Two-loop quantum corrections of scalar QED with nonminimal Chern-Simons coupling

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We investigate two-loop quantum corrections to non-minimally coupled Maxwell-Chern-Simons theory. The non-minimal gauge interaction represents the magnetic moment interaction between the charged scalar and the electromagnetic field. We show that one-loop renormalizability of the theory found in previous work does not survive to the two-loop level. However, with an appropriate choice of the non-minimal coupling constant, it is possible to renormalize the two-loop effective potential and hence render it potentially useful for a detailed analysis of spontaneous symmetry breaking induced by radiative corrections. [S0556-2821(99)05024-9]

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I. INTRODUCTION

Maxwell-Chern-Simons electrodynamics has been studied extensively in recent years for a variety of reasons. The Chern-Simons term gives the photon a topological mass without spontaneously breaking gauge symmetry [1] and allows for the existence of charged particles with fractional statistics [2]. Pure Chern-Simons scalar electrodynamics admits topological and non-topological self-dual solitons, for which many exact solutions to the classical equations of motion are available [3]. Moreover, such theories may also have physical significance. Relativistic three dimensional Chern-Simons theories provide a consistent description of the high temperature limit of four dimensional gauge theories [4] and certain solid state systems with planar dynamics [2]. In addition, the non-relativistic version of Maxwell-Chern-Simons theory has been applied to the fractional Hall effect, and more recently to rotating superfluid ^{3}He-*A* [5].

Recently a version of scalar electrodynamics in three dimensions has been studied in which a non-minimal Chern-Simons type gauge interaction was introduced. The non-minimal coupling in this model represents a magnetic moment interaction between the charged scalar and the electromagnetic field. It is of interest for several reasons. First, it is well known that one of the most important features of scalar quantum electrodynamics (QED) is the occurrence of the Coleman-Weinberg mechanism [6]. In scalar QED with non-minimal coupling, the Chern-Simons term is generated through the Coleman-Weinberg mechanism [7]. In this sense, the non-minimal model is the one in which the Chern-Simons term arises naturally rather than being put in by hand.

Another reason that the non-minimal model is of interest involves the study of vortex solutions. In recent years, the classical vortex solutions of $(2+1)$-dimensional Chern-Simons field theories have received considerable attention [3,8]. To find such a solution exactly, the model must be self-dual. A self-dual theory is one in which the classical equations of motion can be reduced from second- to first-order differential equations. In the absence of a Maxwell term, scalar QED with a Chern-Simons term is self-dual, and the topological and non-topological vortex solutions have been found with an appropriately chosen scalar potential [3]. However, if the Maxwell term is present, a self-dual Maxwell-Chern-Simons theory can be achieved only if a magnetic moment interaction between the scalar and the gauge field, i.e., the non-minimal Chern-Simons coupling, is introduced [8,9].

It is well known that Maxwell-Chern-Simons scalar QED is renormalizable. Non-minimal gauge interactions are, however, notoriously non-renormalizable in four dimensions. There is some hope that the situation might be different in three dimensions. Some time ago it was found by two of us that the non-minimal Chern-Simons coupling in $(2+1)$-dimensional scalar electrodynamics is actually renormalizable at the one-loop level [7]. The renormalizability occurs because the non-minimal gauge interaction contains the three-dimensional antisymmetric tensor. An analysis [7] of the symmetry breaking by induced radiative corrections shows that at the one-loop level the non-minimal model behaves differently from the minimally coupled one. In the usual Maxwell-Chern-Simons scalar QED, symmetry breaking results from quantum corrections but depends on the choice of a renormalization scale [10], whereas in the non-minimal model the symmetry breaking is unambiguous and there is a finite temperature phase transition to the symmetry restored state.

It is clearly of interest to discover whether the renormalizability of this model persists beyond the one-loop level and to compare the symmetry breaking phase transitions in the minimal and non-minimal models at higher orders. The two-loop behavior of the minimal model has recently been analyzed in detail [10] where it was shown that the Coleman-Weinberg mechanism occurs at two-loops. The purpose of the present paper is to commence a detailed analysis of the two-loop behavior of the non-minimally coupled Maxwell-Chern-Simons theory. We will show that the model, not surprisingly, is not renormalizable at the two-loop level. How-

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ever, the two-loop effective potential can be made renormalizable providing certain conditions are satisfied by the coupling constants of the model. Thus we will show that under certain circumstances (i.e. when the lowest order in the momentum expansion is sufficient) the model may yield physically relevant predictions for spontaneous symmetry breaking by radiative corrections at the two-loop level. A detailed analysis of the renormalized effective potential and the consequent symmetry breaking is deferred to a future publication.

The paper is organized as follows. Section II is a brief review, containing an introduction to the model, a discussion of some technical aspects of dimensional regularization in $2+1$ dimensions, and a list of some of the necessary Feynman rules. Section III demonstrates that the full effective action is not renormalizable at the two loop-level. Section IV is devoted to the (somewhat lengthy) calculation of the two-loop effective potential in the modified minimal subtraction $\overline{\text{MS}}$ scheme. In contrast to the one-loop case, the renormalizability of the two-loop effective potential requires a specific choice of the non-minimal coupling constant. Our conclusions are summarized in Sec. V, while some useful formulas are collected in the Appendix.

II. SCALAR QED WITH NON-MINIMAL CHERN-SIMONS COUPLING

A. Lagrangian

The Lagrangian for scalar QED in $2+1$ dimensions with a non-minimal Chern-Simons coupling is [7]

$$\mathcal{L}=\frac{1}{2}(D_\mu\phi)^* D^\mu\phi-\frac{1}{4}F_{\mu\nu}F^{\mu\nu}-\frac{i}{8}\gamma_0\epsilon^{\mu\nu\rho}F_{\nu\rho}\times[\phi^* D_\mu\phi-(D_\mu\phi)^*\phi]-\frac{\lambda}{6!}(\phi^*\phi)^3, \tag{1}$$

where the complex scalar field ϕ can be decomposed into the real and imaginary parts, $\phi=\chi+i\eta$, $F_{\mu\nu}=\partial_\mu A_\nu-\partial_\nu A_\mu$, $D_\mu\phi=\partial_\mu\phi+ieA_\mu\phi$, and γ_0, e, and λ are the non-minimal Chern-Simons coupling, the gauge coupling and the scalar self-interaction coupling constants, respectively. The dimensional assignments for the fields and coupling constants are as follows:

$$[A_\mu]=[\phi]=[e]=M^{1/2},\quad [\gamma_0]=M^{-1/2},\quad [\lambda]=M^0. \tag{2}$$

The negative mass dimension of γ_0 indicates that the theory is not renormalizable in general.

Following the standard technique to calculate the effective potential we first assume the existence of a non-vanishing vacuum expectation value for the scalar field $\langle\phi\rangle=v$ with v real, and shift the real part of the scalar field, $\chi\to\chi+v$ [11]. We look for a non-vanishing value of v by determining the minimum of the effective potential generated by quantum corrections. Note that in the Lagrangian (1), we have put the bare mass of the scalar field, the quartic scalar self-interaction coupling and the statistical parameter of a possible Chern-Simons term equal to zero, even though these terms are allowed by the gauge symmetry. The corresponding counterterms may appear, however, in the counterterm Lagrangian. The physical parameters can be obtained in the usual way, i.e. by choosing renormalization conditions that give zero renormalized parameters at $v=0$. To avoid the infrared divergences we use the Landau-type R_ξ gauge [12]. The gauge-fixing term is

$$\mathcal{L}_{\text{g.f.}}=-\frac{1}{2\xi}(\partial_\mu A^\mu-\xi e v\eta)^2,\quad \xi=0. \tag{3}$$

Up to a total derivative term, the Lagrangian can be divided into the kinetic part and ten interaction terms:

$$\mathcal{L}^{(0)}=-\frac{1}{2}\chi\left(\partial^2+\frac{\lambda}{4!}v^4\right)\chi-\frac{1}{2}\eta\left(\partial^2+\frac{\lambda}{5!}v^4-\xi e^2v^2\right)\eta+\frac{1}{2}A^\mu\left[(\partial^2+e^2v^2)g_{\mu\nu}-\left(1-\frac{1}{\xi}\right)\partial_\mu\partial_\nu-\gamma_0 e v^2\epsilon_{\mu\nu\rho}\partial^\rho\right]A^\nu,$$

$$\mathcal{L}^{(1)}=-\lambda\left(\frac{1}{6!}\chi^6+\frac{1}{5\times4!\times2!}\chi^4\eta^2+\frac{1}{5\times4!\times2!}\chi^2\eta^4+\frac{1}{6!}\eta^6\right),$$

$$\mathcal{L}^{(2)}=-\lambda v\left(\frac{1}{5!}\chi^5+\frac{1}{5\times3!\times2!}\chi^3\eta^2+\frac{1}{5\times4!}\chi\eta^4\right),$$

$$\mathcal{L}^{(3)}=-\lambda v^2\left(\frac{1}{2\times4!}\chi^4+\frac{1}{10\times2!\times2!}\chi^2\eta^2+\frac{1}{10\times4!}\eta^4\right),$$

$$\mathcal{L}^{(4)}=-\frac{1}{3!\times3!}\lambda v^3\chi^3-\frac{1}{5!\times3!\times2!}\lambda v^3\chi\eta^2,$$

$$\mathcal{L}^{(5)}=eA^\mu(\partial_\mu\eta\chi-\partial_\mu\chi\eta),$$

$$\mathcal{L}^{(6)}=\frac{1}{2}\gamma_0\epsilon^{\mu\nu\rho}\partial_\nu A_\rho(\chi\partial_\mu\eta-\eta\partial_\mu\chi),$$

$$\mathcal{L}^{(7)}=\frac{1}{2}e^2A_\mu A^\mu(\chi^2+\eta^2),$$

$$\mathcal{L}^{(8)}=-\frac{1}{2}\gamma_0 e\epsilon^{\mu\nu\rho}A_\mu\partial_\rho A_\nu(\chi^2+\eta^2),$$

$$\mathcal{L}^{(9)}=e^2vA_\mu A^\mu\chi,$$

$$\mathcal{L}^{(10)}=-\gamma_0 v e\epsilon^{\mu\nu\rho}A_\mu\partial_\rho A_\nu\chi. \tag{4}$$

B. Regularization

A regularization scheme must be chosen to handle the ultraviolet divergences of the theory. In this paper we shall use dimensional regularization. The use of dimensional regularization in a theory that explicitly depends on epsilon tensors involves adopting a complicated form for the gauge field propagator, as will be discussed below. In spite of this complication, dimensional regularization is simpler than the Pauli-Villars regularization adopted in a previous paper [7] since it allows us to preserve explicit gauge symmetry.

There are several problems involved with analytic continuation to n dimensions. The first of these is standard. The mass dimensions of the fields and parameters become

$$[\phi]=[A_\mu]=[v]=M^{(n-2)/2}, \quad [e]=M^{(2-n)/2},$$

$$[\gamma_0]=M^{(2-n)/2}, \quad [\lambda]=M^{2(3-n)}, \tag{5}$$

and thus, in order to ensure that the parameters keep their original mass dimensions, one must make the following replacements for the parameters in the regularized Lagrangian:

$$v\to\mu^{(n-3)/2}v, \quad e\to\mu^{(3-n)/2}e, \quad \gamma_0\to\mu^{(3-n)/2}\gamma_0,$$

$$\lambda\to\mu^{2(3-n)}\lambda. \tag{6}$$

The second problem is more complicated. Dimensional regularization in a theory with a three-dimensional antisymmetric tensor $\epsilon_{\mu\nu\rho}$ must be handled carefully. It has been explicitly shown that naive dimensional regularization schemes cannot make the theory well defined when they are applied to a Chern-Simons type model [13]. Therefore, in carrying out dimensional regularization we must adopt the three-dimensional analogue of the consistent definition for γ_5, which was originally proposed by 't Hooft and Veltman [14], and later given a strict mathematical justification by Breitenlohner and Maison [15]. The explicit definition of this dimensional continuation for Chern-Simons-type theory was explained in Ref. [16] where it is shown that this regularization method is indeed compatible with the Slavnov-Taylor identities. The explicit definition for the dimensional continuation of the epsilon and the metric tensors is

$$\epsilon^{\mu\nu\rho}=\begin{cases}\pm 1 & \text{if} \quad (\mu\nu\rho)=\text{permutation of} \quad (0,1,2),\\ 0 & \text{otherwise,}\end{cases}$$

$$g_{\mu\nu}=\begin{cases}+1 & \text{for} \quad \mu=\nu=0,\\ -1 & \text{for} \quad \mu=\nu=1,2,\ldots,n-1,\end{cases}$$

$$g_{\mu\nu}=\begin{cases}+1 & \text{for} \quad \mu=\nu=0,\\ -1 & \text{for} \quad \mu=\nu=1,2.\end{cases} \tag{7}$$

These definitions give rise to the following contractions:

$$\epsilon^{\mu\nu\rho}\epsilon^{\lambda\eta}_{\rho}=\tilde{g}^{\mu\lambda}\tilde{g}^{\nu\eta}-\tilde{g}^{\mu\eta}\tilde{g}^{\nu\lambda}, \quad g^{\mu\nu}\tilde{g}^{\lambda}_{\nu}=\tilde{g}^{\mu\lambda},$$

$$\tilde{p}^{\mu}=\tilde{g}^{\mu\nu}p_{\nu}, \quad \tilde{p}^{2}=\tilde{p}_{\mu}p^{\mu}.$$

C. Feynman rules

The tree-level Feynman rules can be derived by standard functional integration techniques. The propagators for the scalar fields χ and η have the same form as in the four-dimensional case,

$$iS_\chi(p)=\frac{i}{p^2-m_\chi^2}, \quad m_\chi^2=\frac{\lambda}{4!}v^4;$$

$$iS_\eta(p)=\frac{i}{p^2-m_\eta^2}, \quad m_\eta^2=\frac{\lambda}{5!}v^4. \tag{8}$$

Following Ref. [10] we can obtain the dimensional regularized propagator for the gauge field,

$$\begin{aligned}iD_{\mu\nu}(p)=i\Bigg\{&-\left[\frac{(\gamma_0 e v^2)^2}{(p^2-e^2v^2)[(p^2-e^2v^2)^2-p^2(\gamma_0 e v^2)^2]}+\frac{1}{p^2(p^2-e^2v^2)}\right](\tilde{p}^2\tilde{g}_{\mu\nu}-\tilde{p}_\mu\tilde{p}_\nu)\\&+\frac{\gamma_0 e v^2}{(p^2-e^2v^2)^2-p^2(\gamma_0 e v^2)^2}\epsilon_{\mu\nu\rho}ip^\rho-\frac{1}{p^2(p^2-e^2v^2)}[(p^2g_{\mu\nu}-p_\mu p_\nu)-(\tilde{p}^2\tilde{g}_{\mu\nu}-\tilde{p}_\mu\tilde{p}_\nu)]\\&+\frac{(\gamma_0 e v^2)^3(p^2-\tilde{p}^2)}{[(p^2-e^2v^2)^2-\tilde{p}^2(\gamma_0 e v^2)^2][(p^2-e^2v^2)^2-p^2(\gamma_0 e v^2)^2]}\left[\frac{\gamma_0 e v^2}{p^2-e^2v^2}(\tilde{p}^2\tilde{g}_{\mu\nu}-\tilde{p}_\mu\tilde{p}_\nu)-\epsilon_{\mu\nu\rho}ip^\rho\right]\Bigg\}.\end{aligned} \tag{9}$$

The above expression can be rewritten as

$$
\begin{aligned}
iD_{\mu\nu}(p) = i\Bigg\{ & -\left[\frac{1}{m_1+m_2}\left(\frac{1}{m_1}\frac{1}{p^2-m_1^2}+\frac{1}{m_2}\frac{1}{p^2-m_2^2}\right)-\frac{1}{m_3^2}\frac{1}{p^2}\right](\tilde{p}^2\tilde{g}_{\mu\nu}-\tilde{p}_\mu\tilde{p}_\nu)+\frac{1}{m_1+m_2}\left(\frac{1}{p^2-m_2^2}-\frac{1}{p^2-m_1^2}\right)\epsilon_{\mu\nu\rho}ip^\rho \\
& -\frac{1}{m_3^2}\left(\frac{1}{p^2-m_3^2}-\frac{1}{p^2}\right)[(p^2g_{\mu\nu}-p_\mu p_\nu)-(\tilde{p}^2\tilde{g}_{\mu\nu}-\tilde{p}_\mu\tilde{p}_\nu)] \\
& +\frac{(\gamma_0 e v^2)^3(p^2-\tilde{p}^2)}{[d^2-\tilde{p}^2(\gamma_0 e v^2)^2][d^2-p^2(\gamma_0 e v^2)^2]}\left[\frac{\gamma_0 e v^2}{p^2-e^2v^2}(\tilde{p}^2\tilde{g}_{\mu\nu}-\tilde{p}_\mu\tilde{p}_\nu)-\epsilon_{\mu\nu\rho}ip^\rho\right]\Bigg\} \\
\equiv -i\Bigg\{ & A(p)(\tilde{p}^2\tilde{g}_{\mu\nu}-\tilde{p}_\mu\tilde{p}_\nu)-B(p)\epsilon_{\mu\nu\rho}ip^\rho+C(p)[(p^2g_{\mu\nu}-p_\mu p_\nu)-(\tilde{g}^2\tilde{g}_{\mu\nu}-\tilde{p}_\mu\tilde{p}_\nu)] \\
& +\frac{(\gamma_0 e v^2)^3(p^2-\tilde{p}^2)}{[d^2-\tilde{p}^2(\gamma_0 e v^2)^2][d^2-p^2(\gamma_0 e v^2)^2]}\left[\frac{\gamma_0 e v^2}{p^2-e^2v^2}(\tilde{p}^2\tilde{g}_{\mu\nu}-\tilde{p}_\mu\tilde{p}_\nu)-\epsilon_{\mu\nu\rho}ip^\rho\right]\Bigg\}, \qquad (10)
\end{aligned}
$$

where we have defined

$$
\begin{aligned}
m_1 &= \frac{1}{2}ev(\sqrt{(\gamma_0 v)^2+4}+\gamma_0 v), \\
m_2 &= \frac{1}{2}ev(\sqrt{(\gamma_0 v)^2+4}-\gamma_0 v), \\
m_3^2 &= e^2v^2, \quad d = p^2-e^2v^2, \qquad (11)
\end{aligned}
$$

and

$$
\begin{aligned}
A(p) &= \frac{1}{m_1+m_2}\left(\frac{1}{m_1}\frac{1}{p^2-m_1^2}+\frac{1}{m_2}\frac{1}{p^2-m_2^2}\right)-\frac{1}{m_3^2}\frac{1}{p^2}, \\
B(p) &= \frac{1}{m_1+m_2}\left(\frac{1}{p^2-m_1^2}-\frac{1}{p^2-m_2^2}\right), \\
C(p) &= \frac{1}{m_3^2}\left(\frac{1}{p^2-m_3^2}-\frac{1}{p^2}\right). \qquad (12)
\end{aligned}
$$

To further simplify the notation we make the following definitions:

$$
S(p)=\frac{1}{p^2}, \quad S_i(p)=\frac{1}{p^2-m_i^2}, \quad i=1,2,3,
$$

which allow us to write,

$$
\begin{aligned}
A(p) &= \frac{1}{m_1m_2(m_1+m_2)}[m_2S_1+m_1S_2-(m_1+m_2)S], \\
B(p) &= \frac{1}{m_1+m_2}[S_1-S_2], \quad C(p)=\frac{1}{m_1m_2}(S_3-S). \qquad (13)
\end{aligned}
$$

These expressions obey the following identities:

$$
2e^2v^2(A-C)+e\gamma_0v^2B=S_1+S_2+S_3,
$$

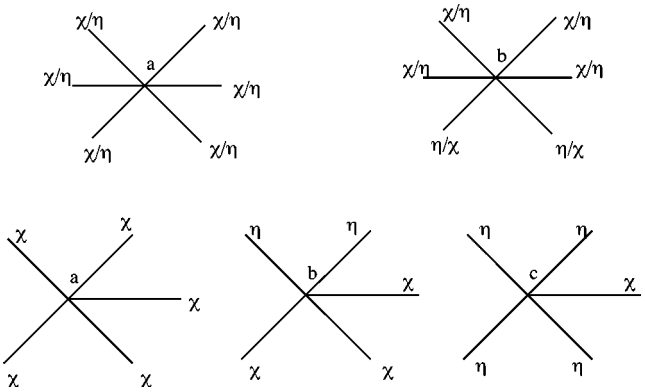

FIG. 1. Interaction vertices $V_{1a}, V_{1b}, V_{2a}, V_{2b}, V_{2c}$.

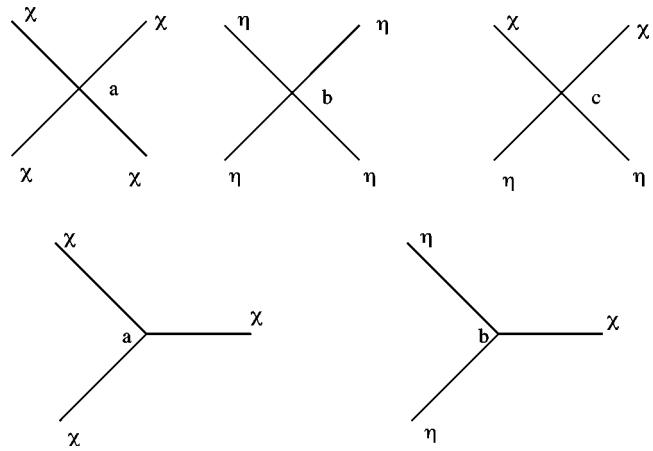

FIG. 2. Interaction vertices $V_{3a}, V_{3b}, V_{3c}, V_{4a}, V_{4b}$.

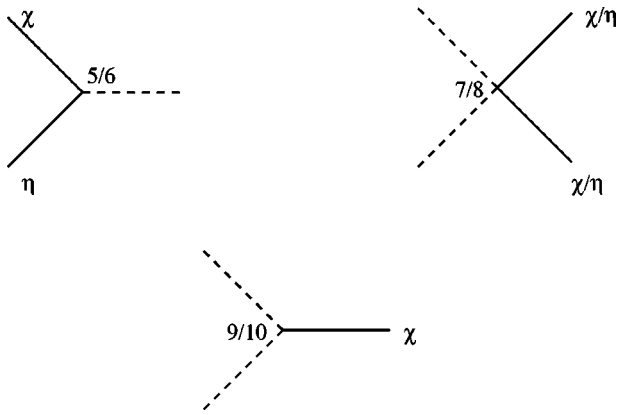

FIG. 3. Interaction vertices $V_5, V_6, V_7, V_8, V_9, V_{10}$.

$$e^2v^2(A-C)+e\gamma_0v^2B = \frac{1}{m_1+m_2}[m_1S_1+m_2S_2-(m_1+m_2)S_3],$$

$$A-C=\frac{1}{m_1m_2(m_1+m_2)}[m_2S_1+m_1S_2-(m_1+m_2)S_3],$$

$$e^2A+\frac{1}{2}e\gamma_0B=\frac{1}{2v^2}(S_1+S_2-2S). \tag{14}$$

These expressions are useful to simplify calculations.

There are several mass poles in the dimensional regularized gauge field propagator. The first two terms in Eq. (10) show that m_1 and m_2 are the photon masses in the original three-dimensional space-time. The third term indicates that m_3 is the mass that photons acquire in $n-3$ dimensional space-time. The last term is proportional to an evanescent quantity, $p^2-\tilde{p}^2$, and the power counting shows that this term behaves as $1/p^5$ for large p. In the two-loop calculation the contribution of this term at the level of regularization is finite and hence vanishes in the limit $n\to 3$.

The interaction vertices of the model were derived in Ref. [7]. These vertices are shown in Figs. 1–3 with dotted lines denoting gauge bosons, while solid lines corresponding to either the η field or the χ field, as labeled. For conciseness, the vertices in the figures are labeled purely by their number, so that for example $3b$ denotes V_{3b}, etc. The Feynman rules for the vertices and the corresponding interaction Lagrangians are as follows:

$$\mathcal{L}^{(1)}\to V_{1a}(\chi^6)=-i\lambda, \quad V_{1b}(\chi^4\eta^2)=V_{1b}(\chi^2\eta^4)=-i\lambda/5,$$

$$\mathcal{L}^{(2)}\to V_{2a}(\chi^5)=-i\lambda v,$$

$$V_{2b}(\chi^3\eta^2)=V_{2c}(\chi\eta^4)=-i\lambda v/5,$$

$$\mathcal{L}^{(3)}\to V_{3a}(\chi^4)=-i\lambda v^2/2,$$

$$V_{3b}(\chi^2\eta^2)=V_{3b}(\eta^4)=-i\lambda v^2/10,$$

$$\mathcal{L}^{(4)}\to V_{4a}(\chi^3)=-i\lambda v^3/6, \quad V_{4b}(\chi\eta^2)=-i\lambda v^3/30,$$

$$\mathcal{L}^{(5)}+\mathcal{L}^{(6)}\to V_5(\eta\chi A)+V_6(\eta\chi A) = e(p-q)_\lambda+i\gamma_0\epsilon^{\tau\alpha\lambda}k_\tau p_\alpha,$$

$$\mathcal{L}^{(7)}+\mathcal{L}^{(8)}\to V_7(\chi^2A^2)+V_8(\chi^2A^2) = V_7(\eta^2A^2)+V_8(\eta^2A^2) = 2ie^2g_{\alpha\beta}-e\gamma_0v\epsilon_{\alpha\lambda\beta}(p-q)_\lambda,$$

$$\mathcal{L}^{(9)}+\mathcal{L}^{(10)}\to V_9(\chi A^2)+V_{10}(\chi A^2) = -e\gamma_0v\epsilon_{\mu\alpha\nu}(k-q)^\alpha+2ie^2vg_{\mu\nu}. \tag{15}$$

III. TWO-LOOP RENORMALIZABILITY

In this section we discuss the renormalizability of the theory at the two-loop level. In order to show that the theory is renormalizable we must show that contributions to the two-loop quantum effective action from terms that do not appear in the original Lagrangian are finite. The superficial degree of divergence (SDD) of a diagram is given by

$$\omega=3L+\sum_v \delta_v v_v-2I, \tag{16}$$

where L is the number of loops, I is the number of internal lines, δ_v is the number of derivatives associated with each vertex, and v_v is the number of vertices of each type. The sum is over all vertices in the diagram. We can rewrite this expression using the following relations:

$$L=I+1-V,$$

$$I=\frac{1}{2}\sum_v i_v v_v,$$

$$l_v=i_v+e_v, \tag{17}$$

where $V=\Sigma_v v_v$ is the number of vertices, l_v is the total number of lines entering vertex v, and i_v and e_v are respectively the number of internal and external lines entering the vertex v. Furthermore, there exists a relation between the number of vertices and lines:

$$\sum_v l_v v_v=2I+E. \tag{18}$$

Using Eqs. (17) with $L=2$ this constraint can be written as

$$\sum_v (l_v-2)v_v=2+E. \tag{19}$$

Consequently, we have the SDD

$$\omega=3+\sum_v \omega_v v_v-\frac{1}{2}E, \tag{20}$$

where $\omega_v\equiv\delta_v+\frac{1}{2}l_v-3$ is the degree of divergence for each vertex, which can be read out from the interaction Lagrangian (4),

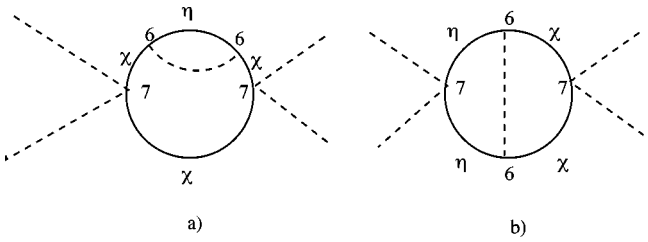

FIG. 4. Two loop contributions to A^4 part of the effective action.

$$\omega_1=0,\quad \omega_2=-\frac{1}{2},\quad \omega_3=-1,\quad \omega_4=-\frac{3}{2},\quad \omega_5=-\frac{1}{2},$$

$$\omega_6=\frac{1}{2},\quad \omega_7=-1,\quad \omega_8=0,\quad \omega_9=-\frac{1}{2},\quad \omega_{10}=-\frac{3}{2}.$$

Substituting in we get

$$\omega=3-\frac{1}{2}E+\left(-\frac{1}{2}\nu_2-\nu_3-\frac{3}{2}\nu_4-\frac{1}{2}\nu_5+\frac{1}{2}\nu_6-\nu_7-\frac{1}{2}\nu_9-\frac{3}{2}\nu_{10}\right). \tag{21}$$

To explicitly verify the renormalizability, we need to look at the one-particle-irreducible (1PI) parts of the Green functions that have positive SDD but no correspondence in the classical Lagrangian. We consider the 1PI part of the four-point function of gauge fields, $A^4\equiv\langle A_\mu A_\nu A_\lambda A_\rho\rangle$. It is straightforward to determine which combinations of vertices satisfy the two conditions (19) and (21). One must then select those combinations from which it is possible to obtain a diagram with four external photon lines. We categorize these diagrams in three groups: diagrams with two internal V_6's, one internal V_6 and no internal V_6.

Note that when calculating the SDD of an A^4 diagram, the following points must be taken into account. If V_6 is an external vertex, then its degree of divergence is $\omega_6=-1/2$ instead of $\omega_6=1/2$. This change occurs because of the fact that V_6 carrys one factor of momentum from the photon line,

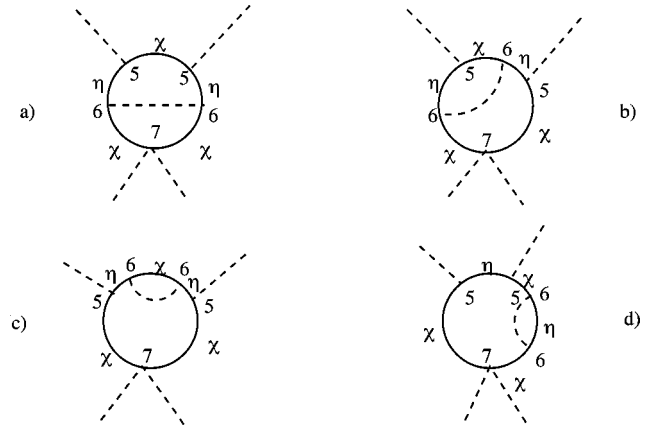

FIG. 5. Two loop contributions to A^4 part of the effective action.

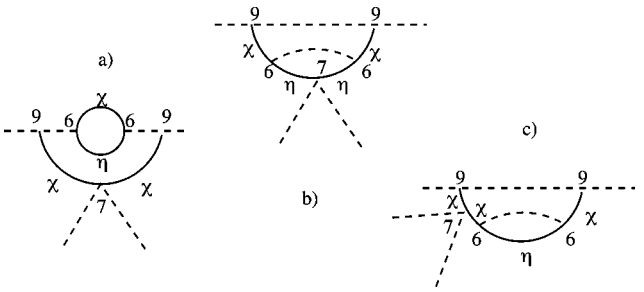

FIG. 6. Two loop contributions to A^4 part of the effective action.

which becomes an external momentum when V_6 is an external vertex. Similarly, the degree of divergence of V_8 is reduced to $\omega_8=-1$ instead of $\omega_8=0$ when V_8 is an external vertex (V_8 is considered an external vertex if both photon lines are external legs). These exceptions must be considered in looking for divergent diagrams.

From Eq. (21) all diagrams with 2 V_6's have SDD = 2, 1, or 0. The diagrams with SDD equal to 0 and no external V_8's are shown in Figs. 4–10. It is straightforward to see that if the SDD is 2, then the graph has two external V_8's. As discussed above, these graphs have factors of the external photon momentum from the vertices V_8 which can be factored out, leaving a two-loop integral with SDD $=0$. The diagrams are the same as those shown in Fig. 4, with the external V_7's replaced by V_8's. If the SDD is 1, then the diagram has one external V_8. These graphs also have a factor of external photon momentum which can be factored out to leave an integral of SDD$=0$. The diagrams are the same as those shown in shown in Figs. 5–7 with the external V_7's replaced by V_8's.

The same thing happens for diagrams with one V_6. The SDD is either 1 or 0. The diagrams with SDD equal to 0 and no external V_8's are shown in Figs. 11, 12. If the SDD is 1, then the diagram has an external V_8 which reduces the SDD to 0, with a prefactor that contains an external photon momentum. These diagrams are the same as those shown in Fig. 11 with the external V_7's replaced by V_8's. For the diagrams with no V_6, all V_8's contribute one photon line which is internal and one which is external, and thus contribute the naive value $\omega_8=0$ to the counting of the diagram's SDD. All divergent diagrams have SDD$=0$ and are shown in Figs. 13–15.

Note that all of the diagrams with external V_5's have partner diagrams with all possible combinations and permuta-

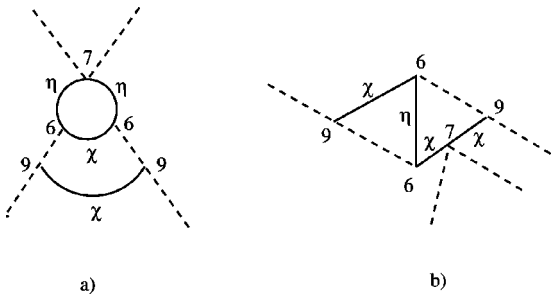

FIG. 7. Two loop contributions to A^4 part of the effective action.

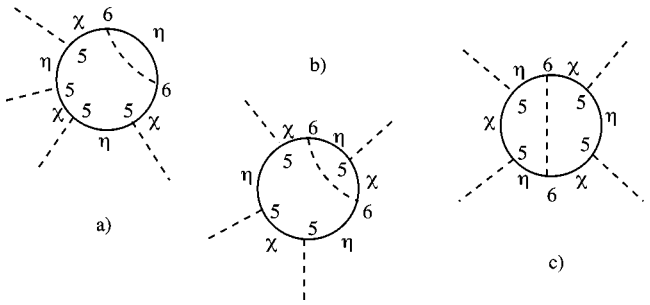

FIG. 8. Two loop contributions to A^4 part of the effective action.

tions of V_5's replaced by V_6's. These partner diagrams will have the same SDD=0 as the original diagrams, but they will contain additional factors of external photon momenta, since they contain external V_6's.

The above analysis shows that all the two-loop divergent diagrams for A^4 have SDD=0. For the purpose of studying renormalizability, we need to extract only the divergent parts and see whether they cancel. There are several simplifications that one can make when extracting the divergent part of a diagram with SDD=0. We start by considering the scalar propagator. When a scalar propagator (8) contains an external momentum it has the form

$$S(q+p,m)=\frac{1}{(q+p)^2-m^2}$$

where p is an external momentum and q is an internal momentum that will be integrated over. To study the UV behavior of this propagator we can employ following decomposition:

$$S(q+p,m)=\frac{1}{(q+p)^2-m^2}=\frac{1}{q^2}-\frac{2p\cdot q+p^2-m^2}{q^2[(q+p)^2-m^2]}. \tag{22}$$

The UV degree of divergence of the second term is one less than that of the first term and thus gives no contribution to the UV divergent part of the integral. Thus, in calculating the divergent part of the integral, we can make the replacement

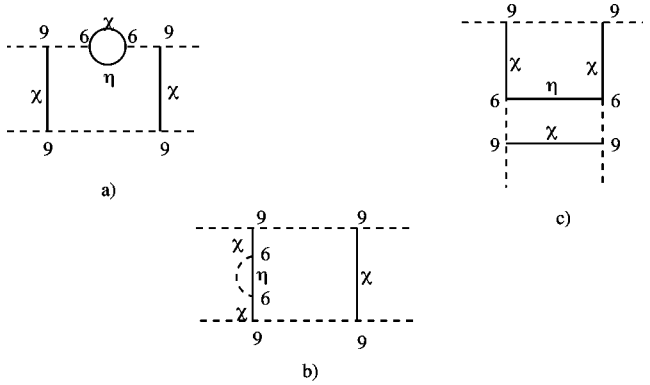

FIG. 9. Two loop contributions to A^4 part of the effective action.

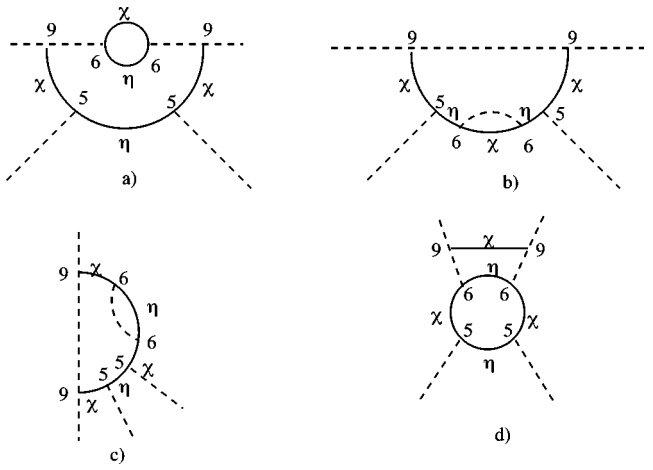

FIG. 10. Two loop contributions to A^4 part of the effective action.

$$S(q+p,m)\rightarrow S(q)\equiv 1/q^2. \tag{23}$$

To avoid the IR divergence induced by this procedure, it is necessary to keep the mass term at least in one propagator, i.e., replace $S(q+p,m)$ by $S(q,m)$. We will keep all three.

Similar simplifications are possible for the gauge propagator (10). First, the evanescent parts of the gauge field propagator will give vanishing contribution to the UV divergent part of the integral in the limit $n\rightarrow 3$ since they have good UV behavior. Second, the UV degree of divergence of the parity odd term in the gauge field propagator is one less than that of the parity even part of the propagator. Thus, there are no contributions to the UV divergent part of the integral from terms in the propagator that are proportional to the epsilon tensor. Last, as explained above for the scalar propagator, factors of external momentum can be dropped.

In addition, the calculation is simplified because of the fact that there are no UV divergences at one-loop in dimensional regularization. This fact is due to the special analytic properties of the one-loop amplitudes in odd dimensional space-time [17]. As a consequence, there is no need for us to consider the subtraction of sub-divergences.

Using the above observations, one can show that all divergent terms will be proportional to the two-loop integral of the form

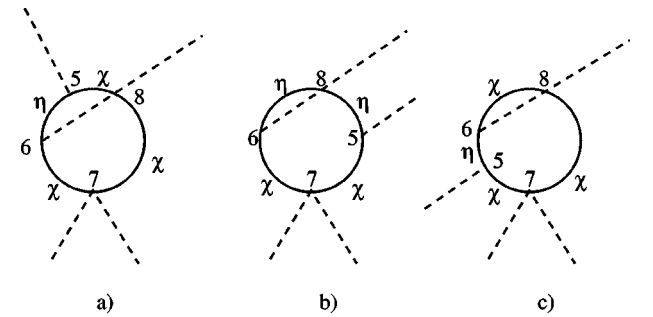

FIG. 11. Two loop contributions to A^4 part of the effective action.

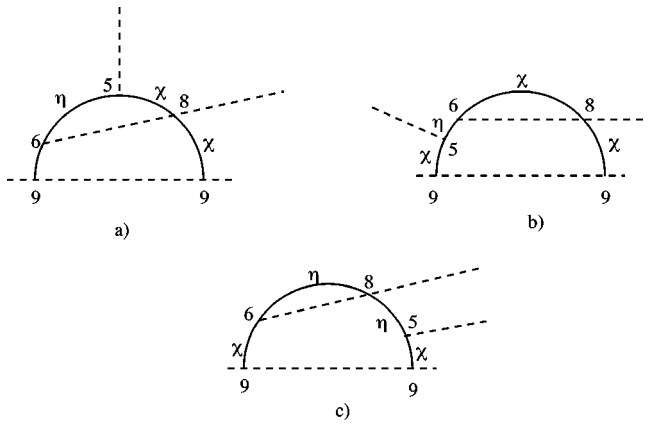

FIG. 12. Two loop contributions to A^4 part of the effective action.

$$I=\int dk \int dq S(q)S(k)S(k+q,m) \tag{24}$$

with $dk \equiv d^3k/(2\pi)^3$. The divergence will appear as a pole of the form $1/(3-n)$.

We concentrate first on diagrams in which the prefactor does not depend on the external photon momentum (Figs. 4–15). These diagrams can be further classified into three groups by coupling constant prefactors. The first group of diagrams is shown in Figs. 4, 5, 8, 11, 13, 14 and is proportional to $e^2\gamma_0^2$; the second group is shown in Figs. 6, 7, 10, 12, 15 and is proportional to $e^4\gamma_0^2(\gamma_0 v)^2$. The third group is shown in Fig. 9 and is proportional to $e^4\gamma_0^2(\gamma_0 v)^4$. Each of these sets of diagrams must be separately finite if the theory is to be renormalizable. We will concentrate on the first group of diagrams, Figs. 4, 5, 8, 11, 13, 14. Direct calculation shows that the divergent pieces cancel between the diagrams in Figs. 4, 5, 8, 11, 13, while Fig. 14 is finite.

As a further check, we calculate the A^4 term with four external momentum factors. Gauge invariance requires that this contribution should take the form $(F_{\mu\nu})^4$, and renormalizability requires that it be finite. We obtain contributions from the diagrams in Fig. 8, with all of the external V_5's replaced by V_6's. Direct calculation shows that the divergent pieces cancel.

Unfortunately, further checking reveals that the action is not in fact renormalizable. We have calculated the χ^4 term with four external momentum factors. The corresponding

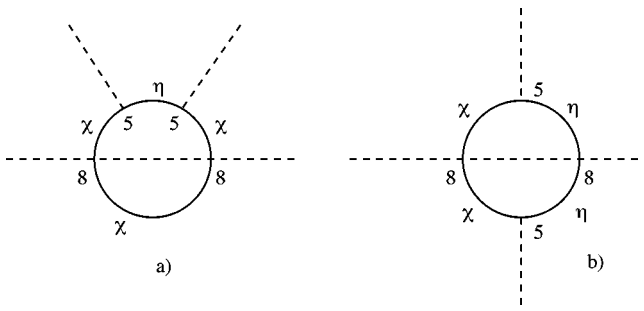

FIG. 13. Two loop contributions to A^4 part of the effective action.

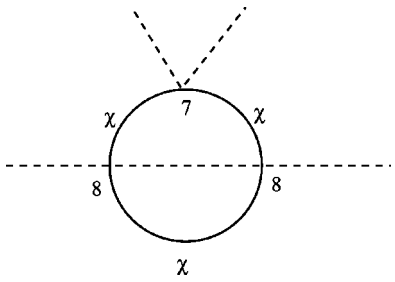

FIG. 14. Two loop contributions to A^4 part of the effective action.

diagrams are shown in Fig. 16. This term is a contribution to the gauge invariant structure $(D_\mu\phi)^4$ and must be finite if the action is to be renormalizable. We find that in this case the divergent terms do not cancel so that the two-loop quantum effective action is not renormalizable.

However, as we will show in the next section, the two-loop effective potential can be made renormalizable providing certain conditions are satisfied by the coupling constants in the model. Therefore, for the lowest order in the momentum expansion of the quantum effective action, the nonminimal model may yield physically relevant predictions for the spontaneous symmetry breaking by the radiative corrections at the two-loop level.

IV. EFFECTIVE POTENTIAL

The effective potential is the energy density of the vacuum in which the expectation value of the scalar field is given by $\langle\phi\rangle = v$ [11]. It can be determined from the effective action $\Gamma[\tilde{\phi}]$ according to

$$\Gamma[\tilde{\phi}=v]=-(2\pi)^n\delta^{(n)}(0)V_{\rm eff}(v), \tag{25}$$

where $\tilde{\phi}$ is the vacuum expectation value of the scalar field in the presence of the external source. Using the fact that $\Gamma[\tilde{\phi}]$ is the generating functional of the proper vertex,

$$\Gamma[\tilde{\phi}]=\sum_{j=1}^{\infty}\frac{1}{j!}\int d^nx_1 d^nx_2\cdots d^nx_j\Gamma^{(j)}(x_1,\ldots,x_j),$$

one has

$$V_{\rm eff}(v)=-\sum_{j=2}^{\infty}\frac{1}{j!}\Gamma^{(j)}(0,0,\ldots,0)v^j, \tag{26}$$

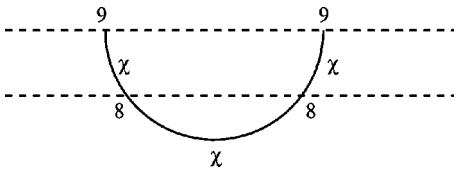

FIG. 15. Two loop contributions to A^4 part of the effective action.

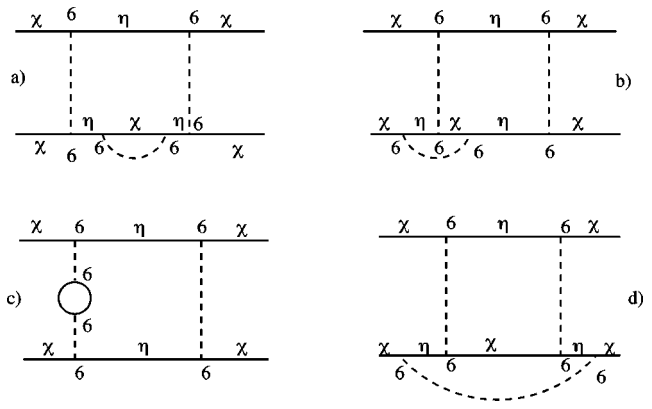

FIG. 16. Two loop contributions to χ^4 part of the effective action.

which means that one can get the effective potential by calculating the 1PI vacuum diagrams. From Eq. (26) we can write [18]

$$V_{\rm eff}(v)=V^{\rm tree}(v)-\frac{i\hbar}{2}\int\frac{d^np}{(2\pi)^n}\ln\det[iD^{-1}(p)] +i\hbar\left\langle\exp\left(\frac{i}{\hbar}d^nx\mathcal{L}_{\rm int}(\hbar^{1/2}\phi,v)\right)\right\rangle_{\rm 1PI}, \quad (27)$$

where $D^{-1}(p)$ is the inverse propagator for each of the bosonic fields in the theory. $V^{\rm tree}$ is the tree potential and can be obtained directly from Eq. (1) as,

$$V^{\rm tree}=\frac{\lambda}{6!}v^6. \quad (28)$$

The second term of Eq. (27) is the one-loop effective potential, and the third term contains the higher order contributions. We shall use this expression to get the two-loop contribution to the effective potential.

A. One-loop effective potential in dimensional regularization

The one-loop effective potential of this model was computed in Ref. [7] in Pauli-Villars regularization. In order to discuss spontaneous symmetry breaking, we now recalculate the one-loop effective potential using dimensional regularization. According to Eq. (27) we have

$$V_{\rm eff}^{\rm one\text{-}loop}=\frac{\hbar}{2}\int\frac{d^np}{i(2\pi)^n}\{\ln[iS_\chi^{-1}(p)]+\ln[iS_\eta^{-1}(p)] +\ln\det[iD_{\mu\nu}^{-1}(p)]\}. \quad (29)$$

From the quadratic part $\mathcal{L}^{(0)}$ of the Lagrangian (4) we have the inverse scalar propagators

$$S_\chi^{(-1)}(p)=p^2-m_\chi^2,$$
$$S_\eta^{(-1)}(p)=p^2-m_\eta^2,$$

and the inverse gauge field propagator

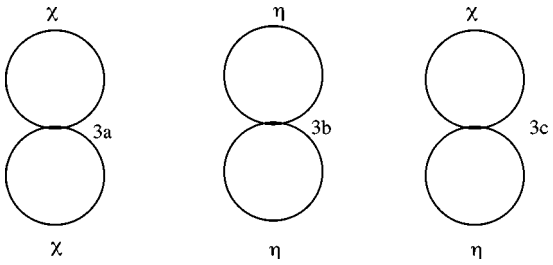

FIG. 17. Two loop contributions to the effective potential from two scalar loops.

$$D_{\mu\nu}^{-1}(p)\equiv K_{\mu\nu}=(-p^2+e^2v^2)g_{\mu\nu} +\left(1-\frac{1}{\xi}\right)p_\mu p_\nu-\gamma_0 ev^2\epsilon_{\mu\nu\rho}ip^\rho, \quad (30)$$

which gives [10]

$$\det K_{\mu\nu}=\exp{\rm Tr}\ln K_{\mu\nu} =(p^2-e^2v^2)\left(e^2v^2-\frac{1}{\xi}p^2\right) \times[(p^2-e^2v^2)-(\gamma_0 ev^2)^2\tilde{p}^2]. \quad (31)$$

Substituting above results into Eq. (29) and making use of the integration formulas given in Ref. [10],

$$\int\frac{d^np}{i(2\pi)^n}\ln(p^2-m^2)=-\frac{\Gamma\left(-\frac{1}{2}n\right)}{(4\pi)^{n/2}}m^n\overset{n\to3}{=}-\frac{1}{6\pi}m^3,$$

$$\int\frac{d^np}{i(2\pi)^n}\ln[(p^2-e^2v^2)^2-(\gamma_0 ev^2)^2\tilde{p}^2] \overset{n\to3}{=}-\frac{1}{6\pi}(ev)^3(4+\gamma_0^2v^2)^{1/2}(1+\gamma_0^2v^2), \quad (32)$$

we obtain the one-loop effective potential in the Landau gauge ($\xi=0$),

$$V_{\rm eff}^{\rm one\text{-}loop}=-\frac{\hbar}{12\pi}[m_\chi^3+m_\eta^3+(ev)^3(4+\gamma_0^2v^2)^{1/2}(1+\gamma_0^2v^2)] =-\frac{\hbar}{12\pi}\left\{\left[\left(\frac{1}{4!}\right)^{3/2}+\left(\frac{1}{5!}\right)^{3/2}\right]\lambda^{3/2}v^6 +(ev)^3(4+\gamma_0^2v^2)^{1/2}(1+\gamma_0^2v^2)\right\}. \quad (33)$$

This result agrees with the one obtained in [7].

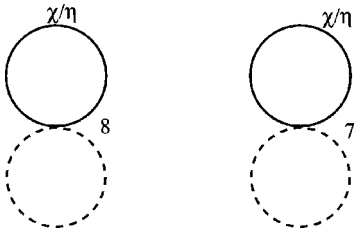

FIG. 18. Two loop contributions to the effective potential from one scalar and one gauge field loop.

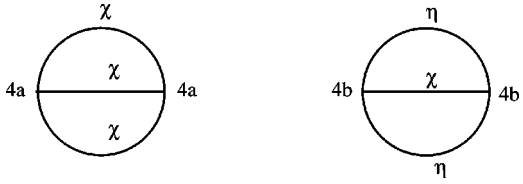

FIG. 19. Theta diagrams constructed from scalar propagators.

B. Two-loop effective potential

It is straightforward to see that the two-loop effective potential is given by diagrams that contain only vertices from the cubic and quartic parts of the interaction Lagrangian. All of these diagrams are either ''theta'' type diagrams or ''figure eight'' type diagrams. For each diagram we give the integral expression for the amplitude. The integral is evaluated by rotating to Euclidean space and using the integrals given in the Appendix. The theta diagrams contain ultraviolet divergent terms and the results are separated into divergent and finite pieces. The divergent piece is represented by the integral I as defined in the Appendix, and is thus proportional to the factor

$$\frac{1}{3-n}-\gamma+1+\ln 4\pi$$

where γ is the Euler constant. As a last step we use Eqs. (11) to write the results in terms of the fundamental parameters e, γ_0, and v.

(1) The ''figure eight'' diagrams containing two scalar loops (Fig. 17). These diagram are produced by the quartic interaction $\mathcal{L}^{(3)}$. The amplitude can be calculated by rotating to Euclidean space. We obtain

$$\begin{aligned}V_{\rm eff}^{(1)}&=-\hbar^2\lambda v^2\mu^{(3-n)}\int\frac{d^nq}{(2\pi)^n}\frac{d^np}{(2\pi)^n}\\&\quad\times\left[\frac{3}{2}\frac{1}{(p^2-m_\chi^2)(q^2-m_\chi^2)}+\frac{3}{10}\frac{1}{(p^2-m_\eta^2)(q^2-m_\eta^2)}+\frac{1}{10}\frac{1}{(p^2-m_\chi^2)(q^2-m_\eta^2)}\right]\\&=\frac{\hbar^2\lambda v^2}{16\pi^2}\left(\frac{3}{2}m_\chi^2+\frac{3}{10}m_\eta^2+\frac{1}{10}m_\chi m_\eta\right)\\&=\frac{\hbar^2\lambda^2v^6}{16\pi^2}\left(\frac{3}{2}\frac{1}{4!}+\frac{3}{10}\frac{1}{5!}+\frac{1}{10}\frac{1}{\sqrt{5}}\frac{1}{4!}\right).\end{aligned}\tag{34}$$

(2) The ''figure eight'' diagram containing one scalar loop and one gauge field loop (Fig. 18). This diagram is produced by the quartic interactions $\mathcal{L}^{(7)}$ and $\mathcal{L}^{(8)}$. Its amplitude is

$$\begin{aligned}V_{\rm eff}^{(2)}&=i\hbar^2\mu^{(3-n)}\int\frac{d^np}{(2\pi)^n}\frac{d^nq}{(2\pi)^n}iD^{\mu\nu}(p)(2ie^2g_{\mu\nu}-2\gamma_0e\epsilon_{\mu\nu\rho}\tilde{p}^\rho)\left(\frac{1}{q^2-m_\chi^2}+\frac{1}{q^2-m_\eta^2}\right)\\&=2e^2\hbar^2\mu^{(3-n)}\int\frac{d^np}{(2\pi)^n}\frac{d^nq}{(2\pi)^n}\left\{-2\tilde{p}^2\left[\frac{1}{m_1+m_2}\left(\frac{1}{m_1}\frac{1}{p^2-m_1^2}+\frac{1}{m_2}\frac{1}{p^2-m_2^2}\right)-\frac{1}{m_3^2}\frac{1}{p^2}\right]-\frac{1}{m_3^2}\left(\frac{1}{p^2-m_3^2}-\frac{1}{p^2}\right)\right.\\&\quad\left.\times[(n-1)p^2-2\tilde{p}^2]\right\}\left(\frac{1}{q^2-m_\chi^2}+\frac{1}{q^2-m_\eta^2}\right)-4\gamma_0e\hbar^2\mu^{(3-n)}\int\frac{d^np}{(2\pi)^n}\frac{d^nq}{(2\pi)^n}\frac{\tilde{p}^2}{m_1+m_2}\left(\frac{1}{p^2-m_1^2}-\frac{1}{p^2-m_2^2}\right)\\&\quad\times\left(\frac{1}{q^2-m_\chi^2}+\frac{1}{q^2-m_\eta^2}\right).\end{aligned}\tag{35}$$

Rotating to Euclidean space we obtain

$$V_{\rm eff}^{(2)}=-2e^2\hbar^2\mu^{(3-n)}\int\frac{d^np}{(2\pi)^n}\frac{d^nq}{(2\pi)^n}\left[\frac{2}{m_1(m_1+m_2)}\frac{\tilde{p}^2}{p^2+m_1^2}+\frac{2}{m_2(m_1+m_2)}\frac{\tilde{p}^2}{p^2+m_2^2}+\frac{n-1}{m_3^2}\frac{p^2}{p^2+m_3^2}-\frac{2}{m_3^2}\frac{\tilde{p}^2}{p^2+m_3^2}\right]$$

$$\times\left(\frac{1}{q^2+m_\chi^2}+\frac{1}{q^2+m_\eta^2}\right)-4\gamma_0 e\hbar^2\mu^{(3-n)}\int\frac{d^np}{(2\pi)^n}\frac{d^nq}{(2\pi)^n}\frac{\tilde{p}^2}{m_1+m_2}\left(\frac{1}{p^2+m_1^2}-\frac{1}{p^2+m_2^2}\right)\left(\frac{1}{q^2+m_\chi^2}+\frac{1}{q^2+m_\eta^2}\right)$$

$$=\frac{\hbar^2}{4\pi^2}\frac{m_\chi+m_\eta}{m_1+m_2}[e^2(m_1^2+m_2^2)+\gamma_0 e(m_1^3-m_2^3)]$$

$$=\frac{\hbar^2}{4\pi^2}\frac{1}{v^2}\frac{m_\chi+m_\eta}{m_1+m_2}(m_1^4+m_2^4)=\frac{\hbar^2}{4\pi^2}\frac{1}{v^2}\frac{m_\chi+m_\eta}{m_1+m_2}[(m_1^2-m_2^2)^2+2m_1^2m_2^2]$$

$$=\frac{\hbar^2}{4\pi^2}\frac{\sqrt{\lambda/4!}+\sqrt{\lambda/5!}}{\sqrt{\gamma_0v^2+4}}(ev)^3[4(\gamma_0v)^4+(\gamma_0v)^2+2]. \tag{36}$$

(3) The ''theta'' diagrams constructed from scalar propagators (Fig. 19). These diagrams come from the interaction $\mathcal{L}^{(4)}$ and contain UV divergences. Their contribution to the effective potential is

$$V_{\rm eff}^{(3)}=-\frac{\hbar^2}{2}\frac{(\lambda v^3)^2\mu^{(3-n)}}{3!^3}\int\frac{d^np}{(2\pi)^n}\frac{d^nq}{(2\pi)^n}\frac{1}{(p^2-m_\chi^2)(q^2-m_\chi^2)[(p+q)^2-m_\chi^2]}$$

$$-\frac{\hbar^2}{2}\frac{(\lambda v^3)^2\mu^{(3-n)}}{(5!\times3!)^2\times2!}\int\frac{d^np}{(2\pi)^n}\frac{d^nq}{(2\pi)^n}\frac{1}{(p^2-m_\eta^2)(q^2-m_\eta^2)[(p+q)^2-m_\chi^2]}$$

$$=\frac{\hbar^2}{2}(\lambda v^3)^2\left(\frac{1}{3!^3}+\frac{1}{(5\times3!)^2\times2!}\right)\frac{\mu^{(n-3)}}{32\pi^2}\left(\frac{1}{3-n}-\gamma+1+\ln4\pi\right)$$

$$-\frac{\hbar^2}{2}(\lambda v^3)^2\frac{1}{32\pi^2}\left[\frac{1}{3!^3}\ln\frac{9m_\chi^2}{\mu^2}+\frac{1}{(5\times3!)^2\times2!}\ln\frac{(2m_\eta+m_\chi)^2}{\mu^2}\right]$$

$$\equiv V^{(3)\rm div}+V^{(3)\rm finite}. \tag{37}$$

(4) The ''theta'' diagram composed of one gauge field propagator, one η propagator and one χ propagator (Fig. 20). This diagram arises from the cubic interaction $\mathcal{L}^{(5)}$ and $\mathcal{L}^{(6)}$ and the UV divergence is present. Its contribution to the effective potential is

$$V_{\rm eff}^{(4)}=\frac{i\hbar^2}{2}\mu^{(3-n)}\int\frac{d^np}{(2\pi)^n}\frac{d^nq}{(2\pi)^n}\{[e(p+2q)_\mu-i\gamma_0\epsilon_{\mu\alpha\beta}q^\alpha p^\beta][e(p+2q)_\nu-i\gamma_0\epsilon_{\nu\gamma\delta}p^\gamma q^\delta]iD^{\mu\nu}(p)iS_\eta(p+q)iS_\chi(q)\}. \tag{38}$$

Inserting the gauge field propagator (10), employing the identities (14) to simplify the expression and performing the contractions:

$$(p+2q)_\mu(p+2q)_\nu(\tilde{p}^2g_{\mu\nu}-\tilde{p}_\mu\tilde{p}_\nu)=4[\tilde{p}^2\tilde{q}^2-4(\tilde{p}\cdot\tilde{q})],$$

$$(p+2q)_\mu(p+2q)_\nu(p^2g^{\mu\nu}-p^\mu p^\nu)=4[p^2q^2-(p\cdot q)^2],$$

$$[(p+2q)_\mu\epsilon_{\nu\gamma\delta}p^\gamma q^\delta+\epsilon_{\mu\alpha\beta}q^\alpha p^\beta(p+2q)_\nu](\tilde{p}^2g_{\mu\nu}-\tilde{p}_\mu\tilde{p}_\nu)=[(p+2q)_\mu\epsilon_{\nu\gamma\delta}p^\gamma q^\delta+\epsilon_{\mu\alpha\beta}q^\alpha p^\beta(p+2q)_\nu]$$
$$\times(p^2g^{\mu\nu}-p^\mu p^\nu)=0,$$

$$[(p+2q)_\mu \epsilon_{\nu\gamma\delta} p^\gamma q^\delta + \epsilon_{\mu\alpha\beta} q^\alpha p^\beta (p+2q)_\nu] \epsilon^{\mu\nu\rho} p_\rho = 4[\tilde{p}^2\tilde{q}^2 - 4(\tilde{p}\cdot\tilde{q})] \epsilon_{\mu\alpha\beta} q^\alpha p^\beta \epsilon_{\nu\gamma\delta} p^\gamma q^\delta (\tilde{p}^2 g_{\mu\nu} - \tilde{p}_\mu \tilde{p}_\nu)$$

$$= \tilde{p}^2[(p\cdot q)^2 - \tilde{p}^2\tilde{q}^2],$$

$$\epsilon_{\mu\alpha\beta} q^\alpha p^\beta \epsilon_{\nu\gamma\delta} p^\gamma q^\delta (p^2 g^{\mu\nu} - p^\mu p^\nu) = p^2[\tilde{p}^2\tilde{q}^2 - 4(\tilde{p}\cdot\tilde{q})],$$

$$\epsilon_{\mu\alpha\beta} q^\alpha p^\beta \epsilon_{\nu\gamma\delta} p^\gamma q^\delta \epsilon^{\mu\nu\rho} p_\rho = 0, \tag{39}$$

we obtain

$$V_{\rm eff}^{(4)} = -\frac{\hbar^2}{2}\int \frac{d^n p}{(2\pi)^n}\frac{d^n q}{(2\pi)^n}\mu^{(3-n)}\left\{\left[-\frac{1}{m_1(m_1+m_2)}S_1(p) - \frac{1}{m_2(m_1+m_2)}S_2(p) + \frac{1}{m_3^2}S(p)\right] S_\chi(q) S_\eta(p+q)(4e^2 \right.$$

$$\left. + \gamma_0^2\tilde{p}^2)[\tilde{p}^2\tilde{q}^2 - (\tilde{p}\cdot\tilde{q})^2]\right\} + \frac{\hbar^2}{2}\frac{4e\gamma_0}{m_1+m_2}\int \frac{d^n p}{(2\pi)^n}\frac{d^n q}{(2\pi)^n}\mu^{(3-n)}[S_1(p) - S_2(p)] S_\chi(q) S_\eta(p+q)[\tilde{p}^2\tilde{q}^2 - (\tilde{p}\cdot\tilde{q})^2]$$

$$-\frac{\hbar^2}{2}\frac{1}{m_3^2}\int \frac{d^n p}{(2\pi)^n}\frac{d^n q}{(2\pi)^n}\mu^{(3-n)}[S_3(p) - S(p)] S_\chi(q) S_\eta(p+q)\{4e^2[p^2q^2 - (p\cdot q)^2]$$

$$-[4e^2 + \gamma_0^2(p^2 - \tilde{p}^2)][\tilde{p}^2\tilde{q}^2 - (\tilde{p}\cdot\tilde{q})^2]\}, \tag{40}$$

where the evanescent terms are thrown away since their contribution vanishes in the limit $n \to 3$. Using the integrals given in the Appendix, we find, for the divergent part,

$$V_{\rm eff}^{(4)(\rm div)} = \frac{\hbar^2\mu^{(n-3)}}{64\pi^2}\left(\frac{1}{3-n} - \gamma + 1 + \ln(4\pi)\right)\left[\left(\frac{4}{v^2} + \gamma_0^2\right)\left(\frac{1}{2}[(m_1-m_2)^2 + m_1 m_2](m_\chi^2 + m_\eta^2) - \frac{1}{4}(m_1^2 - m_2^2)^2 \right.\right.$$

$$\left.\left. + \frac{1}{4}m_1 m_2 (m_1 - m_2)^2 - \frac{1}{4}m_1^2 m_2^2\right) + \frac{1}{4}\gamma_0^2 (m_\chi^2 - m_\eta^2)^2\right]$$

$$= -\frac{\hbar^2\mu^{(n-3)}}{230400\pi^2}\left(\frac{1}{3-n} - \gamma + 1 + \ln(4\pi)\right) v^2(-90e^2\gamma_0^4 v^6\lambda - 450e^2\lambda v^4\gamma_0^2 + 900e^4\gamma_0^6 v^6 + 6300e^4 v^4\gamma_0^4 + 11700e^4\gamma_0^2 v^2$$

$$-360e^2\lambda v^2 + 3600e^4 + \gamma_0^2\lambda^2 v^6) \tag{41}$$

and, for the finite part,

$$V_{\rm eff}^{(4)(\rm finite)} = -\frac{\hbar^2}{64\pi^2 v^2}(m_\chi^2 - m_\eta^2)^2 \ln\left(\frac{(m_\chi + m_\eta)^2}{\mu^2}\right) + \frac{\hbar^2(\gamma_0^2 v^2 + 4)m_2}{256\pi^2 v^2(m_1+m_2)}[(m_\chi + m_\eta)^2 - m_2^2][(m_\chi - m_\eta)^2$$

$$-m_2^2]\ln\left(\frac{(m_\chi + m_2 + m_\eta)^2}{\mu^2}\right) + \frac{\hbar^2(\gamma_0^2 v^2 + 4)m_1}{256\pi^2 v^2(m_1+m_2)}[(m_\chi + m_\eta)^2 - m_1^2][(m_\chi - m_\eta)^2 - m_1^2]\ln\left(\frac{(m_\chi + m_1 + m_\eta)^2}{\mu^2}\right)$$

$$-\frac{\hbar^2(\gamma_0^2 v^2 + 4)}{1536\pi^2 v^2(m_1+m_2)}[12(m_1^2 + m_2^2)(m_\chi^3 - m_\chi^2 m_\eta - m_\eta^2 m_\chi + m_\eta^3) + 5m_1^5 + 5m_2^5 - (m_1^3 + m_2^3)$$

$$\times(10m_\chi^2 + 12m_\chi m_\eta + 10m_\eta^2) + m_3^2(m_1+m_2)(10m_\chi^2 + 10m_\eta^2 - 5m_3^2) + 12(m_1^4 + m_2^4)(m_\chi + m_\eta)]$$

$$-\frac{\hbar^2\gamma_0^2}{7680\pi^2}[6m_1^4 + 6m_2^4 - (12m_\eta^2 + 12m_\chi^2 + 6m_3^2)(m_1^2 + m_2^2) + 25m_3^4 - 26(m_\chi^2 + m_\eta^2)m_3^2 + 60m_\eta m_\chi(m_\chi^2 + m_\eta^2)$$

$$+25(m_\chi^2 - m_\eta^2)^2]. \tag{42}$$

(5) The ''theta'' diagram with one χ propagator and two gauge field propagators (Fig. 21). The interaction Lagrangians yielding this diagram are $\mathcal{L}^{(9)}$ and $\mathcal{L}^{(10)}$. The corresponding contribution to the effective potential is

$$V_{\rm eff}^{(5)}=i\hbar^2\int\frac{d^np}{(2\pi)^n}\frac{d^nq}{(2\pi)^n}\mu^{(3-n)}\{[2ie^2vg_{\mu\lambda}+\gamma_0ve\epsilon_{\mu\lambda\alpha}(\tilde{p}^\alpha-\tilde{q}^\alpha)]iD_{\lambda\rho}(q)iS_\chi(p+q)$$
$$\times[2ie^2vg_{\rho\nu}+\gamma_0ve\epsilon_{\rho\nu\beta}(\tilde{p}^\beta-\tilde{q}^\beta)]iD_{\nu\mu}(p)\}. \tag{43}$$

Denoting

$$V_{\mu\nu}(p,q)\equiv 2ie^2vg_{\mu\nu}+\gamma_0ve\epsilon_{\mu\nu\rho}(\tilde{p}^\rho-\tilde{q}^\rho),$$
$$P^{(T)}_{\mu\nu}\equiv p^2g_{\mu\nu}-p_\mu p_\nu,$$
$$\tilde{P}^{(T)}_{\mu\nu}\equiv\tilde{p}^2\tilde{g}_{\mu\nu}-\tilde{p}_\mu\tilde{p}_\nu, \tag{44}$$

and performing the contractions

$$V_{\mu\lambda}(p,q)V_{\rho\nu}(p,q)\tilde{P}^{(T)\mu\nu}\tilde{P}^{(T)\lambda\rho}=-4e^4v^2[\tilde{p}^2\tilde{q}^2+(\tilde{p}\cdot\tilde{q})^2]+(\gamma_0ve)^2\{-(\tilde{p}^2+\tilde{q}^2)[\tilde{p}^2\tilde{q}^2+(\tilde{p}\cdot\tilde{q})^2]+4\tilde{p}^2\tilde{q}^2(\tilde{p}\cdot\tilde{q})\},$$

$$V_{\mu\lambda}(p,q)V_{\rho\nu}(p,q)\tilde{P}^{(T)\mu\nu}P^{(T)\lambda\rho}=-4e^4v^2[2\tilde{p}^2q^2-\tilde{p}^2\tilde{q}^2+(\tilde{p}\cdot\tilde{q})^2]+(\gamma_0ve)^2[4\tilde{p}^2q^2(\tilde{p}\cdot\tilde{q})-2\tilde{p}^4q^2+\tilde{p}^4\tilde{q}^2-\tilde{p}^2(\tilde{p}\cdot\tilde{q})^2$$
$$-q^2(\tilde{p}\cdot\tilde{q})^2-\tilde{p}^2\tilde{q}^2q^2],$$

$$V_{\mu\lambda}(p,q)V_{\rho\nu}(p,q)P^{(T)\mu\nu}P^{(T)\lambda\rho}=-4e^4v^2[(n-3)p^2q^2+p^2q^2+(p\cdot q)^2]+(\gamma_0ve)^2[-2p^2q^2(\tilde{p}^2+\tilde{q}^2)+4p^2q^2(\tilde{p}\cdot\tilde{q})$$
$$-(p^2+q^2)(\tilde{p}\cdot\tilde{q})^2+(p^2+q^2)\tilde{p}^2\tilde{q}^2],$$

$$V_{\mu\lambda}(p,q)V_{\rho\nu}(p,q)\tilde{P}^{(T)\mu\nu}\epsilon^{\lambda\rho\gamma}iq_\gamma=-2\gamma_0v^2e^3[-4\tilde{p}^2\tilde{p}\cdot\tilde{q}+2(\tilde{p}\cdot\tilde{q})^2+2\tilde{p}^2\tilde{q}^2],$$

$$V_{\mu\lambda}(p,q)V_{\rho\nu}(p,q)P^{(T)\mu\nu}\epsilon^{\lambda\rho\gamma}iq_\gamma=-4\gamma_0v^2e^3[-2p^2\tilde{p}\cdot\tilde{q}+2p^2\tilde{q}^2+(\tilde{p}\cdot\tilde{q})^2-\tilde{p}^2\tilde{q}^2],$$

$$V_{\mu\lambda}(p,q)V_{\rho\nu}(p,q)\epsilon^{\mu\nu\alpha}ip_\alpha\epsilon^{\lambda\rho\beta}iq_\beta=8e^4v^2\tilde{p}\cdot\tilde{q}+2(\gamma_0ve)^2[(\tilde{p}^2+\tilde{q}^2)\tilde{p}\cdot\tilde{q}-\tilde{p}^2\tilde{q}^2-(\tilde{p}\cdot\tilde{q})^2], \tag{45}$$

we get

$$V_{\rm eff}^{(5)}=i\hbar^2\int\frac{d^np}{(2\pi)^n}\frac{d^nq}{(2\pi)^n}\mu^{3-n}e^2v^2S_\chi(p+q)[e^2[-4e^2C(p)C(q)p^2q^2(n-3)-16[A(p)-C(p)]C(q)\tilde{p}^2(q^2-\tilde{q}^2)$$
$$-4C(p)C(q)[p^2q^2+(p\cdot q)^2]-4[A(p)A(q)-C(p)C(q)][\tilde{p}^2\tilde{p}^2+(\tilde{p}\cdot\tilde{q})^2]+8B(p)B(q)\tilde{p}\cdot\tilde{q}]+e\gamma_0[8A(p)B(q)$$
$$\times[\tilde{p}^2\tilde{q}^2+(\tilde{p}\cdot\tilde{q})^2-2\tilde{p}^2\tilde{p}\cdot\tilde{q}]+16C(p)B(q)(p^2-\tilde{p}^2)(\tilde{q}^2-\tilde{p}\cdot\tilde{q})]+\gamma_0^2(-4C(p)C(q)(p^2-\tilde{p}^2)(q^2-\tilde{q}^2)(\tilde{p}^2-\tilde{p}\cdot\tilde{q})$$
$$-4A(p)C(q)(q^2-\tilde{q}^2)\tilde{p}^4-C(p)A(q)(p^2-\tilde{p}^2)2\tilde{p}^2\tilde{q}^2+2(\tilde{p}\cdot\tilde{q})^2-8\tilde{q}^2\tilde{p}\cdot\tilde{q}+2B(p)B(q)[(\tilde{p}^2+\tilde{q}^2)\tilde{p}\cdot\tilde{q}-\tilde{p}^2\tilde{q}^2$$
$$-(\tilde{p}\cdot\tilde{q})^2]-2A(p)A(q)\{2\tilde{p}^2\tilde{q}^2[\tilde{p}^2-(\tilde{p}\cdot\tilde{q})]-\tilde{p}^2[\tilde{p}^2\tilde{q}^2-(\tilde{p}\cdot\tilde{q})^2]\})]. \tag{46}$$

In deriving Eq. (46) we have used the fact that the integrand is invariant under the interchange $p\leftrightarrow q$.

The calculation of this amplitude is quite lengthy. For clarity we divide the result in three parts. We label them V_{e^2}, $V_{e\gamma_0}$, and $V_{\gamma_0^2}$. They correspond to the terms proportional to e^2, $e\gamma_0$, and γ_0^2 respectively in Eq. (46) above. In the expressions that appear below, the couplings e and γ_0 do not explicitly appear because we have used the relations (11) to write

$$e^2=\frac{1}{v^2}m_3^2,$$

$$e\gamma_0=\frac{1}{v^2}(m_1-m_2),$$

$$\gamma_0^2=\frac{1}{v^2}\frac{(m_1-m_2)^2}{m_1m_2}, \tag{47}$$

and eliminate the couplings in favor of the masses. This substitution allows us to combine all three terms to obtain the simplest expression possible for the final result of this diagram. As a last step we use Eqs. (11) to write the result in terms of the fundamental parameters e, γ_0, and v. First, the e^2 part is

$$\begin{aligned}V_{e^2}=&-\frac{3\hbar^2\mu^{(n-3)}}{16\pi^2v^2}\left(\frac{1}{3-n}-\gamma+1+\ln(4\pi)\right)m_3^4+\frac{\hbar^2m_\chi^4}{32\pi^2v_2}\ln\left(\frac{m_\chi^2}{\mu^2}\right)-\frac{\hbar^2m_1(m_2^2-m_\chi^2)^2}{16\pi^2(m_1+m_2)v^2}\ln\left(\frac{(m_2+m_\chi)^2}{\mu^2}\right)\\&-\frac{\hbar^2m_2(m_1^2-m_\chi^2)^2}{16\pi^2(m_1+m_2)v^2}\ln\left(\frac{(m_1+m_\chi)^2}{\mu^2}\right)+\frac{\hbar^2m_1^2(4m_2^2-m_\chi^2)^2}{32(m_1+m_2)^2\pi^2v^2}\ln\left(\frac{(2m_2+m_\chi)^2}{\mu^2}\right)+\frac{\hbar^2m_2^2(4m_1^2-m_\chi^2)^2}{32(m_1+m_2)^2\pi^2v^2}\ln\left(\frac{(2m_1+m_\chi)^2}{\mu^2}\right)\\&+\frac{\hbar^2m_3^2((m_1-m_2)^2-m_\chi^2)^2}{16(m_1+m_2)^2\pi^2v^2}\ln\left(\frac{(m_1+m_2+m_\chi)^2}{\mu^2}\right)+\frac{\hbar^2m_3^4[-3(m_1-m_2)^2+2m_\chi(m_1+m_2+m_\chi)]}{8(m_1+m_2)^2\pi^2v^2}.\end{aligned} \tag{48}$$

The term proportional to $e\gamma_0$ is given by

$$\begin{aligned}V_{e\gamma_0}=&-\frac{5\hbar^2\mu^{(n-3)}}{8\pi^2v^2}\left(\frac{1}{3-n}-\gamma+1+\ln(4\pi)\right)(m_1-m_2)^2m_3^2\\&-\frac{\hbar^2m_3^2(m_1-m_2)^2[49m_1^2+49m_2^2-42(m_1+m_2)m_\chi+26m_3^2-12m_\chi^2]}{48(m_1+m_2)^2\pi^2v^2}-\frac{\hbar^2(4m_1^2-m_\chi^2)^2(m_2-m_1)m_2}{16(m_1+m_2)^2\pi^2v^2}\ln\left(\frac{(2m_1+m_\chi)^2}{\mu^2}\right)\\&+\frac{\hbar^2(4m_2^2-m_\chi^2)^2(m_2-m_1)m_1}{16(m_1+m_2)^2\pi^2v^2}\ln\left(\frac{(2m_2+m_\chi)^2}{\mu^2}\right)+\frac{\hbar^2(m_1-m_2)^2[(m_1-m_2)^2-m_\chi^2]^2}{16(m_1+m_2)^2\pi^2v^2}\ln\left(\frac{(m_1+m_2+m_\chi)^2}{\mu^2}\right)\\&+\frac{\hbar^2(m_1-m_2)(m_2^2-m_\chi^2)^2}{16\pi^2(m_1+m_2)v^2}\ln\left(\frac{(m_2+m_\chi)^2}{\mu^2}\right)-\frac{\hbar^2(m_1-m_2)(m_1^2-m_\chi^2)^2}{16\pi^2(m_1+m_2)v^2}\ln\left(\frac{(m_1+m_\chi)^2}{\mu^2}\right).\end{aligned} \tag{49}$$

The term proportional to γ_0^2 can be calculated in a similar way:

$$\begin{aligned}V_{\gamma_0^2}=&-\frac{\hbar^2(m_1-m_2)^2}{64\pi^2v^2}\Bigg[\mu^{(n-3)}\left(\frac{1}{3-n}-\gamma+1+\ln(4\pi)\right)[25(m_1^2+m_2^2)-35m_3^2-6m_\chi^2]-\frac{2(4m_1^2-m_\chi^2)^2}{(m_1+m_2)^2}\ln\left(\frac{(2m_1+m_\chi)^2}{\mu^2}\right)\\&-\frac{2(4m_2^2-m_\chi^2)^2}{(m_1+m_2)^2}\ln\left(\frac{(2m_2+m_\chi)^2}{\mu^2}\right)+\frac{(m_1^2-m_\chi^2)^2m_1}{(m_1+m_2)m_3^2}\ln\left(\frac{(m_1+m_\chi)^2}{\mu^2}\right)+\frac{(m_2^2-m_\chi^2)^2m_2}{(m_1+m_2)m_3^2}\ln\left(\frac{(m_2+m_\chi)^2}{\mu^2}\right)\\&-\frac{(m_1-m_2)^2[(m_1-m_2)^2-m_\chi^2]^2}{(m_1+m_2)^2m_1m_2}\ln\left(\frac{(m_1+m_2+m_\chi)^2}{\mu^2}\right)+\frac{1}{2310(m_1+m_2)^2}[89981(m_1^4+m_2^4)-110880m_\chi(m_1^3+m_2^3)\\&-(3837m_3^2+28158m_\chi^2)(m_1^2+m_2^2)+(-46200m_\chi m_3^2+9240m_\chi^3)(m_1+m_2)-19356m_\chi^2m_3^2+34124m_3^4]\Bigg].\end{aligned} \tag{50}$$

The final expression for $V_{\rm eff}^{(5)}$ is obtained by combining Eqs. (48), (49), and (50):

$$V_{\rm eff}^{(5)}=V_{e^2}+V_{e\gamma_0}+V_{\gamma_0^2}. \tag{51}$$

To summarize, in the $\overline{\rm MS}$ scheme, the effective potential to two loops is given by

$$V_{\rm eff}=V^{\rm tree}+V_{\rm eff}^{\rm one\text{-}loop}+V_{\rm eff}^{\rm two\text{-}loop},$$

$$V_{\rm eff}^{\rm two\text{-}loop}\equiv V_{\rm eff}^{(1)}+V_{\rm eff}^{(2)}+V_{\rm eff}^{(3)\rm finite}+V_{\rm eff}^{(4)\rm finite}+V_{\rm eff}^{(5)\rm finite}. \tag{52}$$

Having obtained an expression for the effective potential, the next step is to absorb the divergent terms into renormalized coupling constants by choosing certain renormalization conditions. However, from Eq. (41), it is easy to see that $V_{\rm eff}^{(4)\rm div}$ contains terms proportional to v^8. Since these terms would require counterterms that have no correspondence in the classical Lagrangian, we must require that they vanish to ensure

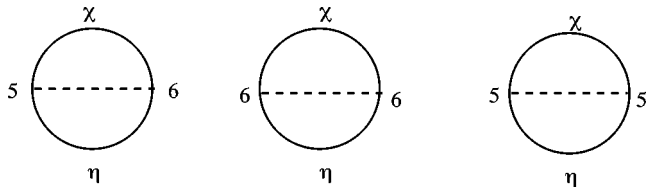

FIG. 20. Theta diagrams from one gauge field propagator and two scalar propagators.

the renormalizability of two-loop effective potential. This can be done by imposing the constraint

$$\gamma_0^2(-90e^2\gamma_0^2\lambda+900e^4\gamma_0^4+\lambda^2)=0. \tag{53}$$

The solutions to this equation are

$$\gamma_0^2=0 \quad \text{or} \quad \gamma_0^2=\frac{(3\pm\sqrt{5})\lambda}{60e^2}. \tag{54}$$

The first, trivial, solution merely verifies that the theory is renormalizable in the limit $\gamma_0\to 0$. For the case of a finite non-minimal Chern-Simons coupling, we must choose one of the second solutions and impose a non-trivial relation between this non-minimal coupling and the other fundamental coupling constants of the theory. Note that our result that the non-minimal coupling depends on the fundamental couplings is consistent with the fact that in a theory without non-minimal Chern-Simons coupling the magnetic moment interaction would be generated through quantum corrections [19]. We should emphasize that Eq. (54) applies to the tree level couplings. Since it is not protected by any symmetry, it will probably be subject to quantum corrections. However, since we cannot go beyond two loops in this model, these corrections, which would affect the effective potential only at three loops, are not relevant.

V. SUMMARY AND CONCLUSIONS

We have studied in some detail the two-loop quantum behavior of $(2+1)$-dimensional scalar QED with a non-minimal Chern-Simons coupling. As expected from dimensional arguments, the complete theory is not renormalizable beyond the one-loop level. This is not a major problem in principle, since the theory is most reasonably considered as an effective field theory describing magnetic moment interactions between the charged scalar and the electromagnetic fields. At this level the renormalizability is useful because it limits the number of parameters that must be introduced at each order in perturbation theory in order to make unambiguous predictions. We find that the effective potential, which is the zeroth order contribution to the quantum effective action in the momentum expansion, can be made renormalizable providing the fundamental couplings are related to the non-minimal one by the condition (54). It is of interest to further examine the effective potential with this condition imposed. The renormalization of the two-loop effective potential using a physical renormalization scheme is a straightforward but lengthy procedure. This, plus an analysis of the resulting symmetry breaking mechanism, will be the subject of a future publication.

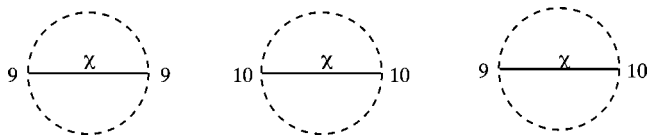

FIG. 21. Theta diagrams from two gauge field propagators and one scalar propagator.

Finally, we remark that the two-loop calculations presented above are extremely lengthy, in part due to the complicated form of the dimensionally regulated gauge field propagator. It would therefore be of interest to examine an alternative regularization scheme that preserves gauge invariance without the need to dimensionally continue the antisymmetric tensor. One excellent candidate is operator regularization, which was shown to greatly facilitate two-loop calculations in Chern-Simons theory [20].

ACKNOWLEDGMENTS

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APPENDIX: TWO-LOOP INTEGRATION FORMULAS

This appendix is a collection of the integration formulas used in the calculation of two-loop effective potential. Denoting $dk=d^nk/(2\pi)^n$ and using the following notation for the Euclidean space propagators,

$$\mathcal{S}_1(q)=\frac{1}{q^2+m_1^2}, \quad \mathcal{S}_2(k)=\frac{1}{k^2+m_2^2},$$

$$\mathcal{S}_3(k+q)=\frac{1}{(k+q)^2+m_3^2},$$

$$S^3\equiv\mathcal{S}_1(q)\mathcal{S}_2(k)\mathcal{S}_3(k+q),$$

we introduce the following integrals:

$$I=\int dkdqS^3,$$

$$N_1=\int dk\mathcal{S}(k), \quad N_2=\int dkk^2\mathcal{S}(k), \quad N_3=\int dkk^4\mathcal{S}(k),$$

$$K^{\mu\nu\lambda\tau}=\int dkdqq^\mu q^\nu k^\lambda k^\tau S^3,$$

$$T^{\mu\nu\lambda\tau}=\int dkdqq^\mu q^\nu q^\lambda k^\tau S^3,$$

$$L^{\mu\nu\alpha\beta\lambda\tau}=\int dkdqq^\mu q^\nu q^\alpha q^\beta k^\lambda k^\tau S^3,$$

$$R^{\mu\nu\alpha\beta\lambda\tau}=\int dkdqq^{\mu}q^{\nu}q^{\alpha}k^{\beta}k^{\lambda}k^{\tau}S^3.$$

Contracting the tensor indices in various combinations with the n- and three-dimensional metric tensors, we get the integrations needed for two-loop calculations:

$$K_1=\int dkdqq^2k^2S^3,\quad K_2=\int dkdq(k\cdot q)^2S^3,$$

$$K_1'=\int dkdq\tilde{q}^2k^2S^3,$$

$$\tilde{K}_1=\int dkdq\tilde{q}^2\tilde{k}^2S^3,\quad \tilde{K}_2=\int dkdq(\tilde{k}\cdot\tilde{q})^2S^3,$$

$$T_1=\int dkdqq^2(k\cdot q)S^3,\quad \tilde{T}_1=\int dkdq\tilde{q}^2(\tilde{k}\cdot\tilde{q})S^3,$$

$$T_2=\int dkdqq^2(\tilde{k}\cdot\tilde{q})S^3,$$

$$L_1=\int dkdqq^4k^2S^3,\quad \tilde{L}_1=\int dkdq\tilde{q}^4\tilde{k}^2S^3,$$

$$L_2=\int dkdqq^2(k\cdot q)^2S^3,$$

$$\tilde{L}_2=\int dkdq\tilde{q}^2(\tilde{k}\cdot\tilde{q})^2S^3,\quad L_3=\int dkdqq^2\tilde{q}^2\tilde{k}^2S^3,$$

$$L_4=\int dkdq\tilde{q}^4k^2S^3,$$

$$L_5=\int dkdqq^2\tilde{q}^2k^2S^3,\quad L_6=\int dkdqq^2(\tilde{k}\cdot\tilde{q})^2S^3,$$

$$R_1=\int dkdqq^2k^2(q\cdot k)S^3,\quad R_2=\int dkdq(q\cdot k)^3S^3,$$

$$R_3=\int dkdq\tilde{q}^2\tilde{k}^2(\tilde{k}\cdot\tilde{q})S^3,$$

$$R_4=\int dkdqq^2\tilde{k}^2(\tilde{k}\cdot\tilde{q})S^3,\quad R_5=\int dkdqq^2k^2(\tilde{k}\cdot\tilde{q})S^3.$$

The results are listed below:

$$I=\frac{\mu^{2(n-3)}}{32\pi^2}\left[\frac{1}{3-n}-\gamma+1-\ln\frac{(m_1+m_2+m_3)^2}{4\pi\mu^2}\right],$$

$$N_1=-\frac{\mu^{2(n-3)}}{4\pi}m,\quad N_2=\frac{\mu^{2(n-3)}}{4\pi}m^3,$$

$$N_3=-\frac{\mu^{2(n-3)}}{4\pi}m^5,$$

$$K_1=-\frac{\mu^{2(n-3)}}{16\pi^2}(m_1^3+m_2^3)m_3+m_1^2m_2^2I,$$

$$K_2=\frac{\mu^{2(n-3)}}{64\pi^2}[m_3^2(m_1m_3+m_2m_3-m_1m_2)-m_3(3m_1^3+3m_2^3+m_1^2m_2+m_2^2m_1)+(m_1^2+m_2^2)m_1m_2]+\frac{1}{4}(m_1^2+m_2^2-m_3^2)^2I,$$

$$K_1'=\frac{3}{n}K_1,$$

$$\tilde{K}_1=\frac{1}{n(n-1)(n+2)}[(9n+3)K_1+(6n-18)K_2],$$

$$\tilde{K}_2=\frac{1}{n(n-1)(n+2)}[3(n-3)K_1+6(2n-1)K_2],$$

$$T_1=\frac{\mu^{2(n-3)}}{32\pi^2}[m_1^3(m_3-m_2)+2m_2^3m_3+m_1^2m_2m_3]-\frac{1}{2}m_1^2(m_1^2+m_2^2-m_3^2)I,$$

$$\tilde{T}_1=\frac{15}{n(n+2)}T_1,\quad T_2=\frac{3}{n}T_1,$$

$$L_1=\frac{1}{16\pi^2}\mu^{2(n-3)}[m_1^5m_3+m_2^3m_3(m_1^2+m_2^2+m_3^2)]-m_1^4m_2^2I,$$

$$L_2=\frac{1}{16\pi^2}\mu^{2(n-3)}\left\{m_2m_3\left[\frac{1}{4}m_1^4+m_2^4+\frac{3}{4}m_1^2m_2^2-\frac{1}{4}m_1^2m_3^2+\frac{1}{3}m_2^2m_3^2\right]-\frac{1}{4}m_1^3m_2[m_1^2+m_2^2-m_3^2]+m_1m_3\left[\frac{1}{2}m_1^4+\frac{1}{4}m_1^2(m_1^2+m_2^2-m_3^2)\right]\right\}-\left[\frac{1}{4}m_1^6+\frac{1}{2}m_1^4(m_2^2-m_3^2)+\frac{1}{4}m_1^2(m_2^2-m_3^2)^2\right]I,$$

$$\tilde{L}_1=\frac{15}{n(n-1)(n+2)(n+8)}[(3n+13)L_1+8(n-3)L_2],$$

$$\tilde{L}_2=\frac{15}{n(n-1)(n+2)(n+8)}[(n-3)L_1+2(5n-4)L_2],$$

$$L_3=\frac{3}{n(n-1)(n+2)(n+8)}[(3n+2)(n+7)L_1+4(n+4)\times(n-3)L_2],$$

$$L_4=\frac{15}{n(n+2)}L_1, \quad L_5=\frac{3}{n}L_1,$$

$$L_6=\frac{3}{n(n-1)(n+2)(n+8)}[(n+6)(n-3)L_1 + 2(3n^2+12n-8)L_2],$$

$$R_1=\frac{\mu^{2(n-3)}}{32\pi^2}[m_1^3m_2^3-m_3^3(m_1^3+m_2^3)-m_3(m_1^5+m_2^5)] + \frac{1}{2}(m_1^2+m_2^2-m_3^2)K_1,$$

$$R_2=\frac{\mu^{2(n-3)}}{32\pi^2}\left[\frac{1}{3}[m_1^3m_2^3-m_3^3(m_1^3+m_2^3)]-m_3(m_1^5+m_2^5)\right] + \frac{1}{2}(m_1^2+m_2^2-m_3^2)K_2,$$

$$R_3=\frac{15}{n(n-1)(n+2)(n+4)}[R_1(5n-1)+2R_2(n-3)],$$

$$R_4=\frac{15}{n(n+2)}R_1,$$

$$R_5=\frac{3}{n}R_1.$$

We have employed the following relations to facilitate the two-loop calculation:

$$\tilde{K}_1-\tilde{K}_2=\frac{6}{n(n-1)}(K_1-K_2),$$

$$\tilde{L}_1-\tilde{L}_2=\frac{30}{n(n-1)(n+2)}(L_1-L_2),$$

$$L_3-\tilde{L}_1=\frac{3(n-3)}{n(n-1)(n+2)(n+8)}[(3n+17)L_1 + 4(n-6)L_2],$$

$$L_4-\tilde{L}_1=\frac{3(n-3)}{n(n-1)(n+2)(n+8)}[(n+7)L_1-8L_2],$$

$$L_5-L_4=\frac{3(n-3)}{n(n+2)}L_1,$$

$$L_6-\tilde{L}_2=\frac{3(n-3)}{n(n-1)(n+2)(n+8)}[(n+1)L_1+2(3n -4)L_2],$$

$$R_4-R_3=\frac{15(n-3)}{n(n-1)(n+2)(n+4)}[(n+1)R_1-2R_2],$$

$$R_5-R_4=\frac{3(n-3)}{n(n+2)}R_1.$$

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