

***CP* violation in $B_d \rightarrow D^+ D^-$, $D^{*+} D^-$, $D^+ D^{*-}$, and $D^{*+} D^{*-}$ decays**

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CP asymmetries in $B_d \rightarrow D^+ D^-$, $D^{*+} D^-$, $D^+ D^{*-}$ and $D^{*+} D^{*-}$ decays are investigated with the help of the factorization approximation and isospin relations. We find that direct *CP* violation is governed only by the short-distance penguin mechanism, while the indirect *CP* asymmetries in $B_d \rightarrow D^\pm D^{*\mp}$ transitions may be modified due to the final-state rescattering effect. An updated numerical analysis shows that the direct *CP* asymmetry in B_d^0 vs $\bar{B}_d^0 \rightarrow D^+ D^-$ decays can be as large as 3%. The *CP*-even and *CP*-odd contributions to the indirect *CP* asymmetry in B_d^0 vs $\bar{B}_d^0 \rightarrow D^{*+} D^{*-}$ decays are found to have the rates 89% and 11%, respectively. Some comments on the possibilities to determine the weak phase β and to test the factorization hypothesis are also given.

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I. INTRODUCTION

A direct measurement of the *CP*-violating parameter $\sin 2\beta$ in B_d^0 vs $\bar{B}_d^0 \rightarrow J/\psi K_S$ decays, where $\beta \equiv \arg[-(V_{tb}^* V_{td})/(V_{cb}^* V_{cd})]$ is known as an inner angle of the Cabibbo-Kobayashi-Maskawa (CKM) unitarity triangle

$$V_{ub}^* V_{ud} + V_{cb}^* V_{cd} + V_{tb}^* V_{td} = 0, \quad (1)$$

has recently been reported by the Collider Detector at Fermilab (CDF) Collaboration [1]. The preliminary result $\sin 2\beta = 0.79_{-0.44}^{+0.41}$ (stat + syst) is consistent very well with the standard-model prediction for $\sin 2\beta$, obtained indirectly from a global analysis of current data on $|V_{ub}/V_{cb}|$, B_d^0 - \bar{B}_d^0 mixing, and *CP* violation in K^0 - \bar{K}^0 mixing [3]. If the CDF measurement is confirmed, *CP* violation of the magnitude $\sin 2\beta$ should also be seen in the decay modes B_d^0 vs $\bar{B}_d^0 \rightarrow D^+ D^-$, $D^{*+} D^-$, $D^+ D^{*-}$, and $D^{*+} D^{*-}$, whose branching ratios are all anticipated to be of $O(10^{-4})$. Indeed the channel $B_d^0 \rightarrow D^{*+} D^{*-}$ has been observed by the CLEO Collaboration [2], and the measured branching ratio $\mathcal{B}(D^{*+} D^{*-}) = [6.2_{-2.9}^{+4.0}$ (stat) ± 1.0 (syst)] $\times 10^{-4}$ is in agreement with the standard-model expectation. Further measurements of neutral and charged *B* decays into $D^{(*)} \bar{D}^{(*)}$ states will soon be available in the first-round experiments of KEK and SLAC *B*-meson factories as well as at other high-luminosity hadron machines (see, e.g., Ref. [4] for a review with extensive references).

In the literature some special attention has been paid to $B \rightarrow D^{(*)} \bar{D}^{(*)}$ transitions and *CP* violation. For example, the *CP* properties of $B_d \rightarrow D^{(*)+} D^{(*)-}$ decays were analyzed in the heavy quark limit in Ref. [5]; the isospin relations and penguin effects in $B \rightarrow D^{(*)} \bar{D}^{(*)}$ decays were explored in Ref. [6]; the possibility of extracting the weak phase β and testing the factorization hypothesis in B_d^0 vs \bar{B}_d^0 decays into the non-*CP* eigenstates $D^\pm D^{*\mp}$ were investigated in Ref.

[7]; and the angular analysis of $B_d \rightarrow D^{*+} D^{*-}$ decays to determine *CP*-even and *CP*-odd amplitudes were presented in Ref. [8]. In addition to those works, numerical estimates of branching ratios and *CP* asymmetries in $B \rightarrow D^{(*)} \bar{D}^{(*)}$ decays have been given in Ref. [9], in which neither electroweak penguin contributions nor final-state rescattering effects were taken into account.

The present paper, different in several aspects from those previous studies, aims at analyzing final-state rescattering effects on direct and indirect *CP* asymmetries in $B \rightarrow D^{(*)} \bar{D}^{(*)}$ decays. We calculate the $I=1$ and $I=0$ isospin amplitudes of these processes by using the factorization approximation and the effective weak Hamiltonian, and account for long-distance interactions at the hadron level by introducing elastic rescattering phases for two isospin channels of the final-state mesons. In this approach we find that direct *CP* asymmetries in both charged and neutral *B* decay modes are governed only by the short-distance penguin mechanism, but indirect *CP* asymmetries in $B_d \rightarrow D^\pm D^{*\mp}$ transitions may be modified due to the final-state rescattering effect. An updated numerical analysis of direct *CP* violation in $B \rightarrow D \bar{D}$, $D^* \bar{D}$, $D \bar{D}^*$, and $D^* \bar{D}^*$ decays is made without neglect of the electroweak penguin effects. We obtain the asymmetry as large as 3% in $B_u^+ \rightarrow D^+ \bar{D}^0$ vs $B_u^- \rightarrow D^- \bar{D}^0$ or B_d^0 vs $\bar{B}_d^0 \rightarrow D^+ D^-$ decays. In the absence of angular analysis we find that the indirect *CP* asymmetry in $B_d \rightarrow D^{*+} D^{*-}$ decays is diluted by a factor 0.89, i.e., 11% of the asymmetry arising from the *P*-wave (*CP*-odd) contribution. We also give some comments on the possibilities to determine the weak phase β and to test the factorization hypothesis in the presence of final-state interactions.

II. ISOSPIN AMPLITUDES

The effective weak Hamiltonian responsible for $B \rightarrow D^{(*)} \bar{D}^{(*)}$ decays can explicitly be written as [10]

$$\mathcal{H}_{\text{eff}} = \frac{G_F}{\sqrt{2}} \sum_q \left[V_{qb} V_{qd}^* \left(\sum_{i=1}^2 c_i Q_i^q + \sum_{i=3}^{10} c_i Q_i \right) \right] + \text{H.c.}, \quad (2)$$

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where V_{qb} and V_{qd} (for $q=u,c$) are the CKM matrix elements, c_i (for $i=1, \dots, 10$) are the Wilson coefficients, and

$$\begin{aligned} Q_1^q &= (\bar{d}_\alpha q_\beta)_{V-A} (\bar{q} b_\alpha)_{V-A}, \\ Q_2^q &= (\bar{d} q)_{V-A} (\bar{q} b)_{V-A}, \\ Q_3 &= (\bar{d} b)_{V-A} (\bar{c} c)_{V-A}, \\ Q_4 &= (\bar{d}_\alpha b_\beta)_{V-A} (\bar{c} \beta c_\alpha)_{V-A}, \\ Q_5 &= (\bar{d} b)_{V-A} (\bar{c} c)_{V+A}, \\ Q_6 &= (\bar{d}_\alpha b_\beta)_{V-A} (\bar{c} \beta c_\alpha)_{V+A}, \end{aligned} \quad (3)$$

as well as $Q_7=Q_5$, $Q_8=Q_6$, $Q_9=Q_3$, and $Q_{10}=Q_4$. Here Q_3, \dots, Q_6 denote the QCD-induced penguin operators, and Q_7, \dots, Q_{10} stand for the electroweak penguin operators.

It is clear that the $\Delta B=+1$ and $\Delta B=-1$ parts of \mathcal{H}_{eff} have the isospin structures $|1/2, +1/2\rangle$ and $|1/2, -1/2\rangle$, respectively. They govern the transitions $B_u^+ \rightarrow D^{(*)+} \bar{D}^{(*)0}$, $B_d^0 \rightarrow D^{(*)+} D^{(*)-}$, $B_d^0 \rightarrow D^{(*)0} \bar{D}^{(*)0}$ and their CP -conjugate processes. The final state of each decay mode can be in either $I=1$ or $I=0$ isospin configuration. For simplicity we denote the amplitudes of six relevant transitions by use of the electric charges of their final- and initial-state mesons, i.e., A^{+0} , A^{+-} , A^{00} (for B_u^+ and B_d^0 decays) and \bar{A}^{-0} , \bar{A}^{+-} , \bar{A}^{00} (for B_u^- and \bar{B}_d^0 decays). These amplitudes can be expressed in terms of the $I=1$ and $I=0$ isospin amplitudes, which include both weak and strong phases. For example¹ [6]

$$\begin{aligned} A^{+0} &= A_1, \\ A^{+-} &= \frac{1}{2}(A_1 + A_0), \\ A^{00} &= \frac{1}{2}(A_1 - A_0) \end{aligned} \quad (4)$$

and the relations between $(\bar{A}^{-0}, \bar{A}^{+-}, \bar{A}^{00})$ and (\bar{A}_1, \bar{A}_0) hold in the same form. In the complex plane two sets of isospin relations form two triangles: $A^{+0}=A^{+-}+A^{00}$ and $\bar{A}^{-0}=\bar{A}^{+-}+\bar{A}^{00}$.

To calculate the magnitudes of $I=1$ and $I=0$ isospin amplitudes, we make use of the effective Hamiltonian \mathcal{H}_{eff} and the factorization approximation. We neglect the contributions of the annihilation-type channels, which are expected to have significant form-factor suppression [11]. It should be noted that in this approach the Wilson coefficients and the relevant hadronic matrix elements of four-quark operators need be evaluated in the same renormalization scheme and at the same energy scale. Following the procedure described in Ref. [12] one can obtain the scale- and renormalization-

scheme-independent transition amplitudes consisting of the CKM factors, the effective Wilson coefficients, the penguin loop-integral functions, and the factorized hadronic matrix elements. Under isospin symmetry, we are only left with two different hadronic matrix elements:

$$\begin{aligned} Z &= \langle D^{(*)+} | (\bar{c} d)_{V-A} | 0 \rangle \langle D^{(*)-} | (\bar{b} c)_{V-A} | B_d^0 \rangle \\ &= \langle D^{(*)+} | (\bar{c} d)_{V-A} | 0 \rangle \langle \bar{D}^{(*)0} | (\bar{b} c)_{V-A} | B_u^+ \rangle, \\ \bar{Z} &= \langle D^{(*)-} | (\bar{d} c)_{V-A} | 0 \rangle \langle D^{(*)+} | (\bar{c} b)_{V-A} | \bar{B}_d^0 \rangle \\ &= \langle D^{(*)-} | (\bar{d} c)_{V-A} | 0 \rangle \langle D^{(*)0} | (\bar{c} b)_{V-A} | B_u^- \rangle. \end{aligned} \quad (5)$$

Note that $|\bar{Z}|=|Z|$ holds for the final states with two pseudoscalar mesons or those with one pseudoscalar and one vector mesons. Only for the final states with two vector mesons $|\bar{Z}|$ and $|Z|$ are different, as the P -wave contributions to Z and \bar{Z} have the opposite signs (see Sec. IV for the detail). Furthermore, we account for final-state interactions at the hadron level by introducing the elastic rescattering phases δ_1 and δ_0 for $I=1$ and $I=0$ isospin channels (a similar treatment can be found, e.g., in Refs. [13,14]). We then arrive at the factorized isospin amplitudes as follows:

$$\begin{aligned} A_1 &= \frac{G_F}{\sqrt{2}} (V_{ud} V_{ub}^* S_u + V_{cd} V_{cb}^* S_c) Z e^{i\delta_1}, \\ A_0 &= \frac{G_F}{\sqrt{2}} (V_{ud} V_{ub}^* S_u + V_{cd} V_{cb}^* S_c) Z e^{i\delta_0}, \end{aligned} \quad (6)$$

in which S_u and S_c are composed of the effective Wilson coefficients and the penguin loop-integral functions (see Sec. III). The expressions of \bar{A}_1 and \bar{A}_0 can be obtained, respectively, from those of A_1 and A_0 in Eq. (6) through the replacements $Z \Rightarrow \bar{Z}$ and $V_{qd} V_{qb}^* \Rightarrow V_{qd}^* V_{qb}$ (for $q=u$ and c). Note that all parameters in the isospin amplitudes, except the CKM factors, are dependent upon the specific final states of B decays.

One can see that $|A_0|=|A_1|$ and $|\bar{A}_0|=|\bar{A}_1|$ hold in the context of our simple factorization scheme. This implies that the $B_d \rightarrow D^{(*)0} \bar{D}^{(*)0}$ transitions would be forbidden, if there were no final-state rescattering effects (i.e., if $\delta_0=\delta_1$). Substituting Eq. (6) into Eq. (4), one obtains

$$\begin{aligned} A^{+-} &= \frac{G_F}{\sqrt{2}} (V_{ud} V_{ub}^* S_u + V_{cd} V_{cb}^* S_c) Z \cos \frac{\delta_1 - \delta_0}{2} e^{i(\delta_1 + \delta_0)/2}, \\ A^{00} &= i \frac{G_F}{\sqrt{2}} (V_{ud} V_{ub}^* S_u + V_{cd} V_{cb}^* S_c) Z \sin \frac{\delta_1 - \delta_0}{2} e^{i(\delta_1 + \delta_0)/2}. \end{aligned} \quad (7)$$

Similarly \bar{A}^{+-} and \bar{A}^{00} can be read off from A^{+-} and A^{00} through the replacements $Z \Rightarrow \bar{Z}$ and $V_{qd} V_{qb}^* \Rightarrow V_{qd}^* V_{qb}$ (for $q=u$ and c). It is easy to find

¹As for $B \rightarrow D^* \bar{D}^*$ decays, the isospin relations hold separately for the transition amplitudes with helicity $\lambda = -1, 0$ or $+1$.

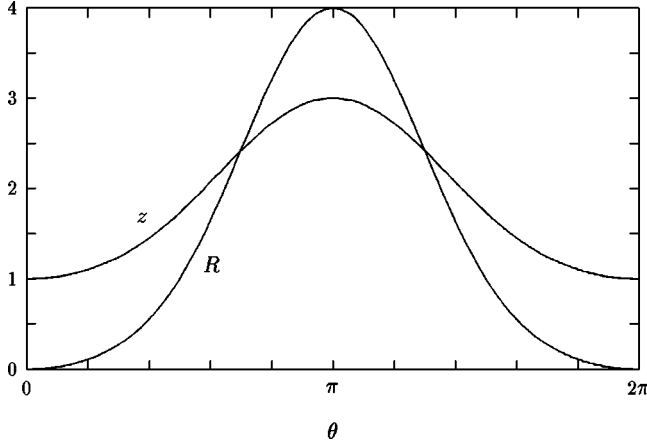


FIG. 1. Behaviors of z and R changing with θ under the condition $|A^{+-}| = |A^{+0}|$.

$$\begin{aligned} |A^{+-}|^2 + |A^{00}|^2 &= |A^{+0}|^2, \\ |\bar{A}^{+-}|^2 + |\bar{A}^{00}|^2 &= |\bar{A}^{-0}|^2, \end{aligned} \quad (8)$$

i.e., the two isospin triangles are right-angled triangles. Whether the relations in Eq. (8) are practically valid or not can be checked, once the experimental data on branching ratios of $B \rightarrow D^{(*)} \bar{D}^{(*)}$ decays are available.

If $|A^{+-}| = |A^{+0}|$ held, $|A^{00}| = 0$ would result within the factorization approach described above. Namely, observation of the (approximate) equality between the decay rates of $B_d^0 \rightarrow D^{(*)+} D^{(*)-}$ and $B_u^+ \rightarrow D^{(*)+} \bar{D}^{(*)0}$ would imply that the decay modes $B_d^0 \rightarrow D^{(*)0} \bar{D}^{(*)0}$ were forbidden or strongly suppressed. This conclusion is in general not true, however. Without any special assumption or approximation, we denote $A_0/A_1 = ze^{i\theta}$, $|A^{00}/A^{+0}|^2 = R$ and obtain consequences of the equality $|A^{+-}| = |A^{+0}|$ as follows:

$$\begin{aligned} z &= \sqrt{3 + \cos^2 \theta} - \cos \theta, \\ R &= 1 + \cos^2 \theta - \cos \theta \sqrt{3 + \cos^2 \theta}. \end{aligned} \quad (9)$$

The behaviors of z and R changing with θ is illustrated in Fig. 1. It is clear that in general $|A^{00}| = 0$ (i.e., $R = 0$) is not necessary to hold even if $|A^{+-}| = |A^{+0}|$ holds. Therefore the detection of $B_d \rightarrow D^{(*)0} \bar{D}^{(*)0}$ transitions is very useful in experiments, in order to demonstrate whether final-state rescattering effects are significant and to test whether the factorization approximation works well.

III. DIRECT CP ASYMMETRIES

We proceed with the factorization scheme to calculate direct CP asymmetries in the decay modes under discussion. As for the final states with two vector mesons, we sum over their polarizations and arrive at $|\bar{Z}|^2 = |Z|^2$, a relationship which apparently holds for other types of final states. With the help of Eqs. (6) and (7) it is easy to show that the decay rate asymmetry between $B_u^+ \rightarrow D^{(*)+} \bar{D}^{(*)0}$ and $B_u^- \rightarrow D^{(*)-} D^{(*)0}$ decays is identical to that between B_d^0 and

$\bar{B}_d^0 \rightarrow D^{(*)+} D^{(*)-}$ or $D^{(*)0} \bar{D}^{(*)0}$ decays. All these CP asymmetries are independent of the rescattering phases and the hadronic matrix elements:²

$$\begin{aligned} \mathcal{A} &= \frac{|\bar{A}^{-0}|^2 - |A^{+0}|^2}{|\bar{A}^{-0}|^2 + |A^{+0}|^2} = \frac{|\bar{A}^{+-}|^2 - |A^{+-}|^2}{|\bar{A}^{+-}|^2 + |A^{+-}|^2} = \frac{|\bar{A}^{00}|^2 - |A^{00}|^2}{|\bar{A}^{00}|^2 + |A^{00}|^2} \\ &= \frac{2r \sin \gamma \operatorname{Im}(\zeta_u \zeta_c^*)}{r^2 |\zeta_u|^2 + |\zeta_c|^2 - 2r \cos \gamma \operatorname{Re}(\zeta_u \zeta_c^*)}, \end{aligned} \quad (10)$$

where r and γ are defined by $re^{i\gamma} \equiv -(V_{ud} V_{ub}^*) / (V_{cd} V_{cb}^*)$. The phase γ corresponds to another inner angle of the unitarity triangle defined in Eq. (1). Equation (10) indicates that direct CP violation arises only from final-state interactions of the quark level (through the penguin mechanism) in $B \rightarrow D^{(*)} \bar{D}^{(*)}$ decays. This result, as a straightforward consequence of the factorization approximation, can directly be confronted with the upcoming experiments at B -meson factories.

Let us evaluate the direct CP asymmetries \mathcal{A} for different final states. As mentioned above, S_u and S_c in Eq. (10) depend on the effective Wilson coefficients \bar{c}_i and the penguin loop-integral functions F_q . The latter can be given, for a momentum-squared transfer k^2 at the $O(m_b)$ scale, as follows [15]:

$$F_q = 4 \int_0^1 dx x(1-x) \ln \left[\frac{m_q^2 - k^2 x(1-x)}{m_b^2} \right]. \quad (11)$$

The absorptive part of F_q , which is a necessary condition for direct CP violation, emerges if $k^2 \geq 4m_q^2$. The concrete expressions of S_u and S_c are found to be

$$\begin{aligned} S_u &= C_1 + C_3 + C_4 \frac{1 + \xi}{9\pi} \left(\frac{10}{3} + F_u \right), \\ S_c &= C_2 + C_3 + C_4 \frac{1 + \xi}{9\pi} \left(\frac{10}{3} + F_c \right), \end{aligned} \quad (12)$$

where

$$C_1 = \frac{\bar{c}_3}{3} + \bar{c}_4 + \frac{\bar{c}_9}{3} + \bar{c}_{10},$$

$$C_2 = \frac{\bar{c}_1}{3} + \bar{c}_2 + C_1,$$

$$C_3 = \frac{\bar{c}_5}{3} + \bar{c}_6 + \frac{\bar{c}_7}{3} + \bar{c}_8,$$

²For time-integrated B_d decays the direct CP asymmetries are diluted by a well-known factor $1/(1+x_d^2)$, where $x_d \approx 0.7$ is the B_d^0 - \bar{B}_d^0 mixing parameter. In this paper we do not take such mixing effects into account.

$$C_4 = \bar{c}_2 \alpha_s + \left(\bar{c}_1 + \frac{\bar{c}_2}{3} \right) \alpha_e. \quad (13)$$

In these equations \bar{c}_i (for $i=1, \dots, 10$) are the renormalization-scheme-independent Wilson coefficients, α_s and α_e stand, respectively, for the strong and electroweak coupling constants, and ξ is a factorization parameter arising from the transformation of $(V-A)(V+A)$ currents into $(V-A)(V-A)$ ones for the penguin operators Q_5, \dots, Q_8 . Note that ξ depends on properties of the final-state mesons [16]

$$\xi = \begin{cases} + \frac{2m_D^2}{(m_c + m_d)(m_b - m_c)} & (D\bar{D}) \\ 0 & (D^*\bar{D}) \\ - \frac{2m_D^2}{(m_c + m_d)(m_b + m_c)} & (D\bar{D}^*) \\ 0 & (D^*\bar{D}^*), \end{cases} \quad (14)$$

where the order of two $D^{(*)}$ mesons corresponds to that in the factorized hadronic matrix element Z or \bar{Z} , as given in Eq. (5).

With the help of Eqs. (11)–(14) we are able to calculate the CP asymmetries \mathcal{A} numerically. Note that $|S_u| \ll |S_c|$, as the former consists only of the penguin contribution and the latter is dominated by the much larger tree-level contribution. This, together with $|r| < 1$, allows an instructive analytical approximation of \mathcal{A} :

$$\mathcal{A} \approx 2r \sin \gamma \operatorname{Im} \left(\frac{S_u}{S_c} \right). \quad (15)$$

For illustration, we typically choose $m_u = 5$ MeV, $m_c = 1.35$ GeV, $m_b = 5$ GeV, and $m_t = 174$ GeV. The strong coupling constant is taken as $\alpha_s = 0.21$ at the $O(m_b)$ scale. Values of the effective coefficients \bar{c}_i read [17] $\bar{c}_1 = -0.313$, $\bar{c}_2 = 1.150$, $\bar{c}_3 = 0.017$, $\bar{c}_4 = -0.037$, $\bar{c}_5 = 0.010$, $\bar{c}_6 = -0.046$, $\bar{c}_7 = -0.001\alpha_e$, $\bar{c}_8 = 0.049\alpha_e$, $\bar{c}_9 = -1.321\alpha_e$, and $\bar{c}_{10} = 0.267\alpha_e$ with $\alpha_e = 1/128$. The CKM factors are taken to be $r = 0.38$ and $\gamma = 60^\circ$, consistent with the latest data on quark mixing and CP violation [3]. The unknown penguin momentum transfer k^2 is treated as a free parameter changing from $0.01m_b^2$ to m_b^2 . Our numerical results are shown in Fig. 2. Some discussions are in order.

(1) All CP asymmetries have the same sign and undergo a change of magnitude at $k^2 = 4m_c^2 \approx 0.3m_b^2$. The asymmetry $\mathcal{A}(D\bar{D})$ is most sensitive to the uncertain penguin momentum transfer k^2 , but its magnitude increases only about 0.3% from $k^2 = 0.01m_b^2$ to $k^2 = m_b^2$. It is found that the strong (gluonic) penguin effect is dominant over the electroweak penguin effect, thus the latter is safely negligible.

(2) The CP asymmetry $\mathcal{A}(D\bar{D})$ can be as large as 3%, while $\mathcal{A}(D\bar{D}^*)$ is only about 2×10^{-3} . The smallness of the latter comes from the cancellation effect, induced by the factor $(1 + \xi)$ with $\xi \sim -0.8$, in S_u and S_c . In our factorization

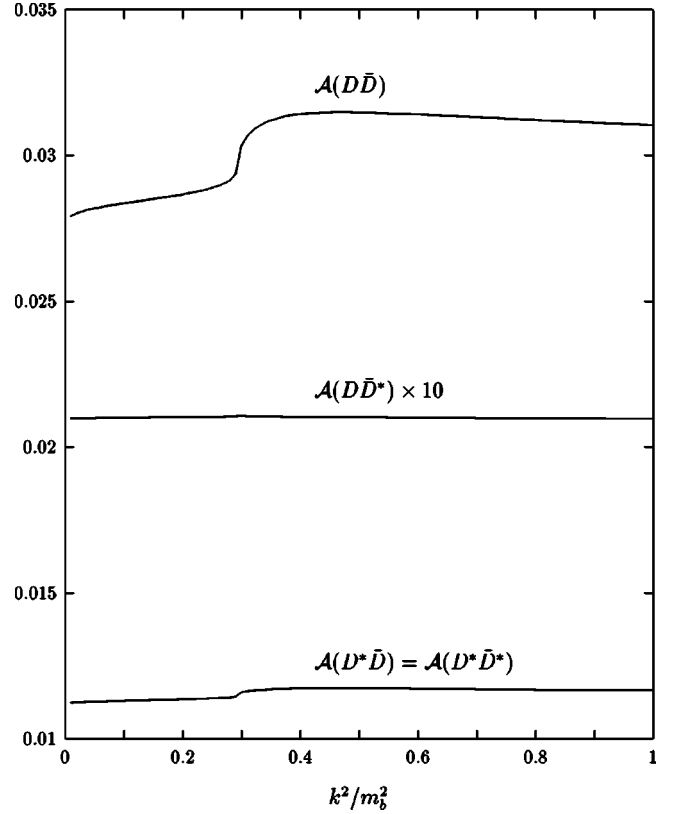


FIG. 2. Direct CP asymmetries of $B \rightarrow D^{(*)}\bar{D}^{(*)}$ in the factorization approximation.

approximation, the asymmetries $\mathcal{A}(D^*\bar{D})$ and $\mathcal{A}(D^*\bar{D}^*)$ are identical and of magnitude 1%.

(3) Observation of the CP asymmetries $\mathcal{A}(D\bar{D})$ and $\mathcal{A}(D^*\bar{D}^*)$ to three standard deviations needs about $10^8 B_u^\pm$ events, if the composite detection efficiency is at the 10% level. More events are required to measure the same CP asymmetries in B_d decays, due to the cost for flavor tagging.

It is therefore worthwhile to search for such direct CP -violating signals in the first-round experiments of B -meson factories.

IV. INDIRECT CP VIOLATION

Although the final-state rescattering phases have no effect on direct CP asymmetries \mathcal{A} in our factorization scheme, they are possible to influence the indirect CP violation arising from the interference between direct B_d transition and $B_d^0\text{--}\bar{B}_d^0$ mixing in the decay modes under consideration. The characteristic measurable of this source of CP violation is in general a difference between two rephasing-invariant quantities defined as [18]

$$\Delta(f) = \operatorname{Im} \left[\frac{q}{p} \cdot \frac{A(\bar{B}_d^0 \rightarrow f)}{A(B_d^0 \rightarrow f)} \right],$$

$$\bar{\Delta}(\bar{f}) = \operatorname{Im} \left[\frac{p}{q} \cdot \frac{A(B_d^0 \rightarrow \bar{f})}{A(\bar{B}_d^0 \rightarrow \bar{f})} \right], \quad (16)$$

where $q/p = (V_{tb}^* V_{td}) / (V_{ub} V_{ud}^*)$ denotes the weak phase of $B_d^0 - \bar{B}_d^0$ mixing,³ and \bar{f} is the CP -conjugate state of f . If f is a CP eigenstate (i.e., $|\bar{f}\rangle = CP|f\rangle = \pm|f\rangle$) and the decay is dominated by the tree-level channel, then $\bar{\Delta}(\bar{f}) = -\Delta(f)$ is a good approximation. In general only the difference $\bar{\Delta}(\bar{f}) - \Delta(f)$, which will vanish if all the CKM factors are real, measures the CP asymmetry. Note that the CP -even and CP -odd components of $f = D^{*+} D^{*-}$ state or $f = D^{*0} \bar{D}^{*0}$ state may cause some dilution in the measurables $\Delta(f)$ and $\bar{\Delta}(\bar{f})$. A proper treatment of indirect CP violation in such modes is to make use of the angular analysis [8]. Alternatively one may evaluate the P -wave contribution to $\Delta(D^{*+} D^{*-})$ and $\Delta(D^{*0} \bar{D}^{*0})$ by use of the factorization approximation and the heavy quark symmetry [20].

As penguin contributions to the transition amplitudes of $B \rightarrow D^{(*)} \bar{D}^{(*)}$ decays have been estimated to be at the percent level, we expect that their effects on $\Delta(f)$ and $\bar{\Delta}(\bar{f})$ are unimportant and negligible. It is obvious, as shown in Eq. (7), that for B_d^0 and \bar{B}_d^0 decays into the CP eigenstates $D^+ D^-$ and $D^0 \bar{D}^0$ the amplitude ratios \bar{A}^{+-} / A^{+-} and \bar{A}^{00} / A^{00} are independent of the rescattering effects. Neglecting small penguin contributions to S_u and S_c (i.e., taking $S_u = 0$ and $S_c = \bar{c}_2 + \bar{c}_1/3$), we arrive at

$$\begin{aligned} \Delta(D^+ D^-) &= \Delta(D^0 \bar{D}^0) = +\sin 2\beta, \\ \bar{\Delta}(D^+ D^-) &= \bar{\Delta}(D^0 \bar{D}^0) = -\sin 2\beta, \end{aligned} \quad (17)$$

where β is just the inner angle of the unitarity triangle defined in Eq. (1). Therefore the measurement of indirect CP asymmetries in $B_d \rightarrow D^+ D^-$ and $B_d \rightarrow D^0 \bar{D}^0$ decays may serve as a cross-check of $\sin 2\beta$ extracted from the CP asymmetry in $B_d \rightarrow J/\psi K_S$ decays.

The channels $B_d \rightarrow D^{*+} D^-$, $D^+ D^{*-}$, and $B_d \rightarrow D^{*0} \bar{D}^0$, $D^0 \bar{D}^{*0}$, whose final states are non- CP eigenstates, are also useful for extraction of the weak angle β . Since the pseudoscalar and vector mesons from B_d^0 and those from \bar{B}_d^0 have different quark-diagram configurations, the hadronic matrix elements and final-state rescattering phases in these two processes should in general be different [7]. As a result,

$$\begin{aligned} \Delta(D^{*+} D^-) &= \zeta R_{+-} \sin(\delta + 2\beta), \\ \bar{\Delta}(D^+ D^{*-}) &= \zeta R_{+-} \sin(\delta - 2\beta) \end{aligned} \quad (18)$$

and

$$\Delta(D^{*0} \bar{D}^0) = \zeta R_{00} \sin(\delta + 2\beta),$$

³The CP violation induced solely by $B_d^0 - \bar{B}_d^0$ mixing (i.e., $|q/p| \neq 1$) is expected to be negligibly small (of order 10^{-3} or smaller [19]) in the standard model.

$$\bar{\Delta}(D^0 \bar{D}^{*0}) = \zeta R_{00} \sin(\delta - 2\beta), \quad (19)$$

where

$$\begin{aligned} \zeta &= \frac{Z_{D\bar{D}^*}}{Z_{D^* \bar{D}}} = \frac{f_D}{f_{D^*}} \frac{A_0^{BD^*}(m_D^2)}{F_1^{BD}(m_{D^*}^2)}, \\ \delta &= \frac{\delta_1^{D\bar{D}^*} + \delta_0^{D\bar{D}^*}}{2} - \frac{\delta_1^{D^* \bar{D}} + \delta_0^{D^* \bar{D}}}{2} \end{aligned} \quad (20)$$

and

$$\begin{aligned} R_{+-} &= \frac{\cos[(\delta_1^{D\bar{D}^*} - \delta_0^{D\bar{D}^*})/2]}{\cos[(\delta_1^{D^* \bar{D}} - \delta_0^{D^* \bar{D}})/2]}, \\ R_{00} &= \frac{\sin[(\delta_1^{D\bar{D}^*} - \delta_0^{D\bar{D}^*})/2]}{\sin[(\delta_1^{D^* \bar{D}} - \delta_0^{D^* \bar{D}})/2]}. \end{aligned} \quad (21)$$

In obtaining these results we have neglected the small penguin effects. The decay constants and form factors in the expression of ζ , coming from decomposition of the hadronic matrix elements $Z_{D\bar{D}^*}$ and $Z_{D^* \bar{D}}$ given in Eq. (5), are self-explanatory. Note that $R_{+-} = 1$ and $\delta = \delta_1^{D\bar{D}^*} - \delta_1^{D^* \bar{D}}$ hold, if one takes the limit $\delta_1^f = \delta_0^f$ (for each final state f), in which the decay modes $B_d \rightarrow D^{(*)0} \bar{D}^{(*)0}$ become forbidden. In the presence of rescattering effects, i.e., $R_{+-} \neq 1$, the extraction of β from $\Delta(D^{*+} D^-)$ and $\bar{\Delta}(D^+ D^{*-})$ seems difficult. However, it is possible to determine the isospin phase difference $\delta_1^f - \delta_0^f$ from the triangle relation in Eq. (4), if the relevant rates of three (one charged B and two neutral B) decay modes are measured in experiments. The observation of $B_d \rightarrow D^{*0} \bar{D}^0$ and $D^0 \bar{D}^{*0}$ transitions turns out to be crucial: (a) if their branching ratios in comparison with those of $B_d \rightarrow D^{*+} D^-$ and $D^+ D^{*-}$ decays are too small to be detected, then the final-state rescattering effects should be negligible and the naive factorization approach with $R_{+-} = 1$ might work well. (b) If their branching ratios are more or less comparable with those of $B_d \rightarrow D^{*+} D^-$ and $D^+ D^{*-}$ decays, then a quantitative isospin analysis should be available, allowing us to extract the isospin phase differences and determine the magnitudes of R_{+-} and R_{00} . In both cases, ζ can experimentally be determined and the result can be confronted with the theoretical value of ζ calculated by inputting relevant decay constants and form factors.

For $B_d \rightarrow D^{*+} D^{*-}$ and $B_d \rightarrow D^{*0} \bar{D}^{*0}$ decay modes the indirect CP asymmetries need a more careful analysis. Note that the transition amplitude of $B_d^0 \rightarrow D^{*+} D^{*-}$ (or $D^{*0} \bar{D}^{*0}$) is a sum of three different components, i.e., the S -, D -, and P -wave amplitudes [21]. Without loss of generality the hadronic matrix elements Z and \bar{Z} for $B_d \rightarrow D^{*+} D^{*-}$ can be written as

$$\begin{aligned}
Z &= \tilde{a}(\boldsymbol{\epsilon}_+ \cdot \boldsymbol{\epsilon}_-) + \frac{\tilde{b}}{m_{D^*}^2}(p_0 \cdot \boldsymbol{\epsilon}_+)(p_0 \cdot \boldsymbol{\epsilon}_-) \\
&\quad + i \frac{\tilde{c}}{m_{D^*}^2}(\boldsymbol{\epsilon}^{\alpha\beta\gamma\delta} \boldsymbol{\epsilon}_{+\alpha} \boldsymbol{\epsilon}_{-\beta} p_{+\gamma} p_{0\delta}), \\
\bar{Z} &= \tilde{a}(\boldsymbol{\epsilon}_+ \cdot \boldsymbol{\epsilon}_-) + \frac{\tilde{b}}{m_{D^*}^2}(p_0 \cdot \boldsymbol{\epsilon}_+)(p_0 \cdot \boldsymbol{\epsilon}_-) \\
&\quad - i \frac{\tilde{c}}{m_{D^*}^2}(\boldsymbol{\epsilon}^{\alpha\beta\gamma\delta} \boldsymbol{\epsilon}_{+\alpha} \boldsymbol{\epsilon}_{-\beta} p_{+\gamma} p_{0\delta}), \tag{22}
\end{aligned}$$

where $\boldsymbol{\epsilon}_\pm$ denotes the polarization of $D^{*\pm}$ meson, p_0 and p_\pm stand, respectively, for the momenta of B_d and $D^{*\pm}$ mesons, and $(\tilde{a}, \tilde{b}, \tilde{c})$ are real scalars without the penguin effects. In terms of the decay constants and form factors, \tilde{a} , \tilde{b} , and \tilde{c} explicitly read

$$\begin{aligned}
\tilde{a} &= m_{D^*} f_{D^*} (m_B + m_{D^*}) A_1^{BD^*}(m_{D^*}^2), \\
\tilde{b} &= -2m_{D^*}^3 f_{D^*} \frac{A_2^{BD^*}(m_{D^*}^2)}{m_B + m_{D^*}}, \\
\tilde{c} &= -2m_{D^*}^3 f_{D^*} \frac{V^{BD^*}(m_{D^*}^2)}{m_B + m_{D^*}}. \tag{23}
\end{aligned}$$

In the absence of angular analysis one may first calculate the ratio \bar{Z}/Z by summing over the polarizations of two final-state vector mesons [21], and then calculate the CP -violating quantities $\Delta(D^{*+}D^{*-})$ and $\bar{\Delta}(D^{*+}\bar{D}^{*-})$ in the neglect of small penguin effects. We finally arrive at

$$\begin{aligned}
\Delta(D^{*+}D^{*-}) &= \Delta(D^{*0}\bar{D}^{*0}) = +\sin 2\beta \frac{1-\chi}{1+\chi}, \\
\bar{\Delta}(D^{*+}D^{*-}) &= \Delta(D^{*0}\bar{D}^{*0}) = -\sin 2\beta \frac{1-\chi}{1+\chi}, \tag{24}
\end{aligned}$$

where

$$\chi = \frac{2(x^2-1)\tilde{c}^2}{(2+x^2)\tilde{a}^2 + (x^2-1)^2\tilde{b}^2 + 2x(x^2-1)\tilde{a}\tilde{b}} \tag{25}$$

with $x = (m_B^2 - 2m_{D^*}^2)/(2m_{D^*}^2) = 2.45$. Clearly the dilution parameter χ results from the P -wave contribution to the overall decay amplitudes. If we adopt the simple monopole model for relevant form factors [22], it turns out that $V^{BD^*}(m_{D^*}^2) = 0.784$, $A_1^{BD^*}(m_{D^*}^2) = 0.715$, and $A_2^{BD^*}(m_{D^*}^2)$

$= 0.753$. Accordingly $\tilde{b}/\tilde{a} = -0.160$ and $\tilde{c}/\tilde{a} = -0.167$. The relationship $\tilde{b}/\tilde{a} \approx \tilde{c}/\tilde{a}$ is indeed guaranteed by the heavy quark symmetry, which makes the form factors appearing in Eq. (23) related to one another. In this symmetry limit we obtain [20]

$$\frac{\tilde{b}}{\tilde{a}} = \frac{\tilde{c}}{\tilde{a}} = -\frac{2m_{D^*}^2}{m_B(m_B + 2m_{D^*})}, \tag{26}$$

amounting to -0.164 . Then we get $(1-\chi)/(1+\chi) \approx 0.89$, a value deviating only about 11% from unity. Note that this dilution factor can also be determined from measuring the ratio $\Delta(D^{*+}D^{*-})/\Delta(D^+D^-)$. From this estimation we find that the P -wave dilution effect is not very significant, therefore extracting the CP -violating parameter $\sin 2\beta$ from $B_d \rightarrow D^{*+}D^{*-}$ decays remains possible even if a delicate angular analysis is not made.

V. SUMMARY

We have analyzed direct and indirect CP asymmetries in B_d^0 vs $\bar{B}_d^0 \rightarrow D^+D^-$, $D^{*+}D^-$, D^+D^{*-} , and $D^{*+}D^{*-}$ decays. The isospin amplitudes of these transitions are calculated with the help of the effective weak Hamiltonian and the factorization approximation, and the long-distance interactions at the hadron level are taken in to account by introducing elastic rescattering phases for two isospin channels of the final-state mesons. We have shown that in this factorization approach the direct CP violation is irrelevant to the final-state rescattering effects, i.e., it is governed only by the short-distance penguin mechanism. The magnitude of direct CP violation is estimated to be 3% in $B_d \rightarrow D^+D^-$ decay modes. The same amount of CP violation can manifest itself in the charged B_u decays into $D^+\bar{D}^0$ and $D^-\bar{D}^0$ states, which are easier to be measured at B -meson factories. We have demonstrated that the penguin effects on indirect CP asymmetries in $B_d \rightarrow D^{(*)+}D^{(*)-}$ decays are insignificant and even negligible. While the long-distance rescattering has no effect on indirect CP violation in $B_d \rightarrow D^+D^-$ and $D^{*+}D^{*-}$ transitions, it may affect that in $B_d \rightarrow D^\pm D^{*\mp}$ modes, whose final states are non- CP eigenstates. We have calculated the P -wave contribution to the indirect CP asymmetry in $B_d \rightarrow D^{*+}D^{*-}$ decays. The corresponding dilution effect is found to be insignificant, therefore observation of large CP violation remains under expectation even without the delicate angular analysis.

It is certainly necessary to test the validity of our factorization hypothesis, on which most of the aforementioned results depend. To do so a measurement of $B_d \rightarrow D^{(*)0}\bar{D}^{(*)0}$ transitions will be particularly helpful. On the one hand, if the branching ratios of these decay modes are too small compared with those of $B_d \rightarrow D^{(*)+}D^{(*)-}$ transitions, then the final-state rescattering effects should be negligible and the naive factorization approximation might work well. On the other hand, if the branching ratios of B_d decays into $D^{(*)0}\bar{D}^{(*)0}$ and $D^{(*)+}D^{(*)-}$ states are found to be more or less comparable in magnitude, then a quantitative isospin

analysis should become available, allowing us to extract the isospin phase differences and control the final-state rescattering effects. In any case much can be learned about the factorization hypothesis and its applicability in B decays into two heavy charmed mesons.

In conclusion, the observation of direct and indirect CP asymmetries in $B_d \rightarrow D^{(*)+} D^{(*)-}$ decays is promising at B -meson factories. They are expected to provide us some valuable information about the weak phase β as well as the penguin and rescattering effects in nonleptonic B decays.

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