Electromagnetic contribution to gravitational mass of a current-conducting channel

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The analysis of electromagnetic contribution to the total mass of a strong current channel is conducted on the basis of Einstein's equations of general theory of relativity. The current dependence of the ratio R of the contribution of the magnetic field to the contribution of the charges has a maximum. While the current is small (from the point of view of its influence on the metric) the ratio R grows with the current. Then R reaches its maximum, and in the region of a strong current up to the collapse boundary it is a decreasing function of the current. If the drift velocity is not too small, the magnetic contribution can be many orders of magnitude higher than the sum of masses of the charges.

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According to the modern point of view the Universe has a cellular and filamentary structure. Within the area accessible for observations 99.999% of its volume consists of plasma, basically of electrons and protons. Filaments of space plasma are current conducting, and according to some estimates [1] intergalactic currents can reach 10^{19}–10^{20} A.

In stationary conditions the interaction of the electromagnetic field with matter depends on the sign of a field invariant $F_{ik}F^{ik}$. (F_{ik}—4-tensor of an electromagnetic field. In the Galilean metric $F_{ik}F^{ik}=H^2-E^2$.) In the case $F_{ik}F^{ik}<0$ one can find a reference frame where the magnetic field is zero, so the field is electrostatic. In this frame of reference there is a nonzero space charge. In order to confine the charged matter, gravitational attraction must be stronger than electrostatic repulsion. For protons the ratio of gravitational forces to electrostatic ones is $Gm_i^2/e^2\sim 10^{-36}$. Therefore [2] in the case $F_{ik}F^{ik}<0$ a fraction of the uncompensated charges in space objects cannot exceed 10^{-18}. Hence, the magnetic field, originated by rotation of a charged celestial object, is very small [3].

In case $F_{ik}F^{ik}>0$ one can find a frame, where electrostatic field and the charge are zeros. However the magnetic field and the current are different from zero. Such a situation takes place, for example, if the current is caused by relative motion of subsystems of electrons and ions. In this case the velocity of relative motion (drift velocity) is different from zero, and there is no frame of reference, where both subsystems are simultaneously at rest. In the frame of reference, where the charge is zero, magnetic forces are the major ones. There are conditions, when the charges of both signs are attracted to the axis by the magnetic forces (pinch effect). In equilibrium the compression is balanced by pressure; however, there is no restriction on the relation between magnetic and gravitational forces of compression. In case $F_{ik}F^{ik}>0$ the magnetic field is not connected with rotation of a charge as a whole. Therefore generally speaking there is no reason to consider the magnetic field of celestial objects to be always weak. The magnetic field of a current channel curves the metric of space-time the same way the charges do, because energy-momentum tensors of particles and magnetic field enter Einstein equations additively.

In this paper we consider a contribution of the proper magnetic field to the total mass of a current-conducting channel. In the most simple geometry we consider a channel with a strong current as a cylindrically symmetric system consisting of two counterstreaming subsystems (ions and electrons) moving in the electromagnetic field of their collective interaction. Our approach to the problem of equilibrium of a high current filament in general relativity [4] is a traditional approach within the electromagnetic hydrodynamics of ideal charged plasma, with or without the account of gravitation. It is clear that the results of the analysis of a cylindrically symmetric current channel cannot be applied directly to the objects with central symmetry—stars, pulsars, and black holes. However the example of a current-conducting channel allows us to reveal some common features, and, in particular, to determine the area of parameters, where the electromagnetic contribution to the total mass of an object is comparable to or exceeds the contribution of particles.

Without gravitation the magnetic field of a long conductor slowly decreases with distance from the axis: $H=2I/cr$, $r_0\lesssim r\lesssim L$, L and r_0—length and radius of a current channel, I—current. The energy per unit length is

$$E_m=\frac{1}{8\pi L}\int H^2dV=\frac{1}{4}\int_0^\infty H^2(r)rdr=\left(\frac{I}{c}\right)^2\ln\left(\frac{L}{r_0}\right). \quad (1)$$

At $r_0\ll L$ the main contribution to the logarithmic integral (1) comes from $r_0\lesssim r\lesssim L$. The ratio of magnetic energy E_m to the energy of particles $E_p=N_im_ic^2$,

$$\frac{E_m}{E_p}=\frac{e^2N\beta^2}{m_ic^2}\ln\left(\frac{L}{r_0}\right), \quad (2)$$

is proportional to the number of particles per unit length of a channel N. In case of nonrelativistic drift velocity $\beta=V/c\ll 1$ the linear densities of electrons and ions are equal: $N_e=N_i=N$. One can see from Eq. (2) that if the energy of magnetic compression per one particle $e^2N\beta^2$ becomes of the order of the rest mass of an ion, it is reasonable to expect that electromagnetic contribution to the curvature of space-time exceeds total mass of the particles. In case of relativistic

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drift velocity $\beta\sim1$ the ratio (2) becomes of the order of unity at a current $I\sim10^7$ A. Respectively, for intergalactic currents of the order $I\sim10^{19}$ A magnetic energy exceeds the rest mass of the charges at $\beta\gtrsim10^{-12}$.

The formula (2) does not take into account the curvature of space-time caused by the matter. It is valid only if the current is weak from the point of view of general relativity. The condition of its applicability is [5]

$$i\equiv G^{1/2}I/c^3\ll\ln^{-1}(L/r_0). \tag{3}$$

The parameter i, introduced in Eq. (3), is a dimensionless value of the current. It reaches unity at the current $I\sim10^{25}$ A. For intergalactic currents that are now under discussion in the literature [1] parameter i is of the order 10^{-5} or even smaller. Nevertheless, it is interesting to trace the current dependence of electromagnetic contribution to the mass through the whole area of equilibrium up to the collapse boundary.

In the region of strong (from the point of view of general relativity) current

$$\ln^{-1}(L/r_0)\ll i\lesssim1 \tag{4}$$

gravitational field of a long ($L\gg r_0$) stationary current channel is located within a small (in comparison with L) distance from the axis [5]. All physical properties of the system depend only on one coordinate. A complete set of equations determining equilibrium structure of a cylindrically symmetric current-conducting filament in general relativity is written down in [4]. It is valid for arbitrary β without any limitations on the strength of a gravitation field. For nonrelativistic drift velocity $\beta\ll1$ the space charge is negligible, the nondiagonal components of the metric tensor are not essential, and the set of Einstein equations simplifies considerably. If one expresses the metric tensor via functions F_i: $g_{ik}=\mathrm{diag}(e^{2F_0},-e^{2F_1},-e^{2F_2},-e^{2F_3})$, and selects the coordinate x^1 so that F_i satisfy a Bronnikov condition [6]

$$F_0+F_2+F_3=F_1, \tag{5}$$

then the Einstein equations take the form [5]

$$\begin{aligned}
F_0''&=\frac{8\pi G}{c^4}e^{2F_1}\left(\sum_a\frac{E_a+3P_a}{2}+\frac{F_{13}F^{13}}{8\pi}\right),\\
2(F_2'F_3'+F_3'F_0'+F_0'F_2')-F_1''&=\frac{8\pi G}{c^4}e^{2F_1}\left(\sum_a\frac{E_a-P_a}{2}+\frac{F_{13}F^{13}}{8\pi}\right),\\
-F_2''&=\frac{8\pi G}{c^4}e^{2F_1}\left(\sum_a\frac{E_a-P_a}{2}-\frac{F_{13}F^{13}}{8\pi}\right),\\
-F_3''&=\frac{8\pi G}{c^4}e^{2F_1}\left(\sum_a\frac{E_a-P_a}{2}+\frac{F_{13}F^{13}}{8\pi}\right).
\end{aligned} \tag{6}$$

Energy E_a, pressure P_a, and density n_a of the type a charges are expressed via the nonzero component A_3 of vector potential of the magnetic field:

$$\begin{aligned}
E_a&=\frac{4\pi g_a}{(2\pi\hbar c)^3}\int_{m_ac^2}^{\infty}dE\,\frac{E^2\sqrt{E^2-m_a^2c^4}}{1+\exp[(E-\mu_a)/T_a]},\\
P_a&=\frac{4\pi g_a}{3(2\pi\hbar c)^3}\int_{m_ac^2}^{\infty}dE\,\frac{(E^2-m_a^2c^4)^{3/2}}{1+\exp[(E-\mu_a)/T_a]},\\
n_a&=\frac{4\pi g_a}{(2\pi\hbar c)^3}\int_{m_ac^2}^{\infty}dE\,\frac{E\sqrt{E^2-m_a^2c^4}}{1+\exp[(E-\mu_a)/T_a]},\\
-\mu_a&=B_aT_a+e_ae^{-F_0}A_3V_a/c.
\end{aligned} \tag{7}$$

Here g_a is a g-factor, μ_a is a chemical potential, A_3 is a nonzero component of vector potential of the magnetic field: $F_{13}=dA_3/dx^1$, V_a is the speed of the subsystem of type a charges in the laboratory frame of reference, B_a is a constant. In case of an electrically neutral filament the Maxwell equation

$$e^{-2F_2}(e^{2F_1}F^{13})'=4\pi e^{-F_0}\sum_a e_an_aV_a/c \tag{8}$$

is a consequence of the Einstein equations (6) and relations (7), containing equations of state for relativistic Fermi gases. In equilibrium $T_ae^{F_0}=\mathrm{const}$, V_a does not depend on x^1, and consequently

$$F_{13}=A_3'=-\frac{c}{e_aV_a}(e^{F_0}\mu_a)'. \tag{9}$$

In the theory of general relativity one can exclude a gravitation field in an arbitrary selected point of space. We consider the space-time to be flat on the axis of a channel, $x^1 \to -\infty$. This is achieved by the selection of boundary conditions:

$$F_0=0, \quad F_0'=0, \quad F_2'=1, \quad F_3=0, \quad F_3'=0, \quad x^1\to-\infty. \tag{10}$$

Actually necessary conditions of regularity at $x^1\to-\infty$ are $F_0'=F_3'=0$, $F_2'=1$. Conditions $F_0=F_3=0$ are a matter of convenience.

Total mass per unit length of a filament can be found using Tolman's formula ([7], page 425):

$$\mathfrak{M}=\frac{c^2}{4\pi GL}\int dVe^{2F_1}R_0^0=\frac{c^2}{2G}F_0'(\infty).$$

Equation (6) allows one to separate the contributions of particles and electromagnetic field into the total mass:

$$\mathfrak{M}=(2E_m+\mathfrak{E}+3\mathfrak{P})/c^2. \tag{11}$$

Here

$$\mathfrak{E}=2\pi\sum_a \int_{-\infty}^{+\infty}dx^1e^{2F_1}E_a$$

and

$$\mathfrak{P}=2\pi\sum_a \int_{-\infty}^{+\infty}dx^1e^{2F_1}P_a.$$

$\mathfrak{E}$ is the energy of particles per unit length and $\mathfrak{P}$ is the pressure, integrated over the cross section of a channel. Both—particles and magnetic field—contribute to the total mass. In the ultrarelativistic case $\mathfrak{E}=3\mathfrak{P}$ the contributions to Eq. (11) from the particle energies and pressures are equal: $\mathfrak{M}=2(E_m+\mathfrak{E})/c^2$. The electromagnetic field is equivalent to ultrarelativistic particles (photons), and for this reason E_m enters in Eq. (11) with the factor 2. Actually this is the sum of energy and tripled pressure of the magnetic field. It looks reasonable to characterize the relative contribution of the magnetic field and the particles to the total mass by the ratio

$$R=\frac{2E_m}{\mathfrak{E}+3\mathfrak{P}}. \tag{12}$$

The estimate (2) shows that the most interesting region of the parameters is the one where the energy per particle is of the order of the proton rest mass. The temperatures of the charges do not play any basic role, so we consider the case $T_a\ll\mu_a\sim m_ic^2$. This allows one to put $T_a=0$ and express the energy, the pressure, and the density of charges (7) via the chemical potentials. At $T_a=0$ the charges are degenerate relativistic Fermi gases. For $E>\mu_a$ expressions under the integrals (7) tend to zero at $T_a\to 0$, and so the interval of integration reduces to (m_ac^2,μ_a), $\mu_a>m_ac^2$. For instance,

$$n_a=\begin{cases}\dfrac{4\pi g_a}{(2\pi\hbar c)^3}\displaystyle\int_{m_ac^2}^{\mu_a}dEE\sqrt{E^2-m_a^2c^4}=\dfrac{4\pi g_a}{3(2\pi\hbar c)^3}(\mu_a^2-m_a^2c^4)^{3/2}, & \mu_a>m_ac^2,\\ 0, \quad \mu_a<m_ac^2. & \end{cases}$$

At $\mu_e\sim m_ic^2$ the electrons are ultrarelativistic. For ultrarelativistic electrons ($\mu_e\sim m_ic^2\gg m_ec^2$, $g_e=2$) at $T=0$ we have

$$n_e=\begin{cases}\dfrac{\mu_e^3}{3\pi^2(\hbar c)^3}, & \mu_e>0,\\ 0, \quad \mu_e<0. & \end{cases}$$

Other integrals in Eqs. (7) are calculated similarly and give the expressions for $\Sigma_a(E_a+P_a)$ and $\Sigma_a(E_a-3P_a)$, used in Eqs. (13) below.

Magnetic forces, acting on the counterstreaming subsystems of the charges, are different. For this reason there is an electric polarization supporting the balance of forces. However at $\beta\ll 1$ in equilibrium the space charge is insignificant in comparison with the electric current: $Q<\beta I/c \ll I/c$. Therefore at $\beta\ll 1$ the space charge can be neglected. The electrical neutrality $n_e=n_i$ allows one to express μ_i in terms of μ_e. In view of $m_i\gg m_e$ and $T_a=0$ we have

$$\mu_i^2=m_i^2c^4+\mu_e^2, \quad m_i\gg m_e.$$

It follows from Eq. (5) that $F_0''+F_2''+F_3''=F_1''$. A linear combination of Eqs. (6) allows one to exclude the second derivatives:

$$\frac{F_{13}F^{13}}{8\pi}=-\sum_a P_a+\frac{c^4}{8\pi G}e^{-2F_1}(F_2'F_3'+F_3'F_0'+F_0'F_2'),$$

and express $F_{13}F^{13}$ in terms of Σ_aP_a and $(F_2'F_3'+F_3'F_0'+F_0'F_2')$. Replacing the second Eq. (6) with Eq. (8), we get (in the rest mass frame of ions)

$$e^{-2F_2}(e^{2F_1}F^{13})'=4\pi e^{-F_0}e_en_eV_e/c,$$

$$F_0''-(F_2'F_3'+F_3'F_0'+F_0'F_2')=\frac{4\pi G}{c^4}e^{2F_1}\sum_a (E_a+P_a),$$

$$F_2''-(F_2'F_3'+F_3'F_0'+F_0'F_2')=-\frac{4\pi G}{c^4}e^{2F_1}\sum_a (E_a+P_a),$$

$$F_3''+(F_2'F_3'+F_3'F_0'+F_0'F_2')=-\frac{4\pi G}{c^4}e^{2F_1}\sum_a (E_a-3P_a).$$

By using Eq. (9), shifting the zero-points of F_2 and x^1,

$$F_2\to F_2-\frac{1}{2}\ln\left(\frac{4}{3\pi}\frac{e^2\beta^2 m_i^2 c}{\hbar^3}\right),$$

$$x^1\to x^1+\frac{1}{2}\ln\left(\frac{4}{3\pi}\frac{e^2\beta^2 m_i^2 c}{\hbar^3}\right),$$

and introducing a dimensionless chemical potential $M=\mu_e/m_i c^2$, we come to the following system of ordinary differential equations:

$$[e^{-2F_3}(e^{F_0}M)']'=\begin{cases}-e^{2F_1-F_0}M^3, & M>0,\\ 0, & M\leqslant 0,\end{cases}$$

$$F_0''-(F_2'F_3'+F_3'F_0'+F_0'F_2')=\begin{cases}Pe^{2F_1}M^3(M+\sqrt{M^2+1}), & M>0,\\ 0, & M\leqslant 0,\end{cases}$$

$$F_2''-(F_2'F_3'+F_3'F_0'+F_0'F_2')=\begin{cases}-Pe^{2F_1}M^3(M+\sqrt{M^2+1}), & M>0,\\ 0, & M\leqslant 0,\end{cases}\tag{13}$$

$$F_3''+F_2'F_3'+F_3'F_0'+F_0'F_2'=\begin{cases}\frac{3}{2}Pe^{2F_1}[M\sqrt{M^2+1}-\ln(M+\sqrt{M^2+1})], & M>0,\\ 0, & M\leqslant 0.\end{cases}$$

All boundary conditions are given on the same left side of the area of integration:

$$F_0=F_0'=F_3=F_3'=M'=0,$$

$$F_2=x^1,\quad F_2'=1,\quad M=M_0,\quad x^1\to-\infty.\tag{14}$$

$\mathfrak{E}$,$\mathfrak{P}$, and E_m in Eq. (12) are expressed via $F_i'(\infty)$:

$$\mathfrak{E}=\frac{c^4}{8G}[F_0'(\infty)-F_3'(\infty)-2F_2'(\infty)+2],$$

$$\mathfrak{P}=\frac{c^4}{8G}[F_0'(\infty)+F_3'(\infty)],$$

$$E_m=\frac{c^4}{8G}[F_0'(\infty)-F_3'(\infty)-1].$$

Total mass per unit length of a channel and the dimensionless value of the current, introduced in Eq. (3), are

$$\mathfrak{M}=\frac{c^2}{2G}F_0'(\infty),\quad i=-\frac{\sqrt{P}}{2}[e^{-2F_3}(e^{F_0}M)']_{x^1=\infty}.$$

There are two dimensionless parameters in the equations (13) and boundary conditions (14):

$$M_0\quad\text{and}\quad P=\frac{Gm_i^2}{e^2\beta^2}.$$

M_0 is the value of a chemical potential of electrons on the axis in the units of $m_i c^2$ (dimensionless Fermi energy). P is the ratio of gravitational energy to the magnetic one. As $Gm_i^2/e^2\sim 10^{-36}$, parameter P, as a rule, is small: $P\ll 1$. Only for $\beta\leqslant 10^{-18}$ parameter P exceeds unity.

Equilibrium structure of a current-conducting filament is determined by four functions $F_i, i=0,2,3$ and M. Each one of them depends on the coordinate x^1 and the two parameters P and M_0. By selecting the coordinate x^1 by the condition (5), we reduced the Einstein equations to a rather simple form. The solution does not contain fictitious singularities. Coordinate x^1 varies from $-\infty$ to $+\infty$, and overlaps the real distance from the axis from zero to infinity. In the limit of a weak curvature of space-time $x^1\to\ln r$. Integrating the system (13) at fixed values of the parameters P and M_0, we find distribution in space for all characteristics of a current channel; energy of the particles, energy stored in the magnetic field, the pressure of each subsystem, as well as the distributions of mass and current density.

Among all the solutions of equations (13) with boundary conditions (14) we have to select only those that satisfy the physical conditions of finiteness of the current and the positiveness of energy densities of the particles and magnetic field. These conditions impose limitations on the area of possible values of the parameters P and M_0. Instead of P and M_0 it is possible to use any two other physical parameters to describe an equilibrium configuration: for example, dimensionless values of the total current i (3) and the total mass per unit length $\mathfrak{m}=G\mathfrak{M}/c^2$. In the variables i and $\mathfrak{m}$ the area of equilibrium configurations is shown in Fig. 1 between solid

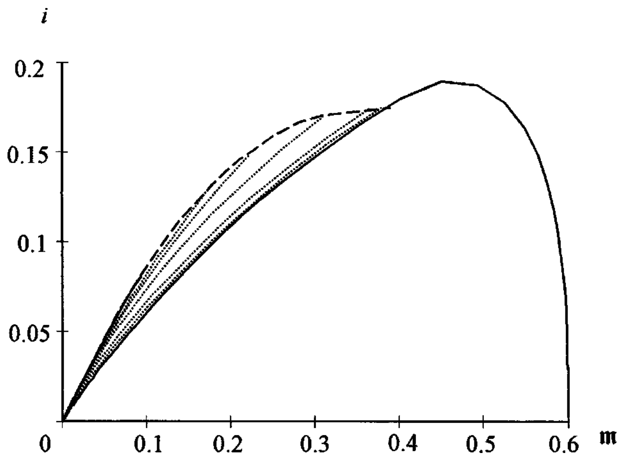

FIG. 1. Area of equilibrium configurations in variables i and $\mathfrak{m}$.

and dashed lines. The condition of positiveness of magnetic energy density is violated on the dashed curve.[1] Another boundary of equilibrium domain—solid line—is the dependence $i(\mathfrak{m})$ for the ultrarelativistic equation of state (see Fig. 3 in [5]) known as the collapse boundary. The dotted lines inside the equilibrium domain correspond to the variation of the parameter M_0 from zero up to the boundary value $M_0(P)$ at fixed values of the parameter P. The smaller P is, the stronger the energy of magnetic compression is, and the closer a dotted line $P=\text{const}$, $0<M_0<M_0(P)$ approaches the collapse boundary. It is necessary to remember, that Eqs. (13) are valid in the region of a strong current, and for this reason the equilibrium domain is limited at $i\rightarrow 0$ by the condition (4).

The dashed and solid boundaries of the equilibrium domain in Fig. 1 intersect at $i_m=0.175$, $\mathfrak{m}_m=0.385$. Equilibrium configurations exist at $\mathfrak{m}<\mathfrak{m}_m$, and at $P\rightarrow 0$ they transform into configurations found in [5] for the ultrarelativistic equation of state of protons. At $\mathfrak{m}>\mathfrak{m}_m$ Eqs. (13) do not have solutions meeting physical requirements for equilibrium configurations. However in the approximation of ultrarelativistic charges the equilibrium configurations exist up to $\mathfrak{m}=0.6$ [5]. Apparently equilibrium configuration with $i>i_m$ and $0.6>\mathfrak{m}>\mathfrak{m}_m$ cannot be realized by real charges with a nonzero rest mass.

[1]Regular solutions meeting physical requirements in the whole interval $-\infty<x^1<\infty$ exist only within the area of the parameters shown in Fig. 1. Outside this area solutions of Eqs. (13) with boundary conditions (14) are singular. Functions F_i become infinite at some point $x^1_m=x^1_m(i,\mathfrak{m})<\infty$. When the parameters $i,\mathfrak{m}$ approach the boundary of equilibrium domain from outside, the point of singularity $x^1_m\Rightarrow\infty$. So in the close vicinity of the dashed line the singular solutions are ''almost regular''—numerical instability starts far outside the channel. For singular solutions the density of magnetic energy $F_{13}F^{13}/8\pi$ changes sign within the interval $-\infty<x^1<x^1_m$. This property provides a sensitive tool to rule out singular solutions in the early stage of integration. This simplifies the numerical procedure of finding the dashed boundary of the equilibrium domain.

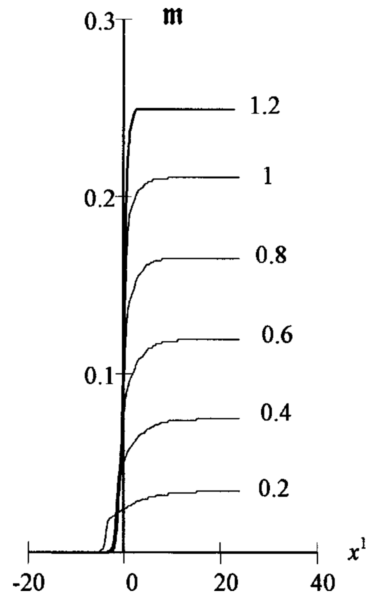

FIG. 2. Coordinate dependence of mass, located at $x<x^1$. $P=0.1846$. The figures indicate appropriate values of M_0 for each curve.

Coordinate dependences of mass per unit length, located at $x<x^1$, are shown in Fig. 2. All the curves are built for $P=0.1846$. This value corresponds to the boundary of equilibrium domain for $M_0=1.2$. The figures at each curve indicate appropriate values of the parameter M_0.

For fixed P the current i is a monotonically growing function of M_0. Figure 2 shows that the smaller M_0 is, the slower $\mathfrak{m}$ reaches its total value. In particular, at $M_0=0.2$ one can see two segments of growth of the mass. There is a fast increase of $\mathfrak{m}$ at $x^1\sim-5$ approximately up to one-half of its total value. Then $\mathfrak{m}$ continues to grow slowly with x^1 and reaches its maximum at $x^1\sim 20$. There are no particles in the field of slow growth, so this slow increase of the mass is stipulated by the magnetic field of the current. At $M_0\ll 1$ (and $i\ll 1$) the magnetic field of the current is located far outside the area occupied by the charges. Therefore the main contribution of the magnetic field to the curvature of space-time comes from the circumferential area of the current channel. With an increase of i the energy, stored in the magnetic field, grows and the curvature of space-time amplifies. As a result the magnetic field of a current becomes more and more confined to the axis of a channel. At $i\sim 1$ the magnetic field appears to be concentrated in the same area as the charges.

The ratio R (12) of magnetic contribution to the contribution of the particles as a function of current i is displayed in Fig. 3. Solid curves are functions $R(i)$ at fixed values of drift velocity. Corresponding values of P for these five curves are: $P=0.2,0.08,0.013,0.0033,0.0002$. The dotted line is the function $R_{ultra}(i)$ for the ultrarelativistic equation of state of the ions. At $i\ll 1$ $R_{ultra}(i)\sim i^{-1}$. The ratio $R(i)$ (12) deviates from $R_{ultra}(i)$ at $i\rightarrow 0$ and tends to a constant. The constant $R(0)$ is proportional to the drift velocity: $R(0)\sim P^{-1/2}=\sqrt{e^2/Gm_p^2}\beta$, $P\rightarrow 0$.

Function $R(i)$ in general is shown schematically in Fig. 4.

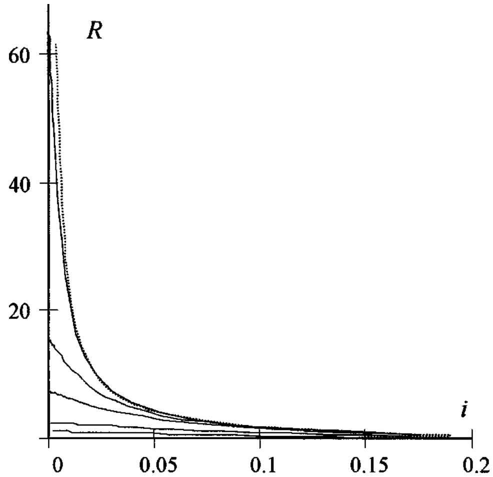

FIG. 3. Current dependence of the relative contribution of the magnetic field to the mass of a current channel. Solid lines are functions $R(i)$ found numerically for $P=0.2$, 0.08, 0.013, 0.0033, 0.0002. The dotted line is the dependence $R_{ultra}(i)$.

In the strong current limit (4) up to the boundary of the collapse the relative contribution of the magnetic field to the gravitational mass decreases with an increase of current. On the contrary, in the limit of a weak current (3), according to Eq. (2), $R(i)$ grows with current:[2]

$$R(i)=2iP^{-1/2}\ln(L/r_0), \quad i\ll\ln^{-1}(L/r_0). \tag{15}$$

[2]In regular units Eq. (15) has a form $R=2(\beta I/I_p)\ln(L/r_0)$. Here I is the current in amperes, $I_p=m_pc^3/e=31.2$ MA, m_p is the rest mass of a proton. Expressing β via I (in amperes) and N (in cm^{-1}), we get $R=13.4(I^2/N)\ln(L/r_0)$, $i\ll\ln^{-1}(L/r_0)$.

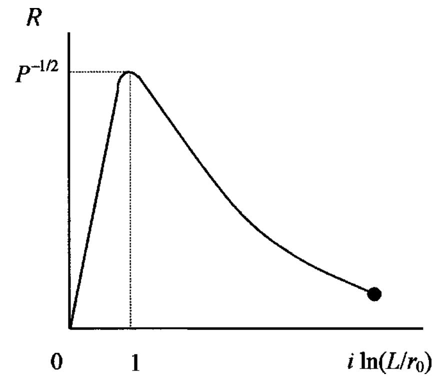

FIG. 4. General view of $R(i)$. Electromagnetic contribution grows with the current in the area (3), has a maximum at $i\sim\ln^{-1}(L/r_0)$, and decreases with the current in the region (4) up to the collapse boundary.

Therefore in the intermediate area $i\sim\ln^{-1}(L/r_0)$ the function $R(i)$ has a maximum, and this maximum is of the order of $P^{-1/2}=\beta\sqrt{e^2/Gm_p^2}\sim10^{18}\beta$.

Thus, if the drift velocity V is not extremely small ($\beta\gg10^{-18}$, i.e., $V\gg10^{-8}$ cm/sec), the electromagnetic contribution to the gravitational mass of a current channel can be many orders of magnitude higher than a simple sum of masses of the charges.

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