

Radiative corrections to the Aharonov-Bohm scattering

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We consider the scattering of relativistic electrons from a thin magnetic flux tube and perturbatively calculate the order α radiative correction to the first order Born approximation. We show also that the second order Born amplitude vanishes, and obtain a finite inclusive cross section for the one-body scattering which incorporates soft photon bremsstrahlung effects. Moreover, we determine the radiatively corrected Aharonov-Bohm potential and, in particular, verify that an induced magnetic field is generated outside of the flux tube.

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I. INTRODUCTION

Much effort has been devoted in recent years to fully understanding the consequences as well as the mathematical subtleties of the Aharonov-Bohm (AB) effect [1]. The motivations range from different areas as low dimensional condensed matter physics (e.g., in the study of anyons) [2] to cosmic string models [3] and have both experimental [4] and theoretical overtones. The conceptual aspects are no less interesting as we briefly illustrate. Basically, due to the use of different boundary conditions to achieve accordance between the exact and the perturbative calculations in the case of spin zero particles, it was necessary to include in the perturbative method a contact deltalike interaction. In 2+1 dimensions, within the Chern-Simons [5] field theory approach, it was shown that the contact interaction may be simulated by a quartic self-interaction of the matter field, with a coupling tuned to eliminate divergences and restore the conformal invariance of the tree approximation [6]. For the scattering of two spin up fermions it was verified [7] that an additional self-interaction was not needed since its role was provided by Pauli's magnetic term. However, if the fermions had antiparallel spins the effect of the magnetic interaction canceled and a divergence showed up. These problems were studied in a relativistic quantum field theory approach [8,9]. Differently from the nonrelativistic calculations previously mentioned, without any additional hypothesis, the scattering amplitudes are finite for both parallel and antiparallel spin fermions [9].

In this paper we extend these investigations by considering the AB scattering of spin-1/2 particles by an external flux tube, directly in 3+1 dimensions. We calculate the elastic cross section, in first order Born approximation in the AB potential, including radiative corrections up to order α . The second order Born approximation is also calculated and found to vanish. Infrared divergences of the elastic cross section, are eliminated from the observable scattering process by considering the "inclusive" cross section, which besides the elastic process, also incorporates soft photon bremsstrahlung contributions. The discussion of vacuum polarization effects allows us to correct the original AB poten-

tial, showing a screening of the magnetic flux and the induction of a magnetic field outside of the original flux tube.

The paper is organized as follows. In Sec. II we introduce the model, calculate the first order Born approximation including radiative corrections up to order α , calculate the second order Born approximation, and the finite inclusive cross section including soft photon bremsstrahlung. In Sec. III we obtain, up to the same order, the radiatively corrected AB potential, and the induced magnetic field outside the flux tube. Some comments are made in Sec. IV.

II. RADIATIVE CORRECTIONS TO THE SCATTERING CROSS SECTION

Our starting point is the standard Feynman gauge QED Lagrangian [10]

$$\begin{aligned} \mathcal{L} = & \bar{\psi}[i\gamma^\mu(\partial_\mu + ie a_\mu + ie A_\mu) - m]\psi - \frac{1}{4}f_{\mu\nu}f^{\mu\nu} \\ & - \frac{1}{2}(\partial_\mu a^\mu)^2, \\ f_{\mu\nu} = & \partial_\mu a_\nu - \partial_\nu a_\mu, \end{aligned} \quad (1)$$

where $a_\mu(x)$ is the radiation field and $A_\mu(x)$ is the external AB potential ($e = -|e|$ is the electron charge). The magnetic flux tube is located at the z axis, and the AB potential can be chosen as (the index t stands for "transverse" to the z axis)

$$\begin{aligned} A^\mu(x) = & [0, \vec{A}_t(\vec{x}_t), 0], \\ A^i(\vec{x}_t) = & -\frac{\Phi}{2\pi} \epsilon^{ij} \frac{x^j}{\rho^2}, \quad \rho = |\vec{x}_t| = \sqrt{x_1^2 + x_2^2} \neq 0, \end{aligned} \quad (2)$$

where Φ is the magnetic flux.¹ The AB potential satisfies the

¹We use natural units ($c = \hbar = 1$), and $\text{diag } g_{\mu\nu} = (+, -, -, -)$. Repeated Greek indices sum from 0 to 3, while repeated Latin indices from the beginning and from the middle of the alphabet sum from 1 to 3 and from 1 to 2, respectively. The antisymmetric tensor ϵ^{ij} is normalized such that $\epsilon^{12} = 1$.

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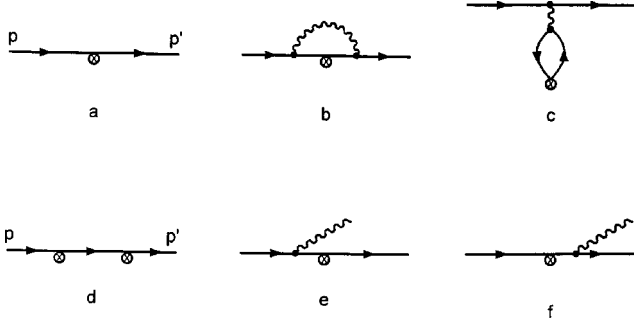


FIG. 1. Diagrams contributing to (a) first order Born approximation, (b) vertex correction, (c) vacuum polarization, (d) second order Born approximation, and (e),(f) bremsstrahlung process. The external field is represented by a cross.

Coulomb gauge condition, $\partial^i A_i = 0$, and in cylindrical polar coordinates it reads $A_\varphi(\rho) = \Phi/2\pi\rho$.

Starting from the Lagrangian (1), there are in principle two ways to proceed: one could try to construct the exact Feynman propagator in the background potential (2), and then evaluate the radiative corrections to the scattering amplitude, or one can resort to a perturbative calculation both in $\beta = e\Phi/2\pi$ and $\alpha = e^2/4\pi$. In the present paper we will restrict ourselves to the second approach. Specifically, we have to calculate the first order Born amplitude in β plus radiative corrections up to order α , hereafter designated by T_1 and add these contributions to the second order Born amplitude in β , designated by T_2 (see Fig. 1):

$$-iT = -iT_1 - iT_2,$$

$$-iT_1 = (-ie)\bar{u}(p', s') \{ \gamma_\rho + \Gamma_\rho(q) + \Pi_{\rho\nu}(q^2) G^{\nu\sigma}(q^2) \gamma_\sigma \} \times A^\rho(q) u(p, s), \quad (3)$$

$$-iT_2 = -ie^2 \bar{u}(p', s') \int \frac{d^4 k}{(2\pi)^4} \mathcal{A}(p' - k) S_F(k) \times \mathcal{A}(k - p) u(p, s). \quad (4)$$

Since the scattering process conserves the energy and the z component of the three-momentum we will consider the case of incidence perpendicular to the solenoid. Then, in the above expressions $p^\mu = (E, \vec{p}_t, 0), s$ and $p'^\mu = (E, \vec{p}'_t, 0), s'$ are the four-momenta and helicities of the incident and outgoing electrons, respectively, with $|\vec{p}| = |\vec{p}'| = p_t$; $q^\mu = (p' - p)^\mu = (0, \vec{q}_t, 0)$ is the momentum transfer from the flux tube. We choose a reference frame in which $\vec{p}_t = p_t(\cos\psi/2, -\sin\psi/2)$, $\vec{p}'_t = p_t(\cos\psi/2, \sin\psi/2)$, and $\vec{q}_t = (0, 2p_t, \sin\psi/2)$. Thus ψ is the scattering angle ($0 < \psi \leq 2\pi$).

In Eqs. (3) and (4) $\Gamma_\rho(q)$ is the renormalized one-loop vertex function, to be defined later [see Eq. (15)],

$$\Pi_{\rho\nu}(q) = -i(q^2 g_{\rho\nu} - q_\rho q_\nu) \Pi(q^2) \quad (5)$$

is the renormalized vacuum polarization tensor, $S_F(k)$ is the fermion propagator, and $G_{\nu\sigma}(q^2) = -ig_{\nu\sigma}(q^2 - \mu^2 + i\epsilon)^{-1}$

is the photon propagator, with an infrared cutoff μ . $A_\mu(q)$ is the Fourier transform of the potential (2) whose nonvanishing components are

$$A^i(q) = (2\pi)^2 \delta(q^0) \delta(q^3) \bar{A}^i(\vec{q}_t),$$

$$\bar{A}^i(\vec{q}_t) = i\Phi \epsilon^{ij} \frac{q^j}{\vec{q}_t^2 + \eta^2}. \quad (6)$$

In Eq. (6) we have introduced an infrared cutoff η to be made zero after the calculations. Integrating over k_0 and k_3 in Eq. (4), and using in the numerator of $S_F(k)$

$$\not{k} + m = 2m \sum_r u(k, r) \bar{u}(k, r), \quad (7)$$

we obtain

$$iT_2 = -i8\pi^2 \beta^2 m \delta(p'_0 - p_0) \delta(p'_3 - p_3) \times \int d^2 k \frac{\sum_r \omega(p' s'; kr) \omega(kr; ps)}{\vec{p}'_t - \vec{k}_t^2 + i\epsilon} \equiv i(2\pi)^2 \delta(p'_0 - p_0) \delta(p'_3 - p_3) \bar{T}_2, \quad (8)$$

$$\omega(p' s'; kr) = \bar{u}(p', s') \epsilon^{ij} \gamma_i \frac{(p' - k)_j}{(\vec{p}'_t - \vec{k}_t)^2 + \eta^2} u(k, r), \quad (9)$$

with an analogous formula for $\omega(kr; ps)$.

The helicity basis for positive energy solutions of the free Dirac equation is

$$u(\vec{k}, r) = \sqrt{\frac{E+m}{2m}} \begin{pmatrix} \chi(r) \\ \frac{rk_t}{E+m} \chi(r) \end{pmatrix}, \quad (10)$$

where $k^\mu = (E, \vec{k}_t, 0)$, $k_t = |\vec{k}_t|$, and $r = \pm$. The Pauli spinor $\chi(r)$ can be parametrized in spherical polar coordinates as

$$\chi(r) = \frac{\sqrt{2}}{2} \begin{pmatrix} 1 \\ e^{i\phi} \end{pmatrix} \delta_{r(+)} + \frac{\sqrt{2}}{2} \begin{pmatrix} -e^{-i\phi} \\ 1 \end{pmatrix} \delta_{r(-)}, \quad (11)$$

where ϕ is the azimuthal angle of \vec{k}_t . The normalizations are $\chi^\dagger(r) \chi(r') = \delta_{rr'}$ and $\bar{u}(\vec{k}, r) u(\vec{k}, r') = \delta_{rr'}$. The helicity basis for $u(\vec{p}, s)$ and $u(\vec{p}', s')$ are constructed in a similar way.

Using Eq. (10) and the similar formula for $u(\vec{p}', s')$, $\omega(p' s'; kr)$ is given by

$$\begin{aligned} \omega(p's';kr) &= \frac{1}{4m} \frac{1}{(\vec{p}'_t - \vec{k}_t)^2 + \eta^2} \\ &\times \left\{ 2(k_t + p_t)^2 e^{-is'[(\psi-2\phi)/4]} \sin\left(\frac{\psi-2\phi}{4}\right) \delta_{s't} \right. \\ &\left. + i(p_t - k_t)^2 (e^{-is'(\psi/2)} + e^{-is'\phi}) \delta_{s'(-r)} \right\}. \end{aligned} \quad (12)$$

The result for $\omega(kr;ps)$ can be obtained from Eq. (12) after some replacements. A straightforward calculation leads to

$$\begin{aligned} \tilde{T}_2 &= i \frac{\pi\beta^2}{2m} \delta_{ss'} e^{-is(\psi/2)} \int_0^{2\pi} d\phi \\ &\times \frac{\sin[(\psi-2\phi)/4] \sin[(\psi+2\phi)/4]}{[\sin^2[(\psi-2\phi)/4] + \nu^2][\sin^2[(\psi+2\phi)/4] + \nu^2]}, \end{aligned} \quad (13)$$

where the integration over k_t was performed via the theorem of residues, and $\nu = \eta/2p_t$. The remaining angular integral can be done as a contour integration, after introducing the variable $x = e^{i\phi}$ (this is possible provided $\eta \rightarrow 0$ only at the end):

$$\begin{aligned} \tilde{T}_2 &= i \frac{\pi^2\beta^2}{2m} \delta_{ss'} e^{-is(\psi/2)} \frac{\sin\psi}{\sin(\psi/2)[\cos\psi - 1 - 8\nu^2 - 8\nu^4]} \\ &\times \frac{1}{\sqrt{1+1/\nu^2}}, \\ \tilde{T}_2(\nu \rightarrow 0) &= 0. \end{aligned} \quad (14)$$

Equation (14) agrees with the exact result for the scattering amplitude without radiative corrections, which involves a factor of $\sin(\pi\beta)$ and thus excludes a contribution of order β^2 [1].

To evaluate T_1 , we introduce form factors $F_1(q^2)$ and $F_2(q^2)$ for the vertex function as

$$\Gamma_\rho(q) = \gamma_\rho F_1(q^2) - \frac{1}{4m} [\gamma_\rho, \gamma_\nu] q^\nu F_2(q^2) \quad (15)$$

and after some standard manipulations one arrives at

$$\begin{aligned} -iT &= -iT_1 = -i(2\pi)^2 \delta(q^0) \delta(q^3) \tilde{T}, \\ \tilde{T} &= -\pi \frac{\beta}{m} \{ [1 + F_1(q^2) - \Pi(q^2)] N(s, s') \\ &\quad + F_2(q^2) M_3(s, s') \}, \end{aligned} \quad (16)$$

where [10]

$$\begin{aligned} F_1(q^2) &= \frac{\alpha}{\pi} \left[\left(\ln \frac{\mu}{m} + 1 \right) (z \coth z - 1) \right. \\ &\quad \left. - 2 \coth z \int_0^{z/2} d\xi \xi \tanh \xi - \frac{z}{4} \tanh \frac{z}{2} \right], \\ F_2(q^2) &= \frac{\alpha}{2\pi} \frac{z}{\sinh z}, \end{aligned}$$

$$\Pi(q^2) = -\frac{\alpha}{3\pi} \left[\frac{1}{3} + 2 \left(1 - \frac{1}{2 \sinh^2(z/2)} \right) \left(\frac{z}{2} \coth \frac{z}{2} - 1 \right) \right], \quad (17)$$

with $\vec{q}_t^2 = 4m^2 \sinh^2(z/2)$ and $p \cdot p' = m^2 \cosh z$. We have also defined the matrix elements

$$\begin{aligned} M_3(s, s') &= \bar{u}(p', s') \frac{i}{2} [\gamma_1, \gamma_2] u(p, s) \\ &= s \left(1 - 2e^{-is\psi/2} \cos \frac{\psi}{2} \sin^2 \frac{\theta}{2} \right) \delta_{ss'} \\ &\quad - \frac{E}{m} \cos \frac{\psi}{2} \sin \theta \delta_{s,-s'}, \end{aligned}$$

$$\begin{aligned} N(s, s') &= i e^{ij} \frac{(p' + p)_i q_j}{\vec{q}_t^2 + \eta^2} \bar{u}(p', s') u(p, s) + M_3(s, s') \\ &= \frac{i e^{-is(\psi/2)}}{\sin(\psi/2)} \delta_{s, s'}, \end{aligned} \quad (18)$$

where we have used the helicity basis previously defined.

Using Eq. (18), the AB elastic scattering cross section is given by

$$\begin{aligned} \left(\frac{d\sigma}{d\psi} \right)_{AB} &= \frac{m^2}{2\pi p} |\tilde{T}|^2 = \left(\frac{d\sigma}{d\psi} \right)_{AB}^{(1)} \left\{ 1 + 2 \left[F_1(q^2) - \Pi(q^2) \right. \right. \\ &\quad \left. \left. + F_2(q^2) \sin^2 \frac{\psi}{2} \right] \right\} + O(\alpha^2), \end{aligned} \quad (19)$$

where

$$\left(\frac{d\sigma}{d\psi} \right)_{AB}^{(1)} = \frac{\pi\beta^2}{2p_t} \frac{1}{\sin^2(\psi/2)} \delta_{ss'} \quad (20)$$

is the scattering cross-section in the first order Born approximation [3,11,12].

As it should, the scattering cross-section up to the order α , Eq. (19), displays helicity conservation. The first order Born approximation agrees with the weak flux limit of the exact quantum-mechanical AB scattering cross section of a spin-1/2 fermion given in Refs. [3,11,12], in contrast with the results obtained for spinless particles [13].

Since the electrical form factor F_1 becomes infrared divergent as $\mu \rightarrow 0$, it is necessary to include the inelastic bremsstrahlung cross-section [see Figs. 1(e) and 1(f)] for soft

photon production in the (photon) energy range $\omega \leq \Delta E$, where $\Delta E \ll E$ is the typical energy resolution of the detector. The calculations go along the same lines as in the Coulomb scattering case [10]. Since we are only interested in the leading order terms in $\Delta E/E \ll 1$, whenever allowed we shall take the limit $E' = E - \omega \rightarrow E$ and $|\vec{p}'| \rightarrow |\vec{p}|$. With these approximations the scattering cross section factorizes in the form ($v = p_t/E$)

$$\begin{aligned} \left[\frac{d\sigma}{d\psi}(\Delta E) \right]_{SB} &= \left(\frac{d\sigma}{d\psi} \right)_{AB}^{(1)} \frac{2\alpha}{\pi} \left\{ (z \coth z - 1) \ln \frac{2\Delta E}{\mu} \right. \\ &+ \frac{1}{2v} \ln \frac{1+v}{1-v} - \frac{1}{2} \cosh z \frac{1-v^2}{v \sin(\psi/2)} \\ &\times \int_{\cos(\psi/2)}^1 d\xi \frac{1}{(1-v^2\xi^2)\sqrt{\xi^2 - \cos^2(\psi/2)}} \\ &\left. \times \ln \frac{1+v\xi}{1-v\xi} \right\}. \end{aligned} \quad (21)$$

The finite inclusive AB cross section is defined by adding Eqs. (19) and (21):

$$\left[\frac{d\sigma}{d\psi}(\Delta E) \right] = \left(\frac{d\sigma}{d\psi} \right)_{AB} + \left[\frac{d\sigma}{d\psi}(\Delta E) \right]_{SB} = \left(\frac{d\sigma}{d\psi} \right)_{AB}^{(1)} (1 - \delta_R), \quad (22)$$

where δ_R is the contribution due to virtual plus real (soft) photon emission, given by

$$\begin{aligned} \delta_R &= \frac{2\alpha}{\pi} \left\{ (1 - z \coth z) \left(1 + \ln \frac{2\Delta E}{m} \right) - \frac{z}{2} \coth z \ln(1 - v^2) \right. \\ &- \frac{1}{9} + \frac{2}{3} \left(1 - \frac{1}{2 \sinh^2(z/2)} \right) \left(1 - \frac{z}{2} \coth \frac{z}{2} \right) \\ &- \frac{1}{2} \frac{z}{\sinh z} \sin^2 \frac{\psi}{2} - \frac{1}{2v} \ln \frac{1+v}{1-v} + \frac{z}{4} \tanh \frac{z}{2} \\ &+ \frac{1-v^2}{2v} \frac{\cosh z}{\sin(\psi/2)} \int_{\cos(\psi/2)}^1 d\xi \frac{1}{\sqrt{\xi^2 - \cos^2(\psi/2)}} \\ &\left. \times \left[\frac{\ln(1+v\xi)}{1-v\xi} - \frac{\ln(1-v\xi)}{1+v\xi} \right] \right\}. \end{aligned} \quad (23)$$

As shown in Fig. 2, we have plotted δ_R for three values of E . In that graph, contributions near $\psi=0$ have been omitted since the AB amplitude is not well defined there [14].

III. RADIATIVE CORRECTIONS TO THE AHARONOV-BOHM POTENTIAL

In this section we shall compute vacuum polarization effects (up to the order α) on the renormalized AB potential. In particular, we will show that, as a result of radiative corrections, a magnetic field is induced outside the flux tube.

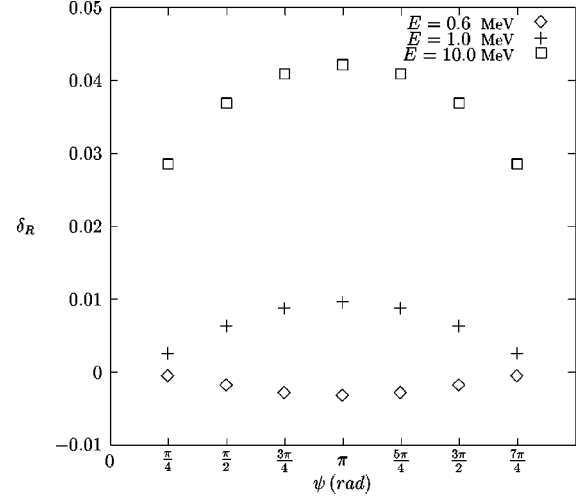


FIG. 2. δ_R for $m=0.5$ MeV and $\Delta E=0.1E$.

Any external electromagnetic potential A_μ is modified due to vacuum polarization effects. If this is properly taken into account the effective AB potential (\mathcal{A}_μ) becomes

$$\mathcal{A}_\mu(q) = \left(g_{\mu\lambda} - i \frac{\Pi_{\mu\lambda}}{q^2} \right) A^\lambda(q) = [1 - \Pi(q^2)] A_\mu(q). \quad (24)$$

As the origin is a highly singular point in the AB potential (2), it is convenient to introduce a regularization which distributes the magnetic flux on the surface of a thin cylindrical shell surrounding the origin, similarly as done in Ref. [11] [$B_3(q) = (2\pi)^2 \delta(q^0) \delta(q^3) \tilde{B}$]:

$$A^i(x_j) = - \frac{\Phi}{2\pi} \epsilon^{ij} \frac{x^j}{\rho^2} \theta(\rho - R) \Rightarrow \tilde{A}^i(q_j) = i\Phi \epsilon^{ij} \frac{q^j}{q_i^2} J_0(Rq_i), \quad (25)$$

$$B_3(x_j) = \frac{\Phi}{2\pi R} \delta(\rho - R) \Rightarrow \tilde{B}(q_j) = \Phi J_0(Rq_i), \quad (26)$$

where R is the radius of the cylindrical shell. As $R \rightarrow 0$ we recover the AB potential of Eq. (2).

From Eqs. (6), (24), and (25) the potential induced by the vacuum polarization reads

$$\begin{aligned} A_i^{\text{VP}}(x_j) &= \left(\frac{\Phi \alpha}{4\pi^3} \epsilon^{ij} \partial_j \right) \int_0^1 dz \frac{z^2}{1-z^2} \left(1 - \frac{1}{3} z^2 \right) \\ &\times \int d^2q e^{i\vec{q}_t \cdot \vec{x}_t} \frac{1}{4m^2/(1-z^2) + q_t^2} J_0(Rq_t), \end{aligned} \quad (27)$$

with $A_0^{\text{VP}} = A_3^{\text{VP}} = 0$. In Eq. (27) we used the integral representation [10]

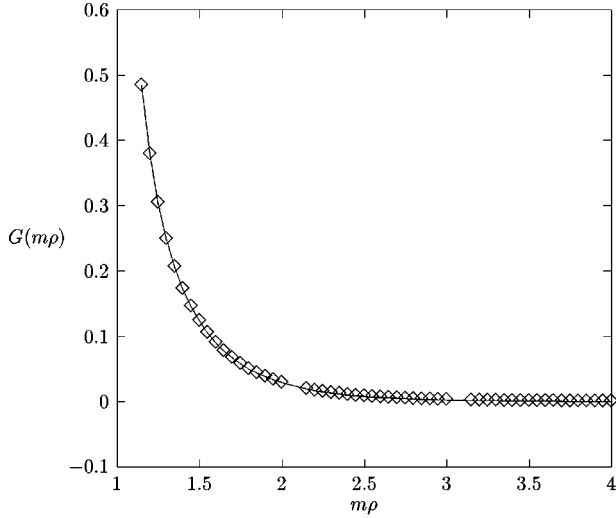


Figure 3

 FIG. 3. The function $G(m\rho)$, plotted for $m\rho > 1$ and $mR = 1$.

$$\Pi(q^2) = \frac{\alpha}{\pi} \int_0^1 dz \frac{z^2}{1-z^2} \left(1 - \frac{1}{3}z^2\right) \frac{q^2}{4m^2/(1-z^2) - q^2}. \quad (28)$$

The integral over d^2q can be done in polar coordinates [15], with the result

$$A_i^{\text{vp}}(x_j) = -\frac{\Phi}{2\pi} \epsilon^{ij} \frac{x^j}{\rho^2} \left[\frac{2\alpha}{3\pi} G(m\rho) \right],$$

$$G(m\rho) = m\rho \int_1^\infty \frac{dz}{z^3} \sqrt{2z^2+1} I_0(2mRz) K_1(2m\rho z). \quad (29)$$

The results may be translated in terms of an effective flux $\Phi(\rho)$. In cylindrical coordinates, we obtain from Eqs. (29) and (24) the result

$$\begin{aligned} \mathcal{A}_\varphi(\rho) &= \frac{1}{2\pi\rho} \Phi(\rho), \\ \Phi(\rho) &= \Phi \left[\theta(\rho-R) + \frac{2\alpha}{3\pi} G(m\rho) \right]. \end{aligned} \quad (30)$$

The integral in $G(m\rho)$ converges for $\rho > R \neq 0$, and is logarithmic divergent for $\rho = R$. A numerical solution is feasible, with the result drawn in Fig. 3. The effective flux picture resembles that of effective charge in QED. Indeed, from the figure we see that the vacuum fluctuations lead to a screening of the flux, as a calculation of the induced magnetic field shows more clearly. The total induced flux at large distance is given by the limit $\rho \rightarrow \infty$ of the formula

$$\Delta\Phi(\rho) = \int_{C(\rho)} d\varphi \rho A_\varphi^{\text{vp}}(\rho) = \frac{2}{3} \frac{\Phi\alpha}{\pi} G(m\rho), \quad (31)$$

and it is found to vanish due to the asymptotic behavior of $G(m\rho)$ for large distances.

As $R \rightarrow 0$, it can be shown that $A_i^{\text{vp}}(x_j)$ has an exact solution in terms of Meijer's G function [15],

$$\begin{aligned} A_i^{\text{vp}}(x_j) &= -\frac{\Phi}{2\pi} \epsilon^{ij} \frac{x^j}{\rho^2} \left[\frac{\alpha}{4\sqrt{\pi}} (m\rho)^5 \right. \\ &\quad \left. \times G_{13}^{30} \left((m\rho)^2 \left| \begin{matrix} 0 \\ -1/2 & -1/2 & -5/2 \end{matrix} \right. \right) \right]. \end{aligned} \quad (32)$$

The asymptotic limits for $m\rho \ll 1$ and $m\rho \gg 1$ can be calculated from Eq. (32) ($R=0$):

$$\Phi(\rho) \approx \Phi \left[1 - \frac{2\alpha}{3\pi} \ln(m\rho) \right] \quad (m\rho \ll 1), \quad (33)$$

$$\Phi(\rho) \approx \Phi \left[1 + \frac{\alpha}{4m\rho} e^{-2m\rho} \right] \quad (m\rho \gg 1). \quad (34)$$

To calculate the induced magnetic field, we use $\vec{B} = \nabla \times \vec{A}$ for $\rho > R$, and suppose that the general expression for $\vec{B} = B_3 \hat{z}$ is

$$B_3(\rho) = \frac{\Phi}{2\pi R} \delta(\rho-R) + B_3^{\text{vp}}(\rho), \quad (35)$$

$$B_3^{\text{vp}}(\rho) = \Phi \gamma(R) \delta(\rho-R) - \frac{2\Phi\alpha}{3\pi^2} m^2 F(m\rho), \quad (36)$$

$$F(m\rho) = \int_1^\infty \frac{dz}{z^2} \sqrt{z^2-1} (2z^2+1) I_0(2mRz) K_0(2m\rho z).$$

The first term on the right-hand side of Eq. (35) is the original magnetic field in the cylindrical shell (B_3), whereas the vacuum polarization part B_3^{vp} contains a highly concentrated contribution on the shell of the same relative sign as B_3 , together with an external contribution in the opposite direction. In a semiclassical picture, this profile is expected since the lines of the induced magnetic field are closed. To determine the function $\gamma(R)$ we use that the total induced flux is zero. This gives $\gamma(R) = (\alpha/3\pi^2 R) G(mR)$. It is possible to use Eq. (32) to deduce the asymptotic limits for $B_3^{\text{vp}}(\rho)$ in the case $R \rightarrow 0$ and $\rho \neq 0$ (at $\rho = 0$ an additional divergent contribution has to be taken into account)

$$B_3^{\text{vp}}(\rho) \approx -\frac{\Phi\alpha}{3\pi^2} \frac{1}{\rho^2} \quad (m\rho \ll 1), \quad (37)$$

$$B_3^{\text{vp}}(\rho) \approx -\frac{\Phi\alpha}{4\pi} \frac{e^{-2m\rho}}{\rho^2} \quad (m\rho \gg 1). \quad (38)$$

IV. CONCLUSIONS

We have calculated the order α radiative corrections, to the scattering cross section of an electron by an external AB

potential, in first Born approximation. The second order Born approximation was also calculated and found to vanish, in agreement with the weak flux limit of the exact quantum mechanical calculation (without radiative corrections). The inclusive cross section, which in addition to the elastic scattering, also includes the soft photon bremsstrahlung was calculated. The “semiclassical,” radiatively corrected AB potential due vacuum polarization currents and using a finite radius flux tube model was also calculated up to the same order. It was shown that an induced magnetic field opposite to the original flux tube is induced outside, which partially screens the original field. We remark that our perturbative results are expected to hold for $|e\Phi/2\pi| \ll 1$.

We would like to mention some literature which has contact with our work. Using the solutions of Dirac equation in the external AB potential, in Ref. [16] the processes of pair production and bremsstrahlung were analyzed. In Ref. [17] the induced vacuum current was obtained. In these works the external AB potential was treated in an exact way, but radiative corrections due to virtual photons were not included. In our approach instead, the AB potential was perturbatively treated but radiative corrections were also included. It must

be observed that our induced external magnetic field can alternatively be obtained from the induced vacuum current of Ref. [17], in the weak flux limit, through the use of the Ampere law. It must also be observed that the soft photon bremsstrahlung cross section, that is part of our inclusive cross section may be obtained from the bremsstrahlung cross section of Ref. [16] in the weak flux and soft photon limits. Our radiative corrections due to virtual photons instead, namely, the contributions of the graphs in Figs. 1(b) and 1(c), are new and not included in these previous papers. A more complete calculation of the AB cross section, in which radiative corrections are perturbatively considered as corrections to the exact treatment of the AB scattering has yet to be done.

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