$(F,\ \mathrm{D5})$ bound state, $\mathrm{SL}(2,Z)$ invariance and the descendant states in type IIB and type IIA string theory

J. X. Lu*
Center for Theoretical Physics, Texas A&M University, College Station, Texas 77843

Shibaji Roy†
Saha Institute of Nuclear Physics, 1/AF Bidhannagar, Calcutta 700 064, India

(Received 8 June 1999; published 15 November 1999)

Recently the space-time configurations of a set of nonthreshold bound states, called the $(F,\ \mathrm{D}p)$ bound states, have been constructed explicitly for every p with $2 \leqslant p \leqslant 7$ in both type IIA (for p even) and type IIB (for p odd) string theories by the present authors. By making use of the $\mathrm{SL}(2,Z)$ symmetry of type IIB theory we construct a more general $\mathrm{SL}(2,Z)$ invariant bound state of the type $((F,\ \mathrm{D1}),\ (\mathrm{NS5},\mathrm{D5}))$ in this theory from the $(F,\ \mathrm{D5})$ bound state. There are actually an infinite number of (m,n) strings forming bound states with (m',n') 5-branes, where strings lie along one of the spatial directions of the 5-branes. By applying T duality along one of the transverse directions we also construct the bound state $((F,\ \mathrm{D2}),\ (\mathrm{KK},\mathrm{D6}))$ in type IIA string theory. Then we give a list of possible bound states which can be obtained from these newly constructed bound states by applying T dualities along the longitudinal directions as well as S dualities to those in type IIB theory. [S0556-2821(99)01522-2]

PACS number(s): 11.25.Sq

I. INTRODUCTION

This is a sequel to our series on the study of a new kind of bound state that exists in both type IIA and type IIB string theories. In [1], we provided arguments from the worldvolume point of view that there exist Bogomol'nyi-Prasad-Sommerfield (BPS) bound states of Dp-branes carrying certain units of quantized constant electric fields, called the $(F,\ \mathrm{D}p)$ bound states, for every p with $1 \leqslant p \leqslant 8$ in type IIA (for p even) and type IIB (for p odd) string theory. The space-time configurations of these bound states have been constructed explicitly for $2 \leqslant p \leqslant 7$ in Ref. [2].[1] Each of these bound states preserves one-half of the space-time supersymmetries. In the worldvolume picture the F in $(F,\ \mathrm{D}p)$ represents the uniform and constant electric field lines flowing along, say, the x^1 axis of the Dp-brane due to the uniform and homogeneous charge distribution on the rest of the $(p-1)$ plane placed at $x^1=-\infty$, originating from an infinite number of open strings ending on this surface. On the other hand, the space-time configuration allows us to identify these field lines with the infinitely long fundamental strings or F-strings in the bulk. To make this identification more concrete, we have calculated the charges carried by F-strings and Dp-branes, as well as the mass per unit p-brane volume, and have shown that they match precisely with what we expect from the worldvolume study. We have noted in [1], that since type IIB theory is conjectured to possess an $\mathrm{SL}(2,Z)$ invariance, there must exist more general bound states than $(F,\ \mathrm{D}p)$ in this theory. By making use of this observation, we have constructed the more general nonthreshold bound state of the type $((F,\ \mathrm{D1}),\ \mathrm{D3})$ and some of its T-dual descendants in Ref. [8].

In this paper, we make an $\mathrm{SL}(2,Z)$ transformation on the nonthreshold bound state $(F,\ \mathrm{D5})$ in type IIB theory to construct $((F,\ \mathrm{D1}),\ (\mathrm{NS5},\mathrm{D5}))$ bound state. The space-time configuration consisting of the metric, the dilaton, the axion and the other nonvanishing gauge fields for this bound state are constructed explicitly. The initial $(F,\ \mathrm{D5})$ configuration consists of an infinite number of Neveu-Schwarz (NS) strings (each NS string is actually q F-strings) distributed uniformly over s D5-branes and lying along one of the spatial directions of D5-brane, where q and s are relatively prime integers as discussed in [2]. We here consider a genuine initial $(F,\ \mathrm{D5})$ bound state, i.e., both q and s are nonzero. In general, we expect that in the bound state $((F,\ \mathrm{D1}),\ (\mathrm{NS5},\mathrm{D5}))$, there are an infinite number of (m,n) strings lying along one of the spatial directions of (m',n') 5-branes. Although for the degenerate case when either the strings or the 5-branes (but not both) are present the integers (m,n) and (m',n') are individually relatively prime, for the general nondegenerate case this is not necessarily so. For the general $((F,\ \mathrm{D1}),\ (\mathrm{NS5},\mathrm{D5}))$ bound state, i.e., when the integers m,n and m',n' are nonzero, we find that a consistent quantization of the charges associated with the NS-NS and Ramond-Ramond (RR) gauge fields of both the strings and the 5-branes relates the charges of the strings with those of the 5-branes. As a result, the integers (m,n) corresponding to the electric charges of the strings and the integers (m',n') corresponding to the magnetic charges of the 5-branes get related as $(m,n)=k(a,b)$, $(m',n')=k'(-b,a)$, where (a,b) and (k,k') are relatively prime integers. This fact in turn tells us that when both k, k'

*Email address: jxlu@rainbow.physics.tamu.edu
†Email address: roy@tnp.saha.ernet.in

[1]The configurations for $p=3,4,6$ were also given previously in [3–5], respectively. Similar nonthreshold bound states in M theory or type IIA or IIB theory for a p'-brane within another p-brane with $p'<p$ were studied in [3–7].

are nonzero, the existence of bound states between m fundamental strings and n D-strings may imply the existence of bound states between m' NS5-branes and n' D5-branes, where the integers (m,n) and (m',n') are related to each other as given before. Thus we find that in general the SL(2, Z) invariant bound state ((F, D1), (NS5, D5)) of type IIB theory is characterized by two pairs of relatively prime integers (a,b) and (k,k'). We can obtain the other bound states, namely, (F, D5) and (D1, NS5) from this general solution by setting $a=1$, $b=0$ and $a=0$, $b=1$, respectively. Also note that the degenerate (NS5, D5) and (F, D1) cases can be obtained from the general ((F, D1), (NS5, D5)) bound state by setting (i) $k=0$, $k'=1$ and (ii) $k'=0$, $k=1$. For the former case we get the SL(2, Z) 5-branes discussed in [9], whereas for the latter case we get SL(2, Z) strings obtained in [6] with four additional isometries. But because the charges of the strings and the 5-branes are related as mentioned above, we cannot have bound states of the form (F, NS5) and (D1, D5) consistent with the fact that these bound states preserve 1/4 rather than 1/2 of the space-time supersymmetries. We have also obtained the expression for the tension of SL(2, Z) invariant nonthreshold bound state ((F, D1), (NS5, D5)) and have shown how it reduces to the tensions for the corresponding special case bound states.

The descendants of this bound state could be obtained by applying T duality along various transverse and longitudinal directions. We give an explicit construction of the bound states ((F, D2), (KK, D6)) in type IIA theory by applying T duality in one of the transverse directions. We also discuss how the bound states (F, D6) and (D2, KK) as well as the degenerate cases (F, D2) and (KK, D6) can be obtained from this general bound state as special cases. The tension expression for the bound state ((F, D2), (KK, D6)) is also given. We point out the problem of taking further T dualities along the transverse directions. Finally, we give a list of possible other descendant bound states which can be obtained from these by T dualities in various longitudinal directions as well as S dualities to those in type IIB theory.

This paper is organized as follows. In Sec. II, we use the SL(2, Z) invariance of type IIB theory to construct the nonthreshold ((F, D1), (NS5, D5)) bound state starting from the (F, D5) one. We show that (F, D5), (D1, NS5), (F, D1), and (NS5, D5) bound states can be obtained from this bound state as special cases. In Sec. III, we apply T duality on ((F, D1), (NS5, D5)) bound state along one of the transverse directions to construct ((F, D2), (KK, D6)) bound state and discuss the special cases as in Sec. II. Since we have shown how to implement S and T dualities in general, we here list all the possible bound states that can be obtained from the above-mentioned bound states by the application of T dualities in various longitudinal directions and S dualities to those belonging to type IIB theory. We conclude this paper in Sec. IV.

II. SL(2, Z) INVARIANCE AND NONTHRESHOLD ((F, D1), (NS5, D5)) BOUND STATE

In this section, we will use the SL(2, Z) symmetry of type IIB theory to construct the nonthreshold bound state ((F, D1), (NS5, D5)) from the explicit solution (F, D5) given in [2]. We will follow the procedure outlined in [6,10,11]. Let us begin with (F, D5) solution [2] given by the metric,

$$\begin{aligned} ds^2 = H'^{1/2}H^{1/4}[&H^{-1}(-(dx^0)^2+(dx^1)^2) \\ &+H'^{-1}((dx^2)^2+(dx^3)^2+(dx^4)^2+(dx^5)^2) \\ &+dy^i dy^i], \end{aligned} \tag{2.1}$$

with $i=1,2,3,4$; the dilaton,

$$e^{\phi}=H^{-1/2}, \tag{2.2}$$

and the remaining nonvanishing fields,

$$\begin{aligned} H_3^{(1)} &= -q\Delta_{(q,s)}^{-1/2} dH^{-1}\wedge dx^0 \wedge dx^1, \\ H_3^{(2)} &= s\frac{\sqrt{2}\kappa_0 Q_0^5}{\Omega_3}\epsilon_3, \\ H_5 &= qs\Delta_{(q,s)}^{-1}H'^{-2}dH\wedge dx^2\wedge dx^3\wedge dx^4\wedge dx^5. \end{aligned} \tag{2.3}$$

Note that in writing this solution we have set the scalars $\phi_{B0}=\chi_{B0}=0$, as they have nothing to do with the dilaton and the axion in the theory [2]. Also in the above $H_3^{(1)}$ and $H_3^{(2)}$ are the NS-NS and RR 3-form field strengths. H_5 is the self-dual 5-form field-strength with $H_5={}^*H_5$, where * denotes the Hodge dual. H is a harmonic function given by

$$H=1+\frac{Q_5}{r^2}, \tag{2.4}$$

where $r^2=y^iy^i$ and $Q_5=\Delta_{(q,s)}^{1/2}\sqrt{2}\kappa_0 Q_0^5/(2\Omega_3)$, H' is another harmonic function defined as

$$H'=\frac{q^2+s^2H}{\Delta_{(q,s)}}=1+\frac{s^2Q_5/\Delta_{(q,s)}}{r^2}, \tag{2.5}$$

with $\Delta_{(q,s)}=q^2+s^2$. Here q and s are two relatively prime integers denoting, respectively, the quantized NS string charge or the number of F-strings per $(2\pi)^4\alpha'^2$ of four-dimensional area perpendicular to the strings in (F, D5) and the D5-brane charge as discussed in [2]. ϵ_n denotes the volume form on an n-sphere whereas Ω_n is the volume of a unit n-sphere and is given as

$$\Omega_n=\frac{(2\pi)^{(n+1)/2}}{\Gamma\left(\frac{n+1}{2}\right)}. \tag{2.6}$$

Also, $\sqrt{2}\kappa_0=(2\pi)^{7/2}\alpha'^2$ and the unit charge for a Dp-brane is $Q_0^p\equiv(2\pi)^{(7-2p)/2}\alpha'^{(3-p)/2}$.

The electric charge of the F-strings in (F, D5) bound state can be calculated as

$$e^{(1)}=\frac{1}{\sqrt{2}\kappa_0}\int_{R^4\times S^3}(e^{-\phi}*H_3^{(1)}+H_3^{(2)}\wedge B_4). \qquad (2.7)$$

But as mentioned in [2], this expression is in fact infinite as there are an infinite number of F-strings in (F, D5). However, we can still define a quantized charge from Eq. (2.7) in the form as given below [2]

$$Q^{(1)}=(2\pi)^4\alpha'^2\frac{e^{(1)}}{\sqrt{2}\kappa_0 A_4}=qT_f, \qquad (2.8)$$

with $T_f=1/(2\pi\alpha')$ as the fundamental string tension. Here $A_4=\int dx^2dx^3dx^4dx^5$ is the coordinate area of $x^2x^3x^4x^5$-plane. The charge $Q^{(1)}$ represents the number of F-strings per $(2\pi)^4\alpha'^2$ area over $x^2x^3x^4x^5$-plane measured in some units (note that the F-strings lie along the x^1 axis). Also the quantized magnetic charge of the D5-brane is given as

$$P^{(2)}=g^{(2)}=\frac{1}{\sqrt{2}\kappa_0}\int_{S^3}H_3^{(2)}=sQ_0^5. \qquad (2.9)$$

It is well known that type IIB supergravity possesses a classical Cremmer-Julia [12] symmetry group SL(2, R). A discrete subgroup SL(2, Z) is now believed [13] to survive in the full quantum type IIB string theory. Under a global SL(2, R) symmetry the Einstein metric $g_{\mu\nu}$ is a singlet, the two 3-form field strengths $H_3^{(1)}$ and $H_3^{(2)}$ transform as a doublet, and the 5-form field strength is also a singlet. So the transformations of the various fields along with the two scalars, the dilaton (ϕ) and the axion (χ, the RR scalar) parametrizing the coset SL(2, R)/SO(2) defined as

$$\mathcal{M}=e^{\phi}\begin{pmatrix}\chi^2+e^{-2\phi} & \chi\\ \chi & 1\end{pmatrix}$$

are

$$g_{\mu\nu}\to g_{\mu\nu},\mathcal{M}\to\Lambda\mathcal{M}\Lambda^T,\begin{pmatrix}H_3^{(1)}\\ H_3^{(2)}\end{pmatrix}\equiv\mathcal{H}\to(\Lambda^{-1})^T\mathcal{H}$$
$$H_5\to H_5, \qquad (2.10)$$

where Λ is a global SL(2, R) transformation matrix and ''T'' denotes the transpose of a matrix.

Let us next look at how the charges would transform under the global SL(2, R) transformation. Since a general SL(2, Z) invariant configuration will have both (F, D1) strings (infinite numbers of them) living on (NS5, D5) branes, the charge expression in Eq. (2.7) will be modified to give electric charges of both F-string and D-string as

$$e^{(i)}=\frac{1}{\sqrt{2}\kappa_0}\int_{R^4\times S^3}(\mathcal{M}^{ij}*H_3^{(j)}+\epsilon^{ij}H_3^{(j)}\wedge B_4), \quad (2.11)$$

where $i,j=1,2$ and ϵ^{ij} is the SL(2, R) invariant totally antisymmetric tensor with $\epsilon^{12}=1$. As before, $e^{(i)}$ is not well defined and we can define the quantized charges as

$$Q^{(i)}=\frac{(2\pi)^4\alpha'^2e^{(i)}}{\sqrt{2}\kappa_0A_4}. \qquad (2.12)$$

The quantized magnetic charges of the NS5-brane and D5-brane can be obtained as

$$P^{(i)}=g^{(i)}=\frac{1}{\sqrt{2}\kappa_0}\int_{S^3}H_3^{(i)}. \qquad (2.13)$$

Note that the electric charge in Eq. (2.11) or Eq. (2.12) is a Noether charge and follows from the equation of motion, whereas the magnetic charge is topological and follows from Bianchi identity. It is clear from Eq. (2.10) that the electric charges of (F, D1) strings and the magnetic charges of (NS5, D5) branes would transform as

$$\begin{pmatrix}Q^{(1)}\\ Q^{(2)}\end{pmatrix}\equiv\mathcal{Q}\to\Lambda\,\mathcal{Q};\begin{pmatrix}P^{(1)}\\ P^{(2)}\end{pmatrix}\equiv\mathcal{P}\to(\Lambda^{-1})^T\mathcal{P}. \qquad (2.14)$$

Now in order to obtain the global SL(2, R) transformation matrix Λ_0, we start with the zero asymptotic values of the dilaton and the axion, i.e., $\mathcal{M}_0^{(initial)}=I$, where I is the identity matrix, and demand that Λ_0 will transform it into a fixed but arbitrary value as

$$\mathcal{M}_0=\Lambda_0 I\Lambda_0^T, \qquad (2.15)$$

where

$$\mathcal{M}_0=e^{\phi_0}\begin{pmatrix}\chi_0^2+e^{-2\phi_0} & \chi_0\\ \chi_0 & 1\end{pmatrix},$$

with ϕ_0 and χ_0 denoting the arbitrary but given asymptotic values of the scalars. Equation (2.15) will fix the SL(2, R) matrix Λ_0 in terms of ϕ_0, and χ_0 and an undetermined SO(2) angle α as

$$\Lambda_0=e^{\phi_0/2}\begin{pmatrix}e^{-\phi_0}\cos\alpha+\chi_0\sin\alpha & -e^{-\phi_0}\sin\alpha\\ \sin\alpha & \cos\alpha\end{pmatrix}. \qquad (2.16)$$

The angle α will be given shortly.

Note that once we apply SL(2, R) transformation on the initial quantized charges (2.8) and (2.9) of (F, D5) by Eq. (2.14), the charges will no longer remain quantized. In order to get around this problem, one needs either to introduce compensating factors in place of both the charges q and s or to replace q and s by arbitrary classical charges as $\tilde{\Delta}_{(m,n)}^{1/2}$ and $\bar{\Delta}_{(m',n')}^{1/2}$, respectively, where Δ's are the arbitrary numbers and will be determined in the process of charge quantization [14]. By imposing that the transformed charges are integers,[2]

[2]When we consider either (m,n) strings [15] or (m',n') 5-branes [9], the integers (m,n) or (m',n') need to be relatively prime in order for the strings or 5-branes to form nonthreshold bound states. For the present nonthreshold bound state consisting of an infinite number of (m,n) strings and a (m',n') 5-brane, (m,n) and (m',n') are not necessarily relatively prime, as we will see later.

namely, (m,n) for strings and (m',n') for the 5-branes we have, from (2.14),

$$\begin{pmatrix} m \\ n \end{pmatrix} = \Lambda_0^1 \begin{pmatrix} \tilde{\Delta}_{(m,n)}^{1/2} \\ 0 \end{pmatrix} \tag{2.17}$$

for the strings and

$$\begin{pmatrix} m' \\ n' \end{pmatrix} = ((\Lambda_0^5)^{-1})^T \begin{pmatrix} 0 \\ \bar{\Delta}_{(m',n')}^{1/2} \end{pmatrix} \tag{2.18}$$

for the 5-branes. Λ_0^1 and Λ_0^5 are the transformation matrices for strings and 5-branes. Equations (2.17) and (2.18) determine the form of Λ_0^1 and Λ_0^5 in terms of the asymptotic values of the dilaton ϕ_0 and the axion χ_0 as follows:

$$\Lambda_0^1 = \frac{1}{\tilde{\Delta}_{(m,n)}^{1/2}} \begin{pmatrix} m & -ne^{-\phi_0} + \chi_0(m-\chi_0 n)e^{\phi_0} \\ n & (m-\chi_0 n)e^{\phi_0} \end{pmatrix}, \tag{2.19}$$

and

$$\Lambda_0^5 = \frac{1}{\bar{\Delta}_{(m',n')}^{1/2}} \begin{pmatrix} n' & m'e^{-\phi_0} + \chi_0(n'+\chi_0 m')e^{\phi_0} \\ -m' & (n'+\chi_0 m')e^{\phi_0} \end{pmatrix}. \tag{2.20}$$

Note that in the process of obtaining Eqs. (2.19) and (2.20), the SO(2) angle got fixed as

$$\begin{aligned} e^{i\alpha} &= [(m-\chi_0 n)e^{\phi_0/2} + ine^{-\phi_0/2}]\tilde{\Delta}_{(m,n)}^{-1/2} \\ &= [(n'+\chi_0 m')e^{\phi_0/2} - im'e^{-\phi_0/2}]\bar{\Delta}_{(m',n')}^{-1/2}. \end{aligned} \tag{2.21}$$

From the above equation we find that the Δ factors associated with the strings and the 5-branes are given as

$$\begin{aligned} \tilde{\Delta}_{(m,n)} &= e^{\phi_0}(m-\chi_0 n)^2 + e^{-\phi_0} n^2, \\ \bar{\Delta}_{(m',n')} &= e^{\phi_0}(n'+\chi_0 m')^2 + e^{-\phi_0} m'^2. \end{aligned} \tag{2.22}$$

Since the strings and 5-branes are transformed simultaneously by the same SL(2, R) matrix, it is clear from Eqs. (2.19) and (2.20) that the corresponding charges must be related as

$$\begin{aligned} (m,n) &= k(a,b), \\ (m',n') &= k'(-b,a). \end{aligned} \tag{2.23}$$

Here (a,b) and (k,k') are two pairs of relatively prime integers, which can be seen either from the general tension expression described later in Eq. (2.31) or when we consider the special case bound states. We, therefore, note that for the general $((F,\ \mathrm{D1}),\ (\mathrm{NS5},\mathrm{D5}))$ configuration, i.e., when none of the integers are zero, (m,n) and (m',n') are not relatively prime in contrast with the case when we consider either the SL(2, Z) strings [6] or the SL(2, Z) 5-branes [9]. It can be easily checked that $\tilde{\Delta}_{(m,n)} = k^2 \tilde{\Delta}_{(a,b)}$ and $\bar{\Delta}_{(m',n')} = k'^2 \bar{\Delta}_{(-b,a)} = k'^2 \tilde{\Delta}_{(a,b)}$ are SL(2, Z) invariant. Now once we find the SL(2, R) transformation matrix Λ_0 given either by Eq. (2.19) or by Eq. (2.20), we can obtain the general $((F,\ \mathrm{D1}),\ (\mathrm{NS5},\mathrm{D5}))$ configuration by applying the SL(2, R) transformation given in Eq. (2.10) on the initial $(F,\ \mathrm{D5})$ configuration. As mentioned earlier, the initial $(F,\ \mathrm{D5})$ configuration as given in Eqs. (2.1)–(2.6) should be modified by the appropriate Δ factors or, more precisely, q should be replaced by $\tilde{\Delta}_{(m,n)}^{1/2}$ and s by $\bar{\Delta}_{(m',n')}^{1/2}$. Keeping this in mind, the final $((F,\ \mathrm{D1}),\ (\mathrm{NS5},\mathrm{D5}))$ nonthreshold bound state is given by the following metric:

$$\begin{aligned} ds^2 = H'^{1/2}H^{1/4}[&H^{-1}(-(dx^0)^2 + (dx^1)^2) \\ &+ H'^{-1}((dx^2)^2 + (dx^3)^2 + (dx^4)^2 + (dx^5)^2) + dy^i dy^i], \end{aligned} \tag{2.24}$$

with $i = 1,2,3,4$; the dilaton,

$$e^{\phi} = e^{\phi_0} H^{-1/2} H'', \tag{2.25}$$

the axion,

$$\chi = \frac{\chi_0 + (H-1)abe^{-\phi_0}/\tilde{\Delta}_{(a,b)}}{H''}, \tag{2.26}$$

and the rest of the nonvanishing fields,

$$\begin{aligned} H_3^{(1)} &= -\frac{k}{\sqrt{k^2+k'^2}} \tilde{\Delta}_{(a,b)}^{-1/2} e^{\phi_0}(a-\chi_0 b)\, dH^{-1} \wedge dx^0 \wedge dx^1 - \frac{k' b\sqrt{2}\kappa_0 Q_0^5}{\Omega_3}\epsilon_3, \\ H_3^{(2)} &= \frac{k}{\sqrt{k^2+k'^2}} \tilde{\Delta}_{(a,b)}^{-1/2} [e^{\phi_0}\chi_0(a-\chi_0 b) - e^{-\phi_0} b]\, dH^{-1} \wedge dx^0 \wedge dx^1 + \frac{k' a\sqrt{2}\kappa_0 Q_0^5}{\Omega_3}\epsilon_3, \\ H_5 &= \frac{kk'}{k^2+k'^2} H'^{-2} dH \wedge dx^2 \wedge dx^3 \wedge dx^4 \wedge dx^5. \end{aligned} \tag{2.27}$$

In the above the harmonic function $H=1+Q_5/r^2$, where

$$Q_5=\sqrt{k^2+k'^2}\tilde{\Delta}_{(a,b)}^{1/2}\frac{\sqrt{2}\kappa_0 Q_0^5}{2\Omega_3}. \tag{2.28}$$

H' is another harmonic function where

$$H'=1+\frac{k'^2Q_5/(k^2+k'^2)}{r^2}, \tag{2.29}$$

and we have introduced a new harmonic function

$$H''=1+\frac{b^2e^{-\phi_0}Q_5/\tilde{\Delta}_{(a,b)}}{r^2}. \tag{2.30}$$

We note that the metric in Eq. (2.24) retains its form after SL(2, R) transformation except for the introduction of the appropriate Δ factors, as expected. The 5-form field strength $H_5={}^*H_5$ in Eq. (2.27) is SL(2, R) invariant. Also, from Eqs. (2.25) and (2.26) we find that as $r\to\infty$, $e^{\phi}\to e^{\phi_0}$ and $\chi\to\chi_0$, the corresponding asymptotic values as it should be. Let us now discuss how the (F, D5), (D1, NS5) as well as the degenerate cases (NS5, D5) and (F, D1) bound states can be obtained from this general configuration as special cases.

From the above solution given by Eqs. (2.24)–(2.30), we can obtain (F, D5) bound state by setting $a=1$ and $b=0$. Note that the charges associated with the F-strings and the D5-branes are k, k', respectively. As shown in [2], (F, D5) will form nonthreshold bound states only when k and k' are relatively prime integers. Similarly, the other bound state (D1, NS5) can be obtained by setting $a=0$ and $b=1$ (also $\chi_0=0$). Here the charges associated with the D-strings and NS5-branes are k and $-k'$, respectively. However, since string charges are related to the 5-brane charges as given in Eq. (2.23), we can get neither (F, NS5) nor (D1, D5) bound states. This is consistent with the fact that these configurations break 1/4 of the space-time supersymmetries as can be inferred from that of the bound state of (D0, D4) discussed in [16,17]. It should be pointed out that for both k, k' nonzero, the existence of (m,n) string bound states seems to imply the existence of (m',n') 5-brane bound states. This is nice since the existence of 5-brane bound states is not easy to establish considering the unrenormalizability of six-dimensional SYM theory and our poor understanding of the solitonic 5-branes. However, an interesting scenario for the existence of 5-brane bound states has been suggested in Ref. [18]. The degenerate (NS5, D5) and (F, D1) nonthreshold bound state configurations can also be obtained from ((F, D1), (NS5, D5)) configuration given in Eqs. (2.24)–(2.30) by simply setting $k=0$, $k'=1$ and $k=1$, $k'=0$, respectively. In the former case, we get the SL(2, Z) multiplet of 5-branes [9] (NS5, D5) with magnetic charges $(-b,a)$ and in the latter case, we get SL(2, Z) strings [6] with electric charges (a,b) having additional isometries in x^2, x^3, x^4, x^5 directions. Here (a,b) are arbitrary co-prime integers as can be shown for the strings [15] and 5-branes [9] to form nonthreshold bound states.

TABLE I. The T Duality rules for various BPS solutions along both longitudinal and transverse directions in type IIA or type IIB theories.

	Parallel	Transverse
Dp	D$(p-1)$	D$(p+1)$
F	W	F
W	F	W
NS5	NS5	KK
KK	KK	NS5

The expression for the string-frame tension of the general SL(2, Z) bound state ((F, D1), (NS5, D5)) can be obtained by calculating the mass per unit 5-brane volume. We can do so by following the steps given in [2] and by generalizing the ADM mass formula given in [19]. For a complete string-frame tension, we need to restore the ϕ_{B0} and χ_{B0}, which are set to zero from the outset for our above configuration. In particular, we need to set $\phi_{B0}=\phi_0$ so that the string-frame metric approaches the Minkowski one asymptotically, as discussed in [2]. With all these considerations, the complete string-frame tension for this bound state takes the form

$$T_5(k,k';a,b)$$
$$=\frac{T_0^5}{g}\sqrt{[(k-\chi_{B0}k')^2g^2+k'^2][(a-\chi_0 b)^2+b^2g^{-2}]}, \tag{2.31}$$

where $T_0^p=1/[(2\pi)^p\alpha'^{(p+1)/2}]$ is the p-brane tension unit and $g=e^{\phi_0}$ is the string coupling constant. This expression also clearly indicates that both pairs of integers, (k,k') and (a,b), would have to be relatively prime if the SL(2, Z) invariant state ((F, D1), (NS5, D5)) has to form a nonthreshold bound state. Now it can be easily checked that the above formula correctly reproduces the tensions of (F, D5) and (D1, NS5) bound states for $a=1$, $b=0$ and $a=0$, $b=1$ respectively (also $\chi_0=0$). Similarly, the tensions for (NS5, D5) and (F, D1)[3] can be obtained for $k=0$, $k'=1$ and $k=1$, $k'=0$, respectively (also $\chi_{B0}=0$).

III. THE DESCENDANTS OF ((F, D1), (NS5, D5)) BOUND STATE

The T duality rules for various BPS solutions along both longitudinal and transverse directions in type IIA or type IIB theories can be described by Table I (for KK monopole, the transverse direction is taken to be the nut direction).

In Table I W, F, NS5, and KK denote waves, fundamental strings, NS five-branes, and KK monopoles, respectively, and they are associated with NS-NS fields. Dp $(-1\leqslant p\leqslant 8)$[4] are the so-called D-branes and they are associated with RR fields.

[3]For (F, D1) case we have to multiply the expression by $(2\pi)^4\alpha'^2$ of four-dimensional area perpendicular to the strings since there are an infinite number of (F, D1) strings in the general bound state [1,2].

[4]We do not consider D9 or the space-time filling branes [20].

Using Table I, we will give in this section, as a further example, the explicit space-time configuration of $((F,\ \mathrm{D2}),\ (\mathrm{KK},\ \mathrm{D6}))$ bound state in type IIA theory by performing T duality on the $((F,\ \mathrm{D1}),\ (\mathrm{NS5},\ \mathrm{D5}))$ bound state along one of its transverse directions and then give the list of other possible bound states towards the end. The general method of performing T duality has already been described at length in [2]. Here we briefly outline the method for completeness. T duality along the transverse direction is performed by the use of the so-called vertical dimensional reduction and the diagonal or double-dimensional oxidation method. Let us start with a p-brane [in our case it is the more complicated $((F,\ \mathrm{D1}),\ (\mathrm{NS5},\ \mathrm{D5}))$ brane] solution in type IIA (IIB) theory. We then use the "no-force" condition of the BPS states to construct a multicenter solution from the single center one with an infinite periodic array of p-branes placed along the transverse direction. Then we take a continuum limit to obtain the p-brane solution with one isometry along the would-be compactified direction where the solution is now independent of this coordinate. This process in turn reduces the dimensionality of the theory to $D=9$, known as the vertical dimensional reduction. Once we have this solution, we perform the T-duality transformation on various fields to write them from IIA (IIB) basis to IIB (IIA) basis. Then, by the so-called double-dimensional oxidation, we can simply read off the $D=10$ $(p+1)$-brane solution from the $D=9$ solution. One can also apply T duality along the longitudinal directions of the p-brane and obtain new bound states by the method of diagonal reduction and vertical oxidation, just opposite to the previous case.

$((F,\ \mathrm{D2}),\ (\mathrm{KK},\ \mathrm{D6}))$ *bound state*. Here we assume that the (NS5, D5) in the original $((F,\ \mathrm{D1}),\ (\mathrm{NS5},\ \mathrm{D5}))$ bound state is aligned along x^1, x^2, x^3, x^4, and x^5 direction and we apply T duality along x^6 direction [assuming $(F,\ \mathrm{D1})$ to be aligned along the x^1 direction]. Then following the procedure just outlined [2], we find that this bound state in type IIA theory is given by the following Einstein metric:

$$\begin{aligned} ds^2 = {} & e^{\phi_0/8}H^{1/4}H'^{5/8}H''^{1/8}[H^{-1}(-(dx^0)^2+(dx^1)^2)+H'^{-1}\{(dx^2)^2+\cdots+(dx^5)^2 \\ & + e^{-\phi_0}H''^{-1}(dx^6+k'b(\sqrt{2}\kappa_0Q_0^6/\Omega_2)(1-\cos\theta)d\varphi)^2\}+dy^idy^i], \end{aligned} \tag{3.1}$$

where θ and φ are the angular coordinates of y^1, y^2, and y^3 and $i=1,2,3$; the dilaton,

$$e^{\phi}=e^{3\phi_0/4}H^{-1/2}H'^{-1/4}H''^{3/4}, \tag{3.2}$$

and the rest of the nonvanishing fields,

$$\begin{aligned} F_2 &= \left(\chi_0-\frac{a}{b}\right)dH''^{-1}\wedge dx^6 - k'a\frac{\sqrt{2}\kappa_0Q_0^6}{\Omega_2}\epsilon_2, \\ F_3 &= -\frac{k}{\sqrt{k^2+k'^2}}\tilde{\Delta}_{(a,b)}^{-1/2}e^{\phi_0}(a-\chi_0 b)dH^{-1}\wedge dx^0\wedge dx^1, \\ F_4' &= \frac{kk'}{\sqrt{k^2+k'^2}}\tilde{\Delta}_{(a,b)}^{1/2}\frac{\sqrt{2}\kappa_0Q_0^6}{\Omega_2}H^{-1}dx^0\wedge dx^1\wedge\epsilon_2 \\ &\quad + \frac{k}{\sqrt{k^2+k'^2}}\tilde{\Delta}_{(a,b)}^{-1/2}be^{-\phi_0}(HH'')^{-1}dH\wedge dx^0\wedge dx^1 \\ &\quad \wedge dx^6. \end{aligned} \tag{3.3}$$

Here the harmonic functions H, H', and H'' are given as

$$H=1+\frac{Q_6}{r},\quad H'=1+\frac{k'^2Q_6/(k^2+k'^2)}{r},$$
$$\text{and}\quad H''=1+\frac{b^2e^{-\phi_0}Q_6/\tilde{\Delta}_{(a,b)}}{r}, \tag{3.4}$$

where $Q_6=\sqrt{k^2+k'^2}\tilde{\Delta}_{(a,b)}^{1/2}\sqrt{2}\kappa_0Q_0^6/\Omega_2$.

As discussed in the previous section, we can obtain the bound states $(F,\ \mathrm{D6})$, (D2, KK) as well as the degenerate cases (F, D2), (KK, D6) from this general bound state as special cases by simply setting $a=1,\ b=0$;[5] $a=0,\ b=1$; $k=1,\ k'=0$ and $k=0,\ k'=1$, respectively, in Eqs. (3.1)–(3.4). Note that for the case of (F, D2) we have additional isometries in x^2, x^3, x^4, and x^5 directions. Again as before, we cannot get the bound states $(F,\ \mathrm{KK})$ and (D2, D6) from the general bound state because of the charge relation Eq. (2.23). This is consistent with the fact that these states preserve 1/4 space-time supersymmetries. Note that in order for the above metric Eq. (3.1) to be free from conical singularity, x^6 should have a period of $4\pi k'b(\sqrt{2}\kappa_0Q_0^6/\Omega_2)$.

A complete string-frame tension formula similar to Eq. (2.31) can also be written for the general bound state $((F,\ \mathrm{D2}),\ (\mathrm{KK},\ \mathrm{D6}))$ in the form

$$T_6(k,k';a,b) = \frac{T_0^6}{g}\sqrt{[(k-\chi_{B0}k')^2g^2+k'^2][(a-\chi_0b)^2+b^2g^{-2}]}, \tag{3.5}$$

with T_0^6 as defined before. This expression reproduces the tensions for the special case bound states $(F,\ \mathrm{D6})$, (D2, KK), $(F,\ \mathrm{D2})$, (KK, D6) by setting $a=1,\ b=0$; $a=0,\ b$

[5] It can be easily checked from Eq. (2.26) that when $b=0$, the first term of F_2 in Eq. (3.3) will not contribute.

$=1$ (also $\chi_0=0$); $k=1$, $k'=0$ and $k=0$, $k'=1$ (also $\chi_{B0}=0$). As in the previous case, in order to get the correct tension expression for (F, D2), we need to multiply the above expression by the area $(2\pi)^4\alpha'^2$.

At the level of supergravity solution as discussed in [2], we may expect that we can make a further T duality on ((F, D2), (KK, D6)) along one of the transverse directions of the D6 branes.[6] It is obvious from the metric Eq. (3.1) that we have an isometry $\partial/\partial\varphi$. But T duality along this direction would result in a complicated metric which depends on the angle θ.[7] We do not have a clear interpretation for the resulting configuration. We therefore do not consider this T duality here. Apart from this possible T duality, it is not obvious to us if we can have any other simple T duality as described above along a transverse direction.

Now we give the list of all possible descendants of (($F1$, D1), (NS5, D5)) and ((F, D2), (KK, D6)) by applying T dualities along various longitudinal directions of each of these two bound states. We will follow the notation of Ref. [8]. For example, ($T_i:\mapsto$) will denote T duality along ith direction. We assume that the bound state ((F, D1), (NS5, D5)) is along x^1, x^2, x^3, x^4, and x^5 directions, and (F, D1) strings are along x^1 direction. Then according to the table given in the beginning of this section, we can T-dualize each of the above [(F, Dp), (NS5/KK, D($p+4$))], with $1\leqslant p\leqslant 2$ (for $p=1$ we have NS5 state and for $p=2$ we have KK state), along longitudinal directions of the original (NS5, D5)-branes to obtain new bound states. But since strings are along x^1 directions, we will get different bound states depending on whether we T-dualize ''1'' direction first or not. T-dualizing along ''1'' we obtain, for example, the following bound state:

TABLE II. Possible bound states obtained by T duality.

Bound states	No. common dir.
((F, D($p+1$)), (NS5, D($5-p$)))	1
((F, D($p+2$)), (KK, D($6-p$)))	2
((W, Dp), (NS5, D($4-p$)))	0
((W, D($p+1$)), (KK, D($5-p$)))	1

$$((\mathrm{F},\ \mathrm{D1}),(\mathrm{NS5},\ \mathrm{D5}))(\mathrm{T}_1:\mapsto)((\mathrm{W},\ \mathrm{D0}),(\mathrm{NS5},\ \mathrm{D4})). \tag{3.6}$$

We can also apply T duality along longitudinal directions other than ''1'' first and then apply T duality along ''1.'' For example, if we T-dualize along ''5'' first and then along ''1'' we obtain,

$$((\mathrm{F},\ \mathrm{D1}),(\mathrm{NS5},\ \mathrm{D5}))(\mathrm{T}_5:\mapsto)((\mathrm{F},\ \mathrm{D2}),(\mathrm{NS5},\ \mathrm{D4}))(\mathrm{T}_1:\mapsto)((\mathrm{W},\ \mathrm{D1}),(\mathrm{NS5},\ \mathrm{D3})), \tag{3.7}$$

where the D2 and D4 in ((F, D2), (NS5, D4)) share only one common direction, while the D1 and D3 in ((W, D1), (NS5, D3)) share no common directions. Repeating the above process with ((F, D2), (NS5, D4)) along ''4'' first then along ''1,'' we end up with ((F, D3), (NS5, D3)) and ((W, D1), (NS5, D3)). Continuing this, we have in general ((W,Dp), (NS5, D($4-p$)) and ((F, D($p+1$)), (NS5, D($5-p$))) for $0\leqslant p\leqslant 4$. Applying the similar process to ((F, D2), (KK, D6)), we have in general ((W, D($p+1$)), (KK, D($5-p$))) and ((F, D($2+p$)), (KK, D($6-p$))) for $0\leqslant p\leqslant 4$.

This exhausts all the possibilities. In summary, Table II lists all of the possible bound states which can be obtained by T duality along one transverse and various longitudinal directions on ((F, D1), (NS5, D5)).

The second column indicates the number of common directions shared by the respective D-branes in the bound states. Also in the above $0\leqslant p\leqslant 4$. If we write the above bound states in the form ((X, Y), (Z, V)), then from the properties of ((F, D1), (NS5, D5)) discussed, we can get (X, V) by setting $a=1$, $b=0$, and (Y, Z) by setting $a=0$, $b=1$. The degenerate (Z, V) and (X, Y) configurations can be obtained by setting $k=0$, $k'=1$ and $k=1$, $k'=0$, respectively. These degenerate bound states have also been discussed, for example, in [3,5]. But we cannot get the bound states (X, Z) and (Y, V) because of the charge relations [see Eq. (2.23)].

Now some of the states above belong to type IIB theory and so, we can apply S duality to those states to obtain new bound states. For example, from ((F, D3), (NS5, D3)) we can have (((F, D1), D3), ((NS5, D5), D3)) as a new bound state. Similarly from other states we can also generate new bound states by S duality of type IIB theory. We can again apply T duality on these newly constructed bound states to obtain more bound states, then S duality again to those belonging to type IIB theory. By continuing this process we can obtain all possible nonthreshold bound states by S and T dualities simply from the original (F, D1) strings. This process obviously will end after a finite number of steps and thus we have finitely many bound states in both type IIA and type IIB theories.[8] Although at this stage we are unable to count the exact number of bound states, since there are finitely many we believe that they may be related to the num-

[6]If we T-dualize along the nut direction, we are back to ((F, D1), (NS5 , D5)).

[7]We would like to thank Chris Pope for pointing this out to us.

[8]Even if we count possible bound states by applying T dualities along the transverse directions of D6 in ((F, D2), (KK, D6)).

ber of generators of the largest finite U-duality group of type II theory, i.e., $E_{8(+8)}$. We speculate as in [8] that these bound states would form multiplets of $E_{8(+8)}$ U-duality symmetry in the yet unknown M or U theory. We will come back to provide more evidence for this in the near future.

IV. CONCLUSION

To summarize, by making use of SL(2, *Z*) symmetry of type IIB string theory in this paper, we have constructed a more general bound state of the type ((*F*, D1), (NS5, D5)) from the known (*F*, D5) configuration. There are an infinite number of (m,n) strings forming bound state with (m',n') 5-branes. (m,n) and (m',n') are, respectively, the integers corresponding to the charges associated with the NS-NS and RR gauge fields of the strings and 5-branes. We have shown that a consistent quantization of charges of the strings and 5-branes relates these integers as $(m,n)=k(a,b)$ and $(m',n')=k'(-b,a)$, where (k,k') and (a,b) are two pairs of relatively prime integers. Thus the bound state ((*F*, D1), (NS5, D5)) is characterized by two pairs of integers (k,k') and (a,b). This seems to indicate that the existence of string bound states implies the existence of 5-brane bound states. From the explicit space-time configuration of ((*F*, D1), (NS5, D5)), we have shown how various bound states appear as special cases. Thus we obtain (*F*, D5) and (D1, NS5) as well as the degenerate cases (*F*, D1) and (NS5, D5) bound states from here, but because of the charge relation between strings and 5-branes we cannot get (*F*, NS5) and (D1, D5) bound states. This result is consistent with the fact that (*F*, NS5) and (D1, D5) preserve 1/4 rather than half of the space-time supersymmetries. We have also given the tension expression for the general ((*F*, D1), (NS5, D5)) nonthreshold bound state, which reduces to the correct expressions for the tensions of the individual special case bound states by the proper choice of the integers $(k,k';a,b)$.

The descendants of this bound state could be obtained by applying *T* dualities along various transverse and longitudinal directions as well as *S* duality of type IIB theory. We have given explicit space-time configuration of ((*F*, D2), (KK, D6)) in type IIA theory by applying *T* duality in one of the transverse directions on ((*F*, D1), (NS5, D5)). How the various bound states can be obtained as special cases is also indicated. As in the previous case, we have given a similar tension expression for this bound state. Then we have given the list of all possible bound states which can be obtained from ((*F*, D1), (NS5, D5)) and ((*F*, D2), (KK, D6)) by *T* dualities. As we have mentioned, this is not the end of the story. We can form new bound states by applying *S* duality to these *T*-dual bound states belonging to type IIB theory. *T* duality can again be applied to these new sets of bound states to generate another new set. Then *S* duality on those in type IIB theory will produce even more bound states. Thus, by continuing this process, we can generate all possible bound states, which will be finitely many, in both type IIA and type IIB theories. All these states preserve one-half of the space-time supersymmetries. We conjecture that these bound states would form multiplets of the largest finite U-duality group $E_{8(+8)}$ of yet unknown M or U theory.

ACKNOWLEDGMENTS

We would like to thank Chris Pope for discussions. J.X.L. acknowledges the support of NSF Grant PHY-9722090.

[1] J. X. Lu and S. Roy, J. High Energy Phys. **08**, 002 (1999).
[2] J. X. Lu and S. Roy, ''Non-Threshold (F, Dp) Bound States,'' hep-th/9904129.
[3] J. G. Russo and A. A. Tseytlin, Nucl. Phys. **B490**, 121 (1997).
[4] M. B. Green, N. D. Lambert, G. Papadopoulos, and P. K. Townsend, Phys. Lett. B **384**, 86 (1996).
[5] M. Costa and G. Papadopoulos, Nucl. Phys. **B510**, 217 (1998).
[6] J. Schwarz, Phys. Lett. B **360**, 13 (1995).
[7] J. Polchinski, *String Theory* (Cambridge University Press, Cambridge, England, 1998), Vol. II; J. Izquierdo, N. Lambert, G. Papadopoulos, and P. K. Townsend, Nucl. Phys. **B460**, 560 (1996); M. Green and M. Gutperle, *ibid.* **B476**, 484 (1996); G. Lifschytz, *ibid.* **B499**, 283 (1997); M. B. Green, N. D. Lambert, G. Papadopoulos, and P. K. Townsend, Phys. Lett. B **384**, 86 (1996); J. Breckenridge, G. Michaud, and R. Myers, Phys. Rev. D **55**, 6438 (1997); H. Arfaei and M. M. Sheikh Jabbari, Nucl. Phys. **B526**, 278 (1998).
[8] J. X. Lu and S. Roy, ''((*F*, D1), D3) Bound State and Its *T*-Dual Daughters,'' hep-th/9905014.
[9] J. X. Lu and S. Roy, Phys. Lett. B **428**, 289 (1998).
[10] S. Roy, Phys. Lett. B **421**, 176 (1998).
[11] J. X. Lu and S. Roy, Nucl. Phys. **B538**, 149 (1999).
[12] E. Cremmer and B. Julia, Phys. Lett. **80B**, 48 (1987); Nucl. Phys. **B156**, 141 (1979); B. Julia, in *Superspace and Supergravity*, edited by S. W. Hawking and M. Rocek (Cambridge University Press, Cambridge, England, 1981); J. Schwarz, Nucl. Phys. **B226**, 269 (1983).
[13] C. M. Hull and P. K. Townsend, Nucl. Phys. **B294**, 196 (1995).
[14] R. I. Nepomechie, Phys. Rev. D **31**, 1921 (1985); C. Teitelboim, Phys. Lett. **167B**, 69 (1986).
[15] J. Schwarz, Nucl. Phys. B (Proc. Suppl.) **55**, 1 (1997).
[16] M. Douglas, D. Kabat, P. Pouliot, and S. Shenker, Nucl. Phys. **B485**, 85 (1997).
[17] G. Papadopoulos and P. Townsend, Phys. Lett. B **393**, 59 (1997).
[18] E. Witten, Nucl. Phys. **B460**, 335 (1995).
[19] J. X. Lu, Phys. Lett. B **313**, 29 (1993).
[20] E. Bergshoeff, E. Eyras, R. Halbersma, J. P. van der Schaar, C. M. Hull, and Y. Lozano, ''Space-Time Filling Branes and Strings with Sixteen Supercharges,'' hep-th/9812224.