Multibaryons as symmetric multi-Skyrmions

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We study the rotational corrections to multibaryon systems within the bound state approach to the SU(3) Skyrme model. We use approximate *Ansätze* for the static background fields based on rational maps which have the same symmetries of the exact solutions. To determine the explicit form of the collective Hamiltonians and wave functions we only make use of these symmetries. Thus, the expressions obtained are also valid in the exact case. On the other hand, the inertia parameters and hyperfine splitting constants we calculate do depend on the detailed form of the *Ansätze* and are, therefore, approximate. Using these values we compute the low lying spectra of multibaryons with $B \leqslant 9$ and strangeness 0, -1, and $-B$. Finally, we show that the rotational corrections do not affect the stability of the tetralambda and heptalambda found in a previous work.

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I. INTRODUCTION

In the last few years there have been several important developments in the determination of the lowest energy Skyrmion configurations [1–3]. These types of solutions are essential for the understanding of multibaryons and, perhaps, nuclei in the framework of the topological chiral soliton models. So far, these models have proven to be useful for the description of quantities such as the masses, strong and electromagnetic properties of the octet and decuplet baryons, baryon-baryon interactions, etc. (see, e.g., Refs. [4,5] and references therein). The knowledge of the properties of the multi-Skyrmion configurations opens the possibility of studying more complex baryonic objects. In fact, several investigations concerning non-strange multi-Skyrmion systems have been reported in the literature (see, e.g., Refs. [6–10]). Of particular interest are, however, the strange multibaryons. Perhaps the most celebrated example is the H dibaryon predicted in the context of the MIT bag model more than twenty years ago [11]. This exotic has been studied in various other models, including the Skyrme model [12–15], but its existence remains controversial both theoretically and experimentally. It has also been speculated that strange matter could be stable [16]. This has lead to numerous investigations of the properties of strange matter in bulk and in finite lumps (for a recent review see Ref. [17]). Moreover, with the new heavy ion colliders there is now the possibility of producing strange multibaryons in the laboratory [18]. In this situation the study of multibaryon systems within the SU(3) Skyrme model appears to be quite interesting. A first step in this direction has been reported in Ref. [19] where the rational map approximation [20] to the multi-Skymion fields was used to describe the multibaryon configurations within the bound state approach [21] to the SU(3) Skyrme model. Within this approach strange (multi)baryons appear as systems of kaons bound to a background Skyrmion configuration. To find the kaon binding energy one has to solve the corresponding eigenvalue problem. For a general background, this is a very hard numerical task since one has to deal with several coupled, partial, differential equations. However, this problem is greatly simplified if one introduces the (approximate) rational maps *Ansätze* for the multi-Skyrmion configurations. The construction of these *Ansätze* is based on the analogy between monopoles and Skyrmions and requires that the approximate solutions have the same symmetries as the exact numerical solutions. In fact, it is now known that up to $B=9$ these configurations are very symmetric. Namely, for $B=2$ the solution corresponds to an axially symmetry torus while configurations with $B=3-9$ possess the symmetries of the platonic polyhedra. In contrast with the exact solution, however, the rational map approximation assumes that the modulus of the static pionic field is radially symmetric while its direction depends only on the polar coordinates. It was shown in Ref. [20] that this represents a very good approximation. Once the rational maps are introduced, the kaon eigenvalue problem reduces for each baryon number to one radial eigenvalue equation. The corresponding results have been given in Ref. [19]. In such reference, however, rotational effects were neglected. These effects appear when one performs the collective quantization of the system. It should be stressed that it is only at this stage when the spin and isospin quantum numbers are well defined and that splitting between the corresponding states appears.

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The purpose of the present work is to carry out the collective quantization of the bound multisoliton-kaon systems. This requires paying special attention to their symmetries, which imposes severe constraints on the possible quantum numbers and wave functions.

This paper is organized as follows. In Sec. II we provide a brief description of the model with special emphasis on the effect of the rotational corrections. In Sec. III we describe in detail how to obtain the collective Hamiltonian for the different baryon numbers, while in Sec. IV we focus on the corresponding wave functions. It should be noticed that since the discussions in these two sections rely only on the symmetries of multi-Skyrmion configuration, the corresponding results also hold true for the exact solutions. In Sec. V we present the numerical results and in Sec. VI our conclusions. Finally, in the Appendix we give the explicit form of the rational maps used in the present work.

II. THE MODEL

We start with the effective action of the SU(3) Skyrme model supplemented with an appropriate symmetry breaking term [5]. Expressed in terms of the SU(3)-valued chiral field $U(x)$ it reads

$$\Gamma=\int d^4x\left\{\frac{f_\pi^2}{4}\mathrm{Tr}[\partial_\mu U\partial^\mu U^\dagger]+\frac{1}{32e^2}\mathrm{Tr}[[U^\dagger\partial_\mu U,U^\dagger\partial_\nu U]^2]\right\}+\Gamma_{WZ}+\Gamma_{SB}, \quad (1)$$

where f_π is the pion decay constant ($=93$ MeV empirically) and e is the so-called Skyrme parameter. In Eq. (1), the symmetry breaking term Γ_{SB} accounts for the different masses and decay constants of the pion and kaon fields while Γ_{WZ} is the usual Wess-Zumino action. Their explicit forms are

$$\Gamma_{SB}=\int d^4x\left\{\frac{f_\pi^2m_\pi^2+2f_K^2m_K^2}{12}\mathrm{Tr}[U+U^\dagger-2]+\frac{f_\pi^2m_\pi^2-f_K^2m_K^2}{6}\mathrm{Tr}[\sqrt{3}\lambda^8(U+U^\dagger)]+\frac{f_K^2-f_\pi^2}{12}\times\mathrm{Tr}[(1-\sqrt{3}\lambda^8)(U\partial_\mu U^\dagger\partial^\mu U+U^\dagger\partial_\mu U\partial^\mu U^\dagger)]\right\}, \quad (2)$$

$$\Gamma_{WZ}=-i\frac{N_c}{240\pi^2}\int d^5x\varepsilon^{\mu\nu\alpha\beta\gamma}\mathrm{Tr}(L_\mu L_\nu L_\alpha L_\beta L_\gamma), \quad (3)$$

where λ^8 is the eighth Gell-Mann matrix and m_π and m_K represent the pion and kaon masses, respectively, and f_K is the kaon decay constant.

We proceed by introducing the Callan-Klebanov *Ansatz* for the chiral field [21]

$$U=\sqrt{U_\pi}U_K\sqrt{U_\pi}. \quad (4)$$

In this *Ansatz*, U_K is the field that carries the strangeness. Its form is

$$U_K=\exp\left[i\frac{\sqrt{2}}{f_K}\begin{pmatrix}0 & K\\ K^\dagger & 0\end{pmatrix}\right], \quad (5)$$

where K is the usual kaon isodoublet $K=\binom{K^+}{K^0}$. The other component U_π is the soliton background field. It is a direct extension to SU(3) of the SU(2) field: i.e.,

$$U_\pi=\begin{pmatrix}\exp\left[\frac{i}{f_\pi}\vec{\tau}\cdot\hat{\pi}\right] & 0\\ 0 & 1\end{pmatrix}. \quad (6)$$

Replacing the *Ansatz* Eq. (4) in the effective action Eq. (1) and expanding up to second order in the kaon fields we obtain the Lagrangian density for the kaon-soliton system. In the spirit of the bound state approach this coupled system is solved by finding first the soliton background configuration. For this purpose we introduce the rational map *Ansätze* [20]

$$\vec{\pi}=f_\pi\hat{n}F, \quad (7)$$

with

$$\hat{n}=\frac{1}{1+|R|^2}[2\Re(R)\hat{\imath}+2\Im(R)\hat{\jmath}+(1-|R|^2)\hat{k}], \quad (8)$$

where we have assumed that $F=F(r)$, and $R=R(z)$ is the rational map corresponding to winding number $B=n$. Here, r is the usual spherical radial coordinate whereas the complex variable z is related to the other two spherical coordinates (θ,ϕ) via stereographic projection, namely, $z=\tan(\theta/2)\exp(i\phi)$. The resulting expression for the soliton mass per unit baryon is (in what follows $s=\sin F$; $c=\cos F$)

$$M_{sol}=\frac{f_\pi^2}{2n}\int drr^2\left[F'^2+2n\frac{s^2}{r^2}\left(1+\frac{F'^2}{e^2f_\pi^2}\right)+\frac{\mathcal{I}}{e^2f_\pi^2}\frac{s^4}{r^2}+8\pi m_\pi^2(1-c)\right]. \quad (9)$$

The profile function $F(r)$ is obtained by minimizing M_{sol} subject to the boundary conditions $F(0)=\pi$ and $F(\infty)=0$. In using these boundary conditions we are assuming that all the extra winding number is obtained from the angular dependence of $\hat{\pi}$. The angular integral $\mathcal{I}$ is

$$\mathcal{I}=\frac{r^4}{16\pi}\int d\Omega(\partial_i\hat{n}\cdot\partial_i\hat{n})^2=\frac{1}{4\pi}\int\frac{2idzd\bar{z}}{(1+|z|^2)^2}\left(\frac{1+|z|^2}{1+|R|^2}\left|\frac{dR}{dz}\right|\right)^4. \quad (10)$$

In order to find the lowest soliton-kaon bound state we write the kaon field as [14,15]

$$K_{T_z}(\vec{r},t)=k(r,t)\vec{\tau}\cdot\hat{n}\chi_{T_z}, \tag{11}$$

where χ is a two-component spinor.

The diagonalization of the corresponding kaon Hamiltonian leads to the eigenvalue equation

$$\left[-\frac{1}{r^2}\partial_r(r^2h_n\partial_r)+m_K^2+V_n^{eff}-f_n\epsilon_n^2-2\lambda_n\epsilon_n\right]k(r)=0. \tag{12}$$

Details on how to obtain this equation as well as the explicit expression of the radial functions f_n, h_n, λ_n, and V_n can be found in Ref. [19].

To obtain the hyperfine corrections to the multibaryons masses we proceed with the semiclassical collective coordinates quantization method, where the isospin and spatial rotations are treated as the zero modes. Then, we introduce the time-dependent spatial rotations R and the isospin rotations A such that

$$u_\pi\rightarrow RAu_\pi A^{-1}, \tag{13}$$

$$K\rightarrow RAK. \tag{14}$$

The angular velocities with respect to the body fixed frame are given by

$$(R^{-1}\dot{R})_{ab}=\epsilon_{abc}\Omega_c, \tag{15}$$

$$A^{-1}\dot{A}=\frac{i}{2}\vec{\tau}\cdot\vec{\omega}. \tag{16}$$

Replacing in the effective action we get the collective Lagrangian

$$L_{coll}=-M_{sol}+\frac{1}{2}[\Theta^J_{ab}\Omega_a\Omega_b+\Theta^I_{ab}\omega_a\omega_b+2\Theta^M_{ab}\Omega_a\omega_b]-(c^J_{ab}\Omega_a+c^I_{ab}\omega_a)T_b, \tag{17}$$

where $a,b=1,2,3$ and $T_b=(\chi^\dagger\tau_b\chi)/2$ is the kaon spin.

The moments of inertia Θ_{ab} and hyperfine splitting constants c_{ab} appearing in Eq. (17) are given by

$$\Theta^J_{ab}=m_1C_{ab}+\frac{m_2}{2}\overline{C}_{ab}, \tag{18}$$

$$\Theta^I_{ab}=m_1(\delta_{ab}-A_{ab})+2m_2(n\delta_{ab}-\overline{A}_{ab}), \tag{19}$$

$$\Theta^M_{ab}=m_1B_{ab}+\frac{m_2}{2}\overline{B}_{ab}, \tag{20}$$

$$c^I_{ab}=\delta_{ab}-3\left[(\delta_{ab}-A_{ab})d_1+\frac{1}{2}(\overline{A}_{ab}+2nA_{ab})d_2\right], \tag{21}$$

$$c^J_{ab}=-3[B_{ab}d_1+(\overline{B}_{ab}-nB_{ab})d_2], \tag{22}$$

where the radial integrals m_1, m_2, d_1, and d_2 are

$$m_1=4\pi f_\pi^2\int drr^2s^2\left(1+\frac{F'^2}{e^2f_\pi^2}\right), \tag{23}$$

$$m_2=4\pi f_\pi^2\int dr\frac{s^4}{e^2f_\pi^2}, \tag{24}$$

$$d_1=2\varepsilon_n\int_0^\infty drk^*k\left[\frac{1}{3}r^2f(1+c)-\frac{1}{e^2\mathrm{F}_K{}^2}\frac{d}{dr}(r^2F's)\right], \tag{25}$$

$$d_2=\frac{2\varepsilon_n}{e^2\mathrm{F}_K{}^2}\int_0^\infty drk^*k\frac{2}{3}(1+c)s^2, \tag{26}$$

and the angular integrals

$$A_{ab}=\int\frac{d\Omega}{4\pi}n^an^b, \tag{27}$$

$$\overline{A}_{ab}=r^2\int\frac{d\Omega}{4\pi}\partial_i\hat{n}\cdot\partial_i\hat{n}n^an^b, \tag{28}$$

$$B_{ab}=\int\frac{d\Omega}{4\pi}\partial_bn^a, \tag{29}$$

$$\overline{B}_{ab}=r^2\int\frac{d\Omega}{4\pi}\partial_i\hat{n}\cdot\partial_i\hat{n}\partial_bn^a, \tag{30}$$

$$C_{ab}=\int\frac{d\Omega}{4\pi}\partial_a\hat{n}\cdot\partial_b\hat{n}, \tag{31}$$

$$\overline{C}_{ab}=r^2\int\frac{d\Omega}{4\pi}\partial_i\hat{n}\cdot\partial_i\hat{n}\partial_a\hat{n}\cdot\partial_b\hat{n}. \tag{32}$$

The numerical values of these angular integrals depend only on the particular form of the *Ansatz* for $\hat{n}$ and not on the detailed form of the effective action and its parameters. For the rational maps listed in the Appendix all the matrices Eqs. (27)–(32) are diagonal. As we shall see in the next section, this is a direct consequence of the symmetries of these *Ansätze*.

Given L_{coll}, the canonical momenta are then defined in the usual way

$$J_a=\frac{\partial L_{coll}}{\partial\Omega_a}=\Theta^J_a\Omega_a+\Theta^M_a\omega_a-c^J_aT_a, \tag{33}$$

$$I_a=\frac{\partial L_{coll}}{\partial\omega_a}=\Theta^M_a\Omega_a+\Theta^I_a\omega_a-c^I_aT_a, \tag{34}$$

where we have used that for the cases we are interested in, all the inertia and hyperfine splitting constants are diagonal and thus denoted with a subindex $a=1,2,3$, the corresponding diagonal elements. Depending on whether $\Delta_a\equiv\Theta^J_a\Theta^I_a-(\Theta^M_a)^2$ vanishes or not, we have to follow a somewhat different procedure to obtain the collective Hamiltonian. We

consider first the case in which $\Delta_a \neq 0$ for all values of a. In this case the relations Eq. (33) can be inverted and the collective Hamiltonian results in

$$H^{coll} = \sum_a H_a^{coll}, \tag{35}$$

where

$$\begin{aligned} H_a^{coll} = &(K_a^J J_a^2 + K_a^I I_a^2 - 2K_a^M J_a I_a) \\ &+ 2(K_a^J \overline{c_a^J} J_a + K_a^I \overline{c_a^I} I_a) T_a + \frac{K_a^I K_a^J}{K_a^I K_a^J - (K_a^M)^2} \\ &\times [K_a^J (\overline{c_a^J})^2 + K_a^I (\overline{c_a^I})^2 + 2K_a^M \overline{c_a^I}\,\overline{c_a^J}] T_a^2 \end{aligned} \tag{36}$$

and

$$\begin{aligned} &K_a^J = \frac{1}{2}\frac{\Theta_a^I}{\Delta_a}, \quad K_a^I = \frac{1}{2}\frac{\Theta_a^J}{\Delta_a}, \quad K_a^M = \frac{1}{2}\frac{\Theta_a^M}{\Delta_a}, \\ &\overline{c_a^J} = c_a^J - c_a^I \frac{\Theta_a^M}{\Theta_a^I}, \quad \overline{c_a^I} = c_a^I - c_a^J \frac{\Theta_a^M}{\Theta_a^J}. \end{aligned} \tag{37}$$

If there exist, however, some values i for which $\Delta_i = 0$ there appears a relation between I_i, J_i, and T_i. It reads

$$J_i = \frac{\Theta_i^M}{\Theta_i^I} I_i - \left(c_i^J - c_i^I \frac{\Theta_i^M}{\Theta_i^I} \right) T_i. \tag{38}$$

Using this relation it is not difficult to show that the collective Hamiltonian becomes

$$H^{coll} = \sum_{a \neq i} H_a^{coll} + \sum_i \frac{(I_i + c_i^I T_i)^2}{2\Theta_i^I} \tag{39}$$

and the total multibaryon mass results

$$M = nM_{sol} + |S|\epsilon_n + E_{rot}, \tag{40}$$

where S is the multibaryon strangeness and E_{rot} the expectation value of H_{rot} in the corresponding wave function.

In the next section we will determine the precise form of the collective Hamiltonians for each baryon number.

III. COLLECTIVE HAMILTONIANS

The minimum energy multi-Skyrmion configurations are symmetric under certain groups of transformations [3]. With the exception of the $B=1$ and $B=2$ cases, where these symmetry groups are continuous [$O(3)$ and $D_{\infty h}$, respectively], these transformation groups have a finite number of elements. In this section we will see how the symmetries of the multi-Skyrmion configurations impose severe constraints on the detailed form of the collective Hamiltonian. For the $B \leqslant 4$ cases this has already been discussed in the literature using various arguments. Here, we will extend such analysis within a unified framework. It is important to notice that all the discussions and results that follow are based only on the symmetries of the multi-Skyrmions. Therefore, they will hold not only for the approximate configurations based on the rational maps but also for the exact ones obtained from numerical minimization.

The task here is to determine the precise structure of the inertia and hyperfine splitting tensors, namely, which elements of those tensors vanish and how many of the remaining non-zero elements are independent for each baryon number. First, we note that each operation of the abstract group G is represented by a pair of operations $\{g, D_g\}$ which act in spin and isospin spaces, respectively. The pion field in Eq. (8) is invariant under these combined operations,

$$\vec{\tau}\cdot\vec{\pi}(\hat{r}) = D_g \vec{\tau}\cdot\vec{\pi}(g^{-1}\hat{r})(D_g)^\dagger. \tag{41}$$

Given the form used for the kaon field,[1] Eq. (7), this invariance implies that the action of the group element on the kaon field is also represented by D_g. In fact,

$$D_g K_{T_z}(\vec{r},t) = K_{T_z}(g\vec{r},t), \tag{42}$$

which means that the symmetry operation acting on the kaon field is just given by the representation of the isospin operation D_g in the T-space. Thus, the $\vec{I}$ and $\vec{T}$ operators transform in the same way under elements of G. This shows that it is enough to perform the explicit analysis only for the inertia tensors. Once this is done, the results for the hyperfine splitting constants can be easily obtained, noting that in Eq. (36), c_{ab}^J plays a role similar to that of K_{ab}^M, while c_{ab}^I to that of K_{ab}^I.

The inertia tensors can be diagonalized by an appropriate choice of the spatial and internal reference frames, and this is in fact what happens for the rational map *Ansätze* given in the Appendix. Consider first the case for the spin. The spin generators J_a transform under G in some (possibly reducible) representation. The number of independent diagonal components of the inertia tensor (moments of inertia) will be equal to the number of irreducible representations[2] (irreps) of G into which this representation breaks, since the combination $K_{ab}^J J_a J_b$ must be a scalar under G. The spin generators belong to the 1^+ irrep of $O(3)$ which, for the cases we will consider below, breaks into either a 3-dim irrep or the sum of 1- and 2-dim irreps of G. In the first case there is only one moment of inertia and the spin Hamiltonian is proportional to $\Sigma_a J_a J_a$, while in the second case there are two moments, and the Hamiltonian contains the terms $J_1^2 + J_1^2$ and J_3^2. The same argument holds for the other collective operators.

An important remark is the following. While there is a one-to-one correspondence between g and the elements of G, this is not necessarily the case for the operations D_g. In other words, it could happen that the same D_g is associated with two (or more) different elements in spin space. In this

[1]This *Ansatz* can be easily generalized if the exact numerical soliton configuration is used instead of the approximation based on rational maps.

[2]The character tables containing the list of irreps of the groups we are interested in can be found, e.g., in Refs. [22] and [23]. We follow the conventions of Ref. [22].

case, the operations D_g do not span the full group G but a subgroup of it. As a consequence, the generators J_a and I_a (T_a) could transform in different representations of G. This would imply that the corresponding mixing inertia would vanish. Below we see that this happens for some values of B.

Let us consider now the multi-Skyrmion configurations case by case. The $B=1$ Skyrmion is spherically symmetric [4]. Thus, the relevant symmetry group G is $O(3)$. In this case, $g=D_g$ and both $\vec{J}$ and $\vec{I}$ are in the 3-dim irrep 1^+. Using the arguments given above we have

$$\Theta_a^J=\Theta^J, \quad \Theta_a^I=\Theta^I, \quad \Theta_a^M=\Theta^M, \quad c_a^J=c^J, \quad c_a^I=c^I. \tag{43}$$

Since in this case we are dealing with a continuous group, the equality between the representation of the group elements in spin and isospin spaces can be written in terms of corresponding generators of the algebra. Namely, we obtain the relation $J_a=-(I_a+T_a)$. From Eq. (33) this implies

$$\Theta^J=\Theta^I=-\Theta^M, \quad c^I=1-c^J, \tag{44}$$

which leads to $\Delta_a=0$ for all values of a. Then, the collective Hamiltonian takes the well-known form

$$H_{B=1}^{coll}=\frac{1}{2\Theta}(I^2+c^2T^2+2c\vec{T}\cdot\vec{I}). \tag{45}$$

As already mentioned, the $B=2$ lowest energy Skyrmion configuration has the symmetry of a torus [1], which implies $G=D_{\infty h}$. Choosing the symmetry axis along the z-direction we obtain that the third components of the momenta are in the 1-dim Σ_g^- while the other two components are in the 2-dim irrep Π_g. Since rotations along the z-axis form a continuous subgroup of $D_{\infty h}$, for the terms containing third components of the momenta we obtain a result similar to that of $B=1$,

$$\Theta_3^J=4\Theta_3^I=-2\Theta_3^M, \quad c_3^I=1-c_{3/2}^J, \tag{46}$$

which leads to $\Delta_3=0$. For the other components $\Delta_{1,2}\neq 0$, since the C_2 along those axes only form finite subgroups of G. Consequently, the corresponding component of the different type of inertia and splitting constants need not be equal and the $B=2$ collective Hamiltonian reads

$$H_{B=2}^{coll}=K_1^J(J^2-J_3^2)+K_1^I(I^2-I_3^2)+K_1^I(\overline{c_1^I})^2(T^2-T_3^2)+K_1^I\overline{c_1^I}(I_+T_-+I_-T_+)+\frac{(I_3+c_3^IT_3)^2}{2\Theta_3^I}. \tag{47}$$

For the rest of the baryon numbers under consideration, $B=3-9$, the symmetry group G is finite [2,3]. Therefore, Δ_a never vanishes for all those baryon numbers and the collective Hamiltonian will have the general form Eq. (35). There can be, however, some further simplifications depending on the way in which the symmetry is realized in spin and isospin spaces.

The symmetry group of the $B=3$ solution is $G=T_d$. In this case, we have that $g=D_g$ for all the elements of G [7]. Thus, the components J_a, I_a, and T_a are in the 3-dim irrep F_2. The collective Hamiltonian reads

$$H_{B=3}^{coll}=K^JJ^2+K^II^2-2K^M\vec{I}\cdot\vec{J}+2K^J\overline{c^J}\vec{J}\cdot\vec{T}+2K^I\overline{c^I}\vec{I}\cdot\vec{T}+\frac{K^IK^J}{K^IK^J-(K^M)^2}[K^J(\overline{c^J})^2+K^I(\overline{c^I})^2+2K^M\overline{c^I}\overline{c^J}]T^2. \tag{48}$$

In the case of $B=4$ the relevant symmetry group is O_h. As discussed in Ref. [2], for the minimum energy configuration this symmetry is realized in such a way that the elements D_g cover four times the D_{3d} subgroup. As a result, I_1 (T_1) and I_2 (T_2) are in the 2-dim irrep E_g, I_3 (T_3) in the A_{2g} irrep and the components of $\vec{J}$ lie in the 3-dim irrep T_{1g}. We see then that the mixing inertia and spin splitting tensors vanish. The resulting form of the corresponding collective Hamiltonian is

$$H_{B=4}^{coll}=K^JJ^2+K_1^I(\vec{I}+\overline{c_1^I}\vec{T})^2+(K_3^I-K_1^I)I_3^2+2(K_3^I\overline{c_3^I}-K_1^I\overline{c_1^I})I_3T_3+[K_3^I(\overline{c_3^I})^2-K_1^I(\overline{c_1^I})^2]T_3^2. \tag{49}$$

The lowest energy multi-Skyrmion with $B=5$ has D_{2d} symmetry. In this case, there is a one-to-one correspondence between the realization of the group in spin and isospin spaces. It is easy to check that the third components of the momenta are in the A_2 irrep while the other two components in the 2-dim one E. The resulting collective Hamiltonian is

$$\begin{aligned}H_{B=5}^{coll}=&K_1^J(J^2-J_3^2)+K_1^I(I^2-I_3^2)-2K_1^M(\vec{I}\cdot\vec{J}-I_3J_3)+2K_1^J\overline{c_1^J}(\vec{J}\cdot\vec{T}-J_3T_3)+2K_1^I\overline{c_1^I}(\vec{I}\cdot\vec{T}-I_3T_3)\\&+K_3^JJ_3^2+K_3^II_3^2-2K_3^MI_3J_3+2K_3^J\overline{c_3^J}J_3T_3+2K_3^2\overline{c_3^I}I_3T_3\\&+\frac{K_1^IK_1^J}{K_1^IK_1^J-(K_1^M)^2}[K_1^J(\overline{c_1^J})^2+K_1^I(\overline{c_1^I})^2+2K_1^M\overline{c_1^I}\overline{c_1^J}](T^2-T_3^2)\\&+\frac{K_3^IK_3^J}{K_3^IK_3^J-(K_3^M)^2}[K_3^J(\overline{c_3^J})^2+K_3^I(\overline{c_3^I})^2+2K_3^M\overline{c_3^I}\overline{c_3^J}]T_3^2.\end{aligned} \tag{50}$$

As found in Ref. [3], for $B=6$ the symmetry group is D_{4d}. Because of the way in which the generators of the group are

realized as pairs of spin-isospin operations it is possible to show that while the spin operations cover the full D_{4d} group, the isospin ones cover twice the D_{2d} subgroup. From the corresponding compatibility tables together with the compatibility table of the full rotational group, we find that J_3, I_3, and T_3 transform as the A_2 irrep, J_1 and J_2 as the E_3 irrep and the rest as E_2 irrep. Therefore,

$$H^{coll}_{B=6} = K_1^J J^2 + K_1^I(\vec{I}+\overline{c_1^I}\vec{T})^2 + (K_3^J - K_1^J)J_3^2 + (K_3^I - K_1^I)I_3^2 - 2K_3^M I_3 J_3 + 2K_3^J \overline{c_3^J} J_3 T_3 + 2(K_3^I \overline{c_3^I} - K_1^I \overline{c_1^I}) I_3 T_3 + \left[\frac{K_3^I K_3^J}{K_3^I K_3^J - (K_3^M)^2} [K_3^J (\overline{c_3^J})^2 + K_3^I (\overline{c_3^I})^2 + 2K_3^M \overline{c_3^I}\,\overline{c_3^J}] - K_1^I (\overline{c_1^I})^2 \right] T_3^2. \tag{51}$$

The $B=7$ configuration has icosahedral symmetry I_h with the symmetry realized in such a way that the components of the spin operators transform like the F_{1g} irrep while those of the isospin operators transform as F_{2g} irrep. Thus, the collective Hamiltonian takes the simple form

$$H^{coll}_{B=7} = K^J J^2 + K^I(\vec{I}+\overline{c^I}\vec{T})^2. \tag{52}$$

For $B=8$ we have to deal with the D_{6d} group. As in the case of lower even baryon numbers, the isospin operations do not span the full group but twice a subgroup, D_{3d} in this case. We find that J_3, I_3, and T_3 transform as the A_2 irrep, J_1 and J_2 as E_5 irrep and the rest as the E_4 irrep. This implies that the collective Hamiltonian for $B=8$ has the same form as the $B=6$ one given in Eq. (51). Finally, the $B=9$ multi-Skyrmion has the same symmetry as the $B=3$ one, T_d. Consequently, we obtain a similar form for the corresponding collective Hamiltonian, Eq. (48).

IV. COLLECTIVE WAVE FUNCTIONS

Having determined the explicit form of the collective Hamiltonian, we have to find the corresponding wave functions. These wave functions have to satisfy some constraints imposed by the symmetries of the background multi-Skyrmion. For nonstrange multi-Skyrmions this problem has been discussed by several authors [6–10]. Here, we will extend such studies for kaon-soliton bound systems.

The quantization of a single Skyrmion as a fermion implies that under certain symmetry operations of the classical multisoliton background the corresponding wave functions can pick up a nontrivial phase. These are known as Finkelstein-Rubinstein (FR) constraints [24]. We can generically write the constraints on the ground state as

$$gD_g|\text{g.s.}\rangle = \gamma_g|\text{g.s.}\rangle, \tag{53}$$

where $\gamma_g = \pm 1$ is determined according to the FR constraints. Using continuity arguments it turns out that the FR phases can be nontrivial only for those operations corresponding to rotations, so for our cases of interest only the proper subgroup of G needs to be considered. For the isospin transformations we have to take into account the fact that the symmetry operation also acts on the kaon field. From Eq. (42), however, we notice that this operation coincides with the one acting on the soliton isospin space. Thus, defining $\vec{N} = \vec{I} + \vec{T}$, the problem basically reduces to that of nonstrange baryons just replacing the collective isospin by $\vec{N}$. The (proper) group generators and their corresponding FR phases for the configurations considered in this work were determined in Refs. [7,10]. They are listed in Table I.

It is clear from Eq. (53) that due to the FR phases, the soliton ground state might transform in a one-dimensional nontrivial irrep of G. Using the FR phases listed in Table I and the group character tables, the relevant 1-dim irrep Γ can be determined. We obtain that, with the exception of the $B=5$ and $B=6$ cases, all the wave functions should transform as the trivial irrep of the corresponding symmetry groups. For $B=5$, Γ is the A_2 irrep of D_{2d} while for $B=6$ the wave functions should transform as the A_2 irrep of D_{4d}.

We now need to determine the collective wave functions. The general procedure for arbitrary soliton backgrounds was discussed in [25]. First we consider the problem without strangeness. In this case we need to determine the functions

$$|JJ_z, II_z\rangle = \sum_{J_3 I_3} \alpha^{JI}_{J_3 I_3} D^J_{J_z J_3} D^I_{I_z I_3}, \tag{54}$$

which transform under the *right* action of G in the irrep Γ of the soliton. This can be done following standard group theoretical methods [26]. The product representation $J \times I$ of SU(2) is in general a reducible representation of G. The projector operator into the irrep Γ is

TABLE I. Symmetry group G, generators of the proper subgroup, their corresponding FR phases, and the parity operations for $B=3-9$. The directions of the 3-fold axes in $B=7$ are defined by the spherical angles $(\phi_\alpha, \theta_\alpha) = (\pi/5, \arccos[\sqrt{(5+2\sqrt{5})/15}])$ and $(\phi_\beta, \theta_\beta) = (3\pi/5, \arccos[1/\sqrt{15+6\sqrt{5}}])$.

		Generators of proper subgroup and FR phases				
B	G	$\{g_1, D_{g_1}\}$	γ_{g_1}	$\{g_2, D_{g_2}\}$	γ_{g_2}	Parity operation
3	T_d	$\{C_3^{xyz}, C_3^{xyz}\}$	1	$\{C_2^z, C_2^z\}$	1	$\{C_4^z, C_4^z\}$
4	O_h	$\{C_3^{xyz}, C_3^z\}$	1	$\{C_4^z, C_2^x\}$	1	$\{E, C_2^z\}$
5	D_{2d}	$\{C_2^z, C_2^z\}$	1	$\{C_2^x, C_2^x\}$	-1	$\{C_4^z, C_4^z\}$
6	D_{4d}	$\{C_2^x, C_2^x\}$	-1	$\{C_2^{xy}, C_2^y\}$	-1	$\{C_8^z, \overline{C_4^z}\}$
7	I_h	$\{C_5^z, (C_5^z)^3\}$	1	$\{C_3^\alpha, (C_3^\beta)^2\}$	1	$\{E, E\}$
8	D_{6d}	$\{C_2^x, C_2^x\}$	1	$\{C_6^z, \overline{C_3^z}\}$	1	$\{C_{12}^z, \overline{C_6^z}\}$
9	T_d	$\{C_3^{xyz}, C_3^{xyz}\}$	1	$\{C_2^z, C_2^z\}$	1	$\{C_4^z, C_4^z\}$

$$P_\Gamma = \frac{1}{|G|} \sum_{g \in G} \chi_\Gamma^*(g)\, \rho(g), \tag{55}$$

where $|G|$ is the rank of the group, $\chi_\Gamma(g)$ the character of operation g, and $\rho(g)$ the representation of g in $J \times I$ [cf. Eq. (41)]

$$\rho(g) = D^J(g) \times D^I(D_g). \tag{56}$$

The eigenvalues of P_Γ can either vanish or be equal to one. The eigenvectors corresponding to each nonvanishing eigenvalue provide precisely the coefficients $\alpha^{JI}_{J_3 I_3}$ of Eq. (54), and there are as many wave functions as nonzero eigenvalues. If all eigenvalues vanish there is no collective state with the given J,I. If there is only one, the wave function is an eigenfunction of the collective Hamiltonian, and if there are more than one, the Hamiltonian has to be diagonalized in the subspace spanned by them.

Let us proceed now to the case with $S \neq 0$. We need to find the functions[3]

$$|JJ_z, II_z, S\rangle = \sum_{J_3 I_3 T_3} \beta^{JIT}_{J_3 I_3 T_3} D^J_{J_z J_3} D^I_{I_z I_3} K^T_{T_3}, \tag{57}$$

which transform in irrep Γ under G. However, as noted above, the action of G in isospin and T-spaces is the same, so it is possible to couple them to $\vec{N} = \vec{I} + \vec{T}$. Our problem then reduces to that of the case without strangeness: for given I and S we have several possible values of N. For each of these we determine the linear combinations [see Eq. (54)] with I replaced by N, and finally we uncouple I and T. We obtain

$$|JJ_z, II_z, S\rangle = \sum_{J_3 N_3 I_3 T_3} \alpha^{JN}_{J_3 N_3} \langle II_3 TT_3 | NN_3 \rangle \times D^J_{J_z J_3} D^I_{I_z I_3} K^T_{T_3}, \tag{58}$$

where $\langle II_3 TT_3 | NN_3 \rangle$ are the SU(2) Clebsch-Gordan coefficients.

There is a further restriction of the possible collective states. Given a certain value of the baryon number B and the strangeness S, not all the values of isospin I are allowed. As discussed in Appendix B of Ref. [15], physical states should have hypercharge and isospin given by

$$Y = B + S/3 = \frac{p+2q}{3}; \quad I = \frac{p}{2}, \tag{59}$$

where p and q should be non-negative integer numbers. The allowed values of isospin I for states with $S = 0, -1$ and $-B$ are given in Table II, together with the corresponding values of T. Such values are obtained by requiring that the kaon wave function has to be completely symmetric under individual kaon exchange.

It should also be noted that in the construction of the projector Eq. (55) all the operations of G have to be taken into account (i.e., not only those of the proper subgroup). For this purpose the representations of the parity operation are also needed. For each baryon number they are given in Table I. Another important comment is that for odd baryon numbers the J and N quantum numbers are half-integers. For those cases one has to deal with the double group of G.

[3]Note that $T = |S|/2$. See below.

TABLE II. Allowed values of I and T for states with different strangeness for $B = 3-9$.

B	S	I	T
3	0	1/2, 3/2, . . . ,9/2	0
	−1	0, 1, . . . ,4	1/2
	−3	0, 1, . . . ,3	3/2
4	0	0, 1, . . . ,6	0
	−1	1/2, 3/2, . . . ,11/2	1/2
	−4	0, 1, . . . ,4	2
5	0	1/2,3/2, . . . ,15/2	0
	−1	0, 1, . . . ,7	1/2
	−5	0, 1, . . . ,5	5/2
6	0	0, 1, . . . ,9	0
	−1	1/2, 3/2, . . . ,17/2	1/2
	−6	0, 1, . . . ,6	3
7	0	1/2, 3/2, . . . ,21/2	0
	−1	0, 1, . . . ,10	1/2
	−7	0, 1, . . . ,7	7/2
8	0	0, 1, . . . ,12	0
	−1	1/2, 3/2, . . . ,23/2	1/2
	−8	0, 1, . . . ,8	4
9	0	1/2, 3/2, . . . ,27/2	0
	−1	0, 1, . . . ,13	1/2
	−9	0, 1, . . . ,9	9/2

V. NUMERICAL RESULTS

In our numerical calculations we will use two standard sets of values for the Skyrme model parameters f_π, e and m_π. Set A corresponds to $f_\pi = 64.5$ MeV, $e = 5.45$, $m_\pi = 0$, while Set B to $f_\pi = 54$ MeV, $e = 4.84$, $m_\pi = 138$ MeV [27]. In both cases we set the ratio f_K/f_π to its empirical ratio $f_K/f_\pi = 1.22$. With these values we can calculate M_{sol}, the kaon eigenenergies ϵ_n, and the radial integrals m_1, m_2, d_1, and d_2, which appear in the expression of the moments of inertia and hyperfine splitting constants. Using these values together with those for the angular integrals, all the parameters appearing in the collective Hamiltonians can be evaluated. For $B = 1$ we find that $\Theta = 1.01$ fm and $c = 0.50$ for Set A and $\Theta = 1.0\,1$fm and $c = 0.39$ for Set B, which provide a quite accurate description of the octet and decuplet baryon spectra [21,28]. The numerical values of the parameters in the $B = 2$ collective Hamiltonian Eq. (47) are given in Table III. It is interesting to compare the values of the inertia parameters with those obtained using the numerically obtained exact axially symmetric $B = 2$ Skyrmion [1]. For example, the corresponding values for Set B are

$$K^J_1 = 30\ \text{MeV}, \quad K^I_1 = 48\ \text{MeV}, \quad \Theta^I_3 = 1.45\ \text{fm}. \tag{60}$$

TABLE III. Parameters for $B=2$.

Set	K_1^J (MeV)	K_1^I (MeV)	Θ_3^I (fm)	$\overline{c_1^I}$	c_3^I
A	33.42	53.68	1.15	0.409	0.631
B	27.63	45.20	1.40	0.306	0.562

As we demonstrate, the differences with the values listed in Table III are of only a few percent. On the other hand, so far, there does not exist any calculation of hyperfine splitting constants using the exact numerical $B=2$ Skyrmion. Nevertheless, we can compare our results with those from a calculation based on an improved variational *Ansatz* [15] which are, for Set B,

$$\overline{c_1^I}=0.334, \quad c_3^I=0.554. \tag{61}$$

These values are also very similar to ours. This is also true for Set A. Taking into account that the corresponding inertia parameters are also very close to those given in Table III, it follows that our predicted dibaryon spectra coincide basically with the ones described in Ref. [15].

Results for the $B=3-9$ inertia parameters and hyperfine splitting constants are listed in Tables IV and V, respectively. As expected, the inertia parameters decrease with increasing baryon number. However, the decrease of the spin inertia appears to be much faster than that of the isospin one. This can be understood in the following way. Since we are interested in the overall behavior of inertias as a function of B, we define, for both spin and isospin, the average value $K=1/3\Sigma_a K_{aa}$. While m_1 is roughly proportional to the baryon number, m_2 is basically independent of B. Therefore, assuming $K\approx 1/\Theta$ and using Eqs. (18) and (19), we have

TABLE IV. Inertia parameters for $B=3-9$.

	Set A			Set B		
B	K^J (MeV)	K^I (MeV)	K^M (MeV)	K^J (MeV)	K^I (MeV)	K^M (MeV)
3	15.23	50.77	9.55	12.11	41.03	7.80
4	8.66	39.70	0	6.72	30.98	0
		39.70			30.98	
		32.88			25.89	
5	5.20	28.29	−1.17	4.03	22.30	−0.96
	5.20	28.29	−1.17	4.03	22.30	−0.96
	5.88	33.91	−0.89	4.57	26.47	−0.73
6	3.67	26.12	0	2.84	20.45	0
	3.67	26.12	0	2.84	20.45	0
	4.25	24.48	1.23	3.31	19.28	1.04
7	3.09	23.06	0	2.38	17.90	0
8	2.39	19.48	0	1.85	15.28	0
	2.39	19.48	0	1.85	15.28	0
	2.11	21.08	−0.61	1.63	16.50	−0.52
9	1.78	17.75	−0.43	1.39	14.02	−0.36

TABLE V. Hyperfine splitting constants for $B=3-9$.

	Set A		Set B	
B	$\overline{c^J}$	$\overline{c^I}$	$\overline{c^J}$	$\overline{c^I}$
3	−0.62	0.55	−0.64	0.48
4	0	0.55	0	0.48
		0.55		0.48
		0.46		0.37
5	0.22	0.48	0.23	0.41
	0.22	0.48	0.23	0.41
	0.15	0.57	0.16	0.51
6	0	0.53	0	0.46
	0	0.53	0	0.46
	−0.28	0.49	−0.30	0.43
7	0	0.53	0	0.46
8	0	0.51	0	0.45
	0	0.51	0	0.45
	0.28	0.55	0.31	0.49
9	0.23	0.52	0.25	0.46

$$1/K^J \approx an\,\mathrm{Tr}\,C + b\,\mathrm{Tr}\,\overline{C}, \quad 1/K^I \approx an\,\mathrm{Tr}\,A + b\,\mathrm{Tr}\,\overline{A}, \tag{62}$$

where a and b are constants roughly independent of n. On the other hand, it is not difficult to prove that the traces of the angular integrals appearing in these relations are given by

$$\mathrm{Tr}\,A=1, \quad \mathrm{Tr}\,\overline{A}=2n, \mathrm{Tr}\,C=2n, \quad \mathrm{Tr}\,\overline{C}=4\mathcal{I}. \tag{63}$$

As shown in Ref. [20], $\mathcal{I}\leqslant n^2$. In fact, $\mathcal{I}$ is basically proportional to n^2. Therefore, replacing Eq. (63) in Eq. (62) we obtain that K^J should decrease as n^2 while K^I goes only like $1/n$. This behavior of the inertia parameters has important consequences in the multibaryon spectra. Namely, as the baryon number increases, low lying nonstrange states are expected to have the lowest possible value of isospin. For strange multibaryons this is not necessarily the case due to the coupling of the isospin to the kaonic spin T.

The rotational energies for the nonstrange multibaryons are given in Table VI while those for $S=-1$ states are given in Table VII and those corresponding to zero-hypercharge states in Table VIII. In all the cases, we have included in the tables the lowest lying state and the first two excited states for each channel. Some general observations can be made. Due to the overall decrease of the inertia parameters, the energy splittings become smaller as B increases. We also note that the ordering of the $S=0$ states is the same for both sets of parameters. For the $S=-1$ states there is, however, one exception which corresponds to the second excited multibaryon with $(B,S)=(6,-1)$. For Set A the second excited state is a 3^+ while for Set B it is a 2^+. It should be noted, however, that the third excited states (not listed in Table VII) are precisely a 2^+ for Set A and 3^+ for Set B and

TABLE VI. Quantum numbers and rotational energies for $S=0$ states.

	Set A				Set B			
B	J^P	I	N	E_{rot} [MeV]	J^P	I	N	E_{rot} [MeV]
3	$1/2^+$	1/2	1/2	64	$1/2^+$	1/2	1/2	52
	$5/2^-$	1/2	1/2	147	$5/2^-$	1/2	1/2	117
	$3/2^-$	3/2	3/2	205	$3/2^-$	3/2	3/2	164
4	0^+	0	0	0	0^+	0	0	0
	4^+	0	0	173	4^+	0	0	134
	0^+	2	2	238	0^+	2	2	186
5	$1/2^+$	1/2	1/2	28	$1/2^+$	1/2	1/2	22
	$3/2^+$	1/2	1/2	40	$3/2^+$	1/2	1/2	31
	$3/2^-$	1/2	1/2	44	$3/2^-$	1/2	1/2	34
6	1^+	0	0	7	1^+	0	0	6
	3^+	0	0	44	3^+	0	0	34
	0^+	1	1	52	0^+	1	1	41
7	$7/2^+$	1/2	1/2	66	$7/2^+$	1/2	1/2	51
	$3/2^+$	3/2	3/2	98	$3/2^+$	3/2	3/2	76
	$9/2^+$	3/2	3/2	163	$9/2^+$	3/2	3/2	126
8	0^+	0	0	0	0^+	0	0	0
	2^+	0	0	14	2^+	0	0	11
	1^+	1	1	44	1^+	1	1	34
9	$1/2^+$	1/2	1/2	14	$1/2^+$	1/2	1/2	11
	$5/2^-$	1/2	1/2	30	$5/2^-$	1/2	1/2	24
	$7/2^-$	1/2	1/2	39	$7/2^-$	1/2	1/2	31

TABLE VII. Quantum numbers and rotational energies for $S=-1$ states.

	Set A				Set B			
B	J^P	I	N	E_{rot} [MeV]	J^P	I	N	E_{rot} [MeV]
3	$1/2^+$	0	1/2	38	$1/2^+$	0	1/2	29
	$1/2^+$	1	1/2	84	$1/2^+$	1	1/2	72
	$5/2^-$	0	1/2	122	$5/2^-$	0	3/2	95
4	0^+	1/2	1/2	6	0^+	1/2	1/2	7
	4^+	1/2	1/2	180	4^+	1/2	1/2	141
	0^+	3/2	3/2	191	0^+	3/2	3/2	144
5	$1/2^+$	0	1/2	11	$1/2^+$	0	1/2	7
	$3/2^+$	0	1/2	23	$3/2^+$	0	1/2	17
	$3/2^-$	0	1/2	28	$3/2^-$	0	1/2	21
6	1^+	1/2	0	12	1^+	1/2	0	10
	0^+	1/2	1	32	0^+	1/2	1	23
	3^+	1/2	0	49	2^+	1/2	1	37
7	$7/2^+$	0	1/2	54	$7/2^+$	0	1/2	40
	$3/2^+$	1	1/2	74	$3/2^+$	1	1/2	56
	$7/2^+$	1	1/2	75	$7/2^+$	1	1/2	60
8	0^+	1/2	0	3	0^+	1/2	0	3
	2^+	1/2	0	18	2^+	1/2	0	14
	1^+	1/2	1	28	1^+	1/2	1	21
9	$1/2^+$	0	1/2	4	$1/2^+$	0	1/2	3
	$5/2^-$	0	1/2	20	$5/2^-$	0	1/2	15
	$1/2^+$	1	1/2	21	$1/2^+$	1	1/2	18

that the energy difference with the second excited state is 1 MeV in both cases. For the $Y=0$ states the situation becomes more complicated as B increases. This is due to the rather small energy splittings between the different states. As a general trend we also note that the rotational energies are slightly smaller for Set B. This can be traced back to the fact that the moments of inertia are smaller for that set of parameters.

As discussed above, the lowest lying states for nonstrange baryons always have the lowest possible value of isospin. The corresponding spins are then given by the lowest value allowed by the symmetry constraints. As remarked in Ref. [10], these values turn out to be consistent with those known for light nuclei with the exception of the odd values $B=5,7,9$. It should be stressed that at this point there is no obvious way to identify these rather compact multi-Skyrmion configurations with normal nuclei. Indeed, even for the $B=2$ case it is not clear to what extent the deuteron wave function in the Skyrme model is represented by the torus configuration. Some analysis in terms of classical periodic orbits indicates that the two Skyrmions spend most of their time at large separation and only a short time near the torus [29]. As the strangeness increases (in absolute value) the quantum numbers of the low-lying states become less obvious. This is a consequence of the interplay between the different terms in the corresponding collective Hamiltonian for nonzero values of T. In fact, the quantum numbers of the $Y=0$ states listed in Table VIII could be determined only after the calculation of the energies of a rather large set of allowed states.

Now we discuss the issue of the stability of the $Y=I=0$ states that we generically call multilambda states. The possible stability of a tetralambda state was first suggested in Ref. [30]. A similar conclusion was reached in Ref. [19] where the existence of a stable heptalambda was also proposed. As already mentioned in the Introduction, the rotational corrections were neglected in that work. We are now in a position to check whether these effects do or do not affect the stability of these states. From Table VIII we observe that for Set B the g.s. $Y=0$ tetrabaryon is indeed a tetralambda state. This differs from the situation for Set A where the tetralambda is the first excited state. In any case, this does not affect the rotational contribution to the $4\Lambda-2\Lambda$ mass difference. Using the energies given in Table VIII together with the values given in Table III for the parameters of $H^{coll}_{B=2}$ [see Eq. (47)], we find that the rotational corrections decrease the binding by 36 MeV for Set A and by 26 MeV for Set B. These values are significantly smaller than the binding energy ≈ 176 MeV obtained for both sets of parameters in the adiabatic approximation [19]. Thus, although the rotational corrections tend to decrease the binding, the tetralambda still turns out to be bound within the present approach. For the heptalambda we consider first its stability with respect to the decay into $3\Lambda+4\Lambda$. The value of the corresponding binding energy is -177 MeV [19]. It

TABLE VIII. Quantum numbers and rotational energies for $Y=0$ states.

	Set A				Set B			
B	J^P	I	N	E_{rot} [MeV]	J^P	I	N	E_{rot} [MeV]
3	$1/2^+$	1	1/2	50	$1/2^+$	1	1/2	45
	$3/2^-$	0	3/2	77	$3/2^-$	0	3/2	52
	$3/2^-$	1	3/2	123	$5/2^+$	0	3/2	89
4	0^+	2	2	51	0^+	0	2	43
	0^+	0	0	72	0^+	2	0	54
	0^+	1	1	109	0^+	1	2	77
5	$1/2^+$	1	3/2	29	$1/2^+$	1	3/2	21
	$1/2^-$	1	3/2	32	$1/2^-$	1	3/2	23
	$1/2^+$	2	1/2	39	$3/2^+$	1	3/2	30
6	0^+	2	1	24	0^-	1	2	16
	0^-	1	2	26	1^-	1	2	22
	1^-	1	2	33	1^+	1	2	23
7	$3/2^+$	2	7/2	32	$3/2^+$	2	7/2	28
	$5/2^+$	1	7/2	65	$5/2^+$	1	7/2	42
	$7/2^+$	1	7/2	87	$7/2^+$	1	7/2	59
8	0^+	2	2	19	0^+	2	2	16
	2^+	2	2	31	2^+	2	2	24
	2^+	2	2	33	2^+	2	2	27
9	$1/2^-$	2	5/2	25	$1/2^-$	2	5/2	18
	$3/2^-$	2	5/2	29	$3/2^-$	2	5/2	21
	$3/2^+$	3	3/2	31	$3/2^+$	2	5/2	24

should be noticed that the rotational energy of the zero-isospin $(B,S)=(7,-7)$ state does not appear in Table VIII. In fact, the lowest lying of such states has $E_{rot}=104$ MeV for Set A and $E_{rot}=61$ MeV for Set B. That is, it shows up as an excited state with higher energy. Nevertheless, taking into account the rather large rotational energies of the $Y=I=0$ states with $B=3$ and 4 it happens that the binding energy of the heptalambda is increased by 45 MeV for both sets of parameters. For the case of the heptalambda ionization energy, one can verify that the values given in Ref. [19] remain basically unaffected by the rotational corrections. For this purpose one has to use the values of the rotational energies of the lowest $Y=I=0$ with $B=6$. Such values (which are not listed in Table VIII) are 87 MeV for Set A and 52 MeV for Set B.

VI. CONCLUSIONS

In this work we have studied the rotational corrections to the masses of the multibaryons within the bound state approach to the SU(3) Skyrme model. To describe the multi-Skyrmion backgrounds we have used *Ansätze* based on rational maps. Such configurations are known to provide a good approximation to the exact numerical ones, and lead to a great simplification in the solution of the kaon eigenvalue equation. An important property of these approximate configurations is that they have the same symmetries as the exact ones. Consequently, the collective Hamiltonians and wave functions determined in this work are also valid in that case. They have been obtained by making extensive use of the properties of the corresponding symmetry groups. In particular, we have shown how the Finkelstein-Rubinstein phases fix, in a unique way, the one dimensional irreducible representations in which each wave function should transform.

Using two standard sets of parameters for the effective SU(3) Skyrme action we have calculated all the inertia parameters and hyperfine splitting constants for $B\leqslant 9$. We have found that, as a general trend, the isospin moments of inertia increase as n^2 while the spin ones as n, where $B=n$. Thus, the low lying nonstrange multibaryons have the lowest possible value of isospin. The situation is more complicated in the case of strange particles, for which there is a quite delicate interplay between the different terms contributing to the rotational energies.

We have also estimated the rotational corrections to the tetralambda and heptalambda binding energies given in Ref. [19]. We found that these corrections are relatively small and do not affect the stability of these particles. This statement can be certainly extended to the recent studies on the stability of heavier flavored multi-Skyrmions [31].

We finish with a comment on the Casimir corrections to the multibaryon masses. Although these corrections are not expected to affect the rotational energies obtained in the present work, in any significant way, they might play some role in the determination of the multibaryon binding energies. Within the SU(2) Skyrme model it has been shown [32] that they are responsible for the reduction of the otherwise large $B=1$ soliton mass to a reasonable value when the empirical value of f_π is used. Here, we have avoided the $B=1$ large mass problem by using the customary method of fitting f_π to reproduce the nucleon mass [27]. A more consistent approach should certainly use the empirical f_π and include the Casimir corrections. In this respect, there have recently been some efforts [33] to evaluate the corrections to the $B=1$ mass in the SU(3) Skyrme model. Unfortunately, even in the SU(2) sector, almost nothing is known for $B>1$. This is, of course, a very difficult task. Already in the SU(2) model, it requires the knowledge of the pion excitation spectrum around the nontrivial multi-Skyrmion up to rather large values of angular momentum. Nevertheless, recent studies of the SU(2) multi-Skyrmion low-lying vibrational spectra [34] could be considered to be the first steps in this direction.

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APPENDIX

In this Appendix we present the explicit expression of the rational maps used in this work. They are

$$R_1 = z, \tag{A1}$$

$$R_2 = z^2, \tag{A2}$$

$$R_3 = \frac{i\sqrt{3}z^2 - 1}{z(z^2 - i\sqrt{3})}, \tag{A3}$$

$$R_4 = \frac{1 + 2i\sqrt{3}z^2 + z^4}{1 - 2i\sqrt{3}z^2 + z^4}, \tag{A4}$$

$$R_5 = \frac{z(z^4 - ib_5 z^2 - a_5)}{a_5 z^4 + ib_5 z^2 - 1}, \tag{A5}$$

$$R_6 = \frac{z^4 + ia_6}{z^2(ia_6 z^4 + 1)}, \tag{A6}$$

$$R_7 = \frac{z^5 - a_7}{z^2(a_7 z^5 + 1)}, \tag{A7}$$

$$R_8 = \frac{z^6 - ia_8}{z^2(ia_8 z^6 - 1)}, \tag{A8}$$

$$R_9 = \frac{z^3(-z^6 + 3i\sqrt{3}z^4 + 9z^2 + 5i\sqrt{3}) + a_9 z(-i\sqrt{3}z^6 - z^4 + i\sqrt{3}z^2 + 1)}{5i\sqrt{3}z^6 + 9z^4 + 3i\sqrt{3}z^2 - 1 + a_9 z^2(z^6 + i\sqrt{3}z^4 - z^2 - i\sqrt{3})}. \tag{A9}$$

The numerical values of the real constants a_i, b_i appearing in these expressions are

$$a_5 = 3.07, \quad a_6 = 0.158, \quad a_7 = 0.143, \quad a_8 = 0.137, \quad a_9 = 1.98, \quad b_5 = 3.94. \tag{A10}$$

The reader can check that in most cases our maps agree with those given in Ref. [20]. There are a few exceptions, however. For $B=7$ we have chosen a different orientation in the spin and isospin spaces in such a way that one of the 5-fold axes coincides with the z-direction. In the case of $B=9$ we have selected the map for which the T_d group operations are realized in exactly the same way in both spin and isospin spaces (namely, $g = D_g$). This is not the case for the $B=9$ map given in Ref. [20].

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