# Vector potential versus color charge density in low-$x$ evolution

Alex Kovner and J. Guilherme Milhano
*Theoretical Physics, University of Oxford, 1 Keble Road, Oxford, OX1 3NP, United Kingdom*
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We reconsider the evolution equations for multigluon correlators derived by Jalilian-Marian, Kovner, and Weigert. We show how to derive these equations directly in terms of vector potentials (or color field strength) avoiding the introduction of the concept of color charge density in the intermediate steps. The two step procedure of deriving the evolution of the charge density correlators followed by the solution of classical equations for the vector potentials is shown to be consistent with direct derivation of evolution for vector potentials. In the process we correct some computational errors of Jalilian-Marian, Kovner, and Weigert and present the corrected evolution equations which have a somewhat simpler appearance.

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## I. INTRODUCTION

With the advent of the DESY $ep$ collider HERA there has been a great increase in the scope of the theoretical effort to understand the physics of hadronic scattering at high energy. This is a challenging subject especially as it might provide a bridge between perturbative partonic physics of short distance processes [e.g., deep inelastic scattering (DIS) at moderate $x_{Bj}$] and soft physics of hadronic states which, presumably, dominate the high energy asymptotics. The borderline between the two — the ''semihard'' physics — is interesting also in its own right. In essence it is the physics of partonic systems which are, on one hand, dense enough for new collective phenomena to play important role but, on the other hand, are perturbative since the average momentum transfer between the partons is high enough. In this semihard regime one expects to see perturbatively controllable nonlinear effects which depart from the standard linear evolution of Dokshitzer-Gribov-Lipatov-Altarelli-Parisi (DGLAP) [1] or Balitskiĭ-Fadin-Kuraev-Lipatov (BFKL) [2] type and subsequently lead to unitarization of hadronic cross sections.

An approach to this high partonic density regime from the partonic side has been spearheaded by Levin and collaborators [3–5] based on earlier work by Mueller [8] and is an ''all twist'' generalization of the Gribov-Levin-Ryskin (GLR) recombination picture [6,7]. It led to the formulation of a nonlinear evolution equation which exhibits a perturbative mechanism of unitarization. Analysis of this equation suggests that already at present HERA energies the nonlinearities in the gluon sector are considerable and linear evolution for gluons should break down. Better experimental data on gluon distributions would be extremely valuable in order to verify or falsify this assessment.

A complementary approach was pioneered some years ago by McLerran and Venugopalan [9]. It was later somewhat reshaped conceptually and considerably developed technically in a series of papers [10–13]. Here the idea is that, in the high density regime, rather than using partonic language, it is more appropriate to use the language of classical fields. The hadron then is considered as an ensemble of configurations of the gluon field. The statistical weight that governs the contributions of different configurations to the ensemble averaging changes when one probes the hadron on different time scales. Decreasing $x_{Bj}$ corresponds to increasing the time resolution and therefore corresponds to probing the hadron on shorter time scales. The change of the statistical weight with $x_{Bj}$ is governed by the evolution equation. As long as the field intensity is large enough this evolution should be perturbative in $\alpha_s$ but essentially nonlinear in the field intensity itself.

This evolution equation to first order in $\alpha_s$ was derived in Refs. [11,12]. We will refer to it as the Jalilian-Marian–Kovner–Leonidov–Weigert (JKLW) equation in the following. In Ref. [13] the double logarithmic limit of this evolution was considered. It was shown that in this limit the evolution of the gluon density becomes unitary at large density. Qualitatively, the evolution is very similar to that discussed in Ref. [3], although the details are different. A detailed numerical study of the doubly logarithmic limit of the JKLW evolution was recently performed in Ref. [14].

Technically the derivation of Ref. [12] is fairly involved. Several consistency checks were performed in Ref. [11] and [13] to make sure that the known results are recovered from the general evolution equation in the weak field limit. This includes the BFKL equation, the doubly logarithmic approximation to the DGLAP equation and the GLR equation. It is, however, desirable to have some additional independent checks on the equation which do not involve the weak field limit. It is the aim of the present paper to provide one such check.

In a nutshell the issue we address is the following. The evolution equation in Refs. [11–13] was derived invoking a two step procedure. Rather than considering directly the evolution of the correlators of the gluon field, one first considers the evolution of the color charge density. In the second step, one reexpresses the evolution equation for the charge density as the evolution equation for the vector potential (chromoelectric field). The evolution of the field correlators is in fact what one is after, since it is the vector potential and not the charge density that couples directly to fermions and, therefore, is more directly related to physical observables. Our observation in this paper is that one can avoid the introduction of the color charge density altogether and derive the evolution equations directly for the field correlators. This procedure has the advantage of being somewhat simpler both technically and conceptually. Nevertheless, the final result

 

for the evolution should be the same as in the two step procedure of Refs. [10–13]. The comparison of our results with the earlier derived formulas provides us with a consistency check on the calculation.

We find, in fact, that the results presented in Ref. [12] are not entirely correct. However, after correcting some algebraic mistakes in Ref. [12] we show explicitly that the two approaches yield identical results. We provide the corrected expressions for the ''kernels'' of the evolution equation, which are somewhat simpler than the expressions found in Ref. [12]. We also clarify the issue of possible Gribov ambiguity and show explicitly that the divergent Jacobians, which appeared in the intermediate steps of the derivations in Ref. [13], cancel completely in the final expressions for the correlators of the chromoelectric field. Therefore, the Gribov ambiguity, although affecting the relation between the color charge density and the chromoectric field, does not affect the evolution of the field correlators, at least to order $\alpha_s$. Since the procedure discussed in the present paper avoids the introduction of the charge density entirely, the whole approach is free from the Gribov problem.

Perhaps somewhat surprisingly the corrections to the results of Ref. [12] that we find do not affect either the weak field limit discussed in Ref. [11] or the doubly logarithmic limit of Ref. [13]. They therefore have no bearing on the derivation of the BFKL equation in our approach and also do not help to reconcile the doubly logarithmic limit of the JKLW equation [13] with the nonlinear equation studied in Ref. [3].

The plan of the paper is the following. In Sec. II we briefly recap the procedure of the derivation of the evolution equation as described in Refs. [11–13] and reformulate it directly in terms of the gluon field correlators. In Sec. III, using some of the results of Ref. [12], we calculate the real and virtual parts of the evolution in terms of the field correlators and provide the corrections to the results of Ref. [12]. Finally, in Sec. IV we discuss our results.

## II. THE JKLW EVOLUTION

First, let us briefly recall the framework and the results of Refs. [11–13]. In this approach the averages of gluonic observables in a hadron are calculated via the following path integral:

$$\langle O(A)\rangle = \int D\rho DA^{\mu} O(A) \exp\left\{ -\int d^2x_{\perp} F[\rho^a(x_{\perp})] - i\int d^4x \frac{1}{4}\mathrm{tr}F^{\mu\nu}F_{\mu\nu} + \frac{1}{N_c}\int d^2x_{\perp}dx^{-}\delta(x^{-})\rho_a(x_{\perp})\mathrm{tr}T_a W_{-\infty,\infty}[A^{-}](x^{-},x_{\perp})\right\}, \tag{1}$$

where the gluon field strength tensor is given by

$$F_a^{\mu\nu} = \partial^{\mu}A_a^{\nu} - \partial^{\nu}A_a^{\mu} - g f_{abc} A_b^{\mu} A_c^{\nu} \tag{2}$$

and $W$ is the Wilson line in the adjoint representation along the $x^+$ axis

$$W_{-\infty,+\infty}[A^{-}](x^{-},x_{\perp}) = P\exp\left[ + ig\int dx^{+} A_a^{-}(x^{+},x^{-},x_{\perp}) T_a\right]. \tag{3}$$

The hadron is represented by an ensemble of color charges localized in the plane $x^{-}=0$ with the (integrated across $x^{-}$) color charge density $\rho(x_{\perp})$. The statistical weight of a configuration $\rho(x_{\perp})$ is

$$Z = \exp\{-F[\rho]\}. \tag{4}$$

In the tree level approximation (in the light cone gauge $A^{+}=0$) the chromoelectric field is determined by the color charge density through the equations

$$F^{+i} = \frac{1}{g}\delta(x^{-})\alpha^{i}(x_{\perp}) \tag{5}$$

and the two-dimensional vector potential $\alpha^{i}(x_{\perp})$ is ''pure gauge,'' related to the color charge density by

$$\partial^{i}\alpha_a^{j} - \partial^{j}\alpha_a^{i} - f_{abc}\alpha_b^{i}\alpha_c^{j} = 0,$$
$$\partial^{i}\alpha_a^{i} = -\rho_a. \tag{6}$$

Integrating out the high longitudinal momentum modes of the vector potential generates the renormalization group equation, which has the form of the evolution equation for the statistical weight $Z$ [11][1]

$$\frac{d}{d\zeta}Z = \alpha_s\left\{\frac{1}{2}\frac{\delta^2}{\delta\rho(u)\delta\rho(v)}[Z\chi(u,v)] - \frac{\delta}{\delta\rho(u)}[Z\sigma(u)]\right\}. \tag{7}$$

In the compact notation used in Eq. (7), both $u$ and $v$ stand for pairs of color index and transverse coordinates, with summation and integration over repeated occurrences implied. The evolution in this equation is with respect to the rapidity $\zeta$, related to the Feynman $x$ by

[1]All the functions in the rest of this paper depend only on the transverse coordinates. For simplicity of notation we drop the subscript $\perp$ in the following.

$$\zeta = \ln 1/x. \tag{8}$$

Technically it arises as a variation of $Z$ with the cutoff imposed on the longitudinal momentum of the fields $A^\mu$. The quantities $\chi[\rho]$ and $\sigma[\rho]$ have the meaning of the mean fluctuation and the average value of the extra charge density induced by the high longitudinal momentum modes of $A^\mu$. They are functionals of the external charge density $\rho$. The explicit expressions have been given in Ref. [12] and it is our aim in this paper to provide a check on these expressions.

Equation (7) can be written directly as an evolution equation for the correlators of the charge density. Multiplying Eq. (7) by $\rho(x_1)\cdots\rho(x_n)$ and integrating over $\rho$ yields

$$\begin{aligned}
&\frac{d}{d\zeta}\langle \rho(x_1)\cdots\rho(x_n)\rangle \\
&\quad = \alpha_s\Bigg[\sum_{0<m<k<n+1} \langle \rho(x_1)\cdots\rho(x_{m-1})\rho(x_{m+1})\cdots\rho(x_{k-1})\rho(x_{k+1})\cdots\rho(x_n)\chi(x_m,x_k)\rangle \\
&\quad + \sum_{0<l<n+1} \langle \rho(x_1)\cdots\rho(x_{l-1})\rho(x_{l+1})\cdots\rho(x_n)\sigma(x_l)\rangle\Bigg].
\end{aligned} \tag{9}$$

This set of equations for the correlators of the color charge density completely specifies the evolution of the hadronic ensemble as one moves to higher energies (or lower values of $x$).

The evolution equations for the correlators of the charge density can be rewritten as equations for the correlators of the vector potential [13]:

$$\begin{aligned}
&\frac{d}{d\zeta}\langle \alpha^{i_1}_{a_1}(x_1)\cdots\alpha^{i_n}_{a_n}(x_n)\rangle \\
&\quad = \alpha_s\Bigg[\sum_{0<l<n+1} \langle \alpha^{i_1}_{a_1}(x_1)\cdots\alpha^{i_{l-1}}_{a_{l-1}}(x_{l-1})\alpha^{i_{l+1}}_{a_{l+1}}(x_{l+1})\cdots\alpha^{i_n}_{a_n}(x_n)\sigma^{i_l}_{a_l}(x_l)\rangle \\
&\quad + \sum_{0<m<k<n+1} \langle \alpha^{i_1}_{a_1}(x_1)\cdots\alpha^{i_{m-1}}_{a_{m-1}}(x_{m-1}) \\
&\quad \times \alpha^{i_{m+1}}_{a_{m+1}}(x_{m+1})\cdots\alpha^{i_{k-1}}_{a_{k-1}}(x_{k-1})\alpha^{i_{k+1}}_{a_{k+1}}(x_{k+1})\cdots\alpha^{i_n}_{a_n}(x_n)\chi^{i_m i_k}_{a_m a_k}(x_m,x_k)\rangle\Bigg].
\end{aligned} \tag{10}$$

The quantities $\chi^{ij}_{ab}$ and $\sigma^i_a$ have a very simple physical meaning. The high momentum modes of the vector field which have been integrated out in order to arrive at the evolution equation induce extra color charge density $\delta\rho$. The average value of this induced density and its mean fluctuation appear in the evolution equations Eq. (9) as $\sigma_a$ and $\chi_{ab}$. The appearance of the induced color charge density leads to the change in the value of the chromoelectric field through the solution of Eq. (6) with $\rho+\delta\rho$ on the right hand side. The quantities $\sigma^i_a$ and $\chi^{ij}_{ab}$ are the average value and the mean fluctuation of the induced field, respectively.

It is perhaps helpful to explain how $\sigma^i_a$ and $\chi^{ij}_{ab}$ were obtained in Ref. [13]. As shown in Ref. [12], the induced charge density can be decomposed into two pieces[2]

$$\delta\rho = \delta\tilde{\rho}_1 + \delta\tilde{\rho}_2. \tag{11}$$

The first piece $\delta\tilde{\rho}_1$ is order $g$ while the second piece $\delta\tilde{\rho}_2$ is order $g^2$. The $\delta\tilde{\rho}_1$ is time dependent, and has zero average value, while its mean fluctuation is order $g^2$. The $\delta\tilde{\rho}_2$ being $O(g^2)$ contributes only to the average value of $\delta\tilde{\rho}$ and not to the mean fluctuation. Assuming that the classical equations Eq. (6) hold not only for the background field but also for the relevant part of the fluctuation field one can solve those equations perturbatively. Writing

$$\delta\alpha^i = \delta\alpha^i_1 + \delta\alpha^i_2 \tag{12}$$

with $\delta\alpha_1$ being $O(g)$ and $\delta\alpha_2$ being $O(g^2)$ and keeping in the classical equations all terms to order $g^2$ we have

$$\begin{aligned}
&D^i_{ab}\delta\alpha^j_{1b} - D^j_{ab}\delta\alpha^i_{1b} + D^i_{ab}\delta\alpha^j_{2b} - D^j_{ab}\delta\alpha^i_{2b} \\
&- f_{abc}\delta\alpha^i_{1b}\delta\alpha^j_{1c} = 0, \\
&\partial^i\delta\alpha^i_{1a} + \partial^i\delta\alpha^i_{2a} = -(\delta\tilde{\rho}_{1a} + \delta\tilde{\rho}_{2a}).
\end{aligned} \tag{13}$$

[2]The reason for the notation $\tilde{\rho}$ rather than simply $\rho$ will be explained in the next section.

We have defined for convenience

$$\alpha^i_{ab}=f_{abc}\alpha^i_c,$$

$$D^i_{ab}=\partial^i\delta_{ab}+\alpha^i_{ab}. \tag{14}$$

To order $g$ we find

$$\delta\alpha^i_1=-D^i\frac{1}{\partial D}\delta\tilde{\rho}_1. \tag{15}$$

Therefore, to order $g^2$

$$\chi^{ij}_{ab}(x,y)=r^i_{ac}(x,u)\chi_{cd}(u,v)r^{\dagger j}_{db}(v,y) \tag{16}$$

with

$$r^i_{ab}(x,y)=-\langle x|\left[D^i\frac{1}{\partial D}\right]_{ab}|y\rangle. \tag{17}$$

Here $\langle x|O|y\rangle$ denotes a configuration space matrix element in the usual sense.

At order $g^2$ we have

$$\delta\alpha^i_2=-D^i\frac{1}{\partial D}\delta\tilde{\rho}_2-\frac{1}{2}\epsilon^{ij}\partial^j\frac{1}{D\partial}\delta\alpha_1\times\delta\alpha_1. \tag{18}$$

Here the cross product is defined as

$$A\times B=f_{abc}\epsilon^{ij}A^i_aB^j_b.$$

We thus have

$$\sigma^i_a(x)=r^i_{ab}(x,u)\sigma_b(u)+p^i_{abc}(x,u,v)\chi_{bc}(u,v) \tag{19}$$

with

$$p^i_{abc}(x,y,z)=-\frac{1}{2}\left(\epsilon^{ij}\partial^j\left[\frac{1}{D\partial}\right]_{ad}\right)(x,u)\times f_{dfe}\epsilon^{kl}r^k_{fb}(u,y)r^l_{ec}(u,z). \tag{20}$$

The procedure of deriving Eq. (10) employed in Refs. [12,13] consists, therefore, of two steps. One first splits the gluon field into the classical background field $\alpha^\mu$ and the fluctuation field $a^\mu$. The modes of the fluctuation field with longitudinal momenta in some range $\alpha_s\ln(\Lambda^+/\Lambda'^+)$ are assumed to be small. One defines operatorialy the induced charge density $\delta\rho$ in terms of the fluctuation fields $a^\mu$ and the quantities $\sigma$ and $\chi$ are calculated by integrating out the fluctuation fields perturbatively. In the second step, one solves classical equations of motion which include the induced charge density and calculates $\sigma^i$ and $\chi^{ij}$.

Clearly, consistency requires that the two step procedure that leads from Eq. (1) through Eqs. (7), (9) to the evolution equations Eqs. (10), (16), (19) must be equivalent to the following. Start with the equivalent of Eq. (1):[3]

$$\langle O(A)\rangle=\int D\alpha^i DA^\mu O(A)Z[\alpha_i(x_\perp)]\times\exp\left\{-i\int d^4x\frac{1}{4}\mathrm{tr}F^{\mu\nu}F_{\mu\nu}-\frac{1}{N_c}\int d^2x_\perp dx^-\delta(x^-)\times\partial^i\alpha^i_a(x_\perp)\mathrm{tr}T_aW_{-\infty,\infty}[A^-](x^-,x_\perp)\right\}. \tag{21}$$

Integrate out the high longitudinal momentum components of $a^\mu$ as before, but instead of calculating the induced charge density $\sigma_a$ and $\chi_{ab}$, calculate directly the induced chromoelectric field $\sigma^i_a$ and $\chi^{ij}_{ab}$. Technically this calculation is somewhat simpler, since there is no need to consider the operator $\delta\rho$, which is nonlinear in the fluctuation field $a^\mu$. Instead, one directly calculates the distribution of the static component of $a^\mu$. The resulting evolution equations should coincide with Eq. (10).

With this formulation one circumvents completely the need to introduce the color charge density $\rho$ and to solve classical equations for $\alpha^i$ in terms of $\rho$. While one may want to introduce $\rho$ for reasons of convenience, our present understanding is that it is not necessary from the point of view of physics. The physics that our approach is meant to address is that of the evolution of the hadronic ensemble. The relation between $\alpha^i$ and $\rho$ on the other hand is supposed to hold at every value of $\zeta$, and therefore itself is unrelated to evolution in $\zeta$. The concept of $\rho$ may be sometimes useful to formulate models for the statistical weight $Z$ at some particular value of $\zeta$ as was the original motivation of Ref. [9]. This could then serve as an initial condition for the evolution. This, however, is a separate question and we do not intend to address it here.

Before we proceed further, we wish to make one more comment about the relation between the chromoelectric field and the color charge density Eqs. (15), (18). Both these equations contain the dangerous factor $(\partial D)^{-1}$. The operator $\partial D$ has zero, as well as negative eigenvalues and is very reminiscent of the operators usually associated with the Gribov ambiguity in non-Abelian gauge theories. In fact, it is quite clear that it has precisely the same origin. The second equation in Eq. (13) has the form of the Lorentz-like gauge fixing condition on the fluctuation field $\delta\alpha$. Since the calculation is performed in a nonvanishing background field, the Lorentz gauge indeed suffers from Gribov ambiguity precisely due to negative eigenvalues of the operator $\partial D$.

[3]We note that the statistical weight $Z[\alpha_i]$ which appears in Eq. (21) is not equal to $Z$ of Eq. (4) since going from Eqs. (1)–(21) involves the change of variables $\rho\to\alpha_i$. The two statistical weights, therefore, differ by an appropriate Jacobian.

Given this, one may worry that our perturbative calculation is plagued with the Gribov ambiguity.[4] However, this is not necessarily the case. The point is that $\delta\rho$ itself is not arbitrary. It is calculated through the fluctuation field and, at the end of the day, is averaged over with some statistical weight $Z[\delta\rho]$. It could well be that the statistical weight is such that it only allows induced charge density of the form $\delta\rho=\partial DX$ with regular $X$. If that is the case, the dangerous denominator cancels and the induced field is well defined and regular. In fact, in our present formulation where the calculation is performed directly in terms of the field, it is almost clear that this should indeed happen. In this setup one calculates directly $\delta\alpha$, and Eqs. (15),(18) should be read from right to left, as equations determining an auxilliary quantity $\delta\rho$ through $\delta\alpha$ rather than the other way round. The operator $\partial D$ then appears in the numerator and all expressions are regular. In fact we will show in the next section by explicit calculation that all ''dangerous factors'' indeed cancel in the final expressions for $\chi^{ij}_{ab}$ and $\sigma^i_a$.

Note that, if one insists on formulating the problem in terms of the color charge density, the absence of the Gribov ambiguity implies a nontrivial consistency condition on the statistical weight $Z[\rho]$. Taking an arbitrary weight $Z$ will render the calculation of chromoelectric field correlators ill defined especially at strong fields (strong coupling). This was indeed observed in the numerical calculation [15] where a simple Gaussian in $\rho$ was used as the weight function.[5] In the next section we will calculate $\sigma^i_a$ and $\chi^{ij}_{ab}$ induced by high longitudinal momentum modes.

## III. THE INDUCED CHROMOELECTRIC FIELD

The main ingredients needed for the calculation of the induced chromoelectric field are the eigenfunctions of the quadratic action for the small fluctuations in the static background $\alpha^i$. Solving the classical equations of motion that follow from the action Eq. (21) at fixed $\alpha^i$ we find [in the gauge $\partial^i A^i(x^+\to-\infty)=0$] the classical solution

$$A^-_{cl}=0, \quad A^i_{cl}=\alpha^i(x_\perp)\,\theta(x^-). \tag{22}$$

Defining the quantum fluctuation field $a^\mu$ by $A^\mu=A^\mu_{cl}+a^\mu$ and expanding the action to second order in $a^\mu$ we have

$$S=\frac{1}{2g^2}\{a^-_x K_{xy} a^-_y+2a^-(\partial^+ Da-2fa)+2\partial^+ a^i\partial^- a^i + a^i[D^2\delta^{ij}-D^iD^j]a^j\} \tag{23}$$

Here we are using the notation

$$[fa]_a(x^+,x^-,x_\perp)=\delta(x^-)\alpha^i_{ab}(x_\perp)a^i_b(x^+,x^-,x_\perp),$$

$$Da=D^i[\alpha]a^i=[\partial^i\delta_{ab}+\theta(x^-)\alpha^i_{ab}]a^i_b \tag{24}$$

and as previously

$$\alpha^i_{ab}=f_{abc}\alpha^i_c. \tag{25}$$

The operator $K$ is

$$K^{xy}_{ab}=-\left[(\partial^+)^2\delta_{ab}+\partial^i\alpha^i_{ab}\delta(x^-)\frac{1}{\partial^-}\right]. \tag{26}$$

Note that there is no ambiguity in the definition of the operator $1/\partial^-$ in this expression. It is defined in the sense of principal value. This follows directly from the fact that the matrix $\alpha^i_{ab}$ is antisymmetric and therefore the term involving $1/\partial^-$ in Eq. (23) vanishes for zero frequency fields.

This eigenfunctions of the quadratic fluctuation operator have been found in Ref. [12] and we cite here the relevant results. The calculation is performed in the lightcone gauge $A^+=0$ with the residual gauge freedom fixed by the condition

$$\partial^i A^i(x^-\to-\infty)=0. \tag{27}$$

It is convenient to define an auxiliary field

$$\tilde{a}^-=a^-+K^{-1}(\partial^+ Da-2fa). \tag{28}$$

This field can be seen to decouple from $a_i$. Its correlator is

$$\langle\tilde{a}^-_x\tilde{a}^-_y\rangle=K^{-1}_{x,y}. \tag{29}$$

The operator $K$ Eq. (26) has zero modes. Defining the projector matrices $\eta$ and $\mu$ by

$$\mu_{ab}\partial^i\alpha^i_{bc}\frac{1}{\partial^-}=0, \quad \eta_{ab}\partial^i\alpha^i_{bc}\frac{1}{\partial^-}=\rho_{ac}\frac{1}{\partial^-}, \tag{30}$$

and

$$\mu+\eta=1, \quad \mu^2=\mu, \quad \eta^2=\eta \tag{31}$$

we can write the normalizable zero modes of $K$ in the form

$$f_a(x_\perp,x^-,p^-)=\mu_{ab}f(x_\perp,p^-). \tag{32}$$

The operator $K$ is therefore, strictly speaking, non invertible. The operator $K^{-1}$ in Eq. (28) has to be understood as the inverse of $K$ on the space of functions which does not include the functions Eq. (32). Further, it is only the nonzero mode part of $a^-$ that enters the definition of $\tilde{a}^-$ in Eq. (28).

[4]In standard perturbation theory, the Gribov ambiguity does not show up in any finite order. This is due to the fact that one expands the operator $\partial D$ and its inverse in powers of the coupling constant. To leading order then the operator does not have any negative eigenvalues, which ensures that no problems arise in finite order perturbative calculations. Our situation is, however, different. Since our background field is not assumed to be $O(g)$, the operator cannot be expanded. Therefore, there is no guarantee that the problem does not show up even in perturbation theory.

[5]This problem does not arise in the more recent numerical work [16] since in effect this work uses a different definition of $\rho$ for which the relations analogous to Eqs. (15),(18) do not involve singular factors.

For our calculation we will need the properly normalized solutions of the equations of motion that follow from the action Eq. (23). The complete set of these solutions was found in Ref. [12]:

$$a^i_{p^-,r}=ge^{ip^-x^+}\int d^2p_\perp\left[\theta(-x^-)\exp\left(i\frac{p_\perp^2}{2p^-}x^- - ip_\perp x_\perp\right)v^i_{-,r}(p_\perp) + \theta(x^-)U(x_\perp)\exp\left(i\frac{p_\perp^2}{2p^-}x^- - ip_\perp x_\perp\right)[U^\dagger v^i_{+,r}](p_\perp)+\theta(x^-)\gamma^i_{+,r}\right]. \tag{33}$$

The frequency $p^-$ is a good quantum number since the background field is static. Here $r$ is the degeneracy label, which labels independent solutions with the frequency $p^-$. In the free case it is conventionally chosen as the transverse momentum, $\{r\}=\{p^i\}$. The matrix $U(x_\perp)$ is the SU($N$) matrix that parametrizes the two-dimensional ''pure gauge'' vector potential $\alpha^i(x_\perp)$

$$\alpha^i(x_\perp)=\frac{i}{g}U(x_\perp)\partial^i U^\dagger(x_\perp).$$

The auxiliary functions $\gamma^i_+, v^i_\pm$ are all determined in terms of one vector function. Choosing this independent function as $v^i_-$ we have

$$v^i_{+,r}=[T^{ij}-L^{ij}][t^{jk}-l^{jk}]v^k_{-,r}, \tag{34}$$

$$\gamma^j_{+,r}=2D^i\left[\frac{D^j}{D^2}-\frac{\partial^j}{\partial^2}\right][t^{jk}-l^{jk}]v^k_{-,r}, \tag{35}$$

where we have defined the projection operators

$$T^{ij}\equiv\delta^{ij}-\frac{D^iD^j}{D^2},\quad L^{ij}\equiv\frac{D^iD^j}{D^2},$$

$$t^{ij}\equiv\delta^{ij}-\frac{\partial^i\partial^j}{\partial^2},\quad l^{ij}\equiv\frac{\partial^i\partial^j}{\partial^2}. \tag{36}$$

The proper normalization of the eigenfunctions requires $v^i_-$ to be chosen as complete set of eigenfunctions of the two-dimensional Hermitian operator $O^{-1}$

$$[(t-l)O^{-1}(t-l)]^{ij}_{ab}(x_\perp,y_\perp) =\langle x_\perp|\delta^{ij}_{ab}-2\left\{\left[\partial^i\frac{1}{\partial^2}-D^i\frac{1}{D^2}\right]S^{-1}\left[\frac{1}{\partial^2}\partial^j-\frac{1}{D^2}D^j\right]\right\}_{ab}|y_\perp\rangle \tag{37}$$

such that

$$\int d^2r_\perp v^i_{-,r,a}(x_\perp)v^{*j}_{-,r,b}(y_\perp)=\frac{1}{4\pi|p^-|}[O^{-1}]^{ij}_{ab}(x_\perp,y_\perp). \tag{38}$$

The rotational scalar operator $S$ is

$$S=\frac{1}{D^2}+2\left[\frac{\partial^i}{\partial^2}-\frac{D^i}{D^2}\right]\left[\frac{\partial^i}{\partial^2}-\frac{D^i}{D^2}\right] =\frac{1}{D^2}-2\frac{1}{\partial^2}\partial\alpha\frac{1}{D^2}+2\frac{1}{D^2}D\alpha\frac{1}{\partial^2}. \tag{39}$$

For further use we also need the expression for the $a^-$ component of the fluctuation field. Using the explicit expression for the operator $K$ from Ref. [12] we get from Eq. (28)

$$a^-(x^-,x_\perp,p^-)=\tilde{a}^- - \theta(x^-)\int_{x^-}^{\infty} dy^- D^i(a^i-\gamma_+^i)$$
$$-\theta(-x^-)\left[\int_0^{\infty} dy^- D^i(a^i-\gamma_+^i)+\int_{x^-}^{0} dy^-\partial^i a^i\right]$$
$$+2ip^-\eta\left[\frac{D^i}{D^2}-\frac{\partial^i}{\partial^2}\right](t-l)^{ij}v_-^j(x_\perp). \tag{40}$$

We note that this expression differs by a $x^-$-independent constant from the one given in Ref. [12]. The reason is that in Ref. [12] a constant has been subtracted from $a^-$ such that $\int_{-\infty}^{+\infty}dx^- a^-(x^-)=0$. This corresponds to the symmetric definition of the integral in Eq. (40). This is incorrect, since it violates the residual gauge fixing $\partial^i a^i(x^-\to-\infty)$ at the one loop level. We will see this explicitly later in this section. At any rate, straightforward albeit somewhat tedious calculation gives the result Eq. (40) and this is the expression that will be used in the rest of this paper.

So far the formulas presented in this section [except for the corrected expression for $a^-$, Eq. (40)] are identical to those that appear in Ref. [12] with the only difference that the background charge density $\rho$ has been substituted by the background field via $\rho=-\partial^i\alpha^i$. Now, however, we will take a different route. Our aim is to calculate the order $O(\alpha_s \ln 1/x)$ correction to the background chromoelectric field Eq. (5) directly, rather than to the background charge density. According to the discussion in the previous section (see also Ref. [12]), we are therefore interested in the following two quantities:

$$\alpha_s \ln 1/x\,\chi_{ab}^{ij}(x_\perp,y_\perp)=\langle a_a^i(x_\perp,x^-\to\infty,x^+)$$
$$\times a_b^j(y_\perp,y^-\to\infty,x^+)\rangle, \tag{41}$$

$$\alpha_s \ln 1/x\,\sigma_a^i(x_\perp)=\langle a_a^i(x_\perp,x^-\to\infty,x^+)\rangle. \tag{42}$$

It should be noted that, since the background is static, none of the quantities defined above depend on $x^+$.

### A. The real part—the mean fluctuation

It is a straightforward matter to calculate $\chi_{ab}^{ij}$. Recall that we need this quantity to order $g^2$. The fluctuation fields $a^\mu$ are formally of order $g$ themselves, and therefore to calculate the mean fluctuation we do not have to include loop corrections. Examining the expression for the general solution Eq. (33) we see that it contains oscillating pieces, which do not contribute to the value of the field at infinity as well as the $\gamma_+$ piece, which does not vanish at infinity and, therefore, determines the distribution of the vector potential there.

$$\chi_{ab}^{ij}(x_\perp,y_\perp)=4\pi\int dp^-\langle\gamma_{+,a}^i(x_\perp,p^-)\gamma_{+,b}^j(y_\perp,-p^-)\rangle. \tag{43}$$

Using the explicit expressions for $\gamma_+^i$ we find after some trivial algebra

$$\chi_{ab}^{ij}(x_\perp,y_\perp)=2\left\langle x_\perp\left|\left\{\frac{D^i}{D^2}[D^2-S^{-1}]\frac{D^j}{D^2}\right\}_{ab}\right|y_\perp\right\rangle. \tag{44}$$

We now want to compare this with the corresponding result of Ref. [12]. The induced charge density $\delta\rho$ in Ref. [12] is

$$\delta\rho=\delta\rho_1+\delta\rho_2, \tag{45}$$

with

$$\delta\rho_{1a}(x_\perp)=f_{abc}\alpha_b^i(x_\perp)\left[a_c^i(x^-=0)-\int_0^{\infty}dx^-\partial^+ a_c^i(x^-)\right]$$
$$-\frac{1}{2}f_{abc}\partial^i\alpha_b^i(x_\perp)\int dy^+[\theta(y^+-x^+)$$
$$-\theta(x^+-y^+)]a_c^-(y^+,x_\perp,x^-=0) \tag{46}$$

and

$$\delta\rho_{2a}(x_\perp)=f_{abc}\int dx^-[\partial^+ a_b^i(x)]a_c^i(x)$$
$$-\frac{1}{2}\partial^i\alpha_b^i(x_\perp)\int dy^+ a_c^-(y^+,x_\perp,x^-=0)$$
$$\times\int dz^+ a_d^-(z^+,x_\perp,x^-=0)$$
$$\times[f_{ace}f_{bde}\theta(z^+-x^+)\theta(x^+-y^+)$$
$$+f_{abe}f_{cde}\theta(x^+-z^+)\theta(z^+-y^+)]. \tag{47}$$

Only $\delta\rho_1$ contributes to $\chi$. Substituting the expressions for $a^i$ and $a^-$ into Eq. (46) we find

$$\delta\rho_1=-2(\partial D)\left[\frac{D}{D^2}-\frac{\partial}{\partial^2}\right](t-l)v_-. \tag{48}$$

Thus, we obviously have

$$\gamma_+^i=-D^i\frac{1}{\partial D}\delta\rho_1. \tag{49}$$

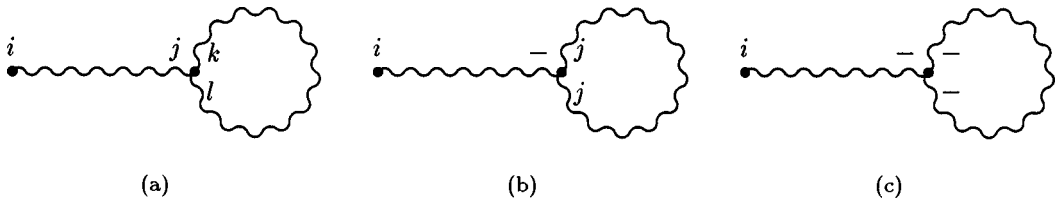


FIG. 1. One loop tadpole diagrams contributing to $\sigma^i_\alpha$. The tadpole is calculated at $x^-\to\infty$.

This reproduces exactly Eq. (15). Obviously the relation between $\chi_{ab}$ and $\chi^{ij}_{ab}$, Eq. (16) is also reproduced by this result.

We note that our result for $\chi_{ab}$ is somewhat different than the one presented in Ref. [12]. As discussed before this is due to an incorrect treatment of the $x^-$-independent component of $a^-$ in Ref. [12].

### B. The virtual part—the average value of the field

We now proceed to calculate the virtual part of the evolution kernel. For this purpose we have to calculate the zero frequency part of the $\{ij\}$ and $\{i-\}$ components of the fluctuation propagator. The calculation of the $\{ij\}$ at zero frequency is straightforward. The result is

$$\begin{aligned}
&\lim_{p^-\to 0} G^{ij}_{ab}(x^-,y^-;x_\perp,y_\perp,p^-)\\
&\quad\equiv \lim_{p^-\to 0}\langle a^i_a(x^-,x_\perp,p^-)a^j_b(y^-,y_\perp,p^-)\rangle\\
&\quad= -i\delta^{ij}\delta(x^- - y^-)\left[\theta(x^-)\langle x_\perp|\left(\frac{1}{D^2}\right)_{ab}|y_\perp\rangle\right.\\
&\qquad\left.+\theta(-x^-)\langle x_\perp|\left(\frac{1}{\partial^2}\right)_{ab}|y_\perp\rangle\right].
\end{aligned}\tag{50}$$

The $\{i-\}$ component is then calculated immediately using this result, Eq. (40) and the fact noted earlier that the field $\tilde{a}$ decouples from $a^i$. The result is

$$\begin{aligned}
&\lim_{p^-\to 0} G^{i-}_{ab}(x^-,y^-;x_\perp,y_\perp,p^-)\\
&\quad\equiv \lim_{p^-\to 0}\langle a^i_a(x^-,x_\perp,p^-)a^-_b(y^-,y_\perp,p^-)\rangle\\
&\quad= i\theta(x^- - y^-)\left[\theta(x^-)\left\langle x_\perp\left|\left(\frac{D^i}{D^2}\right)_{ab}\right|y_\perp\right\rangle\right.\\
&\qquad\left.+\theta(-x^-)\left\langle x_\perp\left|\left(\frac{\partial^i}{\partial^2}\right)_{ab}\right|y_\perp\right\rangle\right].
\end{aligned}\tag{51}$$

We are now ready to calculate $\sigma^i_a$. It is given by the one loop tadpole diagrams of Fig. 1. The vertex 1(c) comes from the expansion of the Wilson line term in the action to third order in the fluctuation. The separate contributions of the diagrams can be written in terms of the fluctuation propagator $G^{\mu\nu}\equiv\langle a^\mu a^\nu\rangle$ in the following form:

$$\begin{aligned}
1(\mathrm{a}) &= \frac{i}{2}\int dy^- d^2y_\perp G^{ij}_{ab}(x^-,y^-,x_\perp,y_\perp,p^-=0)\\
&\quad\times\epsilon^{jk}D^k_{bc}f_{cde}\epsilon^{mn}G^{mn}_{de}(y^-,y^-;y_\perp,y_\perp,y^+,y^+),\\
1(\mathrm{b}) &= -i\int dy^- d^2y_\perp G^{i-}_{ab}(x^-,y^-,x_\perp,y_\perp,p^-=0)\\
&\quad\times f_{bcd}\partial^+_{\tilde{y}^-=y^-}G^{jj}_{cd}(y^-,\tilde{y}^-;y_\perp,y_\perp,y^+,y^+),\\
1(\mathrm{c}) &= \frac{i}{N_c}\int dy^- d^2y_\perp dy^+ dw^+ dz^+\delta(y^-)\\
&\quad\times(\partial^i\alpha^i_b(y_\perp))G^{i-}_{ac}(x^+,y^+,x^-,y^-,x_\perp,y_\perp)\\
&\quad\times G^{--}_{de}(w^+,z^+,y^-,y^-,y_\perp,y_\perp)[\theta(z^+-y^+)\\
&\quad\times\theta(y^+-w^+)f_{bef}f_{cdf}-\theta(y^+-z^+)\theta(z^+-w^+)\\
&\quad\times f_{bcf}f_{def}].
\end{aligned}\tag{52}$$

The diagram Fig. 1(a) corresponds directly to the second term in Eq. (19). For this diagram we immediately find

$$\delta\sigma^i_{a(1)}(x) = -\frac{1}{2}\epsilon^{ij}\left[\frac{D^j}{D^2}\right]_{ab}(x,y)f_{bcd}\epsilon^{kl}\chi^{kl}_{cd}(y,y).\tag{53}$$

The diagrams Fig. 1(b) and 1(c) correspond to the first term in Eq. (19) and can be written as

$$\delta\sigma^i_{a(2)}(x) = -\frac{D^i}{D^2}\langle\delta\rho_2\rangle.\tag{54}$$

with $\delta\rho_2$ [see Eq. (47)]:

$$\begin{aligned}
\langle\delta\rho_2\rangle_a &= f_{abc}\int dx^-\langle[\partial^+a^i_b(x)]a^i_c(x)\rangle\\
&\quad+\frac{1}{2}\left(f_{ace}f_{bde}-\frac{1}{4}f_{abe}f_{cde}\right)\partial^i\alpha^i_b(x_\perp)\\
&\quad\times\int\frac{d\lambda}{\lambda+i\epsilon}dp^-\frac{1}{(p^-)^2}\\
&\quad\times\langle a^-_c(p^-,x_\perp,x^-=0)a^-_d(-p^-,x_\perp,x^-=0)\rangle.
\end{aligned}\tag{55}$$

Using the results for the equal time propagators from [12] we obtain

$$\langle \delta\rho_2\rangle_a = -\frac{1}{2}\left(f_{ace}f_{bde}-\frac{1}{4}f_{abe}f_{cde}\right)\partial^i\alpha_b^i(x_\perp)$$
$$\times\left\langle x_\perp\left|\frac{1}{\partial^2}+\frac{1}{2}\mu\frac{1}{D^2}\mu-2\left[\frac{1}{\partial^2}\alpha D+\frac{\mu}{2}\right]\frac{1}{D^2}\right.\right.$$
$$\left.\times S^{-1}\frac{1}{D^2}\left[D\alpha\frac{1}{\partial^2}+\frac{\mu}{2}\right]\right|x_\perp\Big\rangle_{cd}$$
$$+f_{abc}\left\langle x_\perp\left|\left[t^{ij}-l^{ij}-2\alpha^i\partial^j\frac{1}{\partial^2}\right]\right.\right.$$
$$\times\left[\delta^{jk}-2\left(\frac{\partial^j}{\partial^2}-\frac{D^j}{D^2}\right)S^{-1}\left(\frac{\partial^k}{\partial^2}-\frac{D^k}{D^2}\right)\right]$$
$$\left.\times\left[T^{ki}-L^{ki}-2\frac{1}{\partial^2}\partial^k\alpha^i\right]\right|x_\perp\Big\rangle_{bc}+R^a(x_\perp) \quad (56)$$

with

$$R^a(x_\perp)=f_{abc}\int d^2y_\perp d^2z_\perp\frac{d^2p_\perp d^2k_\perp}{(2\pi)^4}$$
$$\times\frac{p_\perp^2}{p_\perp^2-k_\perp^2}e^{ip_\perp(x_\perp-y_\perp)-ik_\perp(x_\perp-z_\perp)}$$
$$\times\left\{\left\langle y_\perp\left|[t^{ij}-l^{ij}]\left[\delta^{jk}-2\left(\frac{\partial^j}{\partial^2}-\frac{D^j}{D^2}\right)\right.\right.\right.\right.$$
$$\times S^{-1}\left(\frac{\partial^k}{\partial^2}-\frac{D^k}{D^2}\right)\Big][t^{ki}-l^{ki}]\Big|z_\perp\Big\rangle-U(x_\perp)$$
$$\times\left\langle y_\perp\left|U^\dagger[T^{ij}-L^{ij}]\left[\delta^{jk}-2\left(\frac{\partial^j}{\partial^2}-\frac{D^j}{D^2}\right)\right.\right.\right.$$
$$\left.\left.\left.\left.\times S^{-1}\left(\frac{\partial^k}{\partial^2}-\frac{D^k}{D^2}\right)\Big][T^{ki}-L^{ki}]U\Big|z_\perp\Big\rangle U^\dagger(x_\perp)\right\}\right|_{bc}. \quad (57)$$

Here the singularity in the integrand at $p_\perp^2=k_\perp^2$ has to be understood in the sense of the principal value

$$\frac{1}{p_\perp^2-k_\perp^2}=\frac{p_\perp^2-k_\perp^2}{(p_\perp^2-k_\perp^2)^2+\epsilon^2}.$$

Our final result for the induced field is given by the sum of Eqs. (53) and (54) [supplemented by Eqs. (56),(57)].

## IV. CONCLUSIONS

To summarize, the final results of this paper are Eqs. (44) and (54),(56). They supercede the corresponding results of Refs. [12] and [13]. We now want to comment on this result. The first thing to observe is that the dangerous denominator $\partial D$ does not appear in these expressions. The Gribov problem mentioned earlier therefore does not affect our calculation, at least to order $\alpha_s$.

The result for the induced field differs from the corresponding formulas in Refs. [12] and [13] in two ways. One reason is the improved treatment of $a^-$ relative to Ref. [12]. Now we are in the position to understand why the expression for $a^-$ used in Ref. [12] is inconsistent with the residual gauge fixing. In the previous section we have calculated the induced vector potential far at infinity $x^-\to\infty$. It is not much more difficult to calculate it everywhere in space. Diagrammatically it is given by the same diagrams as Fig. 1 except the coordinate on the free end of the propagator is some finite $x^-$. The difference in the analytic expressions Eq. (54) is that the surface charge density $\delta\rho_2$ is substituted by the local charge density integrated up to the longitudinal coordinate $x^-$:

$$-\theta(x^-)\frac{D^i}{D^2}\int_{-\infty}^{x^-}dy^-\langle\delta j_2^+(y^-)\rangle$$
$$-\theta(-x^-)\frac{\partial^i}{\partial^2}\int_{-\infty}^{x^-}dy^-\langle\delta j_2^+(y^-)\rangle. \quad (58)$$

This expression makes it explicit that the induced field vanishes at $x^-\to-\infty$. Therefore, our calculation clearly preserves the residual gauge condition $\partial^i a^i(x^-\to-\infty)=0$. However, if we were to subtract the zero momentum piece from the field $a^-$ as done in Ref. [12], the integration limits in Eq. (40) would become symmetric $\int_x^\infty\to\frac{1}{2}(\int_x^\infty+\int_x^{-\infty})$. The effect of this would be that $G^{i-}(x^-,y^-)$ would not vanish at $x^-\to-\infty$. It is then obvious that we would have $\partial^i a^i(x^-\to-\infty)\neq 0$. The expression obtained in the present paper does not suffer from this problem. It is consistent with the perturbative $i\epsilon$ prescription for regulating the $1/p^+$ gauge pole used in the earlier work [17].

Another difference between our present result and [12] is the appearance of $D^2$ rather than $\partial D$ and $D\partial$ in the denominators in Eqs. (53),(54). This deserves an explanation. This is also related to another point we want to address. Comparing Eq. (54) with Eq. (19) one could wonder whether the present method of calculation of $\chi^{ij}$ is consistent with the two step procedure of Refs. [12,13]. It may look like the relation between the induced field and the induced charge density we obtained here [Eq. (54)] is different from Eq. (19) which was used in the previous work. This, however, is not the case. The reason is that the $O(g^2)$ induced charge density $\delta\tilde{\rho}_2$ which appears in Eq. (19) is not quite the same as $\langle\delta\rho_2\rangle$ in Eq. (54). The $\delta\tilde{\rho}_2$ was defined as complete $O(g^2)$ contribution to the average of induced density. In other words

$$\delta\tilde{\rho}_2=\langle\delta\rho_1+\delta\rho_2\rangle \quad (59)$$

with $\delta\rho_{1,2}$ defined in Eqs. (46),(47). As we discussed above, the fluctuating part of the operator $\delta\rho_1$ is of $O(g)$ and therefore indeed $\delta\tilde{\rho}_1$ can be identified with $\delta\rho_1$. However, the vacuum average of $\delta\rho_1$ is $O(g^2)$ and does contribute in Eq. (59). It can be shown that

$$\langle\delta\rho_{1a}\rangle=f_{abc}\alpha_b^i\langle a_c^i(x^+\to\infty)\rangle. \quad (60)$$

This extra contribution turns $\partial^i\delta\alpha_2^i$ into $D^i\delta\alpha_2^i$ in the second equation in Eq. (13) if we use $\delta\rho_2$ rather than $\delta\tilde{\rho}_2$ on its

right hand side. Taking account of this we see that the procedure described in Sec. II is consistent with Eqs. (44),(54),(56).

In Ref. [12] it was assumed that $\langle \delta\rho_1 \rangle = 0$ and thus the extra contribution of Eq. (60) was overlooked. This led to an apparent noncancellation of spurious factors $1/\partial D$ which as we see now, do indeed cancel in the final result.

Importantly, the corrections we find vanish in the limit of weak field considered in Ref. [11] and also in the double logarithmic limit, where the field is considered not necessarily weak but slowly varying in the transverse plane [13]. This can be seen in the following way. Comparing Eq. (40) to the appropriate expression in Ref. [12] we find that the difference between the two is proportional to $\rho$. In the weak field limit one only needs to know $a^-$ to order 1 and therefore the correction is unimportant. For slowly varying fields all terms proportional to $\rho$ are also negligible. Therefore, the real part ($\chi^{ij}$) in these two limits is insensitive to the correction we found here. The virtual part ($\sigma^i$) does not contribute at all in the DLA limit. In the weak field limit the correction is negligible since $\langle \delta\rho_1 \rangle = O((\alpha^i)^2)$ and one only needs $\delta\rho$ to order $\alpha^i$.

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