Brane junctions in the Randall-Sundrum scenario

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We present static solutions to Einstein's equations corresponding to branes at various angles intersecting in a single 3-brane. Such configurations may be useful for building models with localized gravity via the Randall-Sundrum mechanism. We find that such solutions may exist only if the mechanical forces acting on the junction exactly cancel. In addition to this constraint there are further conditions that the parameters of the theory have to satisfy. We find that at least one of these involves only the brane tensions and cosmological constants, and thus cannot have a dynamical origin. We present these conditions in detail for two simple examples. We discuss the nature of the cosmological constant problem in the framework of these scenarios, and outline the desired features of the brane configurations which may bring us closer towards a resolution of the cosmological constant problem.

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I. INTRODUCTION

The principal challenge facing particle theorists is to understand the physics at energy scales of a few TeV. It seems inevitable that the standard model will be amended at these scales. The most popular scenario is that the world is supersymmetric, with the scale of supersymmetry breaking being around a few hundred GeV. Thus in this scenario all superpartners would become visible around the TeV scale. This possibility would explain why there is such a big hierarchy between the weak and the Planck scales. Thus the bulk of the efforts in the past 20 years has been devoted to modifying particle physics above the weak scale in order to accommodate this huge hierarchy. Very recently it has been understood that there exists a different way towards reconciling particle physics with gravity at high energies, by radically changing our ideas how gravity will work above the TeV scale [1–3]. Most notably, Arkani-Hamed, Dimopoulos and Dvali suggested [1] that in fact the fundamental Planck scale itself could be as low as a few TeV, if there are large extra dimensions. This way the problem of the hierarchy between the Planck and the weak scales is translated into the question of why the size of the extra dimensions is much larger than its natural scale of 1/TeV. The fundamental new ingredient in this idea is that the reason why we do not see the effects of the large extra dimensions is because the standard model fields reside on a 3-brane, and the only fields which can propagate in the extra dimensions are the gravitons.

Recently, Randall and Sundrum (RS) further developed on these ideas by noting that our understanding of Kaluza-Klein (KK) gravity models has been largely limited to factorizable metrics where the components of the metric tensor do not depend on the coordinates of the extra dimension [4,5]. RS noted that if this is not the case, the properties of compactification may change radically. In particular [4], following the idea that the standard model fields may reside on a 3-brane, RS considered two 3-branes embedded into $(4+1)$-dimensional spacetime, with the extra dimension being a compact S^1/Z_2 manifold (this latter motivated by [3]). The bulk cosmological constant was chosen to be negative, while the tensions of the two branes are of opposite signs. RS found that if a particular fine-tuning relation between the cosmological constant and the brane tensions is obeyed, there will be a static solution to Einstein's equations, which is given by two slices of anti–de Sitter (AdS) space glued together at the location of the branes. The metric tensor has a non-trivial exponential dependence on the coordinate y along the extra dimension.[1] This exponential determines the natural mass scale at the location y. Thus it is not inconceivable that while the mass scale at the brane with positive tension is 10^{19} GeV, due to the exponential suppression it might be a few TeV on the brane with negative tension, thereby possibly solving the hierarchy problem [4,7]. RS further noted [5] that the brane with positive tension supports a single bound state (zero mode) of gravitons, thereby ''trapping'' gravity to this wall. This is a very appealing feature of the theory, since in this case one might as well move the second brane with negative tension far away (in fact making the size of the extra dimension infinitely large), while Newton's law of

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[1]Similar domain-wall solutions in the context of supergravity theories have been considered in [6].

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gravity is still correctly reproduced on the brane due to the trapped zero mode. The idea of having non-compact extra dimensions is also explained in Refs. [8,9]. Since the trapping of the zero mode crucially depends on the fact that one has a brane of co-dimension 1, one would think that this feature of trapping gravity on a 3-brane can only hold if one has a (4+1)-dimensional spacetime. However, Arkani-Hamed *et al.* have pointed out [10] that if one considers intersecting branes of co-dimension 1 (intersecting orthogonally in a single 3-brane), one can still find static solutions to Einstein's equations, which will trap gravity to the intersection of the branes. Further solutions to Einstein's equation have been given in [11], within the context of supergravity in [12,13], and the relation to string theory and holography has been explained in [14]. The cosmological aspects of the RS models have been studied in [15,16], while the issue of bulk scalars and stabilization of the radius in [17,18].

In this paper we consider more general intersections of branes. In particular, we discuss ''brane junctions,'' that is intersections of semi-infinite branes intersecting in a single 3-brane. We will mainly concentrate on junctions of 4-branes, but we expect that it will be straightforward to generalize the algorithm of gluing sectors of static AdS spacetimes together to higher dimensions. We find that brane junctions can yield static solutions to Einstein's equations if some fine-tuning conditions between the tensions and the cosmological constants are satisfied. Moreover, the balance of mechanical forces on the junction arising from the brane tensions is a necessary condition for the existence of the static solution. We present these conditions for some simple examples in detail.

A crucial ingredient of the RS solution is the fine-tuning between the brane tension and the bulk cosmological constant, which ensures that there is a static universe with the effective 4-dimensional cosmological constant vanishing. Thus the cosmological constant problem in four dimensions is translated into the problem of the tuning between the brane tension and the fundamental (five dimensional) cosmological constant. In the case of branes intersecting at angles one expects that there will be similar relations, which also involve the angles of the branes. A simple way of understanding the cosmological constant problem would then be to imagine that one starts with a setup of branes whose angles do not satisfy the required tuning relation. Then one lets the system relax, and perhaps it would settle to a configuration where the angles of the branes take the right value, thus providing a flat 4 dimensional universe with a vanishing cosmological constant. For this scenario to be viable, one would need to find a solution of intersecting branes, where all fine-tuning conditions can be satisfied by the choice of angles between the branes. Moreover, this configuration should be a ground state of the system once the dynamics of the branes is included. Unfortunately, as we will see, this is not the case in the solutions based on junctions presented in this paper. There is always at least one remaining fine-tuning involving only the tensions and the cosmological constants. One may hope, however, that a more clever configuration of branes may possess the necessary features and thus provide a dynamical interpretation of the cosmological constant problem.

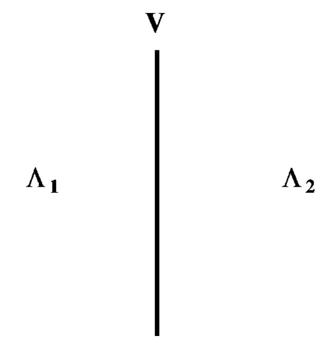

FIG. 1. A single 3-brane with tension V divides the $(4+1)$-dimensional space-time into two domains with different cosmological constants.

This paper is organized as follows: in Sec. II we review the RS solution by considering a 3-brane in $(4+1)$-dimensional spacetime separating two domains with different cosmological constants. In Sec. III we give our general setup for brane junctions in $5+1$ dimensions and discuss the general algorithm of finding the solutions to Einstein's equations and the fine-tuning relations. In Sec IV we work out the solutions and fine-tuning relations in detail for two simple junctions. In Sec. V we summarize our observations about the cosmological constant problem, and we conclude in Sec. VI.

II. REVIEW OF THE RANDALL-SUNDRUM SOLUTION

We first briefly review the original Randall-Sundrum solution by presenting a slightly generalized version of it. In this setup we have a single 3-brane (with positive tension V) embedded into $(4+1)$-dimensional spacetime, where the branes divide the space into two domains: one with cosmological constant Λ_1, the other with Λ_2 (both of them negative). This setup is depicted in Fig. 1. The original RS solution for $\Lambda_1=\Lambda_2=\Lambda$ is given by

$$ds^2=e^{-2m|y|}\eta_{ab}dx^a dx^b-dy^2, \tag{2.1}$$

where $a,b=0,1,2,3$ are the coordinates of the four dimensional spacetime, while y is the coordinate along the (infinite) extra dimension. In order for this to be the solution to the Einstein equations, the parameter m has to satisfy

$$m^2=-\frac{\kappa^2\Lambda}{6}, \tag{2.2}$$

where κ^2 is Newton's constant in five dimensions ($\kappa^2=1/M_*^3$ where M_* is the five dimensional Planck scale), and the tension of the brane has to be tuned to be

$$V=\sqrt{-\frac{6\Lambda}{\kappa^2}}. \tag{2.3}$$

For the generalizations to be presented below it turns out to be useful following [10] to redefine the coordinates such that one obtains a conformally flat metric:

$$dy = e^{-m|y|}dz. \tag{2.4}$$

In these coordinates

$$ds^2 = \omega^2(z)\,\eta_{\mu\nu}dx^\mu dx^\nu, \tag{2.5}$$

where

$$\omega^{-1}(z) = m|z| + 1, \tag{2.6}$$

if one wants to have the location of the brane to be at $z = 0$. In these coordinates it is easy to see why Eq. (2.5) solves the Einstein equations with a negative cosmological constant Λ and a brane with tension V at $z=0$.

The Einstein tensor for a metric of the form $g_{\mu\nu} = \omega^2\tilde{g}_{\mu\nu}$ in d dimensions is given by

$$G_{\mu\nu} = \tilde{G}_{\mu\nu} + (d-2)\Bigg[\tilde{\nabla}_\mu \log\omega \tilde{\nabla}_\mu \log\omega - \tilde{\nabla}_\mu \tilde{\nabla}_\nu \log\omega + \tilde{g}_{\mu\nu}\left(\tilde{\nabla}^2\log\omega + \frac{d-3}{2}(\tilde{\nabla}\log\omega)^2\right)\Bigg], \tag{2.7}$$

where the covariant derivatives $\tilde{\nabla}$ are evaluated with respect to the metric $\tilde{g}$. Since in our case the metric is conformally flat, $\tilde{g}_{\mu\nu} = \eta_{\mu\nu}$, all covariant derivatives can be replaced by ordinary derivatives, and for the same reason $\tilde{G}_{\mu\nu} = 0$. For the case $\omega^{-1}(z) = m|z| + 1$ one can easily see that the Einstein equations at an arbitrary point of the bulk ($z \neq 0$) are satisfied if $6m^2 = -\kappa^2\Lambda$, since the energy-momentum tensor in the bulk is given by $T^{bulk}_{\mu\nu} = \Lambda\,\eta_{\mu\nu}\omega^2(z)$. The singularities in the second derivatives of ω result in the additional term

$$6m\,\omega(z)\,\delta(z)\,\mathrm{diag}(1,-1,-1,-1,0) \tag{2.8}$$

in the Einstein tensor, which must be balanced by the term from the energy-momentum tensor of the brane on the right hand side of Einstein's equations,

$$\kappa^2\omega(z)V\delta(z)\,\mathrm{diag}(1,-1,-1,-1,0), \tag{2.9}$$

thus yielding $6m = \kappa^2 V$.

This solution represents two slices of anti–de Sitter space (the solution of Einstein's equations with negative cosmological constant) glued together at $z=0$. The brane represents the source necessary for fitting the two pieces together. Now it is trivial to generalize this solution to the case with two domains with different cosmological constants. It is a space with two slices of AdS spaces with different m's glued together. Thus one expects a conformally flat metric (2.5) with

$$\omega^{-1}(z) = m_1 z\,\theta(z) - m_2 z\,\theta(-z) + 1, \tag{2.10}$$

where $\theta(z)=1$ for $z>0$ and $\theta(z)=0$ for $z<0$ is the Heaviside step function. Einstein's equations in the bulk require that

$$m_1^2 = -\frac{\kappa^2\Lambda_1}{6}, \quad m_2^2 = -\frac{\kappa^2\Lambda_2}{6}, \tag{2.11}$$

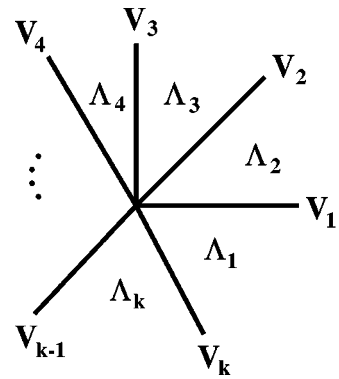

FIG. 2. The setup of semi-infinite 4-branes intersecting in a single 3-brane in $5+1$ dimensions. The brane tensions are denoted by V_i, while the bulk cosmological constants are given by Λ_i.

and the tension of the brane is determined by

$$3(m_1 + m_2) = \kappa^2 V. \tag{2.12}$$

Thus the fine-tuning condition in this case is given by

$$\kappa^2 V^2 = \frac{3}{2}(\sqrt{-\Lambda_1} + \sqrt{-\Lambda_2})^2. \tag{2.13}$$

Clearly by construction the solution we found is static. However, we included the brane as an internal source nailed at $z=0$. The dynamics of the brane is not included in this simple description, and thus it is impossible to determine if the solution is stable against small fluctuations.

The above example already suggests how one can further generalize these solutions by fitting slices of AdS space with different cosmological constants together. Indeed, Arkani-Hamed *et al.* have shown that one can find solutions corresponding to orthogonally intersecting branes. In the next section we show that one can also find solutions corresponding to the junction of semi-infinite branes intersecting in a single 3-brane. We will concentrate on the case of 4-branes embedded in $(5+1)$-dimensional spacetime, but we expect that generalizations to higher dimensions based on the algorithm described below should be straightforward.

III. GENERAL SETUP

We consider a junction of half (semi-infinite in one direction) 4-branes in $5+1$ spacetime dimensions. These branes intersect in a single 3-brane, and the tension of the ith brane is V_i. The bulk cosmological constant in the region between the ith and (i+1)st brane is taken to be Λ_i. This general setup is depicted in Fig. 2. We want to fit slices of static $(5+1)$-dimensional anti–de Sitter space together such that the resulting full solution exactly corresponds to the setup given in Fig. 2. A patch of $(5+1)$-dimensional AdS space can be described by the conformally flat metric

$$ds^2 = \omega^2(x,y)\,\eta_{\mu\nu}dx^\mu dx^\nu, \tag{3.1}$$

where $x_{0,1,2,3}$ are the coordinates of the 4 dimensional space-time, and $x_4 \equiv x$, $x_5 \equiv y$ are the coordinates in the extra dimensions. The conformal factor is given by

$$\omega^{-1}(x,y) = \vec{m}\cdot\vec{x} + 1, \tag{3.2}$$

where $\vec{x} = (x,y)$, the parameters $\vec{m} = (m_x, m_y)$ are related to (negative) the bulk cosmological constant Λ as $m_x^2 + m_y^2 = -(\kappa^2/10)\Lambda$, and κ^2 is Newton's constant in six dimensions ($\kappa^2 = 1/M_*^4$, where M_* is the fundamental Planck scale in six dimensions). Note that the requirement that the conformal factor ω be positive imposes certain inequalities on the possible values of $\vec{m}$ in each AdS patch.

In order to find the full solution to Einstein's equations we need to glue the ω's together such that

(i) the metric tensor is continuous at the location of the branes,

(ii) the discontinuity in the derivatives along the branes reproduces the energy momentum tensor of the brane with given tension V_i rotated into the appropriate direction.

It is convenient to write the conformal factor in a space composed of k AdS patches as

$$\omega^{-1} = 1 + \sum_{i=1}^{k} (\vec{m}_i\cdot\vec{x})\,\theta(\vec{n}_{i-1}\cdot\vec{x})\,\theta(-\vec{n}_i\cdot\vec{x}), \tag{3.3}$$

where $\vec{n}_i = (-\sin\varphi_i, \cos\varphi_i)$ is a unit vector in the x_4-x_5 plane normal to the ith brane, and φ_i is the angle between the brane and the coordinate axis. Clearly, one linear combination of angles is an unphysical parameter corresponding to the overall rotation of the configuration. Thus we can choose the coordinate system such that $\varphi_1 = 0$. We conclude that the ansatz (3.3) depends on k vectors $\vec{m}_i$ and $k-1$ angles, altogether $3k-1$ parameters.

We now turn to the energy-momentum tensor of the configuration of k AdS patches separated by branes. In the bulk of a given patch the energy momentum tensor is given by $T_{\mu\nu}^{bulk,i} = \Lambda_i \omega^2 \eta_{\mu\nu}$. Thus at the generic point the energy-momentum tensor can be written as

$$T_{\mu\nu}^{bulk} = \sum_{i=1}^{k} \Lambda_i \omega^2\, \theta(\vec{n}_{i-1}\cdot\vec{x})\, \theta(-\vec{n}_i\cdot\vec{x})\,\eta_{\mu\nu}. \tag{3.4}$$

The energy-momentum tensor of a 4-brane rotated by an angle φ from the horizontal direction x is given by

$$T_{\mu\nu}^{brane,i} = V_i \omega(x,y)\,\delta(\vec{n}_i\cdot\vec{x}) \begin{pmatrix} 1 &&&&& \\ & -1 &&&& \\ && -1 &&& \\ &&& -1 && \\ &&&& -\cos^2\varphi_i & -\sin\varphi_i\cos\varphi_i \\ &&&& -\sin\varphi_i\cos\varphi_i & -\sin^2\varphi_i \end{pmatrix}. \tag{3.5}$$

Thus the total stress-energy tensor in our space is given by

$$T_{\mu\nu} = T_{\mu\nu}^{bulk} + \sum_{i=1}^{k} T_{\mu\nu}^{brane,i}. \tag{3.6}$$

The Einstein tensor $G_{\mu\nu} = R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R$ for a conformally flat metric $g_{\mu\nu} = \omega^2 \eta_{\mu\nu}$ in d dimensions is given by

$$G_{\mu\nu} = (d-2)\Bigg[\partial_\mu \log\omega\, \partial_\mu \log\omega - \partial_\mu\partial_\nu \log\omega + \eta_{\mu\nu}\left(\partial^2 \log\omega + \frac{d-3}{2}(\partial\log\omega)^2\right)\Bigg]. \tag{3.7}$$

We are now ready to solve the Einstein equations. At a generic point in the bulk we find

$$\vec{m}_i^2 = -\frac{\kappa^2}{10}\Lambda_i. \tag{3.8}$$

The requirements that the singularities in the derivatives at the brane reproduce the brane tension will yield two equations at each brane[2]:

$$\Delta\vec{m}_i = \vec{m}_{i+1} - \vec{m}_i = \frac{\kappa^2 V_i}{4}\vec{n}_i. \tag{3.9}$$

To summarize, we found $3k$ equations on the $3k-1$ parameters of the ansatz (3.1), (3.3). Therefore, generically the k bulk cosmological constants Λ_i and the k brane tensions V_i need to satisfy a single (but quite complicated) fine-tuning condition. We will discuss this fine-tuning condition in more detail in the particular examples in the following section. Once this fine-tuning condition is satisfied a static solution of the form (3.1), (3.3) exists and its parameters are completely determined.

[2]This is easy to see by going to a coordinate system in which the brane under consideration is horizontal, so that the relevant parts of both the energy-momentum and the Einstein tensors are diagonal.

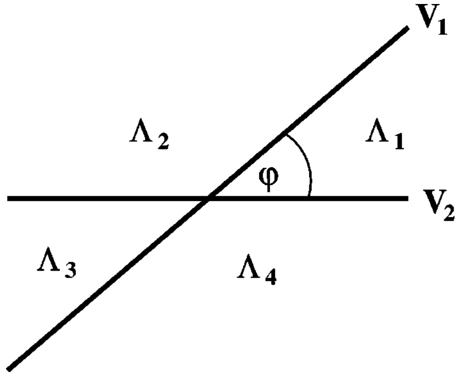

FIG. 3. Two 4-branes with tensions V_1 and V_2 intersecting at an angle φ. The four domains may have different cosmological constants.

It is worth noting that (as should have been expected) the solution satisfies the requirement that (classical) mechanical forces acting at the junction exactly balance. Indeed summing up Eqs. (3.9) we find

$$\sum_{i=1}^{k} V_i \vec{n}_i = 0, \tag{3.10}$$

which can be rewritten as

$$\sum_{i=1}^{k} \vec{V}_i = 0, \tag{3.11}$$

where $\vec{V}_i = (V_{x,i}, V_{y,i}) = (V_i \cos\varphi_i, V_i \sin\varphi_i)$. The latter equation is exactly the condition of vanishing force.

IV. EXAMPLES

Below we will apply the formalism presented in the previous section to discuss two particular examples in detail. The first example will involve two 4-branes intersecting at an angle, with different bulk cosmological constants in the four domains of spacetime, while the second example will involve three semi-infinite 4-branes intersecting in a single 3-brane (a ‘‘triple junction’’). We will give the necessary fine-tuning conditions in detail, and find the metric tensor in every sector of spacetime.

A. Four-branes intersecting at an angle

In our first example we will consider two 4-branes embedded into a $(5+1)$-dimensional spacetime. The tensions of the two branes are given by V_1 and V_2, and the four domains may have different cosmological constants. The setup is given in Fig. 3. Note that since we are considering infinite 4-branes, the condition on the forces balancing at the junction is automatically satisfied; thus at this point the angle φ between the branes is arbitrary.

Following the general formalism of the previous section, we write the metric in the form $g_{\mu\nu} = \omega^2(x,y)\,\eta_{\mu\nu}$, where

$$\begin{aligned}\omega^{-1}(x,y) = {} & f_1(x,y)\,\theta(y)\,\theta(x\cos\varphi - y\sin\varphi) \\ & + f_2(x,y)\,\theta(y)\,\theta(y\sin\varphi - x\cos\varphi) \\ & + f_3(x,y)\,\theta(-y)\,\theta(y\sin\varphi - x\cos\varphi) \\ & + f_4(x,y)\,\theta(-y)\,\theta(x\cos\varphi - y\sin\varphi) + 1,\end{aligned} \tag{4.1}$$

where 1,2,3,4 label the four domains where the value of the cosmological constant is $\Lambda_{1,2,3,4}$, and the $f_i(x,y)$ are functions linear in x,y and positive everywhere inside the domain:

$$f_i(x,y) = m_{i,x}x + m_{i,y}y. \tag{4.2}$$

The Einstein equations in the bulk result in the conditions

$$\begin{aligned} & m_{1x}^2 + m_{1y}^2 = -\lambda_1, \quad m_{2x}^2 + m_{2y}^2 = -\lambda_2, \\ & m_{3x}^2 + m_{3y}^2 = -\lambda_3, \quad m_{4x}^2 + m_{4y}^2 = -\lambda_4, \end{aligned} \tag{4.3}$$

where we have used the notation $\lambda_i = (\kappa^2/10)\Lambda_i$. The Einstein equations at the position of the branes will give the conditions

$$\begin{aligned} & m_{2y} - m_{1y} = v_1 \cos\varphi, \quad m_{1x} - m_{2x} = v_1 \sin\varphi, \\ & m_{2y} - m_{3y} = v_2, \quad m_{3x} - m_{2x} = 0, \\ & m_{3y} - m_{4y} = v_1 \cos\varphi, \quad m_{4x} - m_{3x} = v_1 \sin\varphi, \\ & m_{1y} - m_{4y} = v_2, \quad m_{4x} - m_{1x} = 0, \end{aligned} \tag{4.4}$$

where we have used the notation $v_i = (\kappa^2/4)V_i$. We can express all variables with the help of m_{1x}, m_{1y}, and φ using the discontinuity equations as

$$\begin{aligned} & m_{2x} = m_{1x} - v_1 \sin\varphi, \quad m_{2y} = m_{1y} + v_1 \cos\varphi, \\ & m_{3x} = m_{1x} - v_1 \sin\varphi, \quad m_{3y} = m_{1y} - v_2 + v_1 \cos\varphi, \\ & m_{4x} = m_{1x}, \quad m_{4y} = m_{1y} - v_2. \end{aligned} \tag{4.5}$$

Using these expressions the equations in the bulk can be rewritten as

$$\begin{aligned} & m_{1x}^2 + m_{1y}^2 = -\lambda_1, \\ & (m_{1x} - v_1 \sin\varphi)^2 + (m_{1y} + v_1 \cos\varphi)^2 = -\lambda_2, \\ & m_{1x}^2 + (m_{1y} - v_2)^2 = -\lambda_4, \\ & (m_{1x} - v_1 \sin\varphi)^2 + (m_{1y} - v_2 + v_1 \cos\varphi)^2 = -\lambda_3. \end{aligned} \tag{4.6}$$

From the equations involving λ_1 and λ_4 we learn that

$$m_{1y} = \frac{\lambda_4 - \lambda_1 + v_2^2}{2v_2}. \tag{4.7}$$

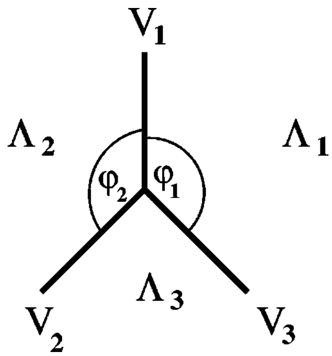

FIG. 4. Three semi-infinite 4-branes intersecting at angles φ_1 and φ_2 in a single 3-brane.

Plugging this back into the other two equations and eliminating m_{1x} we get that

$$\cos\varphi=\frac{\lambda_3-\lambda_2+\lambda_1-\lambda_4}{2v_1v_2}=\frac{2}{5}\,\frac{(\Lambda_3-\Lambda_2+\Lambda_1-\Lambda_4)}{\kappa^2V_1V_2}. \tag{4.8}$$

In particular, this relation implies that in the case when the bulk cosmological constant is isotropic ($\Lambda_1=\Lambda_2=\Lambda_3=\Lambda_4$) the only possible angle between the branes is $\pi/2$. The converse, however, is not true, and branes can be orthogonal with cosmological constants different in each sector. We now have two different expressions for m_{1x} which can be obtained from Eqs. (4.6). Equating them and substituting the values (4.7) for m_{1y} and (4.8) for $\cos\varphi$ we obtain the fine-tuning condition

$$\begin{aligned}(\lambda_1-\lambda_2+\lambda_3-\lambda_4)(\lambda_1\lambda_3-\lambda_2\lambda_4)+v_2^2(\lambda_1-\lambda_2)(\lambda_3-\lambda_4)\\+v_1^2(\lambda_1-\lambda_4)(\lambda_3-\lambda_2)-(\lambda_1+\lambda_2+\lambda_3+\lambda_4)v_1^2v_2^2\\-v_1^2v_2^2(v_1^2+v_2^2)=0.\end{aligned} \tag{4.9}$$

Note that the first three terms vanish if all cosmological constants are set to be equal, and one is left with the fine-tuning equation $-2\lambda=v^2$, implying $\kappa^2V^2=-\frac{16}{5}\Lambda$, which exactly reproduces the fine-tuning condition obtained in [10]. Thus we find that the existence of the static solution determines the angle between branes uniquely, and moreover, there is one fine-tuning condition involving the cosmological constants and the brane tensions. For simplicity in our discussion we considered a specific case of infinite branes. Had we considered semi-infinite branes with different tensions, the solution would still exist subject to a single (although more complicated) fine-tuning condition.

B. Triple junction of semi-infinite 4-branes

In our second example we will consider three semi-infinite 4-branes embedded into a $(5+1)$-dimensional space-time, intersecting in a single 3-brane. The setup is depicted in Fig. 4. Similarly to the previous example, we write the metric in the form $g_{\mu\nu}=\omega^2(x,y)\,\eta_{\mu\nu}$, where

$$\begin{aligned}\omega^{-1}(x,y)=&f_1(x,y)\,\theta(x)\,\theta(y\sin\varphi_1-x\cos\varphi_1)\\&+f_2(x,y)\,\theta(-x)\,\theta(y\sin\varphi_2+x\cos\varphi_2)\\&+f_3(x,y)\,\theta(x\cos\varphi_1-\sin\varphi_1)\\&\times\theta(-y\sin\varphi_2-x\cos\varphi_2)+1,\end{aligned} \tag{4.10}$$

where 1,2,3 label the three domains where the value of the cosmological constant is $\Lambda_{1,2,3}$, and the $f_i(x,y)$ are functions linear in x,y and positive everywhere inside the domain:

$$f_i(x,y)=m_{i,x}x+m_{i,y}y. \tag{4.11}$$

The Einstein equations in the bulk are given by

$$\begin{aligned}m_{1x}^2+m_{1y}^2&=-\lambda_1,\\m_{2x}^2+m_{2y}^2&=-\lambda_2,\\m_{3x}^2+m_{3y}^2&=-\lambda_3,\end{aligned} \tag{4.12}$$

where we have again used the notation $\lambda_i=(\kappa^2/10)\Lambda_i$. The Einstein equations at the position of the branes will give the conditions

$$\begin{aligned}&m_{2y}-m_{1y}=0,\quad m_{1x}-m_{2x}=v_1,\\&m_{2y}-m_{3y}=v_2\sin\varphi_2,\quad m_{2x}-m_{3x}=v_2\cos\varphi_2,\\&m_{1y}-m_{3y}=v_3\sin\varphi_1,\quad m_{3x}-m_{1x}=v_3\cos\varphi_1,\end{aligned} \tag{4.13}$$

where again we have used the notation $v_i=(\kappa^2/4)V_i$. It is convenient to combine the discontinuity equations to obtain the condition for the mechanical balance of the forces at the junction:

$$\begin{aligned}v_2\sin\varphi_2&=v_3\sin\varphi_1,\\v_3\cos\varphi_1+v_2\cos\varphi_2+v_1&=0.\end{aligned} \tag{4.14}$$

These equations completely determine the angles $\varphi_{1,2}$ by the relations

$$\begin{aligned}\cos\varphi_1&=\frac{v_2^2-v_3^2-v_1^2}{2v_1v_3},\\\cos\varphi_2&=\frac{v_3^2-v_1^2-v_2^2}{2v_1v_2}.\end{aligned} \tag{4.15}$$

We can now express the remaining variables with the help of m_{1x} and m_{1y} using the discontinuity equations as

$$\begin{aligned}&m_{2x}=m_{1x}-v_1,\quad m_{2y}=m_{1y},\\&m_{3x}=m_{1x}+v_3\cos\varphi_1,\quad m_{3y}=m_{1y}-v_3\sin\varphi_1.\end{aligned} \tag{4.16}$$

Using these expressions the equations in the bulk can be rewritten as

$$m_{1x}^2+m_{1y}^2=-\lambda_1,$$

$$(m_{1x}-v_1)^2+m_{1y}^2=-\lambda_2,$$

$$(m_{1x}+v_3\cos\varphi_1)^2+(m_{1y}-v_3\sin\varphi_1)^2=-\lambda_3. \quad (4.17)$$

From the first two equations m_{1x} can be expressed as

$$m_{1x}=\frac{\lambda_2-\lambda_1+v_1^2}{2v_1}. \quad (4.18)$$

Using this formula, the expression for m_{1y} from the first equation, and the values of $\cos\varphi$ from Eqs. (4.15) we again obtain a single fine-tuning relation between the tensions and the cosmological constants:

$$v_1^2v_2^2v_3^2+\lambda_1 v_2^2(v_1^2+v_3^2-v_2^2)+\lambda_2 v_3^2(v_1^2+v_2^2-v_3^2)$$
$$+\lambda_3 v_1^2(v_2^2+v_3^2-v_1^2)+v_1^2(\lambda_1-\lambda_3)(\lambda_2-\lambda_3)$$
$$+v_2^2(\lambda_2-\lambda_1)(\lambda_3-\lambda_1)+v_3^2(\lambda_1-\lambda_2)(\lambda_3-\lambda_2)=0. \quad (4.19)$$

In the case of $\Lambda_1=\Lambda_2=\Lambda_3=\Lambda$ and $V_1=V_2=V_3=V$ this relation simplifies to $v^2=-3\lambda$, that is

$$\kappa^2V^2=-\frac{24}{5}\Lambda. \quad (4.20)$$

V. COMMENTS ON THE COSMOLOGICAL CONSTANT PROBLEM

One of the biggest puzzles in particle physics is the vanishing of the cosmological constant (or why its value is at least 120 orders of magnitudes smaller than its natural size of the order M_{Pl}^4 would be). There is no symmetry that could forbid the appearance of the cosmological constant term. Thus the best hope is that there is a dynamical reason behind the vanishing of the cosmological constant. However, within four dimensional theories it is very difficult to find a dynamical adjustment mechanism that would naturally achieve this goal (for a review see [19]).

In the Randall-Sundrum scenario discussed in this paper the vanishing of the effective four-dimensional cosmological constant is a consequence of a fine-tuning between the fundamental (5 dimensional) cosmological constant and the tension of the 3-brane. Thus in the original RS scenario there is no new information gained about how the cosmological constant problem could be solved dynamically.

One can, however, imagine a more complicated scenario like one of the setups presented in this paper, where the 3-brane we live on arises as an intersection of different branes. The effective 4 dimensional cosmological constant is then a function of not only the 5 dimensional cosmological constant and the brane tensions (including the tension of the intersection brane), but also the positions (angles) of the branes. Brane configurations considered in this paper (or their most obvious generalizations) require at least one fine-tuning in addition to the adjustment of the angles to set the effective 4 dimensional cosmological constant to zero. One might hope, however, that brane configurations exist where the effective cosmological constant can be set to zero by adjusting only the orientations of the branes. In order for such a brane setup to be interesting, the values of the angles of the branes at the point where the effective cosmological constant vanishes also have to depend on the tension of the 3-brane at the intersection (a quantity which we did not consider in the models presented in this paper). This is required so that it is possible to cancel the quantum corrections to the effective 4 dimensional cosmological constant due to the fields localized on the intersection by readjusting the angles of the branes. If such a solution indeed existed, then one could translate the cosmological constant problem to a completely dynamical problem in the given brane setup—that is, why the angles of the branes are adjusted such that the effective cosmological constant vanishes. Such a dynamical formulation would be by itself a useful step towards the understanding of the cosmological constant problem. If such a brane configuration indeed existed, one could then furthermore speculate that the reason for the adjustment of the angles to a setup with zero effective cosmological constant is due to the following mechanism: initially, the positions of the branes are not adjusted and the effective 4 dimensional cosmological constant does not vanish. Therefore, the universe is inflating, thereby exerting pressure on the branes, which are slowly relaxing towards the static solution at which the effective 4 dimensional cosmological constant vanishes. Of course, for this speculative picture to hold, one would need to investigate the dynamics of the branes (beyond finding a static brane solution with the described features). In this paper we only looked at the particular static ansatz leading to the flat four-dimensional metric. Therefore, our results only indicate that the point with vanishing cosmological constant is the extremum of the potential for the angles, but not necessarily the minimum.

From a four-dimensional point of view, the angles of the branes appear as scalar fields. Thus one expects that they need to be light to potentially provide a solution to the cosmological constant problem. Even then one is confronted with the usual problem of the adjustment mechanisms for solving the cosmological constant problem. It is difficult to understand why the potential for one or a few scalars is such that at the minimum of the potential the cosmological constant vanishes. Moreover, quantum corrections seem to destroy this tuning even if it was true at the tree level. However, it might be possible that what seems to be a terrible fine-tuning in the effective 4 dimensional theory is a simple consequence of the brane dynamics in higher dimensions, with no tuning required in the full theory of branes (after all, if a solution of the desired type existed, the value of the cosmological constant in the bulk would be generic). If this fine-tuning in the effective theory is indeed the consequence of brane physics in the higher dimensional theory, one might hope that it is stable under radiative corrections, since the quantities that presumably govern the dynamics of the branes are the full quantum corrected ones.

In the setup considered here there is another possibility for improvement on the fine-tuning of the potential in the effective 4 dimensional theory. As we noted, for a given set of parameters the requirement for the existence of the static solutions with the flat four-dimensional metric completely determines the angles. Thus from the four-dimensional point of view, the potential for the angles is determined mostly by their interactions with the metric, in particular with its light KK excitations. The description of the four-dimensional effective theory in RS configurations includes a large number of arbitrarily light KK excitations. Thus it is not inconceivable that their interactions with the angles lead to a situation qualitatively different from the usual considerations.

VI. CONCLUSIONS

In this paper we have presented static solutions to Einstein's equations corresponding to branes at angles intersecting in a single 3-brane. Such solutions might be useful for building models with extra dimensions in the Randall-Sundrum scenario. The solutions are obtained by gluing patches of AdS space together, with the boundaries given by the branes. We find that a static solution of this sort is only possible if the forces from the brane tensions acting on the junction exactly balance. In addition to this condition we find other constraints that the parameters of the theory (the brane tensions, angles of the branes and the bulk cosmological constant) have to satisfy. In all the examples considered in this paper there is one fine-tuning relation which is independent of the angles of the branes and thus cannot have a dynamical origin. It would be very important to understand whether or not static brane configurations of this sort (where all tuning conditions can be satisfied by adjusting the positions of the branes) do exist and, if so, whether they can be minima of the scalar potential of the angles in the effective 4 dimensional theory.

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