Bounds on neutron-star moments of inertia and the evidence for general relativistic frame dragging

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Recent x-ray variability observations of accreting neutron stars may provide the first evidence for frame dragging effects around spinning relativistic objects. Motivated by this possibility and its implications for neutron-star structural properties, we calculate new optimal bounds on the masses, radii, and moments of inertia of slowly rotating neutron stars that show kilohertz quasi-periodic oscillations (QPOs). These bounds are derived under minimal assumptions about the properties of matter at high densities and therefore are largely independent of the unknown equation of state. We further derive a semi-analytical upper bound on the neutron-star moment of inertia without making any assumptions about the equation of state of matter at any density. We use this upper bound to show that the maximum possible nodal precession frequency of an inclined circular orbit around a slowly spinning neutron star is $\nu_{\rm NP} \simeq 45.2(\nu_{\rm s}/300\ \ {\rm Hz})$ Hz, where $\nu_{\rm s}$ is the spin frequency of the neutron star. We conclude that the nodal-precession interpretation of low-frequency QPOs in accreting neutron stars is inconsistent with the beat-frequency interpretation of the kHz QPOs or the identification of the highest-frequency QPO with that of a circular Keplerian orbit in the accretion disk.

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I. INTRODUCTION

Astrophysical observations of general relativistic phenomena are rare and often confined to effects observable within our solar system [1]. Among galactic sources, relativistic compact objects, such as neutron stars and black holes, are the prime candidates for the detection of such phenomena. In these sources, however, the effects of general relativity are often convolved in complex ways with other astrophysical phenomena, making their identification often impossible. The observation, via precise pulsar timing, of orbital shrinkage in double neutron-star systems represents the most successful to date identification of purely general relativistic effects in galactic compact objects [2].

The recent discovery with the *Rossi X-ray Timing Explorer* of quasi-periodic oscillations with variable kilohertz frequencies (hereafter kHz QPOs) from many accreting neutron stars has provided us with a new clock that measures accurately their variability [3]. The centroid frequencies of the highest-frequency kHz QPOs in each source have been identified with Keplerian frequencies of stable orbits in the accretion disks very close to the stars [3–5]. These QPOs can therefore be used as probes of the physical conditions in regions where general relativistic effects in the strong-field regime are non-negligible [5]. This identification has led to new upper bounds on the masses (typically $\lesssim 2.2\ \ M_{\odot}$) and radii (typically $\lesssim 15$ km) of the accreting neutron stars [5,6]. In one specific x-ray source (4U 1820−30), the association of a kHz QPO with the Keplerian orbital frequency *at* the radius of the innermost stable circular orbit around the neutron star may provide us with the first evidence of this prediction of general relativity as well as with the identification of the first relatively massive ($\simeq 2.2 M_{\odot}$) neutron star [7].

In many accreting neutron stars, a third low-frequency ($\simeq 10-70$ Hz) QPO, the so-called horizontal-branch oscillation (HBO), is often observed simultaneously with the kHz QPOs [8,3]. Soon after its discovery the origin of this QPO was linked to the interaction between the accretion disk and the magnetosphere of the neutron star [9]. Recently, however, this interpretation has been challenged [10] by the understanding that the accretion disks appear to penetrate closer to the stellar surface than what is required by the model [3,5,11]. As an alternative model, Stella and Vietri [10] suggested that the frequency of the HBO is the general relativistic Lense-Thirring precession frequency, caused by frame dragging, of an inclined circular orbit that has a Keplerian orbital frequency equal to the frequency of the highest-frequency kHz QPO. The physical mechanism responsible for producing brightness oscillations at the precession frequency of a particular orbit in the accretion disk is still a matter of active research [12]. However, when the effects on the precession frequency of the quadrupole moment of the stellar gravitational field can be neglected, the theoretically predicted correlation between the precession and orbital frequencies is consistent with observations [10,11] (see Ref. [11] for a discussion of the discrepancy between the predicted and observed trends at the high frequencies). The normalization of the observed correlation, though, is relatively high. In other words, the magnitude of the observed HBO frequencies can be accounted by the model, *only if* the moments of inertia of the neutron stars are $\simeq 4-5$ times larger than predicted by any realistic equation of state (EOS) [10,11,13]. Therefore, whether frame-dragging effects predicted by general relativity have actually been observed in accreting neutron-stars depends crucially on our knowledge of the moments of inertia of neutron stars.

Theoretical calculations of neutron-star properties such as their masses, radii, and moments of inertia require knowledge of the equation of state of neutron-star matter up to densities $\gg 10^{15}\ \ {\rm g\,cm^{-3}}$. However, the equation of state at densities higher than the nuclear saturation density ($\simeq 2.7$

$\times 10^{14}$ g cm^{-3}) is still largely unknown and the subject of intense theoretical and experimental research [14]. Therefore accurate predictions for neutron-star structural properties are hampered by the current uncertainty in the equation of state of high-density neutron-star matter [15].

In principle, the equation of state of matter at high densities could be constrained by astrophysical measurements of neutron-star masses and radii, although such measurements are relatively rare. The masses of the compact objects in double neutron-star systems measured via precise pulsar timing of the evolution of their binary orbits [2] were found to cluster around $\simeq 1.35 M_\odot$ [16]. Mass estimates of neutron stars in other binary systems depend strongly on the unknown inclination of the binary orbit and provide little additional information on neutron-star properties [16]. Astrophysical measurements of the radii of neutron stars based on the emitting area of their thermal emission during thermonuclear bursts [17], during the quiescence phase in transient systems [18], or during the cooling phase of isolated neutron stars [19] are also difficult because of the systematic uncertainties in the predicted model spectra. Based on these measurements so far no significant constraints were imposed on any of the current equations of state.

Bypassing the uncertainties of the equation of state at high densities, optimal bounds on the masses, radii, and moments of inertia of neutron stars have been derived under minimal assumptions about the validity of general relativity and the microscopic stability of neutron-star matter [20]. Stricter bounds can be obtained if the so-called causality limit is imposed, i.e., by requiring that the speed of sound be less than the speed of light everywhere in the neutron star [21], although the validity of this requirement has been questioned. Such limits on the macroscopic properties of the neutron stars are largely independent of their unknown equation of state.

In this paper, we obtain optimal bounds on the moments of inertia of the neutron stars that show kHz QPOs under minimal or no assumptions about their structure and equation of state, taking into account the bounds on their masses and radii imposed by the observations of these QPOs. Our aim is to address the possibility that low-frequency QPOs in neutron star sources are related to general relativistic frame dragging which leads to Lense-Thirring precession of inclined orbits.

In Sec. II, we discuss our assumptions and method of solution of the relevant equations. In Sec. III, we present numerical and semi-analytical bounds on neutron-star masses, radii, and moments of inertia and derive an EOS-independent maximum limit on the (Lense-Thirring) nodal precession frequency. In Sec. IV, we discuss the implications of our results for the Lense-Thirring interpretation of the observed HBOs in accreting neutron stars.

II. ASSUMPTIONS AND METHOD OF SOLUTION

In the absence of detailed knowledge of the equation of state of matter at very high densities ($\gtrsim 10^{14}$ g cm^{3}), we shall follow the procedure outlined by Sabbadini and Hartle [20] to obtain bounds on neutron-star masses, radii, and moments of inertia under a *set of minimal assumptions* for the equation of state of neutron-star matter above some fiducial energy density ρ_0. These are (a) the matter is cold, (b) the pressure in a given fluid element is determined uniquely by its energy density, (c) the energy density and pressure are everywhere positive, and (d) $dP/d\rho \geqslant 0$ (microscopic stability). In deriving these new bounds we make use of the recent observations of kHz QPOs in LMXBs to constrain the macroscopic properties of neutron stars in these systems, i.e., their masses and radii [5].

The inferred spin frequencies of the neutron stars in the systems that show kHz QPOs and HBO are $\simeq 250-350$ Hz [3–5], which are significantly smaller than the $\gtrsim 1500$ Hz breakup frequency of a typical neutron star [15]. We therefore assume that the neutron stars in all these systems are slowly rotating. Hereafter, we also set $c = G = 1$, where c is the speed of light and G is the gravitational constant.

The frequencies of the kHz QPOs observed in the x-ray brightness of accreting neutron stars and their dependence on mass-accretion rate have led to the identification of the highest-frequency QPOs with Keplerian frequencies of stable circular orbits in the accretion disks [3–5]. In this interpretation, the observation of kHz QPOs from an accreting neutron star imposes two constraints on its mass and radius [5]: (i) The radius of the orbit responsible for the highest-frequency kHz QPO must be larger than the radius of the innermost stable circular orbit around the neutron star. This leads to an upper bound on the neutron-star mass:

$$M_{\rm NS} \leqslant (\sqrt{864}\pi \nu_{\rm max})^{-1}, \tag{1}$$

where $\nu_{\rm max}$ is the maximum observed frequency of the highest-frequency QPO. (ii) The radius of the orbit responsible for the highest-frequency kHz QPO must be larger than the radius of the neutron-star itself. This leads to a mass-dependent upper limit on the neutron star radius:

$$2M_{\rm NS} \leqslant R_{\rm NS} \leqslant \left(\frac{M_{\rm NS}}{4\pi^2 \nu_{\rm max}^2} \right)^{1/3}, \tag{2}$$

where the lower limit on the neutron star is simply the requirement that the central compact object not be a black hole. In writing Eqs. (1) and (2) we have neglected the fact that the neutron star is slowly spinning; for the inferred spin frequencies of neutron stars in these systems, the correction is only $\lesssim 10\%$ towards increasing these upper bounds.

In calculating the structure of a neutron star with mass and radius consistent with the above bounds, we divide it into two regions, given a value of the fiducial density ρ_0, above which we do not trust the equation of state: the core, with mass $M_{\rm c}$ and radius $R_{\rm c}$ in which $\rho \geqslant \rho_0$, and the envelope exterior to the core. It has been shown [20] that the combined stellar configuration satisfies the set of minimal assumptions for the equation of state for neutron-star matter *if and only if*

$$M_{\rm c} \leqslant \frac{2}{9} R_{\rm c} [1 - 6\pi R_{\rm c}^2 P_0 + (1 + 6\pi R_{\rm c}^2 P_0)^{1/2}] \tag{3}$$

and

$$M_c \geq \frac{4\pi}{3} R_c^3 \rho_0, \tag{4}$$

where P_0 is the pressure that corresponds to the energy density ρ_0 at the edge of the core.

Among all core configurations that satisfy the core mass limits (3) and (4), there is one that maximizes the stellar moment of inertia [20]. The maximizing core configuration is that of constant density $\rho_c \geq \rho_0$. For a given equation of state and fiducial energy density ρ_0 and for a given neutron-star mass and radius, we calculate the structure of the neutron star for the core configuration that maximize its moment of inertia. We then scan the ranges of neutron-star masses and radii allowed by the kHz QPOs and obtain the global optimal bounds on the neutron-star moment of inertia as a function of ρ_0.

To first order in the stellar angular velocity Ω, the metric in and around a slowly rotating star is [22]

$$ds^2 = -e^{\nu(r)}dt^2 + \frac{1}{1-2m(r)/r}dr^2 + r^2(d\theta^2 + \sin^2\theta d\phi^2) - 2\omega(r)r^2\sin^2\theta d\phi dt, \tag{5}$$

where $\omega(r)$ is the angular velocity of a locally non-rotating frame at radius r measured by an asymptotic inertial observer. Under these assumptions, the quantities $\nu(r)$ and $m(r)$ satisfy the Oppenheimer-Volkoff equations

$$\frac{dP}{dr} = -(P+\rho)\frac{m+4\pi r^3 P}{r^2(1-2m/r)} \tag{6}$$

and

$$\frac{dm}{dr} = 4\pi r^2 \rho, \tag{7}$$

as well as the equation

$$\frac{d\nu}{dr} = 2\frac{m+4\pi r^3 P}{r^2(1-2m/r)}, \tag{8}$$

where P is the pressure and ρ the energy density at a radius r inside the neutron star. In order to solve for the quantity $\omega(r)$, and hence for the moment of inertia of the neutron star, we define, following Hartle [22],

$$f(r) \equiv 1 - \frac{\omega(r)}{\Omega} \tag{9}$$

and

$$j \equiv e^{-\nu/2}\left(1 - 2\frac{m}{r}\right)^{1/2}, \tag{10}$$

and solve the equation

$$\frac{d}{dr}\left(r^4 j \frac{df}{dr}\right) + 4r^3 f \frac{dj}{dr} = 0 \tag{11}$$

for the radial dependence of f.

Given a fiducial density ρ_0 and the corresponding pressure P_0, as well as a core mass M_c and radius R_c, we intergrate numerically Eqs. (6) and (7) outwards from the core-envelope interface until the intergration reaches the surface of the neutron star defined by $P(R_{NS}) = 0$. The mass of the neutron star is then simply $m(R_{NS}) = M_{NS}$.

For the core configuration that maximizes the neutron-star moment of inertia (i.e., the one with constant energy density in the core), we integrate analytically the same equations [Eqs. (6), (7)] but inwards from the core-envelope interface. If the resulting neutron-star mass and radius satisfy the constraints imposed by the kHz QPOs, we then solve numerically Eq. (8) with the boundary condition $\nu(R_{NS}) = \ln[1 - 2M_{NS}/R_{NS}]$.

Equation (11) is a second-order partial differential equation with the boundary conditions

$$\left[\frac{df}{dr}\right]_{r=0} = 0 \tag{12}$$

and

$$f(r=R_{NS}) = 1 - 2\frac{I}{R_{NS}^3}, \tag{13}$$

where I is the moment of inertia of the neutron star given in closed form by [23]

$$I = -\frac{2}{3}\int_0^{R_{NS}} \frac{dj(r)}{dr} f(r) r^3 dr = \frac{8\pi}{3}\int_0^{R_{NS}} (\rho + P)\frac{f(r)j(r)r^4}{1-2m(r)/r} dr. \tag{14}$$

We integrate Eq. (11) outwards from the center of the neutron star, using the boundary condition (12) and a trial value f_0 at the inner boundary. We call this trial solution $f_{tr}(r)$ and calculate the corresponding moment of inertia I_{tr} using Eq. (14). We note that Eq. (11) is scale free and therefore assuming a different value of f at the inner boundary, $f(r=0) = \xi f_0$, where ξ is a constant, results in the solution $f(r) = \xi f_{tr}(r)$ and the corresponding moment of inertia $I = \xi I_{tr}$. Given the trial solution, we can then calculate the value of the parameter ξ for which the boundary condition (13) is satisfied, i.e.,

$$\xi = \left[f_{tr}(r=R_{NS}) + \frac{2}{R_{NS}^3} I_{tr}\right]^{-1}. \tag{15}$$

As a result, the solution of Eq. (11) that satisfies boundary conditions (12) and (13) is just $f(r) = \xi f_{tr}(r)$ and the moment of inertia of the neutron star is

$$I=\frac{I_{\rm tr}}{f_{\rm tr}(r=R_{\rm NS})+2I_{\rm tr}/R_{\rm NS}^{3}}. \tag{16}$$

We performed all numerical integrations using a fourth-order Runge-Kutta scheme. We verified our numerical implementation of the procedure outlined above and our integration algorithms by taking the limit $\rho_0\to\infty$, i.e., assuming that we know the equation of state everywhere in the neutron star, and comparing the calculated moments of inertia for different neutron star parameters and equations of state with the values given by Cook, Shapiro, and Teukolsky [15]. We performed an additional test by taking the limit $\rho_0\to 0$, i.e., assuming a constant-density neutron star, and comparing the calculated moments of inertia for different neutron star parameters with the values given by Sabbadini and Hartle [20] and Abramowicz and Wagoner [24]. In all tests the agreement was better than $\sim 0.5\%$.

III. BOUNDS ON NEUTRON-STAR PROPERTIES

A. Optimal bounds on neutron-star masses and radii

The constraints (1) and (2) imposed on the neutron-star mass and radius by the identification of the highest-frequency kHz QPO with a Keplerian orbital frequency are derived without any assumptions regarding the equation of state for neutron-star matter [5]. These constraints can be further optimized assuming an equation of state for the envelope of the neutron star and constraining the properties of the core [Eqs. (3) and (4)] so that they satisfy the minimal set of assumptions discussed in Sec. II. For a given equation of state and fiducial density ρ_0 and for each core mass and radius in the allowed range defined by the limits (3) and (4), we integrate the Oppenheimer-Volkoff equations and calculate the resulting neutron-star mass and radius.

Figure 1 shows the allowed ranges of stellar masses and radii for two representative equations of state (labeled according to Ref. [15]) and for different values of the fiducial density ρ_0. In order to understand the qualitative behavior of the bounds shown in Fig. 1, for a given equation of state, we call $\rho_c(M_{\rm NS})$ the central density and $R_{\rm EOS}(M_{\rm NS})$ the radius of the star of mass $M_{\rm NS}$ for this specific equation of state.

In the limiting case $\rho_0\to 0$, i.e., when the equation of state is assumed to be unknown everywhere in the star, the lower bound on the neutron star radius corresponds to the general relativistic requirement $R_{\rm NS}\geqslant(9/4)M_{\rm NS}$ [20], so that the central pressure in the neutron-star is not infinite. The upper bound on the neutron-star radius corresponds to the bound (2) set by the kHz QPOs.

For a non-zero fiducial density, the lower bound on the stellar radius corresponds to the requirement $R_{\rm c}\geqslant(9/4)M_{\rm c}$ imposed on the *core* (and not the stellar) properties. At high fiducial densities, the mass and radius of the envelope become non-negligible, $R_{\rm NS}>R_{\rm c}$, and hence the lower bound on the neutron-star radius for a given mass becomes tighter (moves to the right in Fig. 1). For a non-zero fiducial density, the upper bound on the stellar radius becomes tighter, as well. The radius of a star of given mass decreases with increasing central density and hence, when $\rho_0>\rho_c(M_{\rm NS})$, the maximum allowed radius of a neutron star with mass $M_{\rm NS}$ is $R_{\rm EOS}(M_{\rm NS})$, which may be smaller than the upper bound on the radius imposed by the kHz QPOs.

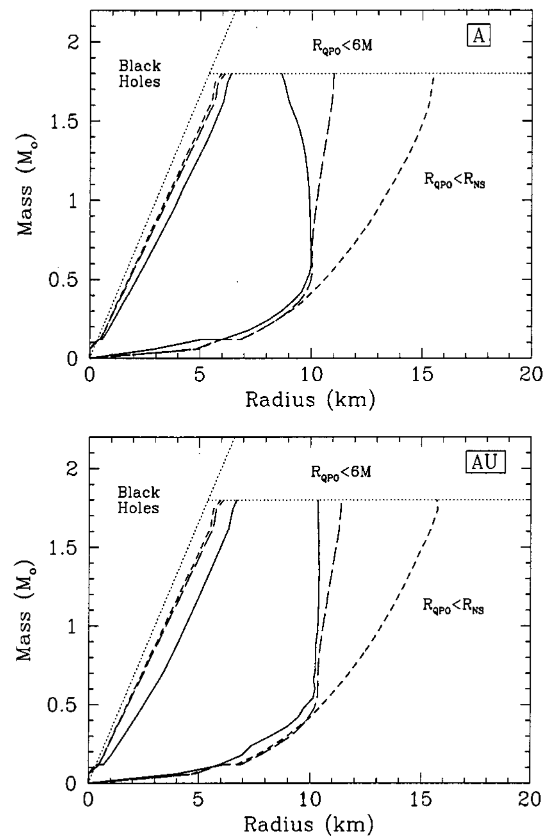

FIG. 1. Bounds on the mass and radius of a neutron star imposed by the identification of a 1220 Hz QPO with a Keplerian orbital frequency. The two panels correspond to different equations of state, which were assumed to be valid up to some fiducial density. Different line types correspond to the constraints imposed for different values of the fiducial density. Dotted lines: equation-of-state independent limits [Eqs. (1) and (2)]. Short-dashed lines: $\rho_0=2.7\times10^{14}$ g cm^{-3}. Long-dashed lines: $\rho_0=7\times10^{14}$ g cm^{-3}, Solid lines: $\rho_0=2\times10^{15}$ g cm^{-3}. The dotted lines for the maximum allowed radii in both panels overlap with the short-dashed lines and have been omitted for clarity.

In Fig. 2 we plot the maximum allowed neutron-star radius as a function of the fiducial density, for different equations of state (labels according to Ref. [15]). As indicated from Fig. 1 already, using any equation of state up to about the nuclear saturation density ($\rho_{\rm ns}\simeq 2.7\times10^{14}$ g cm^{-3}) does not not alter the bounds on the neutron-star radius [Eq. (2)] imposed by the kHz QPOs. Extending the use of any equation of state to higher densities (up to 2 or 4 times $\rho_{\rm ns}$) leads to tighter constraints on the stellar radius that depend strongly on the assumed equation of state. The bound imposed by Eq. (1) on the neutron star mass is not affected by knowledge of the equation of state.

Note here that all physical equations of state satisfy the set of minimal assumptions used in constructing Figs. 1 and 2. As a result, the bounds on the mass and radius of a neutron star plotted in these figures will not lead to any constraints on

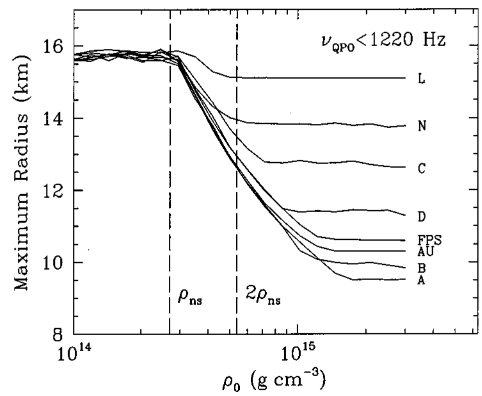

FIG. 2. Maximum neutron-star radius allowed by the identification of a 1220 Hz QPO with a Keplerian orbital frequency, as a function of the fiducial density ρ_0 up to which an equation of state was assumed to be known. Different curves correspond to different equations of state. The nuclear saturation density is $\rho_{ns} \simeq 2.7 \times 10^{14}$ g cm^{-3}.

the microscopic properties of the equation of state at high densities in addition to those already imposed by relations (1) and (2). However, these considerations do impose additional constraints on the macroscopic properties of individual neutron stars, which in principle can be compared with other independent estimates of their masses and radii. Such additional estimates may be obtained from the x-ray spectra [17] or the amplitudes of the coherent oscillations during x-ray bursts [25].

B. Bounds on neutron-star moments of inertia

For a given equation of state and fiducial density ρ_0 and for a neutron star with a mass and radius consistent with the bounds calculated above, we calculate the upper bound on the stellar moment of inertia, as outlined in Sec. II. Figure 3 shows the maximum value of the quantity $I_{45}/(M_{NS}/M_{\odot})$,

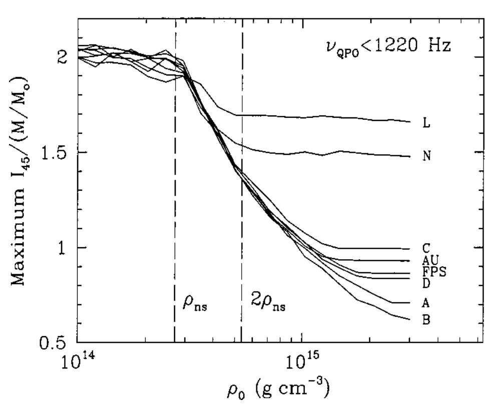

FIG. 3. Maximum neutron-star moment of inertia in units of its mass allowed by the identification of a 1220 Hz QPO with a Keplerian orbital frequency, as a function of the fiducial density ρ_0 up to which the equation of state was assumed to be known. Different curves correspond to different equations of state. The nuclear saturation density is $\rho_{ns} = 2.7 \times 10^{14}$ g cm^{-3}.

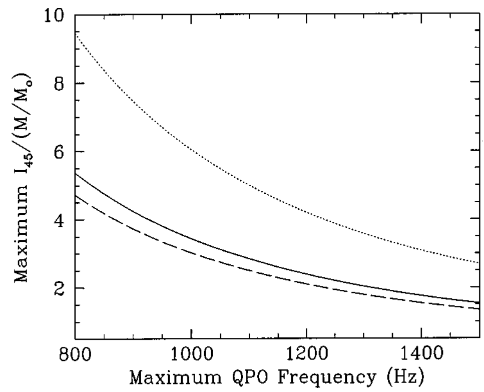

FIG. 4. Maximum neutron-star moment of inertia in units of its mass as a function of the maximum observed kHz QPO frequency. The upper bounds calculated using the numerical method described in Sec. II (solid line) are compared to the ones calculated analytically in the limit of zero (dashed line) and maximum (dotted line) compactness of the neutron star.

where $I \equiv I_{45} 10^{45}$ g cm^3 is the neutron-star moment of inertia, as a function of the fiducial density for different equations of state and for a maximum kHz QPO frequency of 1220 Hz. The maximum ratio $I_{45}/(M_{NS}/M_{\odot})$ decreases with increasing fiducial density ρ_0, in part because $I/M_{NS} \propto R_{NS}^2$, and the maximum allowed neutron-star radius decreases with increasing ρ_0 (see Fig. 2). For fiducial densities higher than the nuclear saturation density, the maximum ratio $I_{45}/(M_{NS}/M_{\odot})$ depends on the assumed equation of state.

For a neutron star showing kHz QPOs with a maximum frequency ν_{max}, we can obtain the maximum value of the ratio $I_{45}/(M_{NS}/M_{\odot})$, independent of the equation of state, by taking the limit $\rho_0 \to 0$. This corresponds to a star with constant density and is the configuration that maximizes its moment of inertia. Figure 4 (solid line) shows the resulting dependence; for a maximum kHz QPO frequency of 1220 Hz, $I_{45}/(M_{NS}/M_{\odot}) \lesssim 2.3$.

We can obtain the dependence of the maximum ratio $I_{45}/(M_{NS}/M_{\odot})$ on ν_{max} at the limit $\rho_0 \to 0$ in the following *analytical* way. For any star, independent of the equation of state, Sabbadini and Hartle [20] showed that

$$I \leqslant \xi M_{NS} R_{NS}^2. \tag{17}$$

The parameter ξ is a function of the compactness of the star (M_{NS}/R_{NS}) and has the limiting values $\xi_{min} = 2/5$ in the Newtonian limit $(M_{NS}/R_{NS} \to 0)$ and $\xi_{max} = 0.799 \simeq 4/5$ in the limit of maximum compactness allowed by general relativity (i.e., $M_{NS}/R_{NS} = 4/9$) [20]. Combining Eq. (17) with the radius bounds (1) and (2), we obtain

$$\frac{I}{M_{NS}} \leqslant \frac{\xi}{24\pi^2 \nu_{max}^2} \leqslant \frac{1}{30\pi^2 \nu_{max}^2}, \tag{18}$$

where ξ corresponds to the compactness of the neutron star that maximizes the ratio I/M_{NS} and in the last inequality we used the fact that $\xi \leqslant 4/5$. In Fig. 4 we compare the analytical upper bound (18) for the two limiting values of ξ ($\xi=2/5$, dashed line; $\xi=4/5$, dotted line) to the one calculated numerically in the limit $\rho_0\to 0$ (solid line).

When $\rho_0\to 0$, the maximum value of the ratio $I/M_{\rm NS}$ corresponds to a neutron star of constant density with typically the maximum radius allowed by constraint (2) imposed by the kHz QPOs. For such a neutron star, $M_{\rm NS}/R_{\rm NS}=1/6$ [see Eqs. (1) and (2)] and hence ξ is closer to the Newtonian limit of 2/5 than to its maximum value of 4/5, as suggested by Fig. 4. Indeed, by comparing the analytical scaling to the numerical result we find that a better approximation to the upper bound of the ratio $I_{45}/(M_{\rm NS}/M_\odot)$ corresponds to $\xi=0.452$ and is (see also Fig. 4)

$$\frac{I_{45}}{M_{\rm NS}/M_\odot}\lesssim 2.3\left(\frac{1220\ \ {\rm Hz}}{\nu_{\rm max}}\right)^2. \tag{19}$$

This is the maximum value of the ratio $I_{45}/(M_{\rm NS}/M_\odot)$ allowed by general relativity for a neutron star that shows a maximum kHz QPO frequency of $\nu_{\rm max}$ in its power spectrum.

C. Maximum nodal precession frequency

The orbital plane of an infinitesimally inclined circular orbit around a slowly spinning neutron star precesses because of general relativistic frame dragging, as well as because of classical effects related to the quadrupole moment of the stellar gravitation field. The nodal precession frequency $\nu_{\rm NP}$ of such an orbit is [10]

$$\nu_{\rm NP}=\nu_{\rm LT}-\nu_{\rm C}=\frac{8\pi^2 I}{M_{\rm NS}}\nu_s\,\nu_{\rm K}^2-\nu_{\rm C}, \tag{20}$$

where $\nu_{\rm LT}$ and $\nu_{\rm C}$ are the contributions of the general relativistic Lense-Thirring precession and of the classical precession, respectively, $\nu_{\rm s}$ is the spin frequency of the neutron star, and $\nu_{\rm K}$ is the Keplerian orbital frequency of the orbit.

The frequency of the general relativistic precession is directly proportional to the ratio I/M, for which we have obtained optimal bounds in the previous section. Given that the effect of classical precession is to reduce the nodal precession frequency and using the upper bound (18) on the ratio $I/M_{\rm NS}$ we obtain

$$\nu_{\rm NP}\leqslant\frac{1}{3}\xi\nu_{\rm s}\simeq 45.2\left(\frac{\nu_{\rm s}}{300\ \ {\rm Hz}}\right)\ \ {\rm Hz}, \tag{21}$$

where we used $\xi\simeq 0.452$, for the reasons discussed in Sec. III B.

Equation (21) shows that there exists an upper limit on the nodal precession frequency of a circular orbit around a slowly spinning neutron star, which we have obtained analytically. It is remarkable that this upper limit is independent of all the other properties of the neutron star, of the unknown equation of state, or of the properties of the circular orbit.

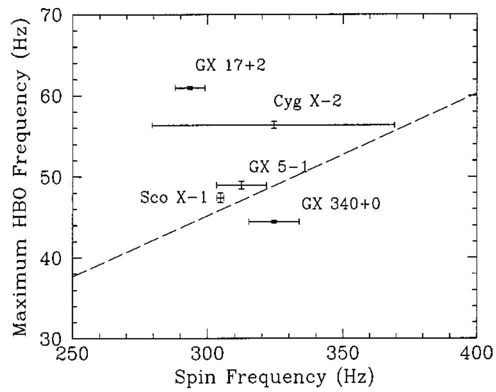

FIG. 5. Maximum HBO frequency versus inferred spin frequency for five bright neutron-star sources. The dashed line is the maximum nodal precession frequency for an inclined circular orbit around a star of a given spin frequency.

IV. DISCUSSION

We have calculated optimal bounds on the masses, radii, and moments of inertia for slowly rotating neutron star in which observed frequencies of kHz QPOs have been identified with Keplerian orbital frequencies. Assuming the validity of an equation of state up to some fiducial density ρ_0, these bounds become tighter as ρ_0 increases. In the limiting case of $\rho_0\to 0$, i.e., when we make no assumption regarding the equation of state, we have derived an analytical upper bound on the neutron-star moment of inertia for a given maximum observed kHz QPO frequency. We also obtain analytically the maximum nodal precession frequency of an inclined circular orbit around a neutron star, which depends *only* on the spin frequency of the star and is independent of the other stellar properties, the equation of state, or the properties of the circular orbit.

In this section we use these constraints to address the possible observational evidence for general relativistic frame-dragging effects in the rapid variability of accreting neutron stars [10]. Many such sources often show three distinct types of QPOs that are not harmonically related [3,5]. The two QPOs at kilohertz frequencies are believed to occur at the Keplerian frequency of a stable circular orbit in the accretion disk and at its beat with the neutron-star spin frequency [4,5,13]. According to Stella and Vietri [10], the third, low-frequency QPO (the HBO) occurs at the nodal precession frequency of the orbit responsible for the kHz QPO.

We can first test quantitatively the suggestion that the HBO occurs at the nodal precession frequency of an inclined circular orbit by comparing the maximum observed HBO frequency in different sources with the maximum possible nodal precession frequency around a slowly spinning neutron star [Eq. (21)]. We use the data of five bright neutron stars, in which all three QPOs can be identified unambiguously, as discussed in detail in Ref. [11]. Figure 5 shows the maximum HBO frequencies in these sources plotted against their spin frequencies inferred from the peak separation of the kHz QPOs. The data points are compared to the maximum possible nodal precession frequency around a neutron star calculated in Sec. III B [Eq. (21)]. Four out of the five sources are inconsistent with our optimal bound at least at the 1σ level. In particular, the observed HBO frequencies in the source GX 17+2 can be excluded at high statistical significance from being nodal precession frequencies, if the peak separation of the kHz QPOs is equal to the spin frequency of the neutron star.

It is possible that the observed HBO occurs at the second harmonic of the nodal precession frequency. A precessing circular orbit has a twofold symmetry that could, in principle, produce even-order harmonics that are stronger than the odd-order harmonics. The observation of a subharmonic of the HBO in neutron-star sources as well as of subharmonics of similar QPOs in black-hole sources [25] gives additional weight to this conjecture. In this case, we address the nodal-precession interpretation of the HBO using the observed correlation between the HBO and kHz QPO frequencies in the same five bright neutron-star sources discussed in Ref. [11].

When the effects of classical precession are negligible, observation of a Keplerian, spin, and nodal precession frequencies leads to a direct measurement of the ratio I/M of the neutron star [see Eq. (20)]. Identification of the HBO frequency in four of these five sources with the second harmonic of the nodal precession frequency requires $I_{45}/(M_{\rm NS}/M_{\odot})\geqslant 2.3$ at the 99% confidence level [11]. However, for the maximum observed kHz QPO frequency of 1220 Hz, $I_{45}/(M_{\rm NS}/M_{\odot})\leqslant 2.3$, independent of the equation of state (extreme case of $\rho_0=0$). Moreover, assuming even the unrealistically stiff equation of state L up to twice nuclear saturation density results in an upper limit of $I_{45}/(M_{\rm NS}/M_{\odot})\leqslant 1.7$ (see Fig. 3). Assuming any other equation of state results in even lower values of the ratio $I_{45}/(M_{\rm NS}/M_{\odot})$ which are inconsistent at high statistical significance with the value required by the nodal-precession interpretation of the HBO.

We therefore conclude that the nodal-precession interpretation of the HBO observed in several neutron-star sources is inconsistent with the identification of the higher-frequency kHz QPO with a Keplerian frequency of a circular orbit and with the identification of the frequency separation of kHz QPOs with the spin frequency of the neutron star [26].

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