Spin-dependent structure functions of the proton in a constituent quark model

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Spin-dependent structure functions $g_1^p(x,Q^2)$ and $g_2^p(x,Q^2)$ for the proton have been derived at the valency quark level in a relativistic independent quark model with an effective potential in equally mixed scalar-vector harmonic form. The level of accuracy of the functional form, so derived at the model scale of a low Q^2, has been tested through a reasonable verification of the relevant sum rules due to Ellis and Jaffe in the case of $g_1^p(x,Q^2)$ as well as Burkhardt and Cottingham in the case of $g_2^p(x,Q^2)$.The twist-2 and the twist-3 components of $g_2^p(x,Q^2)$ have also been separated at this level without effecting any further Q^2 evolution. But $g_1^p(x,Q^2)$ has been evolved from a reference scale of $Q_0^2 \simeq 0.1\,\mathrm{GeV}^2$ to $Q^2 = 10.7\,\mathrm{GeV}^2$ to find a satisfactory comparison with the available experimental data.

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I. INTRODUCTION

Spin-dependent effects in electron-nucleon deep-inelastic scattering are associated with two measurable polarized structure functions $g_1^p(x,Q^2)$ and $g_2^p(x,Q^2)$ defined in the Bjorken limit in a covariant expansion of the antisymmetric part of the relevant hadronic tensor involved in the process. In the naive parton model of the nucleon these structure functions lead to the well known sum rules with important implications on the spin structure of the nucleon. Experimental measurements of longitudinally polarized structure function $g_1^p(x,Q^2)$ [1,2] as well as the possible measurements already planned for the transversely polarized structure function $g_2^p(x,Q^2)$, have initiated a resurgence of interest during the last few years in further theoretical investigation. Starting with the pioneering work of Jaffe [3] there has been many attempts to calculate these polarized structure functions theoretically using the familiar quark models of hadrons such as the MIT-bag model [4], which have been successful in describing much of the low energy phenomenology. Since then, much insight into the theoretical description of nucleon structure functions and parton distributions inside the nucleon realized at very high energy have been gained through the application of such models in their successive improved versions attempting to meet the inherent inadequacies. However, the information now available from experiments in deep-inelastic scattering can provide powerful constraints on all such theoretical models adopted to describe the parton picture of the hadron at high energy scale. Therefore we intend here to derive explicit functional forms of the polarized structure functions $g_1^p(x,Q^2)$ and $g_2^p(x,Q^2)$ in an alternative constituent quark model of relativistic independent quarks confined by an effective equally mixed scalar-vector harmonic potential. The predictive power of this model has been demonstrated successfully in wide ranging low energy hadronic phenomena such as hadron spectroscopy and static hadronic properties [5], weak and electromagnetic decays of the light and heavy mesons [6]. This model has also been quite successful in describing the elastic form factors and charge radii of nucleon, pion, and K mesons and electromagnetic polarizability of proton [5,7]. Although there has been many different constituent quark models based on various potentials or bags with more or less equal success in addressing similar problems in low energy hadronic phenomenology; the present model had singular advantage of simplicity and tractability leading to closed form solution for various relevant expressions with no further free parameters once they had been fixed at the level of describing the spectroscopy and static properties of hadrons. In view of this the motivation of the present work has been to extend the model applicability to the study of the polarized structure functions initially in order to establish ultimately the useful link between the low-energy constituent quark picture of the nucleon with its high energy parton picture as realized from deep-inelastic scattering.

Adopting this model for evaluating the spin-dependent structure functions $g_1^p(x,Q^2)$ and $g_2^p(x,Q^2)$; we take a static no gluon approximation for the nucleon. As a result of this approximation we consider the deep-inelastic scattering of electrons off valency quarks only inside the nucleon. The model solutions for the bound valency quark eigenmodes provide the necessary model input in expressing the quark field operators defining ultimately the electromagnetic current that appears in the relevant hadronic tensor. This enables one to derive the functional forms of the spin dependent structure functions; which is not possible in the naive parton model. There is also another advantage of the present model over other constituent quark models such as MIT-bag model employed earlier for the same in the sense that the explicit functional forms of the structure functions are obtainable here in solvable closed forms. Therefore study of their behavior as functions of the Bjorken variable x becomes straightforward and transparent. It is already well known that the structure functions evaluated in any such constituent quark model neither show the expected Regge behavior at small x nor do they vanish identically beyond the kinematically allowed region in $x \geqslant 1$. These problems are simply the artifacts of the approximations involved within which such models in general are employed. We are therefore fully aware of the limitation of our model here lacking in translational invariance that leads to the support problem beyond the kinematic boundary at $x=1$. However, we find that the support problem encountered in the present derivation is rather minimal since the structure functions falloff very rapidly to zero beyond $x \geqslant 1$. Therefore we prefer to postpone considering appropriate Lorentz boosting to our later works aiming for quantitative precision in the model prediction. Our purpose here is to first realize the structure functions at the model scale as explicit functions of the Bjorken variable x in a closed form so as to provide a model based input for QCD evolution to the experimentally relevant Q^2 region. Apart from their inadequate behavior beyond the kinematic boundaries, overall adequacy of these input functions can be tested through the constraints of the relevant sum rules. If we extend the first moment integrals beyond $x>1$ to infinity we can analytically evaluate the relevant sum rules due to Ellis and Jaffe in case of $g_1^p(x,Q^2)$ and due to Burkhardt and Cottingham in case of $g_2^p(x,Q^2)$. In view of the fact that in the present model SU(2) flavor symmetry is retained through the quark mass input $m_u=m_d$; the neutron structure functions $g_1^n(x,Q^2)$ and $g_2^n(x,Q^2)$ become identically zero.Therefore, within these limitations of the present model the same Ellis Jaffe sum-rule expression also represents the Bjorken sum rule. These analytical verifications provide a useful check of the level of accuracy of the functional forms so derived in the model for the structure functions $g_1^p(x,Q^2)$ and $g_2^p(x,Q^2)$. We can also extract the Wandzura-Wilczek (twist-2) component and the twist-3 component of $g_2^p(x,Q^2)$ at this level. We find a nonzero component of twist-3 which is comparable with other model predictions. We then evolve the $g_1^p(x,Q^2)$ from a model scale of low Q^2 to the experimentally accessible Q^2 region to compare our predictions with the available data.

The work presented here is organized as follows. In Sec. II, we provide the appropriate expression for the hadronic tensor describing the deep-inelastic electron-nucleon scattering with relevant kinematics from which we extract suitable tensor components representing the longitudinally polarized structure function $g_1^p(x,Q^2)$ and transversely polarized structure function $g_2^p(x,Q^2)$ in the Bjorken limit for the target nucleon taken at rest. Section III provides the relevant model inputs and derives the explicit functional forms of the spin dependent structure functions. In Sec. IV we discuss the results of our derivation at the model scale of low Q^2 to establish a reasonable level of accuracy of the functional forms obtained for the structure functions through analytic verification of the relevant sum rules. We also analyze and extract here twist-2 and twist-3 components of $g_2^p(x)$. Finally we provide here the results of the Q^2 evolution of $g_1^p(x,Q^2)$ to experimentally relevant high Q^2 region in comparison with the available data. Section V is a brief summary and conclusion.

II. BASIC FORMALISM

The hadronic tensor relevant for the process of polarized deep-inelastic electron-nucleon scattering is given by

$$W_{\mu\nu}(q,P,S)=\frac{1}{4\pi}\int d^4\xi e^{iq\xi}\langle P,S|[J_\mu(\xi),J_\nu(0)]|P,S\rangle, \tag{2.1}$$

where q is the virtual photon four-momentum and P and S are the four- momentum and spin of the target nucleon (with $P^\mu P_\mu=M^2, S^\mu S_\mu=-M^2$ and $P^\mu S_\mu=0$), respectively. The conventional kinematic variable Q^2 and ν (or Q^2 and x) are defined as $Q^2=-q^2>0$ and $x=Q^2/2\nu$; when $\nu=Pq$ and $0\leqslant x\leqslant 1$. Then in the rest frame of the target nucleon one can have $P\equiv(M,0,0,0)$ and $q\equiv(\nu/M,0,0,\sqrt{\nu^2/M^2+Q^2})$. The nucleon state $|P,S\rangle$ in Eq. (2.1) is normalized as

$$\langle P,S|P',S'\rangle=(2\pi)^3 2E\,\delta^3(\mathbf{P}-\mathbf{P}')\,\delta_{SS'}\,. \tag{2.2}$$

However, in a constituent quark model of target nucleon considered usually at rest; the nucleon state is described in terms of its normalized SU(6) spin flavor configuration denoted shortly by $|N,S\rangle$ so that $|P,S\rangle$ in Eq. (2.1) can be expressed as

$$|P,S\rangle=[(2\pi)^3 2M\,\delta^3(0)]^{1/2}|N,S\rangle. \tag{2.3}$$

The electromagnetic current of the target nucleon is taken here in the form

$$J_\mu(\xi)=\sum_q e_q\bar{\psi}_q(\xi)\gamma_\mu\psi_q(\xi), \tag{2.4}$$

where e_q is the electric charge of the valency quark of flavor q inside the nucleon. It is possible to recast Eq. (2.1) in a more useful form [3] corresponding to the target nucleon at rest as

$$W_{\mu\nu}(q,S)=\frac{M}{2\pi}\int_{-\infty}^{+\infty}dt\,e^{iq_0t}\int d^3\mathbf{r}_1\int d^3\mathbf{r}_2 e^{-i\mathbf{q}\cdot(\mathbf{r_1}-\mathbf{r_2})} \times\langle N,S|[J_\mu(\mathbf{x_1},t),J_\nu(\mathbf{x_2},0)]|N,S\rangle. \tag{2.5}$$

Now expanding the current commutator in Eq. (2.5) and taking the quark propagator appearing in the expansion as the free Dirac propagator under an impulse approximation written in the zero-mass limit as

$$\lim_{m\to 0}S_D(x)=\frac{1}{(2\pi)^3}\int d^4k\,\not{k}\,\epsilon(k_0)\,\delta(k^2)e^{\pm ikx} \tag{2.6}$$

we can extract the antisymmetric part of the hadronic tensor in the form

$$W_{\mu\nu}^{(a)}(q,S)=i\epsilon_{\mu\nu\lambda\sigma}A^{\lambda\sigma}(q,S) \tag{2.7}$$

when

$$A^{\lambda\sigma}(q,S)=\frac{M}{(2\pi)^4}\sum_q e_q^2\int d^4k\, k^\lambda \epsilon(k_0)\,\delta(k^2)\times\int_{-\infty}^{+\infty}dt\, e^{i(q_0+k_0)t}\times\int d^3\mathbf{r_1}d^3\mathbf{r_2}e^{-i(\mathbf{q}+\mathbf{k})\cdot(\mathbf{r_1}-\mathbf{r_2})}\langle N,S|B^\sigma|N,S\rangle \tag{2.8}$$

and

$$\langle N,S|B^\sigma|N,S\rangle=\langle N,S|[\overline{\psi}_q(\mathbf{r_1},t)\gamma^\sigma\gamma^5\psi_q(\mathbf{r_2},0)+\overline{\psi}_q(\mathbf{r_2},0)\gamma^\sigma\gamma^5\psi_q(\mathbf{r_1},t)]|N,S\rangle. \tag{2.9}$$

In polarized electron-nucleon scattering, spin dependent effects are related to this antisymmetric part $W^{(a)}_{\mu\nu}$; which by Lorentz invariance and gauge-invariance can be constructed from scalar functions $g_1(x,Q^2)$, $g_2(x,Q^2)$, and $g_3(x,Q^2)$ in general in the following form:

$$W^{(a)}_{\mu\nu}(q,S)=i\epsilon_{\mu\nu\lambda\sigma}\left(\frac{q^\lambda}{\nu}\right)\left[S^\sigma g_1(x,Q^2)+\left\{S^\sigma-\frac{(q\cdot S)}{\nu}P^\sigma\right\}g_2(x,Q^2)+M^2\frac{(q\cdot S)}{\nu}q^\sigma g_3(x,Q^2)\right]. \tag{2.10}$$

Here $g_1(x,Q^2)$ is the longitudinally polarized structure function, whereas $g_1(x,Q^2)+g_2(x,Q^2)$ gives the transversely polarized structure function. But $g_3(x,Q^2)$ does not contribute to any measurable structure function for which it is ordinarily omitted in the expansion. However, its role in the expansion is essential in the extraction of the correct tensor component $A^{\lambda\sigma}$ for the measurable structure functions $g_1(x,Q^2)$ and $g_2(x,Q^2)$, which in the Bjorken limit in QCD, scale to $g_1(x)$ and $g_2(x)$, respectively. Now comparing Eq. (2.10) with Eq. (2.7) to (2.9) one obtains the relation

$$A^{\lambda\sigma}(q,S)=\left(\frac{q^\lambda}{\nu}\right)\left[S^\sigma g_1(x,Q^2)+\left\{S^\sigma-\frac{(q\cdot S)}{\nu}P^\sigma\right\}g_2(x,Q^2)+M^2\frac{(q\cdot S)}{\nu}q^\sigma g_3(x,Q^2)\right] \tag{2.11}$$

then evaluating the relevant components of $A^{\lambda\sigma}(q,S)$ given in Eq. (2.7) to Eq. (2.9) in the Bjorken limit using the model inputs for the target nucleon at rest polarized longitudinally (L) with $S_L^\mu\equiv(0,0,0,M)$ or transversely (T) with $S_T^\mu\equiv(0,M,0,0)$, respectively, we can obtain explicit functional forms for $g_1(x,Q^2)$ and $g_2(x,Q^2)$ in the following manner:

$$g_1(x,Q^2)=[A^{03}(q,S_L)-A^{30}(q,S_L)], \tag{2.12}$$

$$g_2(x,Q^2)=[A^{01}(q,S_T)+A^{30}(q,S_L)-A^{03}(q,S_L)]. \tag{2.13}$$

III. POLARIZED STRUCTURE FUNCTIONS

The longitudinally polarized structure function $g_1(x,Q^2)$ and the transversely polarized structure function $g_2(x,Q^2)$ for the target nucleon at rest can be obtained in a constituent quark model at the model scale of some low $Q^2\simeq\mathcal{O}(\Lambda^2_{\rm QCD})$ by evaluating the relevant tensor components $A^{\lambda\sigma}(q,S_L)$ or $A^{\lambda\sigma}(q,S_T)$ as per the Eqs. (2.12) and (2.13). In doing so, one can express the quark field operators $\psi_q(\mathbf{r},t)$ and hence $\overline{\psi}_q(\mathbf{r},t)$ in the form of their possible expansions in terms of the bound quark/antiquark eigenmodes derivable in principle from a constituent quark model as

$$\psi_q(\mathbf{r},t)=\sum_\zeta [b_{q\zeta}\Phi^+_{q\zeta}(\mathbf{r})e^{-iE_{q\zeta}t}+\tilde{b}^\dagger_{q\zeta}\Phi^{(-)}_{q\zeta}(\mathbf{r})e^{iE_{q\zeta}t}], \tag{3.1}$$

where $\tilde{b}^\dagger_{q\zeta}$ is antiquark creation operator and $b_{q\zeta}$ is the quark annihilation operator corresponding to flavor q in the eigenmodes (ζ) with $\zeta\equiv(n,k,j)$ representing the set of all the Dirac quantum numbers for the complete set of all possible eigenmodes $\Phi^\pm_{q\zeta}(\mathbf{r})$ with their corresponding energy eigenvalues $E_{q\zeta}$ obtainable in the model. However, in the actual calculation of the relevant expectation value $\langle NS|B^\sigma|NS\rangle$ in Eq. (2.9) over the nucleon ground state (with no gluon approximation) only the ground state positive energy eigenmodes $\Phi^+_{q\zeta_0}(\mathbf{r})$ would effectively contribute.

Now referring to our specific model of relativistic independent quarks with an equally mixed scalar-vector harmonic potential taken phenomenologically in the form [5,6]

$$V(r)=(1/2)(1+\gamma^0)(ar^2+V_0). \tag{3.2}$$

The ground state positive energy eigenmode $\Phi^+_{q\zeta_0}(\mathbf{r})$ is realized in the following form:

$$\Phi^+_{q\zeta_0}(\mathbf{r})=\frac{1}{\sqrt{4\pi}}\begin{pmatrix} ig_q(r)/r \\ \vec{\sigma}\cdot\hat{\mathbf{r}}f_q(r)/r\end{pmatrix}\chi_{\zeta_0}, \tag{3.3}$$

where

$$g_q(r)=N_q(r/r_{0q})e^{-r^2/2r^2_{0q}},$$
$$f_q(r)=-\frac{N_q}{\lambda_q r_{0q}}(r/r_{0q})^2e^{(-r^2/2r^2_{0q})} \tag{3.4}$$

when with $E'_q=(E_{q\zeta_0}-V_0/2)$, $m'_q=(m_q+V_0/2)$, $\lambda_q=(E'_q+m'_q)$, and $r_{0q}=(a\lambda_q)^{-1/4}$ the normalization factor N_q is given by

$$N_q^2=\frac{8\lambda_q}{\sqrt{\pi}r_{0q}}\frac{1}{(3E'_q+m'_q)} \tag{3.5}$$

and the quark binding energy $E_q = E_{q\zeta_0}$ in the ground state is derivable from the bound state condition

$$(\lambda_q/a)^{1/2}(E_q' - m_q') = 3. \tag{3.6}$$

Finally χ_{ζ_0} in the Eq. (3.3) stands for the Pauli spinors $\chi_\uparrow = \binom{1}{0}$ and $\chi_\downarrow = \binom{0}{1}$. This would provide necessary and sufficient model inputs for the calculation of the required tensor components $A^{\lambda\sigma}(q,S)$.

Now substituting Eq. (3.1) in Eq. (2.9) and keeping only the relevant contributing terms in the expansion of $\langle NS|B^\sigma|NS\rangle$, Eq. (2.8) can be simplified to

$$A^{\lambda\sigma}(q,S) = [A_1^{\lambda\sigma}(q,S) + A_2^{\lambda\sigma}(q,S)] \tag{3.7}$$

when

$$\begin{aligned}A_1^{\lambda\sigma}(q,S) = {}& \frac{M}{(2\pi)^3}\sum_{q,\zeta_0} C_{q\zeta_0}\int d^3\mathbf{r_1}d^3\mathbf{r_2}d^4k\,\epsilon(k_0)\,\delta^4(k^2)\\ &\times\delta(k_0+q_0+E_q)\\ &\times[\epsilon(\zeta_0)\overline{\Phi}^{+}_{q\zeta_0}(\mathbf{r_1})k^\lambda\gamma^\sigma\gamma^5\Phi^{(+)}_{q\zeta_0}(\mathbf{r_2})]e^{-i\mathbf{K}\cdot(\mathbf{r_1}-\mathbf{r_2})}\end{aligned} \tag{3.8}$$

and

$$\begin{aligned}A_2^{\lambda\sigma}(q,S) = {}& \frac{M}{(2\pi)^3}\sum_{q,\zeta_0} C_{q\zeta_0}\int d^3\mathbf{r_1}d^3\mathbf{r_2}d^4k\,\epsilon(k_0)\,\delta^4(k^2)\\ &\times\delta(k_0+q_0-E_q)\\ &\times[\epsilon(\zeta_0)\overline{\Phi}^{+}_{q\zeta_0}(\mathbf{r_2})k^\lambda\gamma^\sigma\gamma^5\Phi^{(+)}_{q\zeta_0}(\mathbf{r_1})]\\ &\times e^{-i\mathbf{K}\cdot(\mathbf{r_1}-\mathbf{r_2})},\end{aligned} \tag{3.9}$$

where $\mathbf{K} = (\mathbf{q}+\mathbf{k})$ is the momentum of the struck quark such that $|\mathbf{K}| = K > K_m = (|\mathbf{q}| - |\mathbf{k}|)$ which can be reasonably assumed to be much less than $(q_0, |\mathbf{q}|$ and $|\mathbf{k}|)$ in the Bjorken limit. The delta function appearing in Eqs. (3.8) and (3.9) would imply the respective values for$|\mathbf{k}| = k$ as $k_+ = (q_0 + E_q)$ and $k_- = (q_0 - E_q)$. This would lead to certain relevant kinematic factors necessary to simplify further Eqs. (3.8) and (3.9) in the Bjorken limit as

$$\begin{aligned}K_m = K_m^{\pm}(x) &= |(E_q \mp Mx)|,\\ \cos\theta_K &\simeq (Mx \mp E_q)/K,\\ \cos\theta_K\cos\theta_k &\simeq -(Mx \mp E_q)/K,\\ d(\cos\theta_k)k_\pm^2 &\simeq K\,dK.\end{aligned} \tag{3.10}$$

Finally $C_{q\zeta_0}$ in Eqs. (3.8) and (3.9) stands for

$$C_{q\zeta_0} = e_q^2\langle NS|b^\dagger_{q\zeta_0}b_{q\zeta_0}\epsilon(\zeta_0)|NS\rangle \tag{3.11}$$

when $\epsilon(\zeta_0) = \pm 1$ for ζ_0 representing spin-up and spin-down spinor states respectively. With SU(2) symmetry assumed in the model in the *(u,d)* flavor sector, it can be shown that the terms in square bracket in Eqs. (3.8) and (3.9) would be independent of flavor and spin quantum numbers so as to be decoupled from $\Sigma_{q\zeta_0}C_{q\zeta_0}$ which can be evaluated for the target nucleon as

$$\begin{aligned}\sum_{q\zeta_0} e_q^2\langle NS|b^\dagger_{q\zeta_0}b_{q\zeta_0}\epsilon(\zeta_0)|NS\rangle &= 5/9 \text{ for spin up proton}\\ &= 0 \text{ for neutron with any spin.}\end{aligned} \tag{3.12}$$

This would lead to the fact that in such a model with SU(2) symmetry the spin dependent structure functions $g_1^n(x,Q^2)$ and $g_2^n(x,Q^2)$ for neutron would be identically zero. Then for the target proton with longitudinal polarization and transverse polarization the relevant tensor components appearing in the Eqs. (2.12) and (2.13) can be evaluated in the following form.

$$\begin{aligned}A_L^{03} = {}& \frac{5M}{18\pi}\int_{K_m^+}^{\infty} dK\,K[y_0^2(K) + (2K_m^{+2}/K^2 - 1)y_1^2(K)]\\ &+ (K_m^+ \to K_m^-),\end{aligned} \tag{3.13}$$

$$\begin{aligned}A_L^{30} = {}& -\frac{5M}{18\pi}\int_{K_m^+}^{\infty} dK\,K[2(K_m^+/K)y_0(K)y_1(K)]\\ &+ (K_m^+ \to K_m^-),\end{aligned} \tag{3.14}$$

$$\begin{aligned}A_T^{01} = {}& \frac{5M}{18\pi}\int_{K_m^+}^{\infty} dK\,K[y_0^2(K) - (K_m^{+2}/K^2)y_1^2(K)]\\ &+ (K_m^+ \to K_m^-),\end{aligned} \tag{3.15}$$

where

$$\begin{aligned}y_0(K) &= \int_0^\infty dr\,r g_q(r) j_0(Kr),\\ y_1(K) &= \int_0^\infty dr\,r f_q(r) j_1(Kr).\end{aligned} \tag{3.16}$$

Then using Eqs. (2.12) and (2.13) we can obtain

$$\begin{aligned}g_1^p(x,Q^2) = {}& \frac{5M}{18\pi}\int_{K_m^+}^{\infty} dK\,K[y_0^2(K) + (2K_m^{+2}/K^2 - 1)y_1^2(K)\\ &+ 2(K_m^+/K)y_0(K)y_1(K)] + (K_m^+ \to K_m^-),\end{aligned} \tag{3.17}$$

$$\begin{aligned}g_2^p(x,Q^2) = {}& \frac{5M}{18\pi}\int_{K_m^+}^{\infty} dK\,K[(1 - 3K_m^{+2}/K^2)y_1^2(K)\\ &- 2(K_m^+/K)y_0(K)y_1(K)] + (K_m^+ \to K_m^-).\end{aligned} \tag{3.18}$$

Now using the model solutions for $g_q(r)$ and $f_q(r)$ from Eq. (3.4), we can obtain

$$y_0(K)=\left(\frac{\pi}{2}\right)^{1/2} N_q r_{oq}^2 e^{(-r_{oq}^2 K^2/2)},$$

$$y_1(K)=-\left(\frac{\pi}{2}\right)^{1/2}\left(\frac{N_q}{\lambda_q}\right) r_{0q}^2 K e^{(-r_{oq}^2 K^2/2)}. \tag{3.19}$$

Thus one obtains the explicit functional forms for $g_1^p(x,Q^2)$ and $g_2^p(x,Q^2)$ in analytically closed forms as

$$g_1^p(x,Q^2)=\frac{5MN_q^2 r_{0q}^2}{72}\left[\left(1+\frac{K_m^{+2}}{\lambda_q^2}-\frac{2K_m^+}{\lambda_q}-\frac{1}{\lambda_q^2 r_{oq}^2}\right)\times e^{-r_{oq}^2 K_m^{+2}}+(K_m^+\to K_m^-)\right], \tag{3.20}$$

$$g_2^p(x,Q^2)=\frac{5MN_q^2 r_{oq}^2}{72}\left[\left(\frac{1}{\lambda_q^2 r_{oq}^2}+\frac{2K_m^+}{\lambda_q}-\frac{2K_m^{+2}}{\lambda_q^2}\right)\times e^{-r_{oq}^2 K_m^{+2}}+(K_m^+\to K_m^-)\right], \tag{3.21}$$

where in these final expressions $K_m^{\pm}=(E_q\mp Mx)$.

IV. RESULTS AND DISCUSSIONS

We have thus obtained analytically in closed forms, explicit expressions for the polarized structure functions $g_1^p(x,Q^2)$ and $g_2^p(x,Q^2)$ for the proton as functions of the Bjorken variable x. These functional forms in fact refer to the model scale of a low Q^2 of the order of $\Lambda_{\rm QCD}^2$, which is not explicit in the derivation. Therefore for quantitative comparison with the available experimental data particularly in case of $g_1^p(x,Q^2)$, these results need appropriate Q^2 evolution to experimentally relevant higher Q^2 region according to the QCD evolution equations [8]. However, before applying Q^2 evolution, we would like to discuss certain characteristic features of the explicit functional forms obtained in the model for these structure functions and test their level of accuracy by attempting a verification of the relevant sum rules to a reasonable extent even at the model scale of low Q^2.

First of all we notice that both $g_1^p(x)$ and $g_2^p(x)$ are symmetric functions of the Bjorken variable x and they do not exhibit Regge behavior at $x\to 0$ limit. It is also obvious that they do not vanish identically beyond the physical region for $x\geqslant 1$; thus being plagued with the so called support problem. These features are common artifacts of such constituent quark models lacking in translational invariance. Even then, overlooking the deficiencies in their small and large x behavior, we can check their correctness otherwise by evaluating their first moment integrals to test the well known sum rules. Since both $g_1^p(x,Q^2)$ and $g_2^p(x,Q^2)$ would falloff very rapidly beyond $x=1$ and also they are symmetric in x, the limits of the first moment integrals in $g_1^p(x,Q^2)$ and $g_2^p(x,Q^2)$ can be extended analytically beyond the physical region $(0\leqslant x \leqslant 1)$ to the entire region so as to realize the evaluation of sum-rule integrals $\Gamma_1^p(EJ)$ due to Ellis and Jaffe [9] and $\Gamma_2^p(BC)$ due to Bukhardt and Cottingham [10] in an analytic manner as follows:

$$\Gamma_1^p(EJ)=\int_0^1 dx\, g_1^P(x,Q^2)\simeq 1/2\int_{-\infty}^{+\infty} dx\, g_1^p(x,Q^2) = (1/6)\frac{5(5E_q'+7m_q')}{9(3E_q'+m_q')}, \tag{4.1}$$

$$\Gamma_2^p(BC)=\int_0^1 dx\, g_2^p(x,Q^2)\simeq 1/2\int_{-\infty}^{+\infty} dx\, g_2^p(x,Q^2)=0. \tag{4.2}$$

However, for a numerical evaluation we may take the model parameters and other required model quantities from its previous applications [5–7] as

$$(a,V_0)=(0.017166\,\text{GeV}^3,-0.1375\,\text{GeV}),$$

$$(m_u,m_d)=(0.01\,\text{GeV},0.01\,\text{GeV}),$$

$$(E_q,\lambda_q)=(0.45129\,\text{GeV},0.46129\,\text{GeV}),$$

$$(N_q,r_{0q})=(0.64318\,\text{GeV}^{1/2},3.35227\,\text{GeV}^{-1}). \tag{4.3}$$

But since the physical mass M of the nucleon in this model is realized only after taking into account other possible residual effects [5] in a perturbative manner in obtaining the correct E_q, we prefer here to take $E_q=\frac{1}{3}M_p$ where $M_p=0.938\,\text{GeV}$ is the physical mass of the proton. Then $\Gamma_1^p(EJ)$ and $\Gamma_2^p(BC)$ from its exact numerical integration are obtained as

$$\Gamma_1^p(EJ)=\int_0^1 dx\, g_1^p(x,Q^2)=0.1335, \tag{4.4}$$

$$\Gamma_2^p(BC)=\int_0^1 dx\, g_2^p(x,Q^2)=0.0018. \tag{4.5}$$

$\Gamma_1^p(EJ)$ under the analytic integration in Eq. (4.1) gives the value 0.13499 which is not very different from Eq. (4.4) justifying the extension of the integration limits. This may further be compared with the Ellis-Jaffe theoretical prediction of $\Gamma_1^p(EJ)=0.18$ as against its recent experimental value at $Q^2=10.7\,\text{GeV}^2$ [2]

$$\Gamma_1^p(EJ;EMC)=0.126\pm 0.018,$$

$$\Gamma_1^p(EJ;SMC)=0.142\pm 0.008\pm 0.011. \tag{4.6}$$

Thus we find that both the sum rules due to Ellis-Jaffe as well as that due to Burkhardt-Cottingham are verified here analytically in a very satisfactory manner. We may also note here that since $g_1^n(x,Q^2)$ in the present model is identically zero, the integral in Eq. (4.1) can also be considered as rep-

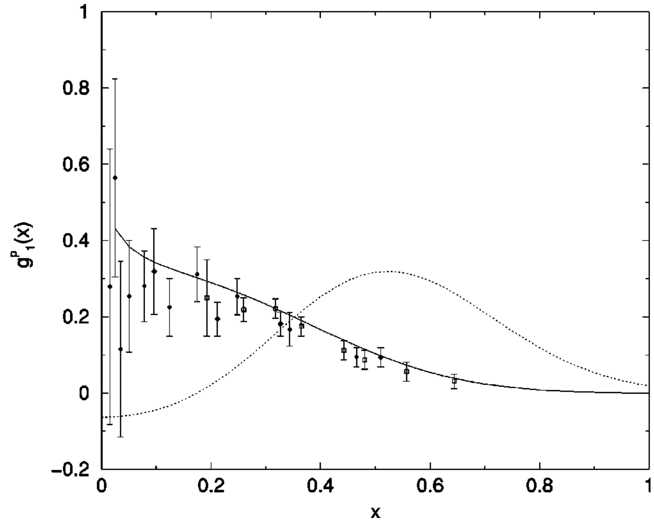

FIG. 1. $g_1^p(x,Q^2)$ (dotted line) at the model scale. $Q^2=Q_0^2=0.1\,(\mathrm{GeV}/c)^2$; QCD evolved $g_1^p(x,Q^2)$ (solid line) at $Q^2=10.7\,(\mathrm{GeV}/c)^2$ is compared with the data from Refs. [1,2].

resenting the Bjorken sum rule expression [11]. Taking into account the fact that in this model one finds [5]

$$(g_A/g_V)=\frac{5(5E_q'+7m_q')}{9(3E_q'+m_q')} \tag{4.7}$$

we find the model prediction here in conformity with the Bjorken sum-rule expression without QCD-radiative correction as

$$\Gamma_{BJ}=\int_0^1 dx[g_1^p(x,Q^2)-g_1^n(x,Q^2)] = \frac{1}{6}(g_A/g_V). \tag{4.8}$$

We may also point out here that the first moment of $g_2^p(x)$ obtained numerically in the present model in Eq. (4.5) as 0.0018 is in good agreement with the analytic result in Eq. (4.2) and is comparable with the corresponding predictions such as 0.0038 in MIT bag model and -0.0009 in a modified bag model [12]. Although Burkhardt-Cottingham sum rule has not been proved in QCD, it is considered as most probably true with the analytical support obtained earlier in MIT-bag model [4].

We now evaluate the structure functions $g_1^p(x,Q^2)$ and $g_2^p(x,Q^2)$ numerically to show explicitly their behavior as functions of the Bjorken variable x at this model scale of low Q^2. These are depicted in Figs. 1 and 2, respectively. We find here that the overall support problem is quite minimal. We also note that like all other models, $g_2^p(x,Q^2)$ starts positive for small x, changes sign at $x=x_0\simeq 0.42$, and after passing through a minimum it tends to zero without a second crossing as in Ref. [12]. It is believed that unlike $g_1^p(x,Q^2)$, $g_2^p(x,Q^2)$ can have contributions from quark-gluon correlations and quark mass effects persisting to even the large Q^2 limit. These contributions can arise out of local operators of

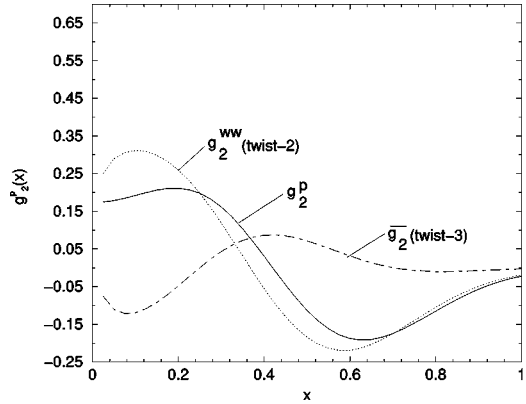

FIG. 2. The calculated model result for $g_2^p(x)$ (solid line), $g_2^{ww}(x)$ (dotted line), and $\overline{g}_2(x)$ (dot-dashed line) at the reference scale $Q_0^2=0.1\,(\mathrm{GeV}/c)^2$.

twist-3 in the formalism of operator product expansion. Thus the structure function $g_2^p(x,Q^2)$ can be decomposed into a twist-2 piece (g_2^{ww}) and a twist-3 piece ($\overline{g}_2$) as

$$g_2^p(x,Q^2)=g_2^{ww}(x,Q^2)+\overline{g}_2(x,Q^2), \tag{4.9}$$

where

$$g_2^{ww}(x,Q^2)=-g_1^p(x,Q^2)+\int_x^1 dy\,\frac{g_1^p(y,Q^2)}{y} \tag{4.10}$$

and hence

$$\overline{g}_2(x,Q^2)=g_2^p(x,Q^2)+g_1^p(x,Q^2)-\int_x^1 dy\,\frac{g_1^p(y,Q^2)}{y}. \tag{4.11}$$

The model results for the Wandzura-Wilczek [13] twist-2 piece g_2^{ww} and the twist-3 piece $\overline{g}_2$ for proton are shown along with $g_2^p(x,Q^2)$ in Fig. 2. We observe that the sign of $g_2^p(x,Q^2)$ is almost similar to that of g_2^{ww}. The twist-3 part $\overline{g}_2(x,Q^2)$ which is mostly sign reversed in relation to g_2^{ww} is comparatively smaller in magnitude and almost vanishing beyond $x=0.64$.

Finally we address the question of appropriate Q^2 evolution of these structure functions to experimentally relevant higher Q^2 region. Leaving aside $g_2^p(x,Q^2)$ for which appropriate evolution equations are so far wanting [14], we take up the case of $g_1^p(x,Q^2)$. Since the model scale of low Q^2 has not been explicit in the derived expression, we fix a reference scale of low $Q_0^2=0.1\,\mathrm{GeV}^2$ corresponding to which the model provides saturation of the momentum sum rule by the valency quarks [15]. With $\Lambda_{QCD}=0.232\,\mathrm{GeV}$, the relevant perturbative expansion parameter $\alpha_s(Q_0^2)/2\pi$ in leading order becomes 0.358, which is well within the reasonable limit to justify the applicability of the QCD-perturbation theory at the leading order even at such low reference scale. The important point to note here is that there is no other choice in any way for choosing an *ad hoc* higher reference scale Q_0^2, since it would require a nonzero initial input of sea quark and

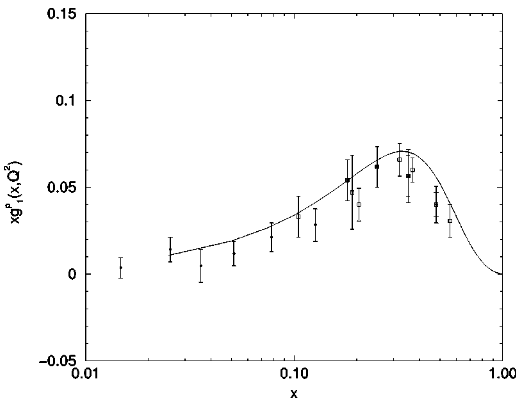

FIG. 3. $xg_1^p(x,Q^2)$ (solid line) at $Q^2=10.7\,(\mathrm{GeV}/c)^2$ compared with the data from Refs. [18,19].

gluon constituents for which one does not have any dynamical information at this scale. We therefore prefer to evolve the $g_2^p(x,Q^2)$ from $Q_0^2=0.1\,\mathrm{GeV}^2$ to higher Q^2 values such as $Q^2=10.7\,\mathrm{GeV}^2$ at which experimental data are available.

For this we use the standard convolution technique based on the nonsinglet evolution equation due to Altarelli and Parisi in its leading order (LO) [8,16]. We are neither very particular here for a high precision analysis nor would we claim quantitative significance to our result near the physical boundary in the variable x. Hence we do not attempt Q^2 evolution to nonleading order at this stage. Since it is believed [17] that nonsinglet evolution converges very fast and can remain stable even for small values of $Q^2/\Lambda^2_{\mathrm{QCD}}$, we expect a reliable interpolation between the reference scale $Q_0^2=0.1\,\mathrm{GeV}^2$ and the experimentally relevant scale of $Q^2 >> Q_0^2$. Our results for $g_1^p(x,Q^2)$ and $xg_1^p(x,Q^2)$ after evolution is shown in comparison with the available experimental data in Figs. 1 and 3, respectively. We observe that if we do not attach much quantitative significance to the result at the small x range near $x=0$ and at the large x range near $x=1$, the agreement is quite encouraging. Finally if we compute the first moment of $g_1^p(x,Q^2)$ corresponding to the evolved result at $Q^2=10.7\,\mathrm{GeV}^2$ we find

$$\int_{0.025}^{1} dx\, g_1^p(x,Q^2)=0.132661, \tag{4.12}$$

which on extrapolation to $x=0$ as the lower limit yields

$$\int_{0}^{1} dx\, g_1^p(x,Q^2)=0.14426. \tag{4.13}$$

This result is in reasonable agreement with the corresponding experimental value in Eq. (4.6).

V. CONCLUSION

Thus within the limitation of the approximations involved in the present model, we find that we have been able to analytically derive explicit functional forms of the spin-dependent structure functions $g_1^p(x,Q^2)$ and $g_2^p(x,Q^2)$ for proton which at the model scale of a low Q^2 provide a reasonable test of the well known sum rules in spite of the inadequacies in their small x and large x behavior. We also extract the twist-3 contribution to $g_2^p(x,Q^2)$ at the model scale which is comparatively smaller in magnitude and almost vanishing beyond $x=0.64$. With appropriate Q^2 evolution for $g_1^p(x,Q^2)$ so derived, we have made a comparison with available experimental data. The overall qualitative features of the structure functions extracted in the model are quite encouraging, in spite of a simple valency quark picture of the nucleon, to start with. This provides an encouraging link between the low energy description of the proton in the model with its high energy behavior in the deep inelastic region without refixing at all the model parameters, taken from its earlier application.

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[1] M. J. Alguard *et al.*, Phys. Rev. Lett. **37**, 1261 (1976); **41**, 70 (1978); G. Baum *et al.*, *ibid.* **45**, 2000 (1980); **51**, 1135 (1983).
[2] EMC, J. Ashman *et al.*, Nucl. Phys. **B328**, 1 (1989); Spin Muon Collaboration, D. Adams *et al.*, Phys. Lett. B **329**, 399 (1994); E-143 Collaboration, K. Abe *et al.*, Phys. Rev. Lett. **74**, 346 (1995).
[3] R. L. Jaffe, Phys. Rev. D **11**, 1953 (1975).
[4] R. J. Hughes, Phys. Rev. D **16**, 622 (1977); J. A. Bartelski, *ibid.* **20**, 1229 (1979); R. L. Jaffe and Xiangdong Ji, *ibid.* **43**, 724 (1991); T. Gehrmann and W. J. Stirling, Z. Phys. C **65**, 461 (1995); X. Song and J. S. McCarthy, Phys. Rev. D **49**, 3169 (1994); Phys. Rev. C **46**, 1077 (1992); A. W. Schriber, A. I. Signal, and A. W. Thomas, Phys. Rev. D **44**, 2653 (1991); A. W. Schriber and A. W. Thomas, Phys. Lett. B **215**, 141 (1988).
[5] N. Barik, B. K. Dash, and M. Das, Phys. Rev. D **32**, 1725 (1985); N. Barik and B. K. Dash, *ibid.* **34**, 2092 (1986); **34**, 2803 (1986); **33**, 1925 (1986).
[6] N. Barik, P. C. Dash, and A. R. Panda, Phys. Rev. D **46**, 3856 (1992); **47**, 1001 (1993); N. Barik and P. C. Dash, *ibid.* **47**, 2788 (1993); **53**, 1366 (1996); N. Barik, S. Tripathy, S. Kar, and P. C. Dash, *ibid.* **56**, 4238 (1997); N. Barik, S. Kar, and P. C. Dash, *ibid.* **57**, 405 (1998).
[7] N. Barik, B. K. Dash, and A. R. Panda, Nucl. Phys. **A605**, 433 (1996); N. Barik *et al.*, Phys. Rev. D **59**, 037301 (1999).
[8] G. Altarelli and G. Parisi, Nucl. Phys. **B126**, 298 (1977).
[9] J. Ellis and R. L. Jaffe, Phys. Rev. D **9**, 1444 (1974); **10**, 1669(E) (1974).
[10] H. Burkhardt and W. N. Cottingham, Ann. Phys. (N.Y.) **56**, 453 (1970).

[11] J. D. Bjorken, Phys. Rev. **148**, 1467 (1966); Phys. Rev. D **1**, 1376 (1970).
[12] M. Stratmann, Z. Phys. C **60**, 763 (1993).
[13] S. Wandzura and F. Wilczek, Phys. Lett. **82B**, 195 (1977).
[14] A. P. Bukhvostov, E. A. Kuraev, and L. N. Lipatov, Zh. Eksp. Teor. Fiz. **87**, 37 (1984) [Sov. Phys. JETP **60**, 22 (1984)]; C. Chou and X. Ji, Phys. Rev. D **42**, 3637 (1990).
[15] Present model applied to the sector of unpolarized structure function yields the valency quark distribution functions $u_v(x,Q^2)$ and $d_v(x,Q^2)$ inside proton. Following the prescription of Ref. [12] we fix the reference scale $Q_0^2=0.1\,\mathrm{GeV}^2$ by incorporating the constraint of momentum saturation by the valency quarks at the model scale. Details of this study would be reported in a separate communication.
[16] A. J. Buras, Rev. Mod. Phys. **52**, 199 (1980); Richard D. Field, in *Application of Perturbative-QCD* (Addison-Wesley, New York, 1989), p. 148.
[17] M. R. Pennington and G. G. Ross, Phys. Lett. **86B**, 371 (1979).
[18] SMC Collaboration, B. Adeva *et al.*, Phys. Lett. B **302**, 534 (1993); **320**, 400 (1994).
[19] E. Henley, Bull. Am. Phys. Soc. **38**, 972 (1993); D. L. Anthony *et al.*, Phys. Rev. Lett. **71**, 959 (1993).