$B \to X_s \tau^+ \tau^-$ in a CP spontaneously broken two Higgs doublet model

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The differential branching ratio, forward-backward asymmetry, CP asymmetry and lepton polarization for a B meson to decay to strange hadronic final states, and a $\tau^+\tau^-$ pair in a CP spontaneously broken two Higgs doublet model are computed. It is shown that contributions of neutral Higgs bosons to the decay are quite significant when $\tan\beta$ is large. And it is proposed to measure the direct CP asymmetry in back-forward asymmetry.

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I. INTRODUCTION

The origin of CP violation has been one of the main issues in high energy physics since the discovery of CP violation in the K_0-$\overline{K}_0$ system in 1964 [1]. The measurements of electric dipole moments of the neutron and electron and the matter-antimatter asymmetry in the Universe indicate that one needs new sources of CP violation in addition to the CP violation coming from the Cabibbo-Koboyashi-Maskawa (CKM) matrix, which has been one of the motivations to search for new theoretical models beyond the standard model (SM).

The minimal extension of the SM is to enlarge the Higgs sectors of the SM [2]. It has been shown that if one adheres to natural flavor conservation (NFC) in the Higgs sector, then a minimum of three Higgs doublets are necessary in order to have spontaneous CP violation [3]. However, the constraint can be avoided if one allows the real and imaginary parts of $\phi_1^+\phi_2$ to have different self-couplings [see below, Eq. (2)]. Then, one can construct a CP spontaneously broken two Higgs doublet model (2HDM), which is the minimal and the most "economical" one[1] among the extensions of the SM that provide new sources of CP violation.

Flavor changing neutral current (FCNC) transitions $B \to X_s\gamma$ and $B \to X_s l^+ l^-$ provide testing grounds for the SM at the loop level and sensitivity to new physics. Rare decays $B \to X_s l^+ l^- (l=e,\mu)$ have been extensively investigated in both SM and the beyond [5,6]. In these processes contributions from exchanging neutral Higgs bosons (NHB) can be safely neglected because of smallness of m_l/m_W $(l=e,\mu)$. The inclusive decay $B \to X_s\tau^+\tau^-$ has also been investigated in the SM, the model II 2HDM and SUSY models with and without including the contributions of NHB [7–10]. In this note we investigate the inclusive decay $B \to X_s\tau^+\tau^-$ with emphasis on CP violation effect in a CP spontaneously broken 2HDM, which we shall call model IV hereafter. We consider the model IV in which the up-type quarks get masses from Yukawa couplings to the one Higgs doublet H_2 and down-type quarks and leptons get masses from Yukawa couplings to the another Higgs doublet H_1. The Higgs boson couplings to down-type quarks and leptons depend on only the CP violated phase ξ which comes from the expectation value of Higgs field and the ratio $tg\beta = v_2/v_1$ in the large $tg\beta$ limit (see next section), which are the free parameters in the model. Because the couplings of the charged Higgs boson to fermions in model IV are the same as those in model II, the constraints on $\tan\beta$ due to effects arising from the charged Higgs bosons are the same as those in model II. Constraints on $\tan\beta$ from K-$\overline{K}$ and B-$\overline{B}$ mixing, $\Gamma(b \to s\gamma)$,$\Gamma(b \to c\tau\overline{\nu}_\tau)$ and R_b have been given [11]

$$0.7 \leqslant \tan\beta \leqslant 0.52\left(\frac{m_{H^\pm}}{1\ \ \text{GeV}}\right) \tag{1}$$

(and the lower limit $m_{H^\pm} \geqslant 200$ GeV has also been given in Ref. [11]). It is obvious that the contributions from exchanging neutral Higgs bosons now is enhanced roughly by a factor of $\tan^2\beta$ and can compete with those from exchanging γ, Z when $\tan\beta$ is large enough. Because the CP violation effects in $B \to X_s\tau^+\tau^-$ come from the couplings of NHB to leptons and quarks, we shall be interested in the large $\tan\beta$ limit in this note. The constraints on ξ can be obtained from the electric dipole moments (EDM) of the neutron and electron, which will be analyzed in the next section.

[1]Comparing the model III 2HDM [4], in which CP is explicitly violated, the CP spontaneously broken 2HDM has only two new parameters besides the masses of the Higgs bosons in the large $\tan\beta$ limit (see below). In this sense it is the most "economical."

II. MODEL DESCRIPTION

Consider two complex $y=1$, $SU(2)_w$ doublet scalar fields, ϕ_1 and ϕ_2. The Higgs potential which spontaneously breaks $SU(2)\times U(1)$ down to $U(1)_{EM}$ can be written in the following form [12]:

$$V(\phi_1,\phi_2)=\sum_{i=1,2}[m_i^2\phi_i^+\phi_i+\lambda_i(\phi_i^+\phi_i)^2]+\lambda_3[(\phi_1^+\phi_1)(\phi_2^+\phi_2)]+\lambda_4[(\phi_1^+\phi_2)(\phi_2^+\phi_1)]+\lambda_5[\mathrm{Re}(\phi_1^+\phi_2)]^2+\lambda_6[\mathrm{Im}(\phi_1^+\phi_2)]^2. \quad (2)$$

Hermiticity requires that all parameters are real so that the potential is CP conservative. It is easy to see that the minimum of the potential is at

$$\langle\phi_1\rangle=\begin{pmatrix}0\\ v_1\end{pmatrix},\quad \langle\phi_2\rangle=\begin{pmatrix}0\\ v_2e^{i\xi}\end{pmatrix}, \quad (3)$$

thus breaking SU(2)$\times$U(1) down to U(1)$_{EM}$ and simultaneously breaking CP, as desired. It should be noticed that only for $\lambda_5\neq\pm\lambda_6$, the phase ξ cannot be rotated away as usual, which breaks the CP conservation.

In the following we will work out the mass spectrum of the Higgs boson. For charged components, the mass-squared matrix for negative states is

$$\lambda_4\begin{pmatrix}v_1^2 & -v_1v_2e^{i\xi}\\ -v_1v_2e^{-i\xi} & v_2^2\end{pmatrix}, \quad (4)$$

Diagonalizing the mass-squared matrix results in one zero-mass Goldstone state:

$$G^-=e^{i\xi}\sin\beta\phi_2^-+\cos\beta\phi_1^-, \quad (5)$$

and one massive charged Higgs boson state:

$$H^-=e^{i\xi}\cos\beta\phi_2^--\sin\beta\phi_1^-, \quad (6)$$

$$m_{H^-}=|\lambda_4|(v_1^2+v_2^2), \quad (7)$$

where $\tan\beta=v_2/v_1$. Correspondingly we could also get the positive states G^+ and H^+ with the same masses zero and $|\lambda_4|(v_1^2+v_2^2)$, respectively.

For neutral Higgs components, because CP conservation is breaking, the mass-squared matrix is 4×4, which could not be simply separated into two 2×2 matrices as usual. However, in the case of large $\tan\beta$ which is what we are interested in, the neutral parts can be written separately as two 2×2 matrices and one of them is

$$v_2^2\begin{pmatrix}\frac{\lambda_5+\lambda_6+(\lambda_6-\lambda_5)\cos(2\xi)}{2} & -\frac{(\lambda_6-\lambda_5)\sin(2\xi)}{2}\\ -\frac{(\lambda_6-\lambda_5)\sin(2\xi)}{2} & \frac{\lambda_5+\lambda_6+(\lambda_5-\lambda_6)\cos(2\xi)}{2}\end{pmatrix}. \quad (8)$$

Diagonalizing the Higgs boson mass-squared matrix results in two eigenstates:

$$\begin{pmatrix}H_1^0\\ H_2^0\end{pmatrix}=\sqrt{2}\begin{pmatrix}c_\xi & -s_\xi\\ s_\xi & c_\xi\end{pmatrix}\begin{pmatrix}\mathrm{Im}\,\phi_1^0\\ \mathrm{Re}\,\phi_1^0\end{pmatrix} \quad (9)$$

with masses

$$m_{H_1^0}^2=\lambda_5v_2^2,$$

$$m_{H_2^0}^2=\lambda_6v_2^2, \quad (10)$$

where $c_\xi=\cos\xi$ and $s_\xi=\sin\xi$. The diagonalizing of the 4×4 neutral Higgs mass-squared matrix has been analytically carried out under some assumptions in Ref. [13] and the results reduce to Eqs. (9) and (10) in the case of large $\tan\beta$.

The other 2×2 matrix can be similarly dealt with. Because the third physical neutral Higgs boson and neutral Goldstone do not couple to down-type quarks and leptons in the large $\tan\beta$ limit in which we are interested, we do not show the explicit results.

Now, we turn to the discussion of the Higgs-fermion-fermion couplings. After completing the transformation from the weak states to the mass states, the couplings of neutral Higgs to fermions which are relevant to our analysis are

$$H_1^0\overline{f}f:\ \frac{igm_f}{2m_w\cos\beta}(s_\xi-ic_\xi\gamma_5),$$

$$H_2^0\overline{f}f:\ -\frac{igm_f}{2m_w\cos\beta}(c_\xi+is_\xi\gamma_5), \quad (11)$$

where f represents down-type quarks and leptons. And the couplings of the charged Higgs bosons to fermions are the same as those in the CP-conservative 2HDM (model II, for examples see Ref. [14]). This is in contrast with the model III in which the couplings of the charged Higgs bosons to fermions are quite different from model II. It is easy to see from Eqs. (11) that the contribution coming from exchanging NHB is proportional to $\sqrt{2}G_Fs_\xi c_\xi m_f^2/\cos^2\beta$, so that the constraints due to EDM translate into the constraints on $\sin2\xi\tan^2\beta$ ($1/\cos\beta\sim\tan\beta$ in the large $\tan\beta$ limit). According to the analysis in Ref. [15], we have the constraint

$$\sqrt{|\sin2\xi|}\tan\beta<50 \quad (12)$$

from the neutron EDM. And the constraint from the electron EDM is not stronger than Eq. (12). It is obvious from Eq. (12) that there is a constraint on ξ only if $\tan\beta > 50$ and the stringent constraint on $\tan\beta$ comes out and is $\tan\beta < 50$ when $\xi = \pi/4$.

III. FORMULA FOR $B \to X_s \tau^+ \tau^-$

Inclusive decay rates of heavy hadrons can be calculated in heavy quark effective theory (HQET) [16] and it has been shown that the leading terms in $1/m_Q$ expansion turn out to be the decay of a free (heavy) quark and corrections stem from the order $1/m_Q^2$ [17]. In what follows we shall calculate the leading term. The transition rate for $b \to s \tau^+ \tau^-$ can be computed in the framework of the QCD corrected effective weak Hamiltonian, obtained by integrating out the top quark, Higgs bosons, and $W^{\pm}, Z$ bosons

$$H_{\mathrm{eff}} = \frac{4G_F}{\sqrt{2}} V_{tb} V_{ts}^* \left(\sum_{i=1}^{10} C_i(\mu) O_i(\mu) + \sum_{i=1}^{10} C_{Q_i}(\mu) Q_i(\mu) \right), \tag{13}$$

where $O_i (i = 1, \ldots, 10)$ is the same as that given in Ref. [5], Q_i's come from exchanging the neutral Higgs bosons and are defined in Ref. [9]. The explicit expressions of the operators governing $B \to X_s \tau^+ \tau^-$ are given as follows:

$$\begin{aligned}
O_7 &= (e/16\pi^2) m_b (\overline{s}_{L\alpha} \sigma^{\mu\nu} b_{R\alpha}) F_{\mu\nu}, \\
O_8 &= (e/16\pi^2) (\overline{s}_{L\alpha} \gamma^{\mu} b_{L\alpha}) \overline{\tau} \gamma_{\mu} \tau, \\
O_9 &= (e/16\pi^2) (\overline{s}_{L\alpha} \gamma^{\mu} b_{L\alpha}) \overline{\tau} \gamma_{\mu} \gamma_5 \tau, \\
Q_1 &= (e^2/16\pi^2) (\overline{s}_{L\alpha} b_{R\alpha}) (\overline{\tau} \tau), \\
Q_2 &= (e^2/16\pi^2) (\overline{s}_{L\alpha} b_{R\alpha}) (\overline{\tau} \gamma_5 \tau).
\end{aligned} \tag{14}$$

At the renormalization point $\mu = m_W$ the coefficients C_i's in the effective Hamiltonian have been given in Ref. [5] and C_{Q_i}'s are [neglecting the $O(tg\beta)$ term]

$$\begin{aligned}
C_{Q_1}(m_W) &= \frac{m_b m_\tau tg^2 \beta x_t}{2 \sin^2 \theta_W} \left\{ \sum_{i=H_1,H_2} \frac{A_i}{m_i^2} (f_1 B_i + f_2 E_i) \right\}, \\
C_{Q_2}(m_W) &= \frac{m_b m_\tau tg^2 \beta x_t}{2 \sin^2 \theta_W} \left\{ \sum_{i=H_1,H_2} \frac{D_i}{m_i^2} (f_1 B_i + f_2 E_i) \right\}, \\
C_{Q_3}(m_W) &= \frac{m_b e^2}{m_\tau g_s^2} [C_{Q_1}(m_W) + C_{Q_2}(m_W)], \\
C_{Q_4}(m_W) &= \frac{m_b e^2}{m_\tau g_s^2} [C_{Q_1}(m_W) - C_{Q_2}(m_W)], \\
C_{Q_i}(m_W) &= 0, \quad i = 5, \ldots, 10,
\end{aligned} \tag{15}$$

where

$$\begin{aligned}
A_{H_1} &= -s_\xi, \quad D_{H_1} = i c_\xi, \\
A_{H_2} &= c_\xi, \quad D_{H_2} = i s_\xi, \\
B_{H_1} &= \frac{i c_\xi - s_\xi}{2}, \quad B_{H_2} = \frac{c_\xi + i s_\xi}{2}, \\
f_1 &= \frac{x_t \ln x_t}{x_t - 1} - \frac{x_{H^\pm} \ln x_{H^\pm} - x_t \ln x_t}{x_{H^\pm} - x_t}, \\
f_2 &= \frac{x_t \ln x_t}{(x_t - 1)(x_{H^\pm} - 1)} - \frac{x_{H^\pm} \ln x_{H^\pm}}{(x_{H^\pm} - x_t)(x_{H^\pm} - 1)}
\end{aligned} \tag{16}$$

with $x_i = m_i^2 / m_w^2$. In Eqs. (15), E_i are given by

$$\begin{aligned}
E_{H_1} &= \tfrac{1}{2} (-s_\xi c_1 + c_\xi c_2), \\
E_{H_2} &= \tfrac{1}{2} (c_\xi c_1 + s_\xi c_2), \\
c_1 &= -x_{H^\pm} + c_\xi x_{H_1} (c_\xi + i s_\xi) + s_\xi x_{H_2} \\
&\quad \times (s_\xi - i c_\xi), \\
c_2 &= i[-x_{H^\pm} + s_\xi x_{H_1} (s_\xi - i c_\xi) \\
&\quad + c_\xi x_{H_2} (c_\xi + i s_\xi)].
\end{aligned} \tag{17}$$

Neglecting the strange quark mass, the effective Hamiltonian (13) leads to the following matrix element for $b \to s \tau^+ \tau^-$:

$$\begin{aligned}
M = \frac{G_F \alpha}{\sqrt{2}\pi} V_{tb} V_{ts}^* \Big[& C_8^{eff} \overline{s}_L \gamma_\mu b_L \overline{\tau} \gamma^\mu \tau + C_9 \overline{s}_L \gamma_\mu b_L \overline{\tau} \gamma^\mu \gamma^5 \tau \\
& + 2 C_7 m_b \overline{s}_L i \sigma^{\mu\nu} \frac{q^\nu}{q^2} b_R \overline{\tau} \gamma^\mu \tau + C_{Q_1} \overline{s}_L b_R \overline{\tau} \tau \\
& + C_{Q_2} \overline{s}_L b_R \overline{\tau} \gamma^5 \tau \Big],
\end{aligned} \tag{18}$$

where [5,7,18]

$$\begin{aligned}
C_8^{eff} = C_8 + \Bigg\{ & g\left(\frac{m_c}{m_b}, \hat{s} \right) + \frac{3}{\alpha^2} k \\
& \times \sum_{V_i = \psi', \psi'' \ldots} \frac{\pi M_{V_i} \Gamma(V_i \to \tau^+ \tau^-)}{M_{V_i}^2 - q^2 - i M_{V_i} \Gamma_{V_i}} \Bigg\} (3C_1 + C_2),
\end{aligned} \tag{19}$$

with $\hat{s} = q^2 / m_b^2$, $q = (p_{\tau^+} + p_{\tau^-})^2$. In Eq. (19) $g(m_c/m_b, \hat{s})$ arises from the one-loop matrix element of the four-quark operators and can be found in Refs. [5,19]. The second term in braces in Eq. (19) estimates the long-distance contribution from the intermediate, ψ', ψ'', . . . [5,18]. In our numerical calculations, we choose $k(3C_1+C_2)=-0.875$ [20].

The QCD corrections to coefficients C_i and C_{Q_i} can be incorporated in the standard way by using the renormalization group equations. Although the C_i at the scale $\mu=O(m_b)$ have been given in the next-to-leading order (NLO) approximation and without including mixing with Q_i, we use the values of C_i only in the leading order approximation (LO) since no C_{Q_i} have been calculated in NLO. The C_i and C_{Q_i} with LO QCD corrections have been given in Ref. [9]:

$$C_7(m_b)=\eta^{-16/23}\left\{C_7(m_W)-\left[\frac{58}{135}(\eta^{10/23}-1)+\frac{29}{189}(\eta^{28/23}-1)\right]C_2(m_W)-0.012C_{Q_3}(m_W)\right\}, \tag{20}$$

$$C_8(m_b)=C_8(m_W)+\frac{4\pi}{\alpha_s(m_W)}\left[-\frac{4}{33}(1-\eta^{-11/23})+\frac{8}{87}(1-\eta^{-29/23})\right]C_2(m_W), \tag{21}$$

$$C_9(m_b)=C_9(m_W), \tag{22}$$

$$C_{Q_i}(m_b)=\eta^{-\gamma_Q/\beta_0}C_{Q_i}(m_W), \quad i=1,2, \tag{23}$$

where $\gamma_Q=-4$ [21] is the anomalous dimension of $\overline{s_L}b_R$, $\beta_0=11-2n_f/3$, and $\eta=\alpha_s(m_b)/\alpha_s(m_W)$.

After a straightforward calculation, we obtain the invariant dilepton mass distribution [9]

$$\frac{\mathrm{d}\Gamma(B\to X_s\tau^+\tau^-)}{\mathrm{d}s}=B(B\to X_c l\bar{\nu})\frac{\alpha^2}{4\pi^2 f(m_c/m_b)}(1-s)^2\times\left(1-\frac{4t^2}{s}\right)^{1/2}\frac{|V_{tb}V_{ts}^*|^2}{|V_{cb}|^2}D(s)$$

$$\begin{aligned}D(s)=&|C_8^{\text{eff}}|^2\left(1+\frac{2t^2}{s}\right)(1+2s)+4|C_7|^2\times\left(1+\frac{2t^2}{s}\right)\left(1+\frac{2}{s}\right)+|C_9|^2\\&\times\left[(1+2s)+\frac{2t^2}{s}(1-4s)\right]\\&+12\,\mathrm{Re}(C_7C_8^{\text{eff}*})\left(1+\frac{2t^2}{s}\right)\\&+\frac{3}{2}|C_{Q_1}|^2(s-4t^2)+\frac{3}{2}|C_{Q_2}|^2 s\\&+6\,\mathrm{Re}(C_9C_{Q_2}^*)t\end{aligned} \tag{24}$$

where $s=q^2/m_b^2$, $t=m_\tau/m_b$, $B(B\to X_c l\bar{\nu})$ is the branching ratio, f is the phase-space factor and $f(x)=1-8x^2+8x^6-x^8-24x^4\ln x$.

The CP asymmetry for the $B\to X_s l^+l^-$ and $\bar{B}\to\bar{X}_s l^+l^-$ is defined as

$$A_{CP}^1(s)=\frac{d\Gamma/ds-d\bar{\Gamma}/ds}{d\Gamma/ds+d\bar{\Gamma}/ds}. \tag{25}$$

We also give the forward-backward asymmetry

$$A(s)=\frac{\int_0^1 dz(d^2\Gamma/dsdz)-\int_{-1}^0 dz(d^2\Gamma/dsdz)}{\int_0^1 dz(d^2\Gamma/dsdz)+\int_{-1}^0 dz(d^2\Gamma/dsdz)}=\frac{E(s)}{D(s)}, \tag{26}$$

where $z=\cos\theta$ and θ is the angle between the momentum of the B meson and that of l^+ in the center of mass frame of the dileptons $\tau^+\tau^-$. Here,

$$E(s)=\mathrm{Re}(C_8^{\text{eff}}C_9^*s+2C_7C_9^*+C_8^{\text{eff}}C_{Q1}^*t+2C_7C_{Q2}^*t). \tag{27}$$

The CP asymmetry in the forward-backward asymmetry for $B\to X_s\tau^+\tau^-$ and $\bar{B}\to\bar{X}_s\tau^+\tau^-$ is defined as

$$A_{CP}^2(s)=\frac{A(s)-\bar{A}(s)}{A(s)+\bar{A}(s)}. \tag{28}$$

It is easy to see from Eq. (24) that the CP asymmetry A_{CP}^1 is very small because the weak phase difference in $C_7C_8^{\text{eff}}$ arises from the small mixing of O_7 with Q_3 [see Eq. (20)]. In contrast with it, A_{CP}^2 can reach a large value when $\tan\beta$ is large, as can be seen from Eqs. (27) and (15). Therefore, we propose to measure A_{CP}^2 in order to search for new CP violation sources.

Let us now discuss the lepton polarization effects. We define three orthogonal unit vectors:

$$\vec{e}_L=\frac{\vec{p}_1}{|\vec{p}_1|},$$

$$\vec{e}_N=\frac{\vec{p}_s\times\vec{p}_1}{|\vec{p}_s\times\vec{p}_1|},$$

$$\vec{e}_T=\vec{e}_N\times\vec{e}_L,$$

where $\vec{p}_1$ and $\vec{p}_s$ are the three momenta of the l^- lepton and the s quark, respectively, in the center of mass of the l^+l^- system. The differential decay rate for any given spin direction $\vec{n}$ of the l^- lepton, where $\vec{n}$ is a unit vector in the l^- lepton rest frame, can be written as

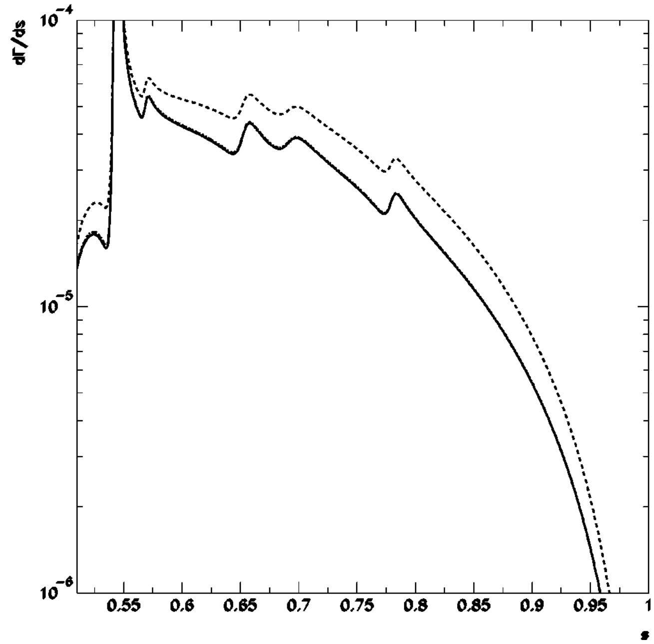

FIG. 1. Differential branching ratio as function of s, where $\xi = \pi/4$, solid and dashed lines represent tan $\beta = 10$ and 30 and the dot-dashed line represents the case of switching off C_{Q_i} contributions.

$$\frac{d\Gamma(\vec{n})}{ds} = \frac{1}{2}\left(\frac{d\Gamma}{ds}\right)_0 [1 + (P_L \vec{e}_L + P_N \vec{e}_N + P_T \vec{e}_T)\cdot \vec{n}], \tag{29}$$

where the subscript ''0'' corresponds to the unpolarized case, and P_L, P_T, and P_N, which correspond to the longitudinal, transverse, and normal projections of the lepton spin, respectively, are functions of s. From Eq. (29), one has

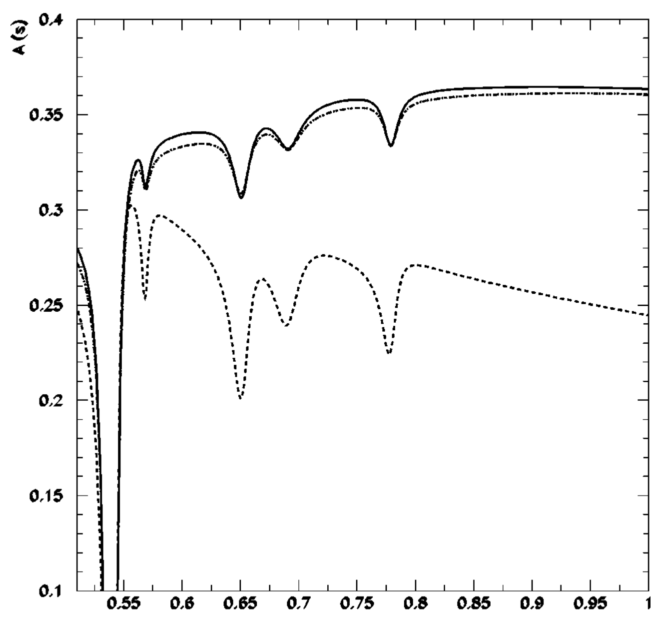

FIG. 2. Backward-forward asymmetry as function of s, where $\xi = \pi/4$. Solid and dashed lines represent tan $\beta = 10$ and 30 and the dot-dashed line represents the case of switching off C_{Q_i} contributions.

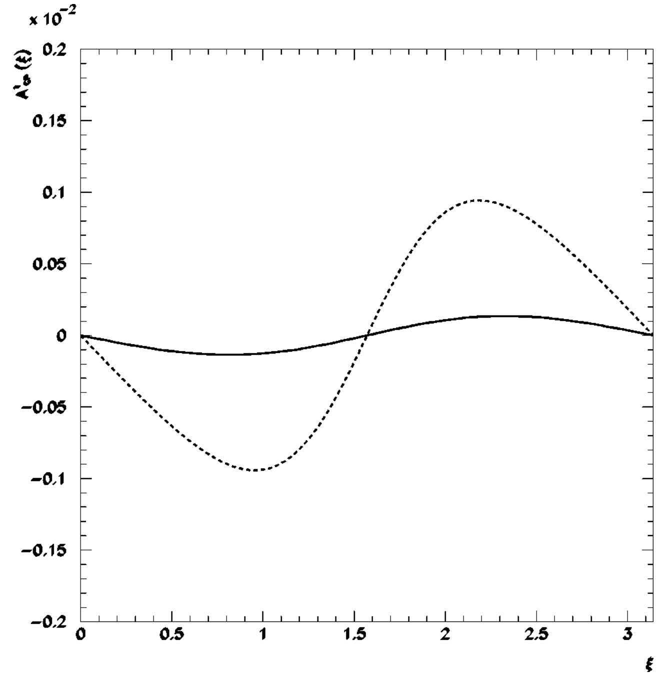

FIG. 3. A^1_{CP} as function of ξ, where $s = 0.8$. Solid and dashed lines represent tan $\beta = 10$ and 30.

$$P_i(s) = \frac{(d\Gamma/ds)(\vec{n}=\vec{e}_i) - (d\Gamma/ds)(\vec{n}=-\vec{e}_i)}{(d\Gamma/ds)(\vec{n}=\vec{e}_i) + (d\Gamma/ds)(\vec{n}=-\vec{e}_i)}. \tag{30}$$

The calculations for the P_i's ($i = L,\ T,\ N$) lead to the following results:

$$P_L = \left(1 - \frac{4t^2}{s}\right)^{1/2} \frac{D_L(s)}{D(s)},$$

$$P_N = \frac{3\pi}{4s^{1/2}}\left(1 - \frac{4t^2}{s}\right)^{1/2} \frac{D_N(s)}{D(s)},$$

$$P_T = -\frac{3\pi t}{2s^{1/2}} \frac{D_T(s)}{D(s)}, \tag{31}$$

where

$$D_L(s) = \mathrm{Re}[2(1+2s) C_8^{\mathrm{eff}} C_9^* + 12 C_7 C_9^* - 6t C_{Q_1} C_9^* - 3s C_{Q_1} C_{Q_2}^*],$$

$$D_N(s) = \mathrm{Im}(2s C_{Q_1} C_7^* + s C_{Q_1} C_8^{\mathrm{eff}\,*} + s C_{Q_2} C_9^* + 4t C_9 C_7^* + 2ts C_8^{\mathrm{eff}\,*} C_9),$$

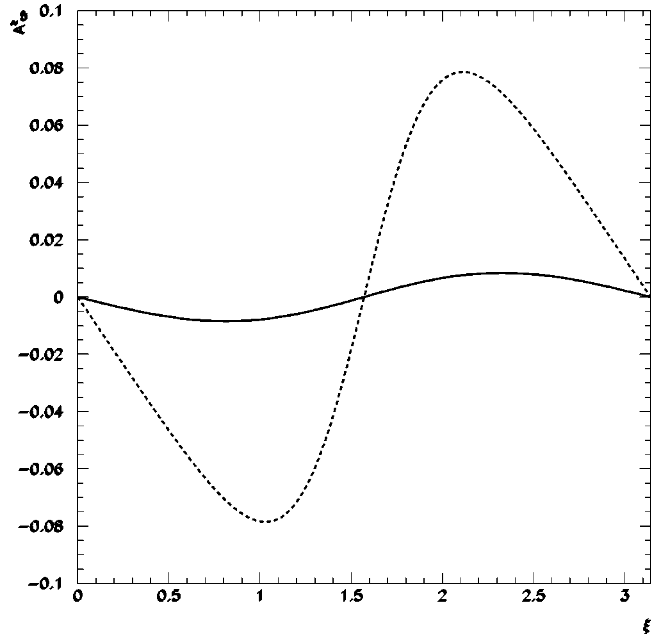

FIG. 4. A_{CP}^2 as function of ξ, where $s=0.8$. Solid and dashed lines represent tan $\beta=10$ and 30.

$$D_T(s)=\mathrm{Re}\Bigg(-2C_7C_9^*+4C_8^{\mathrm{eff}}C_7^*+\frac{4}{s}|C_7|^2-C_8^{\mathrm{eff}}C_9^* +s|C_8^{\mathrm{eff}}|^2-\frac{s-4t^2}{2t}C_{Q_1}C_9^*-\frac{s}{t}C_{Q_2}C_7^* -\frac{s}{2t}C_8^{\mathrm{eff}}C_{Q_2}^*\Bigg). \tag{32}$$

P_i ($i=L$, T, N) have been given in the Ref. [9], where there are some errors in P_T and they gave only two terms in

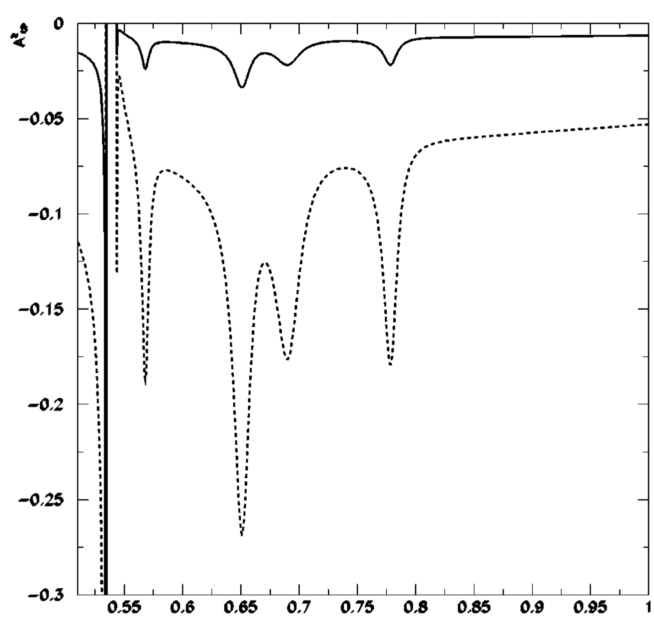

FIG. 5. A_{CP}^2 as function of s, where $\xi=\pi/4$. Solid and dashed lines represent tan $\beta=10$ and 30.

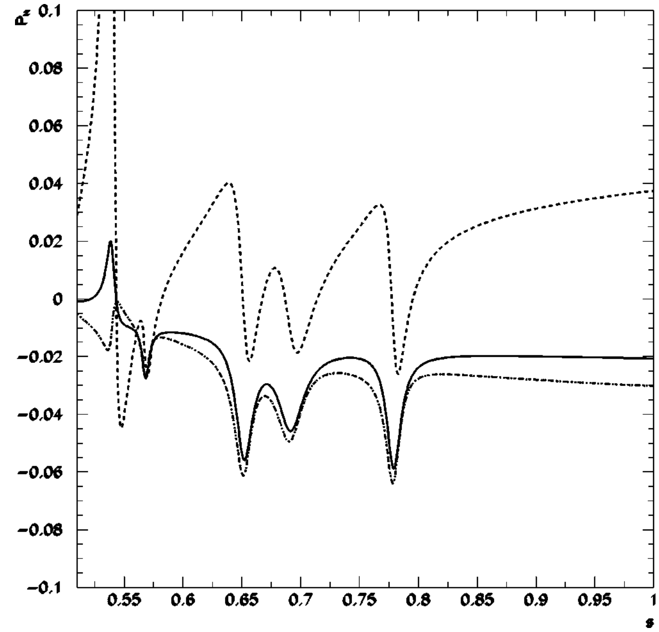

FIG. 6. P_N as function of s, where $\xi=\pi/4$. Solid and dashed lines represent tan $\beta=10$ and 30 and the dot-dashed line represents the case of switching off C_{Q_i} contributions.

D_N, the numerator of P_N. We remind that P_N is the CP-violating projection of the lepton spin onto the normal of the decay plane. Because P_N in $B\to X_s l^+ l^-$ comes from both the quark and lepton sectors, purely hadronic and leptonic CP-violating observables, such as d_n or d_e, do not necessarily strongly constrain P_N [22]. So it is advantageous to use P_N to investigate CP violation effects in some extensions of SM [23]. In the model IV 2HDM, as pointed out

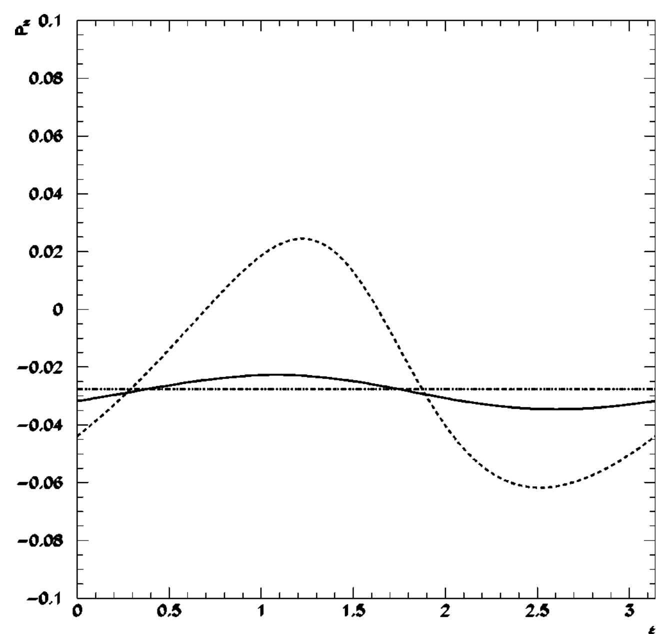

FIG. 7. P_N as function of ξ, where $s=0.8$. Solid and dashed lines represent tan $\beta=10$ and 30 and dot-dashed line represents the case of switching off C_{Q_i} contributions.

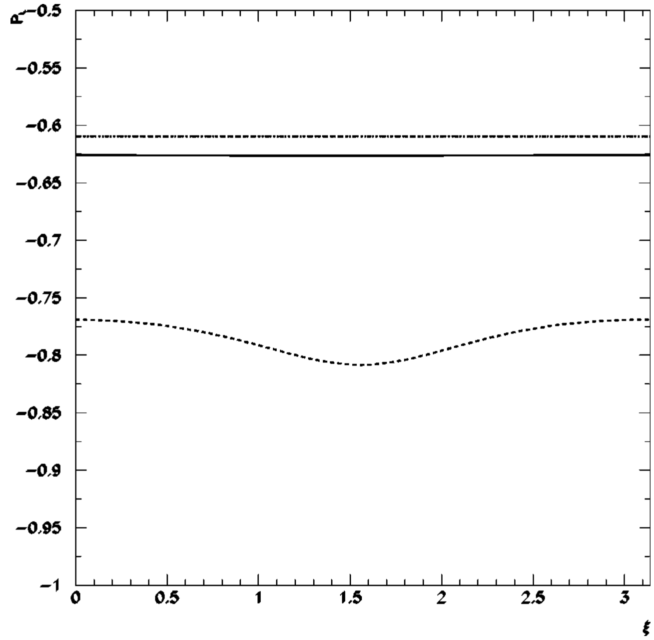

FIG. 8. P_L as function of ξ, where $s=0.8$. Solid and dashed lines represent $\tan\beta=10$ and 30 and the dot-dashed line represents the case of switching off C_{Q_i} contributions.

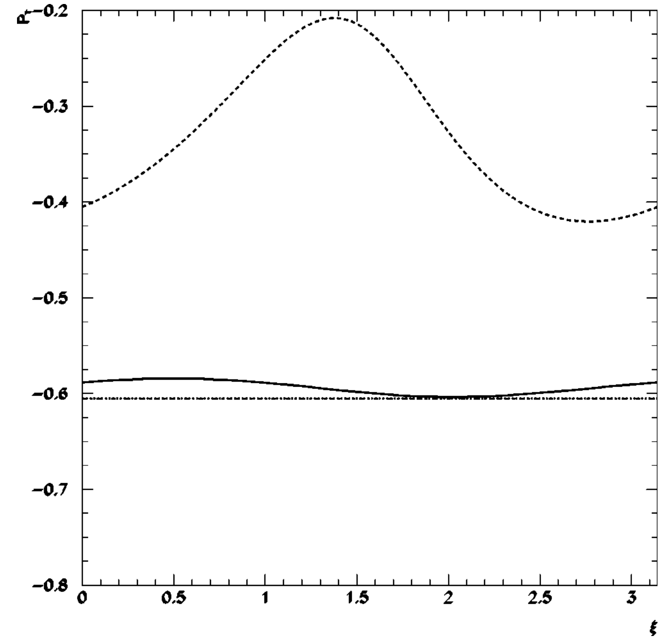

FIG. 9. P_T as function of ξ, where $s=0.8$. Solid and dashed lines represent $\tan\beta=10$ and 30 and the dot-dashed line represents the case of switching off C_{Q_i} contributions.

above, d_n and d_e constrain $\sqrt{|\sin 2\xi|}\tan\beta$ and consequently P_N through C_{Q_i} $(i=1,2)$ [see Eq. (32)].

IV. NUMERICAL RESULTS

The following parameters have been used in the numerical calculations:

$$m_t=175\ \text{GeV},\quad m_b=5.0\ \text{GeV},\quad m_c=1.6\ \text{GeV},$$

$$m_\tau=1.77\ \text{GeV},\quad \eta=1.724,$$

$$m_{H_1}=100\ \text{GeV},\quad m_{H_2}=m_{H^\pm}=200\ \text{GeV}.$$

Numerical results are shown in Figs. 1–9. From Figs. 1 and 2, we can see that the contributions of NHB to the differential branching ratio $d\Gamma/ds$ are significant when $\tan\beta$ is not smaller than 30 and the masses of NHB are in the reasonable region, and the forward-backward asymmetry $A(s)$ is more sensitive to $\tan\beta$ than $d\Gamma/ds$, which is similar to the case of the normal 2HDM without CP violation [9].

The direct CP violation A^i_{CP} $(i=1,2)$ and CP-violating polarization P_N of $B\to X_s\tau^+\tau^-$ are presented in Figs. 3–7, respectively. As expected, A^1_{CP} is about 0.1% and hard to be measured. However, A^2_{CP} can reach about 10%. A^2_{CP} is strongly dependent on the CP-violation phase ξ and comes mainly from exchanging NHBs as expected. From Figs. 6 and 7, one can see that P_N is also strongly dependent on the CP-violation phase ξ and can be as large as 5% for some values of ξ, which should be within the luminosity reach of coming B factories, and comes mainly from NHB contributions in most of range ξ.

Figures 8 and 9 show the longitudinal and transverse polarizations, respectively. It is obvious that the contributions of NHB can change the polarization greatly, especially when $\tan\beta$ is large, and the dependence of P_L on CP violation phase ξ is not significant in most of range ξ. The longitudinal polarization of $B\to X_s\tau^+\tau^-$ has been calculated in SM and several new physics scenarios [7]. Switching off the NHB contributions, our results are in agreement with those in Ref. [7].

In summary, we have calculated the differential branching ratio, back-forward asymmetry, lepton polarizations and some CP-violated observables for $B\to X_s\tau^+\tau^-$ in model IV 2HDM. As the main features of the model, NHB play an important role in inducing CP violations, in particular, for large $\tan\beta$. We propose to measure A^2_{CP}, the direct CP asymmetry in back-forward asymmetry, instead of A^1_{CP}, the usual direct CP violation in branching ratio, because the former could be observed if $\tan\beta$ is large enough (say, $\geqslant 30$) and the latter is too small to be observed. It is possible to distinguish model IV from the other 2HDMs by measuring the CP-violated observables such as A^2_{CP}, P_N if nature chooses large $\tan\beta$.

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