

Electroweak constraints on extended models with extra dimensions

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Electroweak measurements place significant bounds on higher-dimensional versions of the standard model in which the gauge and Higgs fields have Kaluza-Klein excitations. These bounds may be altered quantitatively if chiral matter is also allowed to propagate in the higher-dimensional “bulk.” We determine the electroweak constraints on a number of models of this type, including scenarios in which only the leptons or only the first two generations of matter fields propagate in the bulk. We also consider the possibility that different factors of the electroweak gauge group may be distinguished by their bulk or three-brane assignment, and study a minimal extra-dimensional Z' model. We find typical bounds on the compactification scale between 1.5 and 4 TeV, and comment on models in which these bounds might be significantly relaxed.

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I. INTRODUCTION

The possibility that nature may reveal the presence of extra spacetime dimensions at distance scales as large as an inverse TeV [1,2] has fueled considerable interest in extra-dimensional embeddings of the standard model [3–5]. In the minimal approach, chiral matter fields are confined to three spatial dimensions, or a three-brane, while the gauge and one or both Higgs fields are allowed to propagate in the higher, $4 + \delta$ dimensional bulk spacetime, where δ dimensions are compactified on an orbifold of radius R .¹ This picture has many desirable features, including the possibility of understanding the breaking of supersymmetry at low energies (as well as the origin of the μ parameter) via the Scherk-Schwartz mechanism [3,4,7], and the potential of achieving an accelerated gauge unification [5,8–15]. Bounds on the scale of compactification have been determined in effective four-dimensional theories through the effects of Kaluza-Klein (KK) excitations on precisely measured low-energy electroweak observables, and found to be typically of order a few TeV [16–19]. Hence, the possibility exists that KK excitations of the standard model gauge fields might be produced and studied at a range of future colliders experiments [18,20,21].

Within this framework, it is possible to construct models in which matter that is chiral under the standard model gauge group also propagates in the higher dimensional bulk. This can be arranged if chiral conjugate mirror fields are introduced so that KK mass terms can be formed. In the Z_2 orbifold models of interest to us in this paper, these mirror fields are taken to be Z_2 odd, so that they have no effect on the spectrum of light states. One interesting feature of these models is that a coupling between any number of bulk fields respects a conservation of KK number. For example, a coupling between three Z_2 even fields $\phi_1\phi_2\phi_3$, with $\phi_i = \sum_{n=0} \phi_i^{(n)} \cos(nx_5/R)$ in the case of one extra dimension x_5 , leads to a coupling between the different modes

$$\mathcal{L}_{4D} = c_{ijk} \phi_1^{(i)} \phi_2^{(j)} \phi_3^{(k)}$$

$$\text{with } c_{ijk} = \int dx_5 \cos\left(\frac{ix_5}{R}\right) \cos\left(\frac{jx_5}{R}\right) \cos\left(\frac{kx_5}{R}\right) \quad (1.1)$$

that vanishes if, for example, $i=j=0$ and $k \neq 0$. Clearly, models with chiral matter in the bulk will exhibit a different pattern of couplings between ordinary “zero-mode” particles and the KK excitations of the gauge fields. This suggests that bounds from precision electroweak measurements will be affected and therefore merit a reexamination in this context.

Which extended models should we consider? While there are admittedly a large number of possibilities, we will focus on three cases that are plausible from the point of view of simplicity, and that also have potentially interesting low-energy phenomenologies. The first model, the “bulk lepton” scenario [9], is somewhat similar in spirit to the “unified” standard model [22], though unification is in fact one of its strong points. In the unified standard model, quarks and leptons are distinguished at short distance scales by having completely independent electroweak gauge groups. In the bulk lepton scenario, quarks and leptons are distinguished at short distance scales by the fact that only leptons can propagate into the higher dimensional space. As we point out in Sec. II, this model leads to an improvement in accelerated gauge unification, and thus presents a plausible alternative to the minimal approach. The second case that we consider, the “bulk generations” scenario, has similarities in spirit to topcolor [23] or topflavor models [24]. In these models, the third generation is distinguished from the first two at short distance scales by having its own independent set of standard-model-like gauge factors. In the bulk generations scenario, the third generation is distinguished at short distances by the fact that it is the only generation that cannot propagate into the bulk. We will describe later why this selection of bulk generations is preferred for TeV-scale compactifications. In both the bulk lepton and bulk generations scenarios, the KK excitations of the standard model gauge fields cannot couple to leptons of the light generations, so one might suspect that the form of the corrections to electroweak observables would be significantly affected. The

¹There is also the possibility of large extra gravitational dimensions, which we do not consider here. See, for example, Ref. [6].

third case that we consider is one in which the $SU(2)$ gauge multiplet is confined to the three-brane, while the others are not. As far as the electroweak sector of this model is concerned, one might make the comparison to four-dimensional models in which a Z' boson is obtained by minimally extending the standard model gauge group by a $U(1)$ factor. In the ‘‘ $SU(2)$ -brane’’ scenario, a Z' boson is obtained by minimally extending the electroweak gauge group into extra dimensions, allowing only hypercharge to propagate in the bulk. One might regard the lightest KK state as a minimal extra-dimensional Z' , Z'_{XD} , that could be placed among many others ($Z'_\eta, Z'_\chi, Z'_{SM}, Z'_{LR}$, etc.) that have been studied in the literature. We consider the electroweak bounds on each of these scenarios in Secs. II, III, and IV, respectively. In the final section we summarize our conclusions.

Before proceeding to the analysis, however, it is important to point out that we will determine bounds on the compactification scale, $M_c = 1/R$, only in the five dimensional version of each of the models described above. With only one additional spatial dimension, sums over intermediate KK states are convergent, and results can be obtained without ambiguity. For more extra dimensions, the sums over KK states diverge if the gauge couplings are naively assumed to be independent of KK number. In reality, one expects that there is a suppression of the couplings to higher KK modes, and that this effect physically regulates the sums [20]. The dependence of the couplings on KK number, $g(|\vec{n}|^2)$, follows from string theory considerations, and is model dependent. Thus, we will relegate a quantitative treatment of the $\delta > 1$ models to a time when this dependence is more reliably known.

II. BULK LEPTONS

The possibility that we consider in this section is Scenario 1 of Ref. [9]: The standard model gauge multiplets as well as leptons of all three generations live in the higher dimensional bulk, while the quarks and both Higgs fields are confined to the three-brane. This choice leads to a marked improvement in gauge unification compared to the minimal scenarios discussed extensively in the literature. In addition, this ‘‘bulk lepton scenario’’ predicts that the KK excitations of the W and Z bosons will be leptophobic, as a consequence of the conservation of KK number. The fact that this scenario is a viable alternative to the minimal one, and may yield relatively exotic collider signatures (such as a leptophobic W' with otherwise standard model couplings) is motivation for considering the indirect constraints on the model. It is worth mentioning that the choice of placing both Higgs fields on the three-brane implies that the μ parameter is not generated through compactification as, for example, in Ref. [3]. However, there may be other natural ways of obtaining a μ parameter of the desired magnitude. We comment on this issue at the end of this section.

The essential phenomenological features of this model can be appreciated by considering first a five-dimensional $U(1)$ gauge theory, spontaneously broken by two Higgs fields both confined to a three-brane. The relevant four-dimensional Lagrangian is given by

$$\begin{aligned} \mathcal{L}_4 = \sum_{n=0} \left[-\frac{1}{4} F^{\mu\nu(n)} F_{\mu\nu}^{(n)} + \frac{1}{2} \frac{n^2}{R^2} A_\mu^{(n)} A^{\mu(n)} \right] + g^2 (v_1^2 + v_2^2) \\ \times \left(A_\mu^{(0)} + \sqrt{2} \sum_{n=1} A_\mu^{(n)} \right)^2 \\ + i\bar{q} \left(\sigma^\mu \partial_\mu + ig A_\mu^{(0)} + ig \sqrt{2} \sum_{n=1} A_\mu^{(n)} \right) q \end{aligned} \quad (2.1)$$

where q represents any quark field, and the v_i are Higgs vacuum expectation values (VEVs). The fact that both Higgs fields are stuck on a brane leads to the mixing term between the zero-mode gauge field and its KK excitations. The gauge boson mass matrix takes the form

$$\begin{pmatrix} m_z^2 & \sqrt{2}m_z^2 & \sqrt{2}m_z^2 & \cdots \\ \sqrt{2}m_z^2 & M_c^2 & & \\ \sqrt{2}m_z^2 & & (2M_c)^2 & \\ \vdots & & & \ddots \end{pmatrix} \quad (2.2)$$

where $m_z^2 = 2g^2(v_1^2 + v_2^2)$. Working to lowest order in m_z^2/M_c^2 , it is straightforward to show that this matrix is diagonalized by the rotation

$$R = \begin{pmatrix} 1 & \theta_1 & \theta_2 & \cdots \\ -\theta_1 & 1 & & \\ -\theta_2 & & 1 & \\ \vdots & & & \ddots \end{pmatrix}, \quad \theta_n = -\frac{\sqrt{2}m_z^2}{n^2 M_c^2} \quad (2.3)$$

and that the lowest eigenvalue is given by

$$m_z^{(ph)2} = m_z^2 \left(1 - 2 \sum_{n=1} \frac{m_z^2}{n^2 M_c^2} \right). \quad (2.4)$$

Unlike the case in which one Higgs field lives in the bulk [3], here there is no dependence on the ratio of the Higgs vacuum expectation values (VEVs). The rotation in Eq. (2.3) leads to a shift in the coupling of the zero-mode gauge field to zero-mode brane fermions (in this case the quarks and Higgs field)

$$g_{brane}^{(ph)} = g \left(1 - 2 \sum_{n=1} \frac{m_z^2}{n^2 M_c^2} \right), \quad (2.5)$$

but no change in the coupling to zero-mode bulk fermions (the leptons)

$$g_{bulk}^{(ph)} = g. \quad (2.6)$$

The generalization of these results to an $SU(2) \times U(1)$ gauge theory is straightforward: one breaks electroweak symmetry in the five dimensional theory and rotates from the weak to mass eigenstate basis before integrating over x^5 . Then one obtains mass matrices for the W and Z bosons that are of the same form as Eq. (2.2), with $m_z \rightarrow m_Z, m_W$, respectively. With these results in hand, we may now consider the correc-

tions to electroweak observables. We will denote the SU(2) and U(1) gauge couplings by g and g' , respectively. For clarity, we will express our analytic results in terms of sums over KK modes. To obtain numerical results, we will use $\sum_{n=1} 1/n^2 = \pi^2/6$ for $\delta=1$.

Let us begin with the Fermi constant. The KK excitations of the W do not contribute to muon decay at order m_W^2/M_c^2 . However, G_F is still affected by the shift of the W and Z mass eigenvalues. Given the standard model relation,

$$G_F^{SM} = \frac{\pi\alpha}{\sqrt{2}m_W^{(ph)2} \left(1 - \frac{m_W^{(ph)2}}{m_Z^{(ph)2}}\right) (1 - \Delta r)}, \quad (2.7)$$

we find

$$G_F = G_F^{SM} \left[1 - 2 \sum_{n=1} \frac{m_W^{(ph)2}}{n^2 M_c^2} \right]. \quad (2.8)$$

Note that the standard model radiative corrections are subsumed into Δr , so that the expression in Eq. (2.8) is accurate to order Δr , but not order $\Delta r(m_W^2/M_c^2)$. Using the experimental value $G_f = 1.16639 \pm 0.00001 \times 10^{-5} \text{ GeV}^{-2}$ [25], and $G_F^{SM} = 1.16775 \pm 0.0049 \times 10^{-5} \text{ GeV}^{-2}$ computed from the W and Z masses [16], we obtain the bound

$$M_c > 1.49 \text{ TeV } 95\% \text{ C.L.} \quad (2.9)$$

The bounds from the Z leptonic width and from the ρ parameter are similar to the one from G_F , in that both arise only through the shifts in the gauge boson masses. In the case of the leptonic width, we know that $\Gamma(l^+l^-)$ is proportional to $g^2 m_Z^{(ph)2} / \cos^2 \theta_w$, and that the Z coupling is unaffected by the presence of extra dimensions, since the leptons are in the bulk. Hence, if we choose to express $\Gamma(l^+l^-)$ in terms of G_F and m_Z we find

$$\Gamma(l^+l^-) = \Gamma(l^+l^-)^{SM} \left(\frac{m_W^{(ph)2}}{m_Z^{(ph)2} \cos^2 \theta_w} \right), \quad (2.10)$$

or using the result in Eq. (2.4),

$$\Gamma(l^+l^-) = \Gamma(l^+l^-)^{SM} \left[1 + 2 \sin^2 \theta_w \sum_{n=1} \frac{m_Z^{(ph)2}}{n^2 M_c^2} \right]. \quad (2.11)$$

Assuming the values $\Gamma = 83.91 \pm 0.10$ and $\Gamma^{SM} = 84.00 \pm 0.03$ given in the Review of Particle Physics [25], we find

$$M_c > 1.83 \text{ TeV } 95\% \text{ C.L.} \quad (2.12)$$

The corrections to the ρ parameter are also straightforward to compute

$$\rho = \rho^{SM} \left[1 + 2 \sin^2 \theta_w \sum_{n=1} \frac{m_Z^{(ph)2}}{n^2 M_c^2} \right]. \quad (2.13)$$

Setting $\rho^{SM} = 1.0109 \pm 0.0006$ [25] and computing ρ from the measured W and Z masses, as well as the modified minimal subtraction scheme (\overline{MS}) value of $\cos^2 \theta_w, \rho=1.0114 \pm 0.0023$, and we obtain the bound

$$M_c > 1.11 \text{ TeV } 95\% \text{ C.L.} \quad (2.14)$$

This is superseded by the bounds that we have already obtained.

Atomic parity violation yields an even weaker bound. The relevant four-fermion operator $\bar{e} \gamma_\mu \gamma^5 e \bar{u} \gamma^\mu u$ has the same dependence on G_F and on the physical gauge boson masses as in the standard model, but an additional factor of $1 - 2 \sum_{n=1} m_Z^2 / n^2 M_c^2$ arising from the shift in the Z boson coupling to quarks. Thus, the weak charge is proportional to $\rho(1 - 2 \sum_{n=1} m_Z^2 / n^2 M_c^2)$, or using Eq. (2.13)

$$Q_W = Q_W^{SM} \left[1 - 2 \cos^2 \theta_w \sum_{n=1} \frac{m_Z^{(ph)2}}{n^2 M_c^2} \right]. \quad (2.15)$$

Using the weak charge for cesium $Q_W = -72.4 \pm 0.84$, and $Q_W^{SM} = -73.11 \pm 0.06$ [25], we obtain the bound

$$M_c > 802 \text{ GeV } 95\% \text{ C.L.} \quad (2.16)$$

Finally, we consider the Z hadronic width, $\Gamma(q\bar{q})$. The difference between the derivation of $\Gamma(q\bar{q})$ and $\Gamma(l^+l^-)$ is the additional shift in Z -quark coupling by the factor given in Eq. (2.5). Thus, by comparison to Eq. (2.11),

$$\Gamma(q\bar{q}) = \Gamma(q\bar{q})^{SM} \left(1 + 2 \sin^2 \theta_w \sum_{n=1} \frac{m_Z^{(ph)2}}{n^2 M_c^2} \right) \times \left(1 - 2 \sum_{n=1} \frac{m_Z^{(ph)2}}{n^2 M_c^2} \right)^2 \quad (2.17)$$

or

$$\Gamma(q\bar{q}) = \Gamma(q\bar{q})^{SM} \left[1 - 2(2 - \sin^2 \theta_w) \sum_{n=1} \frac{m_Z^{(ph)2}}{n^2 M_c^2} \right]. \quad (2.18)$$

Assuming the values $\Gamma(q\bar{q}) = 1.7432 \pm 0.0023 \text{ GeV}$, and $\Gamma(q\bar{q})^{SM} = 1.7433 \pm 0.0016 \text{ GeV}$, we obtain the bound

$$M_c > 3.85 \text{ TeV } 95\% \text{ C.L.} \quad (2.19)$$

which clearly supersedes all the other bounds, and places direct production of KK modes outside of the reach of the Fermilab Tevatron. Although the lepton-gauge boson couplings were not affected in this model, the choice of three-brane quarks and Higgs fields suggested by gauge unification was sufficient to assure comparably stringent bounds.

Finally, we comment on the origin of the μ parameter in this model. Since both Higgs fields live on the three-brane, the μ parameter does not arise through compactification, but is simply present as an allowed term in the Lagrangian. Thus it seems at first glance that the μ problem is no better than in

the minimal supersymmetric standard model (MSSM). However, if the fundamental cutoff of the theory Λ_s (the string scale) coincides with the unification point, then we find $\Lambda_s \approx 56$ TeV for $M_c \approx 4$ TeV, assuming the beta functions given in Ref. [9]. To obtain a μ parameter below one TeV, we therefore would like $\mu \approx 10^{-2} \Lambda_s$. Such a suppression seems completely natural from the point of view of horizontal flavor symmetries. For example, in any flavor model in which the third generation fields are trivial singlets, and in which the ratio of Higgs vacuum expectation values (VEVs) is of order unity, one might obtain the desired ratio between bottom and top mass by assuming an additional approximate global symmetry under which only the down-type Higgs doublet transforms nontrivially. Given the breaking of this symmetry in the quark Yukawa interactions, one would then estimate that the μ parameter is of order $h_b \Lambda_s$, where h_b is the bottom quark Yukawa coupling. This is precisely of the desired magnitude. There are presumably many ways in which a modest suppression factor such as this one can be obtained, so we will content ourselves with the observation that the μ problem seems less than problematic in this model given the greatly reduced ultraviolet cutoff of the theory.

III. BULK GENERATIONS

The possibility of complete generations living in the bulk was suggested in the work of Dienes, Dudas and Gherghetta [5] as a plausible variation on the minimal scenario. Here we will consider the bounds on a model in which the first two generations live in the bulk, together with the gauge multiplets and one of the two MSSM Higgs fields. This choice is preferred for a number of reasons. First, we note that if only one of the first two generations lived in the bulk, then conservation of KK number would prevent the KK gauge bosons from coupling to zero-mode fields of that particular generation. The resulting violation of the Glashow-Iliopoulos-Miani (GIM) mechanism provides a much stronger constraint on the compactification scale, leading its decoupling from the weak scale or the scale of superparticle masses. Let us perform some simple estimates. Consider the upper two-by-two block of the Cabibbo-Kobayashi-Maskawa (CKM) matrix in Wolfenstein parametrization

$$V_{CKM} = U_L^{u\dagger} U_L^d \approx \begin{pmatrix} 1 & \lambda \\ \lambda & 1 \end{pmatrix}, \quad (3.1)$$

where $\lambda \approx 0.2$ is the Cabibbo angle. If we make a reasonable assumption that the Yukawa matrices and the biunitary matrices that diagonalize them are hierarchical in form, then we may parametrize

$$U_L^u = \begin{pmatrix} 1 & a\lambda \\ -a\lambda & 1 \end{pmatrix} \quad \text{and} \quad U_L^d = \begin{pmatrix} 1 & b\lambda \\ -b\lambda & 1 \end{pmatrix}, \quad (3.2)$$

with $b - a = 1$. Now consider the interaction between KK gluons and quarks of the first two generations q , assuming that the first generation lives in the bulk. In the gauge basis, the interaction vertex is given by

$$\mathcal{L} = \sum_{n=1} g_s \sqrt{2} \bar{q} G_\mu^{(n)} \gamma^\mu \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} q \quad (3.3)$$

where $G \equiv G^a T^a$ is the gluon field, and g_s is the SU(3) gauge coupling. This leads to flavor-changing interactions in the mass eigenstate basis, including

$$\mathcal{L} = \sum_{n=1} g_s \sqrt{2} \lambda [(1-a) \bar{d}_L \mathcal{G}^{(n)} s_L + a \bar{u}_L \mathcal{G}^{(n)} c_L + \dots]. \quad (3.4)$$

Thus, we have at the very least the following two operators that contribute to $K-\bar{K}$ and $D-\bar{D}$ mixing:

$$\mathcal{L}^{\text{eff}} = \frac{1}{M_c^2} \frac{\pi^2}{6} g_s^2 \lambda^2 [(1-a)^2 (\bar{d}_L T^a \gamma^\mu s_L)^2 + a^2 (\bar{u}_L T^a \gamma^\mu c_L)^2], \quad (3.5)$$

where we have evaluated $\sum_{n=1} 1/n^2 = \pi^2/6$ for $\delta=1$. Using the vacuum insertion approximation, it is straightforward to compute bounds from the splitting of the neutral meson mass eigenstates. We find

$$M_c > 300(1-a) \text{ TeV} \quad \text{and} \quad M_c > 120a \text{ TeV} \quad (3.6)$$

from $K-\bar{K}$ and $D-\bar{D}$ mixing, respectively. This implies an absolute lower bound of $M_c > 85$ TeV (for $a \approx 0.71$) which places the lowest KK mode well above the electroweak scale, and beyond the reach of any proposed collider experiment.

On the other hand, the possibility that the third generation is distinguished by its bulk or brane assignment is far less constrained. The relevant term in the effective Lagrangian, assuming CKM-like mixing angles is

$$\mathcal{L}^{\text{eff}} = \frac{1}{M_c^2} \frac{\pi^2}{6} g_s^2 \lambda^8 c (\bar{b}_L T^a \gamma^\mu d_L)^2 \quad (3.7)$$

and yields a bound of $M_c > 1.33c$ TeV, where c is an operator coefficient. This bound can be evaded, however, if the third generation CKM angles originate only from rotations on the left-handed up quarks. Notice that even the modest choice of $c = 1/2$ renders the bound from Eq. (3.7) weaker than the typical bounds we encountered in the previous section. Other third generation flavor-changing processes may be interesting as signals for this type of model, but at present do not provide any meaningful constraints [26].

The observations above hold true if either the third generation is in the bulk and the first two generations are on the three-brane, or vice versa. However, the first choice seems disfavored by the largeness of the top quark Yukawa coupling. The difficulty originates from the rescalings that one must perform to relate Yukawa couplings in the $4 + \delta$ dimensional theory to the Yukawa couplings we know and love. For example, in the case where the Higgs boson and the top quark both live in the bulk, then a Yukawa coupling of the higher dimensional theory has mass dimension $-\delta/2$. By a

naturalness argument, we might expect this dimensionful coupling to be of the same order as the cutoff of the theory, so that $h \sim h_0/\Lambda_s^{\delta/2}$, where h is the Yukawa coupling, Λ_s is the string scale, and h_0 is dimensionless and of order unity. However, when one derives the effective four dimensional Lagrangian in terms of 4D fields with canonical mass dimension, one finds that

$$h_{4D} = h_0 \left(\frac{\Lambda_c}{\Lambda_s} \right)^{\delta/2}, \quad (3.8)$$

where $\Lambda_c = 1/(2\pi R)$. We find the same result for bulk Higgs bosons and brane fermions, while in the case of bulk fermions and Higgs brane the exponent changes from $\delta/2$ to δ . In light of this result, an order one top quark Yukawa coupling suggests that all of the associated fields live on the three-brane. This leads us to the scenario of interest, in which only the first two generations, gauge fields and down-type Higgs boson live in the bulk.

With all the gauge fields and one Higgs boson in the bulk, the W and Z mass eigenvalues, as well as the shift in their couplings to three-brane fermions are the same as those presented in Ref. [17]:

$$m_{W,Z}^{(ph)2} = m_{W,Z}^{(ph)2} \left[1 - 2 \sin^4 \beta \sum_{n=1} \frac{m_{W,Z}^2}{n^2 M_c^2} \right] \quad (3.9)$$

$$g_{brane}^{(ph)} = g \left[1 - 2 \sin^2 \beta \sum_{n=1} \frac{m_{W,Z}^2}{n^2 M_c^2} \right] \quad (3.10)$$

where $\tan \beta$ is the ratio of brane to bulk Higgs VEVs. As in the bulk lepton scenario, however, conservation of KK number prevents couplings between the KK gauge bosons and *any* zero mode of the first two generations. As we will see, this has a significant impact on the form of the electroweak constraints. In the case of G_F , $\Gamma(e^+e^-)$, and ρ , the analysis differs only trivially from that described in Sec. II, so we will simply state the results:

$$G_F = G_F^{SM} \left[1 - 2 \sin^4 \beta \sum_{n=1} \frac{m_W^{(ph)2}}{n^2 M_c^2} \right] \quad (3.11)$$

$$\Gamma(e^+e^-) = \Gamma(e^+e^-)^{SM} \left[1 + 2 \sin^2 \theta_w \sin^4 \beta \sum_{n=1} \frac{m_Z^{(ph)2}}{n^2 M_c^2} \right] \quad (3.12)$$

$$\rho = \rho^{SM} \left[1 + 2 \sin^2 \theta_w \sin^4 \beta \sum_{n=1} \frac{m_Z^{(ph)2}}{n^2 M_c^2} \right]. \quad (3.13)$$

Note that for the sake of simplicity we have chosen to study $\Gamma(e^+e^-)$ rather than the full leptonic width, which has a more complicated form given the differing Z coupling to third generation leptons; we expect the bounds to be similar. The correction to the weak charge Q_W does not follow directly from the result in Sec. II, since in this case Q_W is only altered by the shift in gauge boson masses:

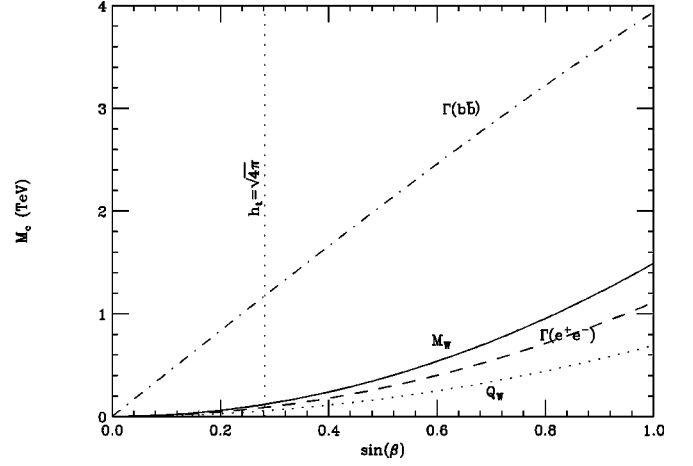


FIG. 1. Bounds on the bulk generations scenario. The h_t line indicates where the top quark Yukawa coupling becomes nonperturbative. The bound from ρ is indistinguishable from the $\Gamma(e^+e^-)$ result.

$$Q_W = Q_W^{SM} \left[1 + 2 \sin^2 \theta_w \sin^4 \beta \sum_{n=1} \frac{m_Z^{(ph)2}}{n^2 M_c^2} \right]. \quad (3.14)$$

Equation (3.14) reflects the fact that $Q_W \propto \rho$ as given in Eq. (3.13). Finally, we consider the Z decay width to $b\bar{b}$. We find

$$\begin{aligned} \Gamma(b\bar{b}) &= \Gamma(b\bar{b})^{SM} \\ &\times \left[1 - 2 \sin^2 \beta (2 - \sin^2 \beta \sin^2 \theta_w) \sum_{n=1} \frac{m_Z^{(ph)2}}{n^2 M_c^2} \right]. \end{aligned} \quad (3.15)$$

For G_F , ρ and Q_W , we use the experimental and standard model values stated in Sec. II. In addition, we assume $\Gamma(e^+e^-) = 83.8134 \pm 0.3085$ MeV, $\Gamma(e^+e^-)^{SM} = 84.01 \pm 0.05$ MeV, $\Gamma(b\bar{b}) = 0.3783 \pm 0.0016$, and $\Gamma(b\bar{b})^{SM} = 0.3762 \pm 0.0004$, computed from branching fractions given in the Review of Particle Physics [25]. We then obtain the bounds shown in Fig. 1.

The most significant bound shown is the one from $Z \rightarrow b\bar{b}$ and is as high as ~ 3.9 TeV in the case where $\sin \beta \approx 1$. However, an interesting feature that these bounds share is that they vanish (at least to the order we are working) in the opposite limit, $\sin \beta \rightarrow 0$. This result is easy to understand qualitatively: Since the first two generations live in the bulk, corrections to electroweak observables that arise from KK exchange diagrams (which are independent of $\sin \beta$) are suppressed. The remaining contributions arise from shifts in the gauge boson masses and couplings, and are all proportional to powers of $\sin \beta$ since the gauge boson mixing depends on the magnitude of the brane Higgs VEV.

How small then can we reasonably take $\sin \beta$? In the MSSM, this issue is normally settled by consideration of the perturbativity of the top quark Yukawa coupling, renormalized up to high energy scales. As pointed out in Ref. [5], the

Yukawa couplings in the minimal scenario are actually driven to weaker values above the compactification scale, and this observation should carry over to the top quark Yukawa coupling in the model of interest here. However, even taking the top quark Yukawa to be as large as ~ 3 at the weak scale, we can only bring the tightest bound from $\Gamma(b\bar{b})$ down to ~ 1 TeV. Conservation of KK number for the light generations implies that KK modes are primarily pair produced in this model, so direct production channels would remain out of the reach of the Tevatron. A possible, albeit speculative, loophole is that $\sin\beta$ could be reduced much further if the top quark mass were generated largely via other (strong) dynamics. This is interesting in that it has been suggested that models of dynamical electroweak symmetry breaking involving top condensation may have a natural origin in an extra dimensional framework [27]. In this case, the results described here would suggest that KK excitations could be brought down to sub-TeV energy scales, without running afoul of precision electroweak constraints.²

IV. AN SU(2)-BRANE SCENARIO

In this section, we consider an even more unconventional possibility, that different factors of the electroweak gauge group have different bulk or brane assignments. In Refs. [9,11] it was pointed out that assigning the SU(2) gauge multiplet to the three-brane is consistent with gauge unification, at least from a bottom-up point of view, if the other gauge multiplets, the right-handed leptons, and one generation of right-handed up and down quarks are assigned to the bulk. However, it was noted in Ref. [15] that a unified boundary condition for the couplings at the high scale may not be expected generically if the gauge groups are differen-

tiated in this way. Here we will simply allow for the possibility of a unified boundary condition, and focus on low-energy phenomenology. As we suggested in the Introduction, any model whose electroweak sector is extended by allowing only the hypercharge multiplet to propagate in the bulk, might be thought of as possessing a minimal extra-dimensional Z' , the first KK mode of the hypercharge gauge boson. While we will assume the bulk or brane assignments of the matter fields stated in Ref. [9] and above, we will describe to what extent our results carry over to any model of this type.

If we assign SU(2) to the three-brane, then we must do the same for all the SU(2) doublet fields. With two three-brane Higgs doublets, we will have, as in Sec. II, mixing between the zero-mode and KK gauge bosons that is independent of $\sin\beta$. However, the form of the mixing matrix is quite different. The neutral gauge boson mass terms in the four dimensional Lagrangian are given by

$$\mathcal{L} = \frac{1}{4} (v_1^2 + v_2^2) \left[g^2 W_3^\mu W_\mu^3 - 2g g' W_\mu^3 \left(B^{\mu(0)} + \sqrt{2} \sum_{n=1} B^{\mu(n)} \right) + g'^2 \left(B^{\mu(0)} + \sqrt{2} \sum_{n=1} B^{\mu(n)} \right)^2 \right] + \sum_{n=1} \frac{1}{2} \frac{n^2}{R^2} B^{\mu(n)} B_\mu^{(n)}, \quad (4.1)$$

where W and B are the SU(2) and U(1) gauge fields, respectively. Rewriting the zero-mode fields in terms of conventionally defined photon and Z fields, we then obtain a mixing matrix between the Z boson, and the KK excitations of the hypercharge gauge field B . In the basis $(Z, B^{(1)}, B^{(2)} \dots)$, we obtain the mass matrix

$$\begin{pmatrix} m_Z^2 & -\sqrt{2}s_w m_Z^2 & -\sqrt{2}s_w m_Z^2 & \cdots \\ -\sqrt{2}s_w m_Z^2 & M_c^2 & 2s_w^2 m_Z^2 & \cdots \\ -\sqrt{2}s_w m_Z^2 & 2s_w^2 m_Z^2 & (2M_c)^2 & \\ \vdots & \vdots & & \ddots \end{pmatrix}, \quad (4.2)$$

where we have written $\sin\theta_w$ as s_w for shorthand. Working again to lowest order in m_Z^2/M_c^2 , this matrix is diagonalized by the rotation

$$R = \begin{pmatrix} 1 & \theta_1 & \theta_2 & \cdots & \theta_n \\ -\theta_1 & 1 & \theta_{12} & \cdots & \theta_{1n} \\ -\theta_2 & -\theta_{12} & 1 & \cdots & \theta_{2n} \\ \vdots & \vdots & & \ddots & \\ -\theta_n & -\theta_{1n} & & & \theta_{nn} \end{pmatrix} \quad \begin{aligned} \theta_n &= \frac{\sqrt{2}s_w m_Z^2}{n^2 M_c^2} \\ \theta_{ij} &= -\frac{1}{|i^2 - j^2|} \frac{2s_w^2 m_Z^2}{M_c^2} \end{aligned} \quad (4.3)$$

²The bulk generation scenario described here would also provide a natural means of restricting extra-dimensional strong dynamics to the third generation.

yielding the lightest eigenvalue

$$m_Z^{(ph)2} = m_Z^2 \left[1 - 2 \sin^2 \theta_w \sum_{n=1} \frac{m_Z^2}{(nM_c)^2} \right]. \quad (4.4)$$

The shift in the Z coupling to brane fermions is slightly more complicated than in the other models we have considered. Writing the vertex in terms of the third component of isospin T^3 and hypercharge Y , we obtain

$$- \frac{e}{s_w c_w} \left[c_w^2 T^3 - s_w^2 Y \left(1 + 2 \sum_{n=1} \frac{m_Z^2}{n^2 M_c^2} \right) \right], \quad (4.5)$$

which reflects the fact that only the $U(1)$ gauge field has KK excitations. The Z coupling to zero-mode bulk fermions remains the same as in the standard model. We are now ready to determine the electroweak constraints.

In this scenario, the W has no KK excitations, and its mass eigenvalue remains unaffected by the presence of extra dimensions. However, the shift in the Z mass affects G_F through the on-shell definition of $\sin^2 \theta$.

$$G_F = G_F^{SM} \left[1 - 2 \sin^2 \theta_w \frac{m_W^{(ph)2}}{m_Z^{(ph)2} - m_W^{(ph)2}} \sum_{n=1} \frac{m_Z^{(ph)2}}{n^2 M_c^2} \right]. \quad (4.6)$$

From this we obtain the bound

$$M_c > 1.52 \text{ TeV } 95\% \text{ C.L.} \quad (4.7)$$

The Z leptonic width on the other hand receives corrections from two sources in this scenario: the shift in the Z mass, and the altered coupling to left-handed (three-brane) leptons. If we write the $Z\bar{e}e$ coupling in terms of its vector and axial vector components, g_V and g_A , then the shift in the Z coupling to the left-handed component gives us a correction

$$\Delta g_V = \Delta g_A \equiv \Delta g = - \frac{e}{2s_w c_w} \left(s_w^2 \sum_{n=1} \frac{m_Z^2}{n^2 M_c^2} \right). \quad (4.8)$$

Thus the two effects described above lead to the form

$$\Gamma(l^+ l^-) = \Gamma(l^+ l^-)^{SM} \left(1 + 2 \Delta g \frac{g_V + g_A}{g_V^2 + g_A^2} \right) \left(\frac{m_W^{(ph)2}}{m_Z^{(ph)2} \cos^2 \theta_w} \right) \quad (4.9)$$

or after some algebra

$$\begin{aligned} \Gamma(l^+ l^-) &= \Gamma(l^+ l^-)^{SM} \\ &\times \left[1 - 2 \sin^2 \theta_w \left(\frac{1 - 8 \sin^4 \theta_w}{1 - 4 \sin^2 \theta_w + 8 \sin^4 \theta_w} \right) \right. \\ &\times \left. \sum_{n=1} \frac{m_Z^{(ph)2}}{n^2 M_c^2} \right]. \end{aligned} \quad (4.10)$$

This gives us a bound comparable to Eq. (4.7)

$$M_c > 1.53 \text{ TeV } 95\% \text{ C.L.} \quad (4.11)$$

Given the breaking of custodial isospin in this scenario, one might expect a significant bound from the ρ parameter. Since the ratio of g_V/g_A is shifted away from the standard model value, we must take into account the effect on $\sin^2 \theta^{eff}$ (from which we determine the corresponding \overline{MS} value) as well the shift in the Z mass in computing ρ . We find

$$\sin^2 \theta^{eff} = \sin^2 \theta_w \left[1 + 2 \sin^2 \theta \sum_{n=1} \frac{m_Z^{(ph)2}}{n^2 M_c^2} \right], \quad (4.12)$$

and

$$\rho = \rho^{SM} \left[1 + 2 \tan^2 \theta_w \sum_{n=1} \frac{m_Z^{(ph)2}}{n^2 M_c^2} \right] \quad (4.13)$$

from which we conclude

$$M_c > 1.26 \text{ TeV } 95\% \text{ C.L.} \quad (4.14)$$

While this is a stronger bound than we obtained from consideration of the ρ parameter in the bulk lepton and bulk generation scenarios, it does not supersede Eq. (4.11). Unlike the other scenarios, we obtain a competitive bound from atomic parity violation,

$$Q_W = Q_W^{SM} \left[1 + \frac{10 \sin^2 \theta_w}{3 - 8 \sin^2 \theta_w} \sum_{n=1} \frac{m_Z^{(ph)2}}{n^2 M_c^2} \right], \quad (4.15)$$

yielding

$$M_c > 1.44 \text{ TeV } 95\% \text{ C.L.}, \quad (4.16)$$

assuming three-brane quarks. If we allow ourselves the freedom to stray from the bulk or brane assignments of the matter fields given in Ref. [9], then we would expect the bounds to vary in a model-dependent way. The exception, however, is the bound from G_F : in any variant of this model, the left-handed fields are again located on the three-brane, so that the form of G_F as determined in muon decay remains unchanged. Since the gauge boson mass matrix is also the same, the bound $M(Z'_{XD}) > 1.52 \text{ TeV}$ is model independent.

V. CONCLUSIONS

In each of the extended models considered in this paper we have found that electroweak constraints lead to typical bounds of order a few TeV. In the bulk lepton scenario, KK excitations of the gauge fields cannot couple at lowest order to the lepton zero modes. Nevertheless, the fact that gauge unification required that we place both Higgs fields on the three-brane yielded unavoidable tree-level Z -KK mixing, resulting in bounds as large as 3.85 TeV. Such mixing was also inherent to the $SU(2)$ -brane scenario, forcing $M_c > 1.52 \text{ TeV}$, from consideration of the Fermi constant. Given the typical results of recent collider studies [18], we conclude that both the bulk lepton and the $SU(2)$ -brane scenarios are outside the reach of the Tevatron for direct pro-

duction of KK states, but nonetheless could be discovered at the CERN Large Hadron Collider (LHC). The bulk generation scenario is interesting in that the lowest order bounds weaken monotonically as $\sin\beta$ is decreased, the same limit in which the brane Higgs VEV vanishes. Since this is the Higgs field that is responsible for giving the top quark its mass, we found that the requirement of perturbativity of the top quark Yukawa coupling leads to bounds of the same order as those in the other two scenarios. However, we noted that in models where the top quark mass has an additional dynamical component (which have been suggested in the extra dimensional context [27]) that $\sin\beta$ could be reduced and the electroweak bounds weakened, allowing the possi-

bility of sub-TeV KK excitations. Finally, we point out that the extended models considered here have collider signatures that differ noticeably from the minimal scenario, ranging from leptophobic W' bosons in the bulk lepton scenario, to flavor-changing neutral current KK interactions involving third generation fields in the bulk generation scenario. Thus, some aspects of the collider phenomenology of these models may be worthy of further study.

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- [1] I. Antoniadis, Phys. Lett. B **246**, 377 (1990).
 - [2] J.D. Lykken, Phys. Rev. D **54**, 3693 (1996).
 - [3] A. Pomarol and M. Quiros, Phys. Lett. B **438**, 255 (1998); A. Delgado, A. Pomarol, and M. Quiros, Phys. Rev. D **60**, 095008 (1999).
 - [4] Z. Kakushadze and S.H.H. Tye, Nucl. Phys. **B548**, 180 (1999); Z. Kakushadze, *ibid.* **B552**, 3 (1999).
 - [5] K.R. Dienes, E. Dudas, and T. Gherghetta, Phys. Lett. B **436**, 55 (1998); Nucl. Phys. **B537**, 47 (1999).
 - [6] N. Arkani-Hamed, S. Dimopoulos, and G. Dvali, Phys. Lett. B **429**, 263 (1998); Phys. Rev. D **59**, 086004 (1999).
 - [7] I. Antoniadis, S. Dimopoulos, A. Pomarol, and M. Quiros, Nucl. Phys. **B544**, 503 (1999).
 - [8] Z. Kakushadze, Nucl. Phys. **B548**, 205 (1999).
 - [9] C.D. Carone, Phys. Lett. B **454**, 70 (1999).
 - [10] D. Ghilencea and G.G. Ross, Phys. Lett. B **442**, 165 (1998).
 - [11] A. Delgado and M. Quiros, hep-ph/9903400.
 - [12] P.H. Frampton and A. Rasin, Phys. Lett. B **460**, 313 (1990).
 - [13] A. Perez-Lorenzana and R.N. Mohapatra, hep-ph/9904504.
 - [14] H-C Cheng, B.A. Dobrescu, and C.T. Hill, hep-ph/9906327.
 - [15] Z. Kakushadze and T.R. Taylor, hep-th/9905137.
 - [16] P. Nath and M. Yamaguchi, Phys. Rev. D **60**, 116004 (1999).
 - [17] M. Masip and A. Pomarol, Phys. Rev. D **60**, 096005 (1999).
 - [18] T. G. Rizzo and J. D. Wells, Phys. Rev. D (to be published), hep-ph/9906234.
 - [19] A. Strumia, hep-ph/9906266.
 - [20] I. Antoniadis, K. Benakli, and M. Quiros, Phys. Lett. B **331**, 313 (1994); hep-ph/9905311.
 - [21] P. Nath, Y. Yamada, and M. Yamaguchi, hep-ph/9905415.
 - [22] H. Georgi, E.E. Jenkins, and E.H. Simmons, Nucl. Phys. **B331**, 541 (1990).
 - [23] C.T. Hill, Phys. Lett. B **266**, 419 (1991).
 - [24] D.J. Muller and S. Nandi, Proceedings of ICHEP 96 1409 hep-ph/9610404.
 - [25] Particle Data Group, C. Caso, *et al.*, Eur. Phys. J. C **3**, 1 (1998).
 - [26] See, for example, G. Buchalla, G. Burdman, C.T. Hill, and D. Kominis, Phys. Rev. D **53**, 5185 (1996).
 - [27] B.A. Dobrescu, hep-ph/9903407; hep-ph/9812349.