
$$\times\int\frac{d^2\omega}{\Delta^{(d+2)/2}}\int\frac{\omega\bar{\omega}\, d^2\omega}{\Delta^{(d+2)/2}}$$
$$=(\text{const})\frac{1}{\alpha_0\bar{\alpha}_0|A|^6}(x^\gamma x^\gamma+\Omega^2\delta\bar{\delta})^{-2(d-1)} \tag{A13}$$

to the same order as in Eq. (A12). Note that A depends only on $\beta,\bar{\beta}$. Let us define $\delta_0=\Omega\delta$. Then $K^{(2)}$ involves the integral of the above expression (A13) with respect to the measure $\Omega^{-2}d^2\beta d^2\delta_0 d^d x_\perp$. The integral over $\beta,\bar{\beta}$ converges and clearly gives a result proportional to $(\delta_0\bar{\delta}_0+x^\gamma x^\gamma)^{-2(d-1)}$. We therefore see that the integral over $\delta_0,\bar{\delta}_0,x^\gamma$ converges if $d+2>2(d-1)$; i.e., for $d\leqslant 3$. On the other hand, for $d\geqslant 4$, this contribution to $K^{(2)}$ diverges at every point in the spacetime.

We have now shown that, for $d\geqslant 4$, the terms that arise from the order zero piece of $P^{(2)}$ cause a divergence in $K^{(2)}$ at order $k=2$ [in the counting of (A10)]. To conclude that $K^{(2)}$ is in fact divergent, we need only show that higher order terms in $P^{(2)}$ cannot cancel this divergence. This is not hard. Let $P^{(2)(1)}$ be the collection of first order terms in $P^{(2)}$, proportional to either $|\epsilon|$ or $|\delta|$. A compensating divergence could only come from the interaction of $P^{(2)(1)}$ with a term of order ϵ or $\bar{\epsilon}$ in $P^{(1)}_{m\bar{n}}$. Let $P^{(1)(1)}_{m\bar{n}}$ denote the first order terms in $P^{(1)}_{m\bar{n}}$. Due to the structure of our system, $P^{(2)(1)}$ always appears with either $P^{(1)(1)}_{s\bar{s}}+P^{(1)(1)}_{v\bar{v}}$ or $P^{(1)(1)}_{s\bar{v}}+P^{(1)(1)}_{v\bar{s}}$. However, both of these vanish. That a higher order divergence does not arise from the interaction of $P^{(2)(1)}$ with the zero order term $P^{(1)(0)}_{m\bar{n}}$ in $P^{(1)}_{m\bar{n}}$ follows from the fact that $P^{(1)(0)}_{m\bar{n}}$ is independent of $m,\bar{n}$. Thus, $K^{(2)}(x_0)$ does indeed diverge for all x_0 when $d\geqslant 4$; i.e., for $M2$, $D2$, and $D3$-branes.

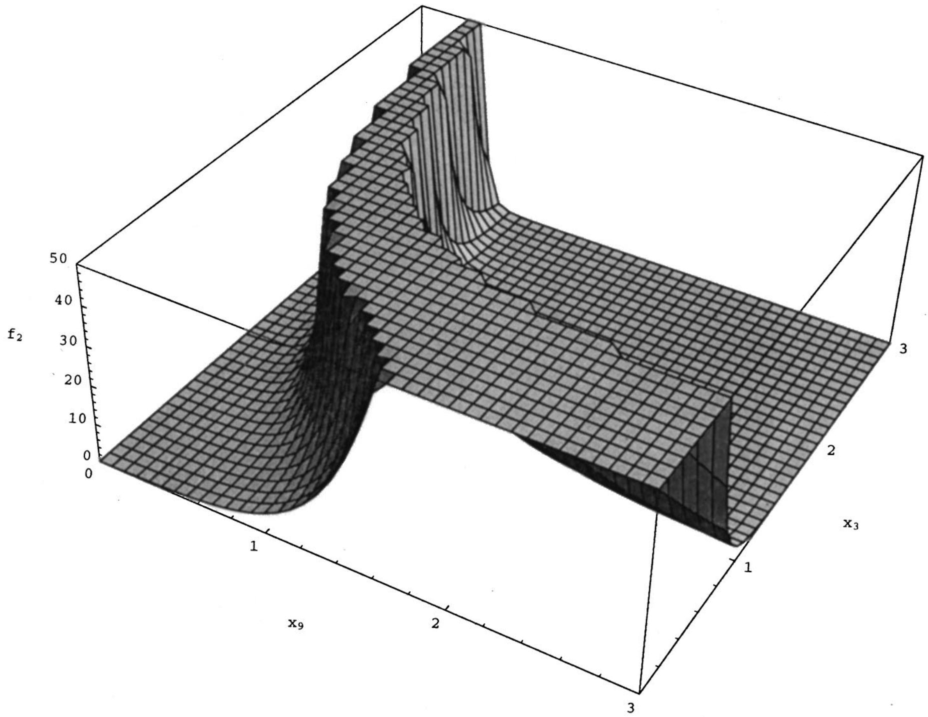

FIG. 1. Plot of $7\omega_8 f_2$ (which corresponds to the dilaton) with fixed $x_8=0$ and $\alpha=1$. This plot was made by evaluating the integral (B12) numerically.

Once again, one may consider replacing the localized intersecting brane with a smooth dust of branes concentrated in a region of size L in the transverse directions. This leads to smooth metric functions $g_{mn}^{(1)}$ which converge to the localized brane first order fields (3.11) in the $L\to 0$ limit. The analysis proceeds much as in the case of a delta-function source, but with extra integrals over α_0 and the location of the brane in the x^γ directions. In particular, $H^{(1)}$ has a similar structure. Thus, in the limit where the source becomes a delta function, $K^{(2)}(x_0)$ diverges for all x_0 for $d\geqslant 4$. As before, one can also show that $g_{mn}^{(2)}$ and $\|\partial_t\|^2$ diverge as well.

Thus, the second order perturbations are infinite and perturbation theory breaks down at second order for $d\geqslant 4$, though not for $d\geqslant 3$. This suggests that the full nonlinear localized solutions are quite different for $d\leqslant 3$ than for $d\geqslant 4$. In particular, it is consistent with the prediction of [30] that localized solutions should not exist, at least for small α_0. It is interesting that the divergence encountered here does not in fact depend on the value of α_0, but it is not clear if such a feature of the full solutions should be apparent at this level of analysis.

APPENDIX B: STRINGS ENDING ON $D3$-BRANES

In this appendix we compute weak coupling solutions to the coupled $D=10$ type-IIB supergravity $D3$-brane Dirac-Born-Infeld system, starting from the world-volume spike soliton describing a fundamental string ending on the $D3$-brane [37–39]. For this case, a useful ansatz for the full nonlinear metric is not known, but we hope that our work below will help to motivate one. The total action is given by $S=S_{\text{bulk}}+S_{\text{kinetic}}+S_{\text{WZ}}$, where these terms are given in the Einstein frame by [44,45]

$$S_{\text{bulk}}=\frac{1}{g_s^2}\int d^{10}x\sqrt{g}\bigg[R-\frac{1}{2}\partial_\mu\phi\partial^\mu\phi-\frac{1}{12}e^{-\phi}H^2 -\sum_n \frac{1}{2n!}e^{a_n\phi}F_{[n]}^{(R)^2}\bigg]$$

$$S_{\text{kinetic}}=-\frac{1}{g_s}\int d^4\xi\sqrt{-\det(G_{ab}+e^{-\phi/2}\mathcal{F}_{ab})}$$

$$S_{\text{WZ}}=-\frac{1}{6g_s}\int D-\frac{1}{4g_s}\int B^R\wedge\mathcal{F}-\frac{1}{8g_s}\int B^{NS}\wedge B^R -\frac{1}{16g_s}\int l\mathcal{F}\wedge\mathcal{F}. \tag{B1}$$

Here, B^{NS} is the NS-NS 2-form field, D,B^R,l are the Ramond-Ramond (RR) 4, 2, and 0-form fields, and ϕ is the dilaton. We have also defined $G_{ab}=\partial_a X^\mu\partial_b X^\nu g_{\mu\nu}$ and $\mathcal{F}_{ab}=F_{ab}-\partial_a X^\mu\partial_b X^\nu B^{NS}_{\mu\nu}$, where F_{ab} is the field strength associated with the U(1) connection A_a living on the brane. The field $H=dB^{(\text{NS})}$ is the NS-NS field strength, $F_{[n]}$ are the field strengths of the corresponding RR gauge potentials, and $a_1=2$, $a_3=1$, $a_5=0$. We have also set $l_s=1$. Recall that $F_{[5]}$ is a self-dual field strength. This information cannot be inserted in a covariant action, and therefore we must keep in mind that the complete solution for $F_{[5]}$ in terms of the field D in our equations is $F_{[5]}=dD+{}^*dD$.

In the weak coupling limit $g_s\to 0$, the field equations to zeroth order in g_s are satisfied by the world-volume spike soliton [37–39], representing a fixed number N_F of fundamental strings ending on the $D3$-brane in the flat background $g_{\mu\nu}=\eta_{\mu\nu}$, with all the other bulk fields equal to zero, given in static gauge $\xi^a=X^a=x^a$, $a=0,1,2,3$ by

$$X^9=\frac{\alpha^2}{r},\quad A_0=\frac{\alpha^2}{r}, \tag{B2}$$

where $r^2=x_1^2+x_2^2+x_3^2$ and $\alpha^2=g_sN_F$. Although it looks much like the parameter α_0 associated with the intersecting brane solutions of Secs. II and III, the parameter α appearing here is physically much different. It does not correspond to a modulus in the field theory and in fact is quantized since the number N_F of fundamental string charge must be an integer.

Our aim is to linearize the bulk field equations and compute the first order corrections in g_s. The form (B2) solves the Born-Infeld equations only in the limit of small α [37], but this is achieved for $g_s\to 0$ with N_F fixed. Now, small α will in fact mean that, for example, the extrinsic curvature of the embedded $(3+1)$ surface will be large. In general, we would not expect even the exact Born-Infeld description to be valid in this domain. Luckily, for intersections of this form, it was shown in [40] that the Born-Infeld description is in fact exact.

The nonzero components of the brane stress tensor are given by

$$T^{00}_{\rm brane}=\frac{1}{2g_s}\left(1+\frac{\alpha^4}{r^4}\right)\delta^{(6)},\quad T^{ii}_{\rm brane}=-\frac{1}{2g_s}\delta^{(6)}$$
$$T^{i9}_{\rm brane}=T^{9i}_{\rm brane}=\frac{\alpha^2}{2g_s}\frac{x^i}{r^3}\delta^{(6)},\quad T^{99}_{\rm brane}=-\frac{1}{2g_s}\frac{\alpha^4}{r^4}\delta^{(6)}, \tag{B3}$$

where the index i ranges over 1, 2, 3 and $\delta^{(6)}=\delta(x^4)\delta(x^5)\cdots\delta(x^8)\delta(x^9-(\alpha^2/r))$. The final expression for $h_{\mu\nu}$ can be given in terms of the following three integrals:

$$f_0(x)=\frac{1}{7\omega_8}\int\frac{d^3x'}{\left[(x_1-x_1')^2+\cdots+(x_3-x_3')^2+x_4^2+\cdots x_8^2+\left(x_9-\frac{\alpha^2}{r'}\right)^2\right]^{7/2}},$$
$$f_1^i(x)=\frac{1}{7\omega_8}\int\frac{x'^i d^3x'}{r'^3\left[(x_1-x_1')^2+\cdots+(x_3-x_3')^2+x_4^2+\cdots x_8^2+\left(x_9-\frac{\alpha^2}{r'}\right)^2\right]^{7/2}},$$
$$f_2(x)=\frac{1}{7\omega_8}\int\frac{d^3x'}{r'^4\left[(x_1-x_1')^2+\cdots+(x_3-x_3')^2+x_4^2+\cdots x_8^2+\left(x_9-\frac{\alpha^2}{r'}\right)^2\right]^{7/2}}, \tag{B4}$$

with ω_8 being the area of the 8-sphere. The solution for the linearized Einstein metric is

$$h_{00}=\frac{g_s}{2}\left(f_0+\frac{3\alpha^4}{2}f_2\right),\quad h_{ii}=-\frac{g_s}{2}\left(f_0-\frac{\alpha^4}{2}f_2\right)$$
$$h_{AA}=\frac{g_s}{2}\left(f_0+\frac{\alpha^4}{2}f_2\right),\quad h_{99}=\frac{g_s}{2}\left(f_0-\frac{3\alpha^4}{2}f_2\right), \tag{B5}$$
$$h_{9i}=h_{i9}=g_s\alpha^2 f_1^i,$$

where $A=4,\ldots,8$.

Varying the action with respect to the dilaton and keeping only terms that are first order in g_s yields to $\partial^\mu\partial_\mu\phi=(g_s\alpha^4/2r^4)\delta^{(6)}$ which has the solution

$$\phi=-\frac{1}{2}g_s\alpha^4 f_2. \tag{B6}$$

For the Neveu-Schwarz (NS) 3-form field strength we have the linearized equation $g_s^{-2}\partial_\mu H^{\mu\alpha\beta}=J^{\alpha\beta}_{\rm (NS)}$ with nonzero current components

$$J^{0i}_{\rm NS}=-\frac{\alpha^2x^i}{g_sr^3}\delta^{(6)},\quad J^{09}_{\rm (NS)}=\frac{\alpha^4}{g_sr^4}\delta^{(6)}. \tag{B7}$$

In the ''Lorentz Gauge,'' these equations read simply as $g_s^{-2}\partial^\mu\partial_\mu B^{\alpha\beta}_{\rm (NS)}=J^{\alpha\beta}_{\rm (NS)}$ and have the solution

$$B^{\rm (NS)}_{0i}=-\alpha^2g_sf_1^i,\quad B^{\rm (NS)}_{09}=\alpha^4g_sf_2, \tag{B8}$$

with all other components vanishing. These are exactly the bulk gauge fields that would be excited by a fundamental string aligned in the x^9 direction.

The first order equations for the Ramond-Ramond (RR) fields are

$$\frac{1}{g_s^2}\nabla_\mu F^{(R)^{\mu\alpha\cdots\gamma}}_{[n]}=J^{(R)^{\alpha\cdots\gamma}}, \tag{B9}$$

where the only nonzero currents are

$$J^{ij}_{(R)} = -\frac{\alpha^2 x_k \epsilon^{ijk}}{32 g_s r^3}\,\delta^{(6)}, \quad J^{0ijk} = \frac{e^{ijk}}{144 g_s}\,\delta^{(6)},$$
$$J^{09jk} = -\frac{\alpha^2 \epsilon^{ijk} x_i}{144 g_s r^3}\,\delta^{(6)}, \tag{B10}$$

and the components obtained by permutations of their indices. The current associated with the 0-form l vanishes. Again we use the ''Lorentz Gauge'' to solve the equations, and obtain

$$B^{(R)}_{ij} = \frac{\alpha^2 g_s}{32}\,\epsilon_{ijk} f^k_1, \quad D_{0ijk} = \frac{g_s}{144}\,\epsilon_{ijk} f_0,$$
$$D_{09jk} = -\frac{g_s \alpha^2}{144}\,\epsilon_{ijk} f^i_1. \tag{B11}$$

Let us now explore the form of the integrals f in Eq. (B4). Consider f_q with $q=0,2$. The symmetries of all the expressions show that we can rotate the x^i plane and the $(x_4,\ldots,x_8)$ plane in such a way that any point in spacetime is equivalent to one such that the only nonzero components are x_3, x_8, and x_9. In that situation we integrate over θ obtaining

$$f_q(x_3,x_8,x_9) = \frac{4\pi}{5 x_3 \omega_8}\int_0^\infty dr\, r^{2(3-q)} \times\left[\frac{1}{(r^2[(r-x_3)^2+x_8^2]+[\alpha^2 - r x_9]^2)^{5/2}} - \frac{1}{(r^2[(r+x_3)^2+x_8^2]+[\alpha^2 - r x_9]^2)^{5/2}}\right]. \tag{B12}$$

It is easy to see that these integrals will be convergent. For $r\to\infty$ they go like $\int dr\, r^{-2(q+2)}$. Note that when $x_3\to 0$ both (infinite) terms in (B12) cancel each other. The only singularity occurs when x is located over the source, that is, when

$$x_8 = 0, \quad x_3 x_9 = \pm\alpha^2. \tag{B13}$$

This was expected, and means that our perturbative analysis is not valid near the source. Figure 1 shows a plot of $7\omega_8 f_2$ (which corresponds to the dilaton) with fixed $x_8=0$ and $\alpha=1$. The plot was made by evaluating the integral (B12) numerically.

The flat region is a ''numerical cutoff'' near the singularity at the source; i.e., it is just the region where $\omega_8 f_2 \geqslant 50$. Note how the singular region narrows, indicating a weaker singularity, far from $x_3=0$. Recall that large x_3 is far from the fundamental string. This behavior is therefore expected, since we know that a pure $D3$-brane by itself is not a source for the dilaton. The other functions f_q and k_q show similar behavior.

Here we have studied only the lowest order bulk fields in the limit of small α. It would be interesting both to understand the first order fields produced by the exact bion solution [37] and to study higher order contributions to the bulk fields. For the case where the string passes through the $D3$-brane (and does not end on it), [30] would again predict that a fully localized intersecting brane solution does not exist. The argument involves considering the S-dual system of a $D1$-brane intersecting a $D3$-brane and identifying a set of moduli which live on the $(0+1)$-dimensional intersection manifold and which are T dual to the moduli that determine the delocalization of the $D2\perp D2(0)$ intersection. In this case, these moduli are not associated with the parameter α, but rather with the fact that the two halves of the string on opposite sides of the $D3$-brane can separate. Note, however, that the case considered here is somewhat different since we only have a string on a single side of the $D3$-brane. In particular, we cannot consider this solution as a limit of solutions in which the branes are separated in a transverse direction. Therefore, it appears possible that the present case may have different behavior.

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