Semileptonic $B\to\pi$ decay in a constituent quark-meson model

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(Received 6 July 1999; published 8 December 1999)

We evaluate the form factors describing the exclusive decay $B\to\pi l\nu$ by using a constituent quark-meson model based on an effective quark-meson Lagrangian. The model allows for an expansion in the pion momenta and we consider terms up to the first order in the pion field derivatives. We compute the leading terms in the soft pion limit and consider corrections to this limit.

PACS number(s): 13.20.He, 12.39.Fe, 12.39.Hg

The investigation of the semileptonic decay $B\to\pi l\nu$ is relevant for the extraction of the Kobayashi-Maskawa matrix element V_{ub}. The analysis of this exclusive decay mode would offer a method alternative to the inclusive semileptonic B decay for the study of the $b\to u$ transition. A precise measurement of this process is one of the main aims of the future B factories.

On the theoretical side this decay process has received a lot of attention in the literature (see, for example, the review in [1,2]) since it offers an example of a heavy-to-light quark transition computable by the presently available theoretical methods.

The present paper is devoted to the study of the $B\to\pi l\nu$ decay mode in the framework of the constituent-quark-meson (CQM) model [3]. In this model the transition amplitudes are evaluated by computing diagrams in which heavy and light mesons are attached to quark loops. Moreover, the light chiral symmetry restrictions and the heavy quark spin-flavor symmetry dictated by the heavy-quark effective theory (HQET) are both implemented. The advantage of such a description is the reduced number of free parameters with respect to an effective Lagrangian at the meson level with no dynamical assumptions [4].

A short glossary, useful to go through the results reported here, is in order. We call H the field representing the low-lying heavy meson doublet $(0^-,1^-)$ [5], Z_H the heavy field renormalization constant, induced by loop effects, and Δ_H the difference between the H meson doublet mass and the mass of the constituent heavy quark. Δ_H is an adjustable parameter of the model and we restrict to $\Delta_H=0.4\pm0.1$ GeV since only this range of values allows for a good phenomenology of semileptonic weak decays (for a discussion see [3]). For the definition of the model, it is important to fix the regularization procedure allowing to calculate explicitly the quark loop integrals. We use the Schwinger proper time regularization method, assuming, as ultraviolet (UV) and infrared (IR) cutoff, $\Lambda\simeq1.25$ GeV and $\mu\simeq0.3$ GeV, respectively. Another parameter is the constituent light quark mass m that we have fixed in [3] to the value: $m=0.3$ GeV for u and d flavors.

We consider the weak current matrix element for the semileptonic $B\to\pi$ transition which is given by $(q=p-q_\pi)$

$$\langle\pi(q_\pi)|V^\mu(q)|B(p)\rangle=\left[(p+q_\pi)^\mu+\frac{m_\pi^2-m_B^2}{q^2}q^\mu\right]F_1(q^2)-\left[\frac{m_\pi^2-m_B^2}{q^2}q^\mu\right]F_0(q^2) \quad (1)$$

with $F_1(0)=F_0(0)$. The calculation of the semileptonic process proceeds through the evaluation of the diagrams in Figs. 1, 2, and 3. Figure 1 gives rise to a nonderivative coupling. Figures 2(a) and 2(b) are polar diagrams where the pion is introduced through a derivative interaction term: in Fig. 2(a) the intermediate particle is the vector meson particle belonging to the H heavy meson multiplet; in Fig. 2(b) the intermediate particle is the scalar ($J^P=0^+$) meson particle belonging to the positive parity S heavy meson multiplet (this multiplet is built similarly to H and contains also an axial vector meson $J^P=1^+$ state). The diagram in Fig. 2(b) represents only a correction to the chiral symmetry limit. To obtain the contribution of Fig. 1, an expansion of the chiral rotated light quark field χ up to the first order in π is needed [3,6]. In the case of Fig. 2 and Fig. 3 the same expansion is truncated at the zero order. The χ field, as defined in [6], is given by $\chi=\xi q$ being q the usual spinor field describing the light degrees of freedom and $\xi=e^{i\pi/f_\pi}$, with $f_\pi=130$ MeV.

The diagram in Fig. 1 produces a result proportional to the leptonic B-decay constant; its predictions are expected to

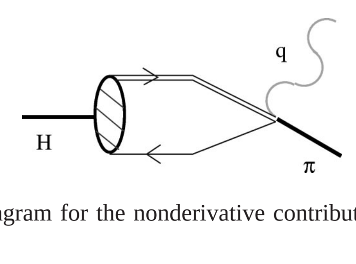

FIG. 1. Diagram for the nonderivative contribution to the form factor $B\to\pi$.

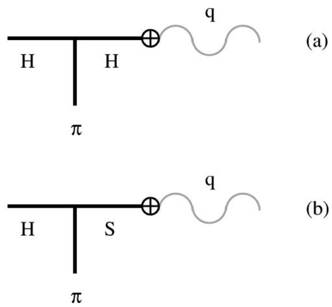

FIG. 2. Diagram for the polar contribution to the form factor $B\to\pi$.

be valid at small pion momenta, near the zero recoil point: $q_0^2 \simeq (m_B - m_\pi)^2$. One obtains, from this nonderivative (ND) coupling, the contributions

$$F_0^{\mathrm{ND}} = f_B/f_\pi, \tag{2}$$

$$F_1^{\mathrm{ND}} = f_B/2f_\pi, \tag{3}$$

where f_B is the B leptonic decay constant. The CQM-model evaluation of f_B is given in [3]. Neglecting a smooth logarithmic dependence, the heavy meson mass dependence of f_B, as predicted by the HQET, is as follows:

$$f_B = \hat{F}/\sqrt{m_B}, \tag{4}$$

where $\hat{F}$ parametrizes the leading term in the decay constant f_B (see, for example, [1]). Next we consider the polar diagrams in Fig. 2. First of all let us consider the diagram in Fig. 2(a) which gives a contribution proportional to $g\hat{F}$, where g is the $HH\pi$-strong coupling constant. One obtains the following contribution to F_1 [1,7]:

$$F_1^{\mathrm{Pol}}(q^2) = \frac{\hat{F}g}{f_\pi\sqrt{m_B}} \frac{1}{1-q^2/m_{B*}^2}, \tag{5}$$

where g is the $HH\pi$ strong coupling constant evaluated in [3] (see also [8] for an evaluation in the framework of the QCD sum-rule approach and [9] in the framework of the effective meson Lagrangian approach). The diagram in Fig. 2(b) contributes to the other form factor:

$$F_0^{\mathrm{Pol}}(q^2) = \frac{1}{m_B^2 - m_\pi^2}\left(\frac{h m_\pi \sqrt{m_B}\hat{F}^+}{f_\pi}\right)\frac{1}{1-q^2/m_{B**}^2}, \tag{6}$$

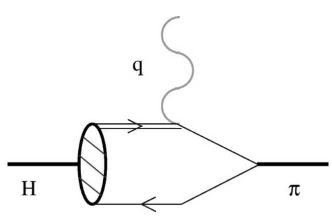

FIG. 3. Diagram for the direct contribution to the form factor $B\to\pi$.

where h is the $HS\pi$ strong coupling constant evaluated in [3], B^{**} is the 0^+ state in the S multiplet $(0^+,1^+)$, and $\hat{F}^+$ is the B^{**} leptonic decay constant analogous to $\hat{F}$. All the coupling constants appearing in these equations can be computed in the CQM. The polar results (5) and (6) should be reliable near the poles, i.e., again for q^2 large, around q_0^2, the zero recoil point. We shall discuss below a procedure to extrapolate these results to lower q^2 values. For future reference we quote here the values of expressions (5) and (6) at $q^2=0$:

$$F_1^{\mathrm{Pol}}(0) = 0.52 \pm 0.01, \tag{7}$$

$$F_0^{\mathrm{Pol}}(0) = 0.012 \pm 0.001. \tag{8}$$

They are obtained with the values $\hat{F} = 0.34 \pm 0.02$, $\hat{F}^+ = 0.24 \pm 0.03$, $g = 0.46 \pm 0.04$, $h = -0.76 \pm 0.13$ [3].

The results obtained so far are not new; they have been obtained by several groups and our contribution consists here only in the calculation, within the CQM model, of the various parameters appearing in the previous equations. The model however allows one to consider a new contribution, depicted in Fig. 3: it differs from the ND (nonderivative) term since it is derivative and from the polar term because it does not contain couplings to resonances. The current directly couples to the quark in this case rather than to the heavy meson as in the polar contribution. Both the polar and the new ''direct'' contributions can be reliably calculated only at large q^2, note however for polar terms different from Eqs. (7) and (8), the relation $F_1(0) = F_0(0)$ will be automatically satisfied by the new contributions to be described in the following. We compute them by a straightforward application of the basic rules of the CQM. As the current will transform a heavy meson into a light one in this case, we need the interaction of the pion with light quarks. We recall here the corresponding Lagrangian. The term relevant for the calculation will be the one containing an odd number of pions. The following term defines the Feynman rule we follow to insert the pion in our CQM diagram:

$$\mathcal{L} = \overline{\chi}(iD^\mu\gamma_\mu + \mathcal{A}^\mu\gamma_\mu\gamma_5)\chi - m\overline{\chi}\chi + \frac{f_\pi^2}{8}\partial_\mu\Sigma^\dagger\partial^\mu\Sigma. \tag{9}$$

Apart from the mass term, $\mathcal{L}$ is chiral invariant. Here χ is the chiral rotated light quark field quoted above, $\Sigma = \xi^2$ and π is the 3×3 matrix representing the flavor SU(3) octet of pseudoscalar mesons. Moreover, $D_\mu = \partial_\mu - i\mathcal{V}_\mu$ and

$$\mathcal{V}^\mu = \tfrac{1}{2}(\xi^\dagger\partial^\mu\xi + \xi\partial^\mu\xi^\dagger), \tag{10}$$

$$\mathcal{A}^\mu = \tfrac{1}{2}(\xi^\dagger\partial^\mu\xi - \xi\partial^\mu\xi^\dagger), \tag{11}$$

where Eq. (10) generates couplings of an even number of mesons to the $\overline{\chi}\chi$ pair, while Eq. (11) gives an odd number of π fields.

An explicit calculation of the mentioned diagram gives

$$F_1^{\mathrm{Dir}}(q^2) = \frac{2}{f_\pi}\sqrt{\frac{Z_H}{m_H}}[q_\pi(C - mA) + m_H B], \tag{12}$$

TABLE I. Form factors F_1 and F_0 at high q^2 values, near ($q^2_{\rm max}\simeq 26.4$ GeV^2) for $B\to\pi$ semileptonic decays in CQM model and comparison with other calculations. The error quoted for our result comes only from a 20% variation in the parameter controlling the evolution from large q^2 values (where the calculation is more reliable) to smaller ones.

q^2	14.9 GeV^2	17.2 GeV^2	20 GeV^2	26.4 GeV^2
CQM (this work)				
$F_1^{B\pi}$	$1.58^{+0.28}_{-0.52}$	$2.06^{+0.27}_{-0.50}$	$2.96^{+0.26}_{-0.47}$	$13.78^{+0.13}_{-0.31}$
$F_0^{B\pi}$	$0.59^{+0.10}_{-0.18}$	$0.62^{+0.08}_{-0.14}$	$0.65^{+0.05}_{-0.10}$	0.83 ± 0.01
IS [20] (quark model)				
$F_1^{B\pi}$	0.83	0.96	1.19	3.14
$F_0^{B\pi}$	0.48	0.48	0.48	0.47
GNS [11] (quark model)				
$F_1^{B\pi}$	0.82	1.05	1.45	2.31
$F_0^{B\pi}$	0.38	0.40	0.40	0.07
LNS [12] (quark model)				
$F_1^{B\pi}$	0.53	0.57	–	–
$F_0^{B\pi}$	0.69	0.76	–	–
Ball [16] (QCD light cone)				
$F_1^{B\pi}$	0.85 ± 0.15	1.1 ± 0.2	1.6	–
$F_0^{B\pi}$	0.5 ± 0.1	0.55 ± 0.15	0.7	–
Lattice [19] (UKQCD)				
$F_1^{B\pi}$	0.85 ± 0.20	1.10 ± 0.27	1.72 ± 0.50	–
$F_0^{B\pi}$	0.46 ± 0.10	0.49 ± 0.10	0.56 ± 0.12	–

$$F_0^{\rm Dir}(q^2)=\frac{2}{f_\pi}\sqrt{\frac{Z_H}{m_H}}\left[\left(1-\frac{q^2}{m_\pi^2-m_B^2}\right)q_\pi(C-mA)+m_HB\left(1+\frac{q^2}{m_\pi^2-m_B^2}\right)\right], \quad (13)$$

where

$$A=(2q_\pi)^{-1}[I_3(\Delta_H-q_\pi)-I_3(\Delta_H)], \quad (14)$$

$$B=mA-m^2Z(\Delta_H), \quad (15)$$

$$C=(2q_\pi)^{-1}[\Delta_H I_3(\Delta_H)-(\Delta_H-q_\pi)I_3(\Delta_H-q_\pi)], \quad (16)$$

and

$$Z(\Delta)=\frac{N_c}{16\pi^{3/2}}\int_{1/\Lambda^2}^{1/\mu^2}\frac{ds}{s^{1/2}}e^{-sm^2}\int_0^1 dx \times e^{s\Delta^2(x)}[1+{\rm erf}(\Delta(x)\sqrt{s})], \quad (17)$$

$$I_3(\Delta)=\frac{N_c}{16\pi^{3/2}}\int_{1/\Lambda^2}^{1/\mu^2}\frac{ds}{s^{3/2}}e^{-s(m^2-\Delta^2)}[1+{\rm erf}(\Delta\sqrt{s})], \quad (18)$$

where $q_\pi^\mu=(q_\pi,0,0,q_\pi)$ is the pion 4-momentum ($m_\pi\to 0$) and $\Delta(x)=\Delta-xq_\pi$. We observe that the soft pion limit of the previous expression brings $Z(\Delta)\to I_4(\Delta)$ defined in [3]. Notice that in Eqs. (12) and (13) the q^2 dependence arises because of $q_\pi=(m_B^2-q^2)/2m_B$ and can be computed numerically. We find

$$F_1^{\rm Dir}(q^2=0)=F_0^{\rm Dir}(q^2=0)=0.13\pm0.05. \quad (19)$$

The error in the numerical evaluations is due to the variation of Δ_H in the range of values 0.3–0.5. The contribution of the direct diagram in Fig. 3 is an appreciable 10% to 30% correction, depending on the region in q^2.

Since the three contributions to the form factors F_0 and F_1 are independent, we can sum them up, with the result

$$\hat F_j(q^2)=\frac{f_B}{(j+1)f_\pi}+F_j^{\rm Dir}(q^2)+\frac{F_j^{\rm Pol}(0)}{1-q^2/m_j^2}, \quad (20)$$

where $m_1=m_{B^*}$, $m_0=m_{B^{**}}$ and we have marked the form factors with a caret to stress that this formula does not hold in the whole q^2 interval. As a matter of fact, as discussed above, the q^2 range in which Eq. (20) is expected to approximate reliably the form factors is around the zero recoil point: $q^2\simeq q_0^2\simeq m_B^2$. This follows from the fact that the model allows for a systematic derivative expansion, whose first terms are represented by $\hat F_j$; terms of higher order in the pion derivatives, which can be important at small q^2, are suppressed for large q^2. This observation gives us a hint to extrapolate to smaller values of q^2. Writing

$$F_j(q^2)=\hat F_j(q^2)G_j(q^2) \quad (21)$$

where $j\in\{0,1\}$; this parametrization has to satisfy

$$G_j(q_0^2)=1 \quad (22)$$

as $q^2\sim q_0^2$ is the region where $\hat F_j$ are a good approximation of the form factors. Another condition that has to be satisfied is the constraint

$$\hat F_1(0)G_1(0)=\hat F_0(0)G_0(0), \quad (23)$$

which follows from $F_1(0)=F_0(0)$. It is reasonable to assume that the corrections to $G_j(q^2)\equiv 1$ arise from terms with extra pion derivatives. Therefore we put

$$G_j(q^2)=1-\frac{E_\pi}{\alpha_j\Lambda_\chi}=1-\frac{(q_\pi\cdot p)}{\alpha_j m_B\Lambda_\chi}, \quad (24)$$

where E_π is the pion energy in the B rest frame, $\Lambda_\chi=1$ GeV, and α_j are free parameters. Since Eq. (24) is equivalent to (under the assumption that $q_\pi^2\ll m_B^2$):

$$G_j(q^2)=1+(q^2-m_B^2)/2m_B\Lambda_\chi\alpha_j \quad (25)$$

condition (22) is automatically satisfied. Formula (23) implies that α_0 and α_1 are related,

$$\frac{m_B}{2\Lambda_\chi \alpha_1} = 1 - \left(1 - \frac{m_B}{2\Lambda_\chi \alpha_0} \right) \frac{\hat{F}_0(0)}{\hat{F}_1(0)}. \tag{26}$$

We could fix one of the two parameters from experimental data, were they available. For the time being, in absence of such information, we have to use some theoretical input. There exist many theoretical calculations of the $B \to \pi$ couplings; for example, quark models [10–12] predict $F_0(0) = 0.20$ to 0.50 with the exception of [13] which gives a very small value $F_0(0) = 0.09$. Chiral perturbation theory together with heavy quark effective theory gives $F_0(0) = 0.38$ [1,4], QCD sum rules give $F_0(0) = 0.25$ to 0.40 [14–16]. Finally lattice results are $F_0(0) = 0.27$ to 0.35 [17–19]. We take as an input the result of the QCD sum rules calculation of [15] that gives $F_0(0) = 0.30 \pm 0.04$, in this way we obtain $\alpha_0 = 3.6$. It is interesting to note that the rather large value of α_0 obtained by this procedure indicates that the effective parameter of the derivative expansion is not of the order of 1 GeV ($\simeq \Lambda_\chi$), but larger, which means that, in spite of the fact that this approximation should hold only at zero recoil point, it gives reasonable estimates also at lower q^2. This conclusion is corroborated by our estimate of the first correction to the leading terms of the form factors, i.e., $F_j^{\rm Dir}(q^2)$, which is appreciable, but not very large (10% to 30% of the total).

Letting the parameter α_0 vary by 20% allows to see how the parametrization affects the result. In Table I the two form factors F_1 and F_0, including the CQM correction, are given for few q^2 values near the maximum value $q^2_{\rm max} = 26.4\ \text{GeV}^2$. The error refer a variation of the values of α_0 in the range (2.9–4.3).

We have also reported results from other theoretical approaches. Let us note, as a concluding remark, that our calculation includes, differently from other approaches based on the derivative expansion, some deviations from the leading behavior expected in the soft pion limit. These extra terms, while sizable, are not such to change qualitatively the simple pole behavior predicted by the chiral effective theory.

A.D. acknowledges the support of the EC-TMR (European Community Training and Mobility of Researchers) Program on ''Hadronic Physics with High Energy Electromagnetic Probes.''

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