Critical look at rescattering effects on γ from $B^+ \to K\pi$

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Three ways of dealing with rescattering effects in $B^\pm \to K^0\pi^\pm$ are compared, in order to determine the weak phase γ from these processes and $B^\pm \to K^\pm\pi^0$. We find that neglecting these contributions altogether may involve sizeable errors in γ, depending on the rescattering amplitude and on the value of a certain measurable strong phase. We show that an attempt to eliminate these effects by using the charge-averaged rate of $B^\pm \to K^\pm K^0$ suffers from a large theoretical error due to SU(3) breaking, which may be resolved when using also the processes $B^\pm \to \pi^\pm \eta_8$.

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I. INTRODUCTION

The weak phase $\gamma = \mathrm{Arg}(V_{ub}^*)$ is presently the least well known quantity among the four parameters (three angles and a phase) of the Cabibbo-Kobayashi-Maskawa (CKM) matrix. Its determination, which is regarded to be more difficult than that of the other two angles of the CKM unitarity triangle [1], can provide a crucial test of the CKM mechanism for CP violation in the standard model. Several methods have been proposed to determine γ from hadronic two-body B decays. The methods which seem to be experimentally most feasible in the near future are based on applications of SU(3) flavor symmetry in B decays into two light charmless pseudoscalars [2]. These methods involve certain theoretical uncertainties, which are expected to be reduced when more data become available and when better theoretical understanding of hadronic B decays is achieved.

In a first paper in a series, Gronau, London, and Rosner (GLR) [3] proposed to extract γ by combining decay rate measurements of $B^+ \to K\pi$, $B^+ \to \pi\pi$ with their charge-conjugates. SU(3) breaking, occurring in a relation between $B \to \pi\pi$ $I=2$ and $B \to K\pi$ $I=3/2$ amplitudes, was introduced through a factor f_K/f_π when assuming that these amplitudes factorize. In its original version, suggested before the observation of the heavy top quark, the method of Ref. [3] neglected electroweak penguin (EWP) contributions and certain rescattering effects. Subsequently, model-calculations showed that due to the heavy top quark the neglected EWP terms were significant [4]; and recently these terms were related by SU(3) to the $B \to K\pi$ $I=3/2$ current-current amplitudes [5,6]. This led to a modification [7] of the GLR method, to be referred to as the Gronau-London-Rosner-Neubert (GLRN) method, which in the limit of flavor SU(3) symmetry includes EWP effects in a model-independent way. Corrections from SU(3) breaking, affecting the relation between EWP terms and current-current terms, were argued to be small [5,8].

Assuming that the above SU(3) breaking effects are indeed under control, there is still an uncertainty due to rescattering effects. To determine γ from the above rates, one takes the $B^+ \to K^0\pi^+$ amplitude to be pure penguin, involving no term with weak phase γ. This assumption, which neglects quark annihilation and rescattering contributions from charmless intermediate states, was challenged by a large number of authors [9]. Several authors proposed ways of controlling rescattering effects in $B^\pm \to K^0\pi^\pm$ by relating them through SU(3) to the much enhanced effects in $B^\pm \to K^\pm\bar{K}^0$ [10–12] (see also Refs. [13–15]). The charge-averaged rate of the latter processes can be used to set an upper limit on the rescattering amplitude in $B^\pm \to K^0\pi^\pm$. While present limits are at the level of $20-30\,\%$ of the dominant penguin amplitude [6,8] (depending somewhat on the value of γ), they are expected to be improved in the future. The smaller the rescattering amplitude is, the more precisely can γ be determined from the GLRN method. A recent demonstration [8], based on a few possible rate measurements, seems to show that if the rescattering amplitude is an order of magnitude smaller than the dominant penguin amplitude in $B^+ \to K^0\pi^+$, the uncertainty in γ is only about $5°$.

In the present Letter we reexamine in detail the uncertainty in γ due to rescattering effects. Using a geometrical interpretation for the extraction of γ, we perform in Sec. II numerical simulations which cover the entire parameter space of the two relevant strong phases, the rescattering phase ϕ_A and the relative phase ϕ between $I=3/2$ current-current and penguin amplitudes. We find that, contrary to the demonstration made in Ref. [8], a 10% rescattering amplitude leads to an uncertainty in γ as large as about $14°$ around $\phi \sim 90°$. For certain singular cases no solution can be found for γ. We show that ϕ can be determined rather precisely from the $B^\pm \to K\pi$ rate measurements [8], which could reduce substantially the error in γ if values far apart from $\phi = 90°$ were found.

It has been suggested [12] to go one step beyond setting limits on rescattering contributions in $A(B^\pm \to K^0\pi^\pm)$ and to completely eliminate them by using the charge-averaged rate measurement of $B^\pm \to K^\pm K^0$. Applying our geometrical formulation, we will show in Sec. III that the resulting determination of γ is unstable under SU(3) breaking which can introduce very large uncertainties in γ.

Finally, in order to overcome these uncertainties, we have

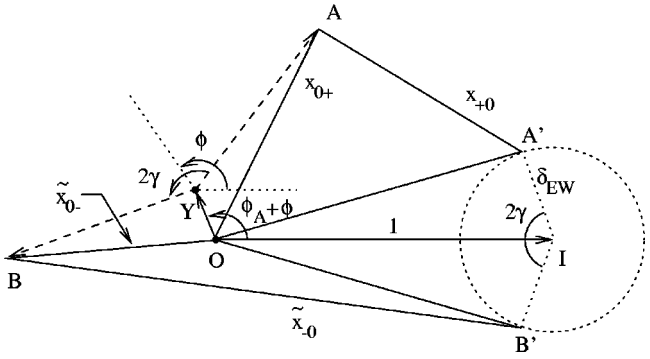

FIG. 1. Relative orientation of the $B^+\to K\pi$ amplitude triangles (4), normalized as described in the text. The triangle OAA' (OBB') corresponds to the $B^{+(-)}\to K\pi$ decays. The rescattering amplitude in $B^\pm\to K^0\pi^\pm$ is described by the line OY.

recently proposed to use in addition to $B^\pm\to K^\pm\bar{K}^0$ also the processes $B^\pm\to\pi^\pm\eta_8$ [16]. Although this may be considered an academic exercise, mainly due to complicating $\eta-\eta'$ mixing effects, we will examine in Sec. IV the precision of this method. We will show that when neglecting $\eta-\eta'$ mixing, the theoretical error in γ is reduced to a few degrees. We conclude in Sec. V. An algebraic condition, used in Sec. III to eliminate rescattering effects by $B^\pm\to K^\pm K^0$ decays, is derived in an appendix.

II. RESCATTERING UNCERTAINTY IN γ FROM $B^\pm\to K\pi$

The amplitudes for charged B decays can be parametrized in terms of graphical contributions representing SU(3) amplitudes (we use the notations of Ref. [6]):

$$A(B^+\to K^0\pi^+)=|\lambda_u^{(s)}|e^{i\gamma}(A+P_{uc})+\lambda_t^{(s)}(P_{ct}+P_3^{\rm EW}), \tag{1}$$

$$\sqrt{2}A(B^+\to K^+\pi^0)=|\lambda_u^{(s)}|e^{i\gamma}(-T-C-A-P_{uc}) +\lambda_t^{(s)}(-P_{ct}+\sqrt{2}P_4^{\rm EW}), \tag{2}$$

$$\sqrt{2}A(B^+\to\pi^+\pi^0)=|\lambda_u^{(s)}|e^{i\gamma}(-T-C), \tag{3}$$

where $\lambda_{q'}^{(q)}=V_{q'b}^*V_{q'q}$ are the corresponding CKM factors. These amplitudes satisfy a triangle relation [3,7]

$$\sqrt{2}A(B^+\to K^+\pi^0)+A(B^+\to K^0\pi^+) =\sqrt{2}\tilde{r}_u|A(B^+\to\pi^+\pi^0)|e^{i(\gamma+\xi)}(1-\delta_{\rm EW}e^{-i\gamma}). \tag{4}$$

Here we denote $\tilde{r}_u=(f_K/f_\pi)\lambda/(1-\lambda^2/2)\simeq 0.28$, $\delta_{\rm EW}=-(3/2)|\lambda_t^{(s)}/\lambda_u^{(s)}|\kappa\simeq 0.66$ [$\kappa\equiv(c_9+c_{10})/(c_1+c_2)=-8.8\times10^{-3}$], while ξ is an unknown strong phase. The second term in the brackets represents the sum of EWP contributions to the amplitudes on the left-hand side [5,6]. The factor f_K/f_π accounts for factorizable SU(3) breaking effects.

The relation (4), together with its charge-conjugate counterpart, written for $\tilde{A}(\bar{B}\to\bar{f})\equiv e^{2i\gamma}A(\bar{B}\to\bar{f})$, are represented graphically by the two triangles OAA' and OBB' in Fig. 1. Here all amplitudes are divided by a common factor $\mathcal{A}\equiv\sqrt{2}\tilde{r}_u|A(B^+\to\pi^+\pi^0)|e^{i(\gamma+\xi)}$, such that the horizontal line OI is of unit length and the radius of the circle is $\delta_{\rm EW}$. Four of the sides of the two triangles are given by

$$x_{0+}=\frac{1}{\sqrt{2}\tilde{r}_u}\frac{|A(B^+\to K^0\pi^+)|}{|A(B^+\to\pi^+\pi^0)|},$$

$$x_{+0}=\frac{1}{\tilde{r}_u}\frac{|A(B^+\to K^+\pi^0)|}{|A(B^+\to\pi^+\pi^0)|},$$

$$\tilde{x}_{0-}=\frac{1}{\sqrt{2}\tilde{r}_u}\frac{|A(\bar{B}^-\to\bar{K}^0\pi^-)|}{|A(B^+\to\pi^+\pi^0)|},$$

$$\tilde{x}_{-0}=\frac{1}{\tilde{r}_u}\frac{|A(\bar{B}^-\to K^-\pi^0)|}{|A(B^+\to\pi^+\pi^0)|}. \tag{5}$$

The relative orientation of the two triangles depends on γ and is not determined from measurements of the sides alone. *Assuming that the rescattering amplitude with weak phase γ in $B^+\to K^0\pi^+$ can be neglected*, one takes the amplitude (1) to be given approximately by the second (penguin) term [3,7], which implies $OB=e^{2i\gamma}OA$ in Fig. 1. In this approximation, the weak phase γ is determined by requiring that the angle (2γ) between OA and OB is equal to the angle (2γ) at the center of the circle [7].

In order to study the precision of determining in this way the phase γ as function of the rescattering contribution which is being neglected, let us rewrite Eq. (1) in the form

$$A(B^+\to K^0\pi^+)=-V_{cb}\left(1-\frac{\lambda^2}{2}\right)p(1+\epsilon_A e^{i\phi_A}e^{i\gamma}),$$

$$p\equiv P_{ct}+P_3^{\rm EW}, \tag{6}$$

where ϵ_A measures the magnitude of rescattering effects. In Fig. 1 the magnitude of these effects has a simple geometrical interpretation in terms of the distance of the point Y from the origin O, $\epsilon_A=|YO|/|YA|$, where YO and YA are the two components in the $B^+\to K^0\pi^+$ amplitude carrying weak phases γ and zero, respectively,

$$YO=|\lambda_u^{(s)}|e^{i\gamma}[(A+P_{uc})-p]/\mathcal{A},\quad YA=V_{cb}\left(1-\frac{\lambda^2}{2}\right)p/\mathcal{A}. \tag{7}$$

The rescattering phase ϕ_A is given by $\phi_A=\mathrm{Arg}(YO/YZ)$, where Z is any point on the line bisecting the angle AYB. A second strong phase which affects the determination of γ is ϕ, the relative strong phase between the penguin amplitude p and the $I=3/2$ current-current amplitude $T+C$. In Fig. 1 this phase is given by $\phi=\mathrm{Arg}(YZ/OI)$.

Let us now investigate the dependence of the error in γ when neglecting rescattering on the relevant hadronic parameters. Our procedure will be as follows. First we generate a set of amplitudes based on the geometry of Fig. 1 and on given values of the parameters γ, ϵ, ϵ_A, ϕ_A, and ϕ; then we

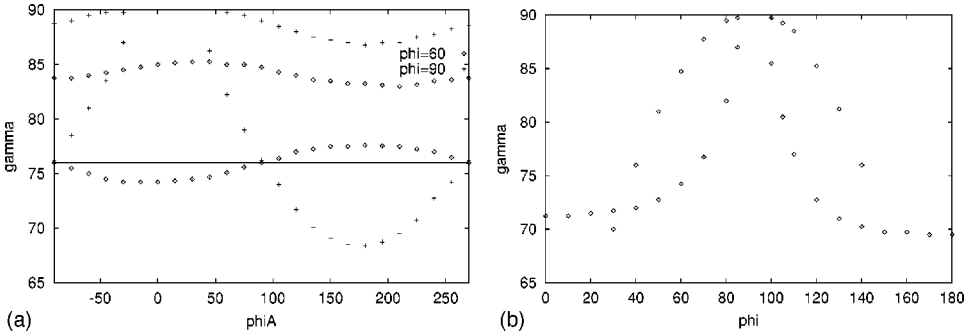

FIG. 2. The weak phase γ is obtained as the solution to the equation $\cos(2\gamma)=\cos(BOA)$. (a) the dependence of the solution on ϕ_A, for two values of $\phi=60°$ and $\phi=90°$; (b) the dependence of the solution on ϕ, for $\phi_A=0°$ (both graphs correspond to $\epsilon_A=0.1, \gamma=76°$).

solve the equation $\cos 2\gamma=\cos(BOA)$ and compare the output value of γ with its input value. Here ϵ is given in terms of the ratio of charge-averaged branching ratios [3,7]

$$\epsilon \equiv \frac{\lambda}{1-\lambda^2/2}\frac{f_K}{f_\pi}\sqrt{\frac{2B(B^\pm\to\pi^\pm\pi^0)}{B(B^\pm\to K^0\pi^\pm)}}, \tag{8}$$

The geometrical construction in Fig. 1 is described by

$$YA=\frac{e^{i(\phi-\gamma)}}{\epsilon\sqrt{1+2\epsilon_A\cos\phi_A\cos\gamma+\epsilon_A^2}}OI,$$

$$OY=\epsilon_A e^{i(\phi_A+\gamma)}YA, \tag{9}$$

implying a rate asymmetry between $B^+\to K^0\pi^+$ and $B^-\to\overline{K^0}\pi^-$.

For illustration, we take $\gamma=76°, \epsilon=0.24$ [5], $\epsilon_A=0.1$ (which is a reasonable guess [10,11]), and we vary ϕ and ϕ_A in the range $0°\leqslant\phi\leqslant 180°$, $-90°\leqslant\phi_A\leqslant 270°$. The results of a search for solutions in the interval $65°\leqslant\gamma\leqslant 90°$ are presented in Fig. 2 which displays a twofold ambiguity. Figure 2(a) shows the solution as function of ϕ_A for two values of ϕ, $\phi=60°$ and $\phi=90°$. Whereas for $\phi_A=90°$ the solution is very close to the input value, the deviation becomes maximal for $\phi_A=0°,180°$. This agrees with the geometry of Fig. 1, in which the largest rescattering effects are expected when YO is parallel or antiparallel to the line bisecting the angle BYA.

In a second plot, Fig. 2(b), we fix $\phi_A=0°$ and vary ϕ over its entire range, which illustrates the maximal rescattering effect. We find two branches of the solution for γ, both of which deviate strongly from the input value $\gamma=76°$ for values of ϕ around 90°. At $\phi=90°$ there is no solution for $\epsilon_A=0.1$ in the considered interval. We checked that the solution is restored and approaches the input value as the magnitude of ϵ_A decreases to zero, as it should. Thus, the uncertainty in γ, seen both in Fig. 2(a) and Fig. 2(b) at $\phi_A=0°$ and around $\phi=90°$, is about 14°. It can even be worse in the singular cases where no solution for γ can be found.

A variant of this method for determining γ, proposed recently in Ref. [8], was formulated in terms of two quantities R_* and $\tilde{A}$ defined by

$$R_*\equiv\frac{B(B^\pm\to K^0\pi^\pm)}{2B(B^\pm\to K^\pm\pi^0)},$$

$$\tilde{A}\equiv\frac{B(B^+\to K^+\pi^0)-B(B^-\to K^-\pi^0)}{B(B^\pm\to K^0\pi^\pm)} - \frac{B(B^+\to K^0\pi^+)-B(B^-\to\overline{K^0}\pi^-)}{2B(B^\pm\to K^0\pi^\pm)}. \tag{10}$$

These quantities do not contain $\mathcal{O}(\epsilon_A)$ terms; their dependence on the rescattering parameter ϵ_A appears only at order $\mathcal{O}(\epsilon\epsilon_A)$. Therefore, it was argued in Ref. [8], the determination of γ, by setting $\epsilon_A=0$ in the expressions for R_* and $\tilde{A}$, is insensitive to rescattering effects. This procedure gives two equations for γ and ϕ which can be solved simultaneously from R_* and $\tilde{A}$. Using two pairs of input values for $(R_*,\tilde{A})$ (corresponding to a restricted range for ϕ_A and ϕ) seemed to indicate that the error in γ for $\epsilon_A=0.08$ is only about 5°. (The relations between the parameters used in Ref. [8] and ours are $\phi=-\phi$, $\eta=\phi_A+\pi$, $\overline{\epsilon_{3/2}}=\epsilon$, and $\epsilon_a=\epsilon_A$.)

In Fig. 3 we show the results of such an analysis carried out for the entire parameter space of ϕ_A and ϕ. Whereas the

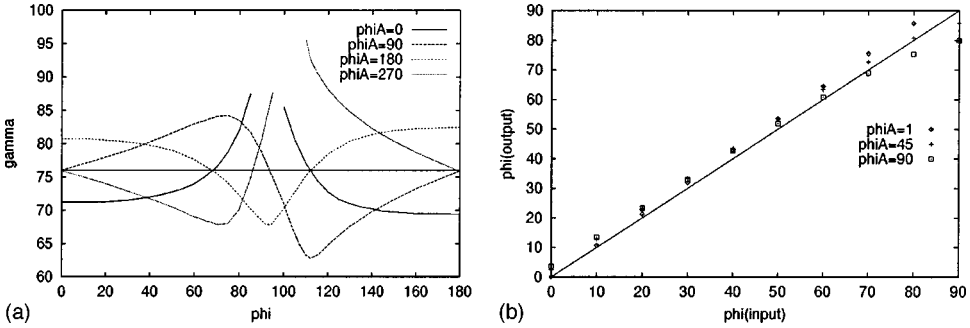

FIG. 3. (a) the weak phase γ extracted from the method using the parameters $(R_*,\tilde{A})$, as a function of the strong phase ϕ for several values of $\phi_A(\epsilon_A=0.1)$. The horizontal line shows the assumed physical value of $\gamma=76°$. (b) the strong phase ϕ can be reconstructed using the $(R_*,\tilde{A})$ data.

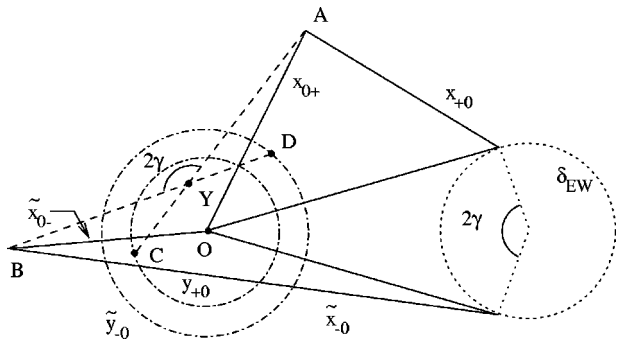

FIG. 4. Geometric construction for the method described in Sec. III. The lines OC and OD denote the amplitudes $A(B^{\pm}\to K^{\pm}K^0)$, normalized as described in text. The positions of the points C and D are found as intersection points of the lines AY and BY with the two circles of radii given by $|A(B^{\pm}\to K^{\pm}K^0)|$.

angle ϕ can be recovered with small errors, the results for γ show the same large rescattering effects for values of ϕ around 90° as in Fig. 2. (A slight improvement is the absence of a discrete ambiguity in the value of γ.) These results show that the large deviation of γ from its physical value for $\phi=90°$ is a general phenomenon, common to all variants of this method. Some information about the size of the expected error can be obtained by first determining ϕ. Values not too close to 90° would be an indication for a small error.

III. ELIMINATING RESCATTERING BY $B^{\pm}\to K^{\pm}K^0$

The amplitude for $B^+\to K^+\bar{K}^0$ is obtained from $A(B^+\to K^0\pi^+)$ in Eq. (1) by a U-spin rotation [10]

$$A(B^+\to K^+\bar{K}^0)=|\lambda_u^{(d)}|e^{i\gamma}(A+P_{uc}) +|\lambda_t^{(d)}|e^{-i\beta}(P_{ct}+P_3^{\rm EW}). \qquad (11)$$

In the limit of SU(3) symmetry the amplitudes in Eq. (11) are exactly the same as those appearing in Eq. (1). In Fig. 4 $A(B^+\to K^+\bar{K}^0)$, scaled by the factor $\lambda/(1-\lambda^2/2)$ (and divided by $\mathcal{A}$ as in Fig. 1), is given by the line OC and its charge-conjugate is given by OD. We have shown in Ref. [16] that knowledge of these two amplitudes allows one to completely eliminate the rescattering contribution $A+P_{uc}$ from the determination of γ. This is achieved by effectively replacing in the GLRN method the origin O by the intersection Y of the lines AC and BD. γ is determined by requiring that the angle (2γ) between YA and YB is equal to the angle (2γ) at the center of the circle.

The amplitude (11) can be decomposed into two terms carrying definite weak phases in form very similar to Eq. (6),

$$\frac{\lambda}{1-\lambda^2/2}A(B^+\to K^+\bar{K}^0)=-V_{cb}\left(1-\frac{\lambda^2}{2}\right)p \times\left(-\frac{\lambda^2}{(1-\lambda^2/2)^2}+\epsilon_A e^{i\phi_A}e^{i\gamma}\right), \qquad (12)$$

The ratio $|CY|/|AY|=\lambda^2/(1-\lambda^2/2)^2$ implies that the triangle AYB is about 25 times larger than the triangle CYD. This will result in a large uncertainty in γ also when the equality between the corresponding terms in $B^+\to K^0\pi^+$ and $B^+\to K^+\bar{K}^0$ amplitudes involves relatively small SU(3) violation.

The geometrical construction by which rescattering amplitudes can be completely eliminated in the SU(3) limit consists of three steps. (See Fig. 4. For an alternative suggestion, see Ref. [12].)

(a) Determine the position of the point Y as a function of the variable angle 2γ and the decay rates of $B^{\pm}\to K\pi$ and $B^+\to\pi^+\pi^0$. The point Y is chosen on the midperpendicular of AB such that the equality of the angles marked 2γ is preserved for any value of γ.

(b) Draw two circles of radii $\lambda/(1-\lambda^2/2)|A(B^{\pm}\to K^0K^{\pm})|$ centered at the origin O (dashed-dotted circles in Fig. 4). The intersections of the lines AY and BY with these circles determine C and D, respectively (up to a twofold ambiguity), again as functions of γ.

(c) The physical value of γ is determined by the requirement $|AC|=|BD|$ [16]. This condition on γ can be formulated in an algebraic form, showing that only the charge-averaged rate of $B^{\pm}\to K^{\pm}K^0$ is needed. The condition is given by Eq. (A1) in the Appendix.

Let us examine the precision of this method for $\epsilon_A=0.1$ at $\phi\simeq 90°$, for which the simpler method of Sec. II receives large rescattering corrections. In Fig. 5(a) we show the left-hand side of Eq. (A1) as a function of variable γ at $\phi=90°$ for several values of ϕ_A. The value of γ is obtained from the condition that the left-hand side of this equation vanishes. In the absence of SU(3) breaking this method re-

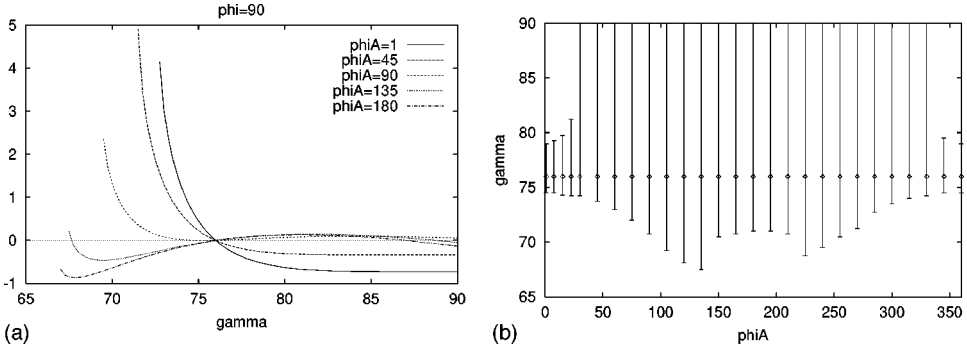

FIG. 5. (a) The left-hand of Eq. (A1) as a function of variable γ for $\phi=90°$ and for different values of ϕ_A. All these curves intersect at $\gamma=76°$, which is the assumed physical value. (b) SU(3) breaking effects introduce an error on the extracted value of γ, here shown as function of ϕ_A at $\phi=90°$.

produces precisely the physical value of γ ($\gamma=76°$) for all values of ϕ_A. However, SU(3) breaking effects can become important, to the point of completely spoiling this method. We simulate these effects by taking the amplitudes p and $a \equiv A+P_{uc}-p$ in $B^{\pm}\to K^{\pm}K^0$ [Eq. (11)] to differ by at most 30% from those in $B^{\pm}\to K^0\pi^{\pm}$ [Eq. (1)]. This expands the lines of Fig. 5(a) into bands of finite width, which give a range for the output value of γ.

In Fig. 5(b) we show the effects of SU(3) breaking on the determination of γ as a function of ϕ_A for $\phi=90°$. We see that for values of $|\phi_A|$ larger than about 25° the error on γ is quite large. Thus, we conclude that for certain values of the strong phases the determination of γ using this method is unstable under SU(3) breaking in the relation between $B^+ \to K^0\pi^+$ and $B^+\to K^+\overline{K^0}$.

IV. THE USE OF $B^{\pm}\to\pi^{\pm}\eta_8$

In Ref. [16] we proposed to use in addition to $B^+ \to K^+\overline{K^0}$ also $B^+\to\pi^+\eta_8$ and their charge conjugates. Writing

$$A(B^+\to\pi^+\eta_8)=|\lambda_u^{(d)}|e^{i\gamma}(-T-C-2A-2P_{uc}) + |\lambda_t^{(d)}|e^{-i\beta}(-P_{ct}+P_5^{\mathrm{EW}}), \qquad (13)$$

we find the triangle relation

$$A(B^+\to K^+\overline{K^0})+\sqrt{\frac{3}{2}}A(B^+\to\pi^+\eta_8) = \frac{1}{\sqrt{2}}A(B^+\to\pi^+\pi^0). \qquad (14)$$

This relation and its charge conjugate provide another condition which determines the positions of the points C and D. As in Sec. III, the phase γ is determined by the equation $\cos(BYA)=\cos 2\gamma$, where the point Y is fixed by the intersection of the lines AC and BD. General considerations, based on the relative sizes of the amplitudes involved, suggest that this method is relatively insensitive to SU(3) breaking effects [16].

We illustrate this in Fig. 6 where we show on the same plot the two sides of the equation $\cos(BYA)=\cos 2\gamma$ as functions of the variable γ. As in the method of Sec. III, SU(3) breaking is simulated by taking the penguin (p) and annihilation (a) amplitudes in $B^{\pm}\to K^{\pm}K^0$ to differ by at most 30% (separately for their real and imaginary parts) from those in $B^{\pm}\to K^0\pi^{\pm}$. The latter are used to construct the positions of the points C and D. In the example of Fig. 6 we take $\epsilon_A=0.1, \phi=90°, \phi_A=45°$, for which the two methods described in Secs. II and III were shown to lead to large errors in γ. For an input value $\gamma=76°$, the output is given by the range $74°<\gamma<78°$, obtained by the intersection of the solid line with the band formed by the diamond points. We see that the error in γ due to SU(3) breaking is less than $\pm 2°$, which confirms the general arguments of Ref. [16]. This scheme, or rather its analogous version using B^0 and B_s decay [16], may prove useful for a determination of γ in case

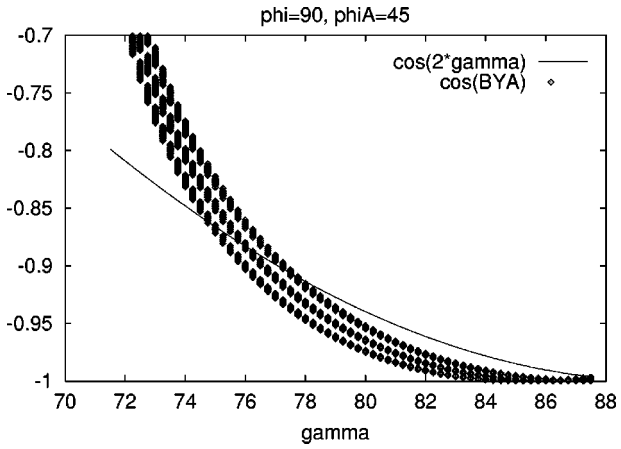

FIG. 6. Numerical results for the method of Sec. IV. The two sides of the equation $\cos(BYA)=2\gamma$ as function of variable γ, including 30% SU(3) breaking effects in the p and a amplitudes. The physical value of γ is determined by the intersection of the solid line with the wide band. The strong phases are taken as $(\phi,\phi_A)=(90°,45°)$.

that the strong phases (ϕ,ϕ_A) turn out to have values which preclude the use of the two simpler methods.

V. CONCLUSION

We compared three ways of dealing with rescattering effects in $B^{\pm}\to K^0\pi^{\pm}$, in order to achieve a precise determination for the weak phase γ from these processes and $B^{\pm} \to K^{\pm}\pi^0$. In the simplest GLRN method which neglects rescattering we find that large errors in γ are possible for a particular region of the strong phases, $\phi\sim 90°$, even when the rescattering term is only at a level of 10%. Limits on rescattering at this level are attainable from $B^{\pm}\to K^{\pm}K^0$. B^+ and B^- decay rate measurements into $K\pi$ are expected to provide rather precise information on ϕ. In the likely case that ϕ turns out to be far away from 90° small errors in γ would be implied.

On the other hand, in the less likely case that values of ϕ are measured near 90° one may try to eliminate the rescattering effects by using also the charge-averaged $B^{\pm} \to K^{\pm}K^0$ rate. This method suffers from a sizable uncertainty due to SU(3) breaking. These uncertainties could be resolved by also measuring $B^{\pm}\to\pi^{\pm}\eta_8$. The effect of $\eta-\eta'$ mixing requires further study. Alternatively, to avoid this effect, one can apply the same method using B^0 and B_s decays.

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APPENDIX

The weak angle γ is fixed in the method described in Sec. III by the condition $|AC|=|BD|$, or equivalently $|YC|$

$=|YD|$. Explicitly, this can be written after some algebra as an equation in γ

$$2(1-x_0)^2\vec{Y}^2+2x_0(1-x_0)\vec{Y}\cdot(\vec{A}+\vec{B})+x_0^2(x_{0+}^2+\tilde{x}_{0-}^2) -(y_{+0}^2+\tilde{y}_{-0}^2)=0, \tag{A1}$$

where x_0 is defined as the ratio of two CP rate differences

$$x_0\equiv(y_{+0}^2-\tilde{y}_{-0}^2)/(x_{0+}^2-\tilde{x}_{0-}^2)\underset{\mathrm{SU(3)}}{\rightarrow}-\lambda^2/(1-\lambda^2/2)^2. \tag{A2}$$

Here

$$y_{+0}=\frac{1}{\sqrt{2}}\frac{f_\pi}{f_K}\frac{|A(\overline{B}^+\rightarrow K^+\overline{K}^0)|}{|A(B^+\rightarrow\pi^+\pi^0)|},$$

$$\tilde{y}_{-0}=\frac{1}{\sqrt{2}}\frac{f_\pi}{f_K}\frac{|A(\overline{B}^-\rightarrow K^-K^0)|}{|A(B^+\rightarrow\pi^+\pi^0)|} \tag{A3}$$

obey an SU(3) relation with the amplitudes (5) of $B^\pm\rightarrow K\pi^\pm$

$$y_{+0}^2-\tilde{y}_{-0}^2=-\frac{\lambda^2}{(1-\lambda^2/2)^2}(x_{0+}^2-\tilde{x}_{0-}^2). \tag{A4}$$

This implies that CP rate differences in $B^\pm\rightarrow K^0\pi^\pm$ and $B^\pm\rightarrow K^\pm K^0$ are equal and of opposite sign [12]. We see that in the SU(3) limit the condition (A1), which eliminates rescattering effects, requires only a measurement of the charge-averaged rate of $B^\pm\rightarrow K^\pm K^0$ and not the CP asymmetry in these processes [12].

To prove Eq. (A1), let us consider two lines AY and BY cutting two circles of radii R_1, R_2 (centered at the origin) at points C and D, respectively. The intersection points can be written as $\vec{C}=\vec{Y}+x_1(\vec{A}-\vec{Y})$ and $\vec{D}=\vec{Y}+x_2(\vec{B}-\vec{Y})$, where x_1,x_2 are solutions of the equations

$$(\vec{A}-\vec{Y})^2x_1^2+2x_1\vec{Y}\cdot(\vec{A}-\vec{Y})+(\vec{Y}^2-R_1^2)=0, \tag{A5}$$

$$(\vec{B}-\vec{Y})^2x_2^2+2x_2\vec{Y}\cdot(\vec{B}-\vec{Y})+(\vec{Y}^2-R_2^2)=0. \tag{A6}$$

The condition $|YC|=|YD|$ is equivalent to requiring that these two equations have a common solution $x_1=x_2$. Obviously, if such a solution exists, it is given by

$$x_0=\frac{R_1^2-R_2^2}{2\vec{Y}\cdot(\vec{A}-\vec{B})}=\frac{R_1^2-R_2^2}{\vec{A}^2-\vec{B}^2}, \tag{A7}$$

where we used the equality $(\vec{A}-\vec{Y})^2=(\vec{B}-\vec{Y})^2$. Taking the sum of Eqs. (A5) and (A6) with the value (A7) for x leads immediately to the condition (A1).

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