Spin-1 massive particles coupled to a Chern-Simons field

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We study spin-one particles interacting through a Chern-Simons field. In the Born approximation, we calculate the two body scattering amplitude considering three possible ways to introduce the interaction: (a) a Proca-like model minimally coupled to a Chern-Simons field, (b) the model obtained from (a) by replacing the Proca's mass by a Chern-Simons term, and (c) a complex Maxwell-Chern-Simons model minimally coupled to a Chern-Simons field. In the low energy regime the results show similarities with the Aharonov-Bohm scattering for spin-1/2 particles. We discuss the one loop renormalization program for the Proca model. In spite of the bad ultraviolet behavior of the matter field propagator, we show that, up to one loop, the model is power counting renormalizable thanks to the Ward identities satisfied by the interaction vertices.
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I. INTRODUCTION

In recent years much work has been devoted to the study of the properties of the Chern-Simons (CS) field [1]. This was motivated not only by its potential applications to condensed matter physics but also because these studies have unveiled some new and interesting aspects of the dynamics of relativistic quantum physics. In particular, it has been noticed that in some circumstances the CS field plays a stabilizing role providing theories with improved ultraviolet behavior [2]. However, most of these investigations have been restricted to the cases of spinless and spin-1/2 particles. The reasons behind this fact are the notorious difficulties found in the conventional treatment for higher spin fields in four dimensions. The troublesome aspects include noncausal propagation and lack of renormalizability. It is certainly worthwhile to study the interaction of a CS and spin one matter field so that the origin of the difficulties could be better understood and perhaps new and safer routes could be found. With this in mind, we would like to present here the results of some investigations concerning the dynamics of spin one fields interacting through a CS term.

As a first observation, we note that a free, spin-one particle of mass M can be described alternatively by the Proca Lagrangian

$$\mathcal{L}_P=-\frac{1}{2}F^{\dagger}_{\mu\nu}F^{\mu\nu}-M^2\phi^{\dagger}_{\mu}\phi_{\mu}, \tag{1}$$

where $F_{\mu\nu}=\partial_\mu\phi_\nu-\partial_\nu\phi_\mu$, or by the Maxwell-Chern-Simons (MCS) Lagrangian,

$$\mathcal{L}_{\rm MCS}=-\frac{1}{2}F^{\dagger}_{\mu\nu}F^{\mu\nu}+\frac{M}{2}\epsilon^{\mu\nu\rho}[\phi^{\dagger}_{\mu}\partial_\nu\phi_\rho+\phi_\mu(\partial_\nu\phi_\rho)^{\dagger}]. \tag{2}$$

Whereas the first formulation encompasses both modes of spin ± 1, the latter, Eq. (2), represents only a single mode of spin $M/|M|$. The two formulations are not entirely inequivalent, however. The model (2) is equivalent to the self-dual model, [3,4] and this last model is similar to a square root of Proca's [5]. Nevertheless, such equivalence does not in general persist whenever the models are coupled to other dynamical fields [6]. In this investigation we will study the two body scattering amplitude for the cases of minimal coupling of Eq. (1) to a CS field and when in Eq. (1) the Proca mass $M^2\phi^{\dagger}_{\mu}\phi^{\mu}$ is replaced by a complex CS term. We will also consider the case of minimal coupling of Eq. (2) to a CS field. Analogously to the scattering of lower spin particles we will expect to find similarities with the Aharonov-Bohm (AB) scattering [7]. We may recall that for spinless particles the Born approximation found in the perturbative method only agrees with the expansion of the exact result if a contact interaction, simulated in the field theory approach by a quartic $(\phi^*\phi)^2$ interaction, is included from the beginning [8,9]. It is also known that in the spin-1/2 case no new interaction is needed, the role of the quartic interaction being played by the magnetic Pauli term [10].

We will pursue the investigation of the spin effects on the perturbative AB scattering by considering spin one particles. Previous work in this direction started either with a complex Proca field minimally coupled to the electromagnetic field [11] or with a linearized Yang Mills equation [12]. In both approaches the AB scattering was discussed from a first quantized viewpoint. Here we consider the problem from the perspective of the theory of quantum fields, i.e., as the low energy limit of a fully quantized relativistic theory of spin one particles interacting through a CS field. One advantage of such procedure is that it incorporates some purely quantum field effects, as vacuum polarization and anomalous magnetic momentum.

For the Proca model minimally coupled to a CS field we will discuss the one loop renormalization program and calculate the anomalous magnetic moment of the matter field. As we will show, thanks to the Ward identities satisfied by the basic interaction vertices, up to one loop the model is renormalizable, in spite of the bad behavior of the propagator of the matter field. The MCS model, on the other hand, turns out to be power counting renormalizable to all orders of perturbation.

Our work is organized as follows. In Sec. II we present the polarization vectors and Feynman rules for the models mentioned above. There, we also study the Born approximation for the two body scattering amplitudes. In Sec. III we discuss in detail the one loop renormalization parts for the Proca model and also determine the anomalous magnetic moment. A discussion of our results is presented in Sec. IV. The paper contains also an appendix with details of the calculations.

II. POLARIZATION VECTORS AND FEYNMAN RULES

As a preliminary step toward our study of the AB scattering of two spin-one particles, let us examine some of the kinematic aspects of the asymptotic theories. First of all, being a transversal field, $\partial_\mu \phi^\mu = 0$, the Proca field described by Eq. (1) can be expanded in plane waves as

$$\phi^\mu = \frac{1}{2\pi}\int \frac{d^2p}{2w_p}\sum_{\lambda=1}^{2}\epsilon_\lambda^\mu [a_\lambda e^{-ipx} + b_\lambda^\dagger e^{ipx}], \tag{3}$$

where $w_p = \sqrt{\vec{p}^2 + M^2}$ and the polarization vectors satisfy the transversality condition $p_\mu \epsilon_\lambda^\mu = 0$. A convenient choice is

$$\epsilon_1^\mu = \left(0, \epsilon^{ij}\frac{p_j}{|\vec{p}|}\right), \qquad \epsilon_2^\mu = \left(\frac{|\vec{p}|}{M}, \frac{w_p}{M}\frac{p^i}{|\vec{p}|}\right). \tag{4}$$

The creation operators $a^\dagger$ and $b^\dagger$ allow us to construct the Fock space of the asymptotic states. In this space, we found that the spin part of the angular momentum operator

$$J = \int d^2x\, \varepsilon_{ij} x^i :T^{0j}:, \tag{5}$$

where $T^{\mu\nu} = F_\rho^{\dagger\mu} F^{\rho\nu} + F_\rho^\mu F^{\dagger\rho\nu} - g^{\mu\nu}\mathcal{L}_P$ is given by

$$J_S = -i\int \frac{d^2p}{2w_p}\sum_{\lambda,\lambda'=1}^{2}\epsilon_{ij}\epsilon_{\lambda'}^i(p)\epsilon_\lambda^j(p) \times [a_{\lambda'}^\dagger(p)a_\lambda(p) + b_{\lambda'}^\dagger(p)b_\lambda(p)]. \tag{6}$$

In the particle's rest frame we can see that

$$|p=0, s=\pm 1\rangle = \frac{a_1^\dagger(0) \pm i a_2^\dagger(0)}{\sqrt{2}}|0\rangle \tag{7}$$

are eigenstates of J_S.

As discussed in Ref. [13], in the case of the MCS model, Eq. (2) one has just one polarization which can be taken as

$$\varepsilon_\alpha(k) = (\varepsilon_0(k), \varepsilon_i(k)), \tag{8}$$

where $\varepsilon_0(k) = \vec{k}\cdot\vec{\varepsilon}(0)/|M|$ and $\varepsilon_i(k) = \varepsilon_i(0) + [\vec{k}\cdot\vec{\varepsilon}(0)/|M|(w_p + |M|)]k_i$ with $\varepsilon^\mu(0) = 1/\sqrt{2}[0,1,i(M/|M|)]$ being the polarization vector in the particle's rest frame.

Minimally coupling the Proca field to a CS field, A^μ, leads us to the Lagrangian

$$\mathcal{L}_P = -\frac{1}{2}G_{\mu\nu}^\dagger G^{\mu\nu} - M^2\phi_\mu^\dagger\phi^\mu + \frac{\theta}{2}\varepsilon_{\mu\nu\rho}A^\mu\partial^\nu A^\rho + \frac{\lambda}{2}(\partial_\mu A^\mu)^2, \tag{9}$$

where $G^{\mu\nu} = D^\mu\phi^\nu - D^\nu\phi^\mu$ and $D^\mu = \partial^\mu - ieA^\mu$. The Feynman rules associated with the above Lagrangian are depicted in Fig. 1. In the Landau gauge ($\lambda \to \infty$), the analytic expressions accompanying these rules are the CS field propagator

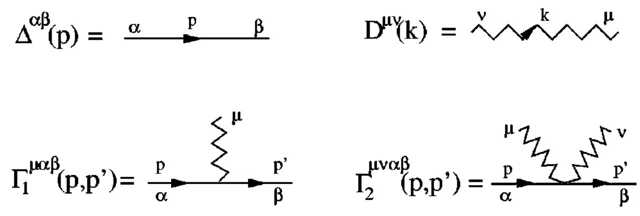

FIG. 1. Feynman rules for the Proca model minimally coupled to a CS field.

$$D_{\mu\nu}(k) = -\frac{1}{\theta}\epsilon_{\mu\nu\rho}\frac{k^\rho}{k^2 + i\epsilon}, \tag{10}$$

matter field propagator

$$\Delta^{\alpha\beta}(p) = \frac{-i}{p^2 - M^2 + i\epsilon}\left[g^{\alpha\beta} - \frac{p^\alpha p^\beta}{M^2}\right], \tag{11}$$

and interaction vertices (p and p' denote the matter field's momenta)

$$\Gamma_1^{\mu\alpha\beta}(p,p') = -ie[(p+p')^\mu g^{\alpha\beta} - p^\beta g^{\mu\alpha} - p'^\alpha g^{\mu\beta}] \tag{12}$$

$$\Gamma_2^{\mu\nu\alpha\beta} = ie^2[g^{\mu\beta}g^{\nu\alpha} + g^{\mu\alpha}g^{\nu\beta} - 2g^{\mu\nu}g^{\alpha\beta}]. \tag{13}$$

The above propagators and vertices obey the identities

$$e\frac{d\Delta_{\alpha\beta}(p)}{dp_\mu} = \Delta_{\alpha\rho}(p)\Gamma_1^{\mu\rho\sigma}(p,p)\Delta_{\sigma\beta}(p), \tag{14}$$

$$e\frac{d}{dp_\nu}\Gamma_1^{\mu\alpha\beta}(p,p-q) = \Gamma_2^{\mu\nu\alpha\beta}, \tag{15}$$

and

$$p'_\beta\Gamma_1^{\mu\alpha\beta}(0,p') = p_\alpha\Gamma_1^{\mu\alpha\beta}(p,0) = 0. \tag{16}$$

The expressions (14) and (15) are typical of gauge theories being similar to the ones found in scalar QED. These properties will be helpful to discuss the ultraviolet behavior of the Green functions.

To make contact with the Aharonov-Bohm scattering, let us study the low energy approximation for the scattering of two vector particles. We assume that in the center of mass frame the incoming particles have momenta $p_1=(w_p,\vec{p})$ and $p_2=(w_p,-\vec{p})$ and spins s_1 and s_2, respectively. We will then denote the momenta and spins of the outgoing particles by $p_3=(w_p,\vec{p}\,')$, $p_4=(w_p,-\vec{p}\,')$ and s_3, s_4. The energy of the incoming particle is $w_p=\sqrt{m^2+\vec{p}^2}$, the spins s_i can either take the values ± 1 and $|\vec{p}|=|\vec{p}\,'|$. The tree approximation for this process is

$$\begin{aligned} M_{fi}=&[\varepsilon^*_\beta(p_3,s_3)\Gamma_1^{\mu\alpha\beta}(p_1,p_3)\varepsilon_\alpha(p_1,s_1)]\\ &\times D_{\mu\nu}(q)[\varepsilon^*_\rho(p_4,s_4)\Gamma_1^{\nu\sigma\rho}(p_2,p_4)\varepsilon_\sigma(p_2,s_2)]\\ &+(p_3\leftrightarrow p_4,s_3\leftrightarrow s_4), \end{aligned} \tag{17}$$

where $q=p_1-p_3$ and

$$\varepsilon^\alpha(p,s)=\frac{\epsilon_1^\alpha(p)+is\,\epsilon_2^\alpha(p)}{\sqrt{2}}, \tag{18}$$

with $\epsilon_1^\alpha(p)$ and $\epsilon_2^\alpha(p)$ as in Eq. (4), are circularly polarized vectors. From the above expressions we can verify that the scattering amplitude vanishes unless spin is conserved, i.e., $s_1=s_3$ and $s_2=s_4$ or $s_1=s_4$ and $s_2=s_3$.

After expanding in powers of $|\vec{p}|/M$, we get, in leading order,

$$M_{fi}(s,\vec{p},\vec{p}\,')=\frac{4ie^2|M|}{\theta}e^{-is\alpha/2}[s+2i\cot(\alpha)], \tag{19}$$

where α is the scattering angle and $s=s_1+s_2$ is the total spin of the incoming particles. Similarly to the spin-1/2 case, the origin of the constant term in Eq. (19) is a Pauli interaction between each vector particle and the magnetic field produced by the other. In the antiparallel case these effects cancel each other.

Let us now consider a model in which the mass of the vector particles has a topological origin. In such situation, one should use the polarization vector given in Eq. (8). One can then envisage two possibilities to introduce the coupling to the CS field. One could use (9) but with the Proca mass replaced by a topological one, i.e.,

$$M^2\phi^\dagger_\mu\phi^\mu\rightarrow\frac{M}{2}\varepsilon^{\mu\nu\rho}\phi^\dagger_\mu\partial_\nu\phi_\rho+\frac{M}{2}\varepsilon^{\mu\nu\rho}\phi_\mu(\partial_\nu\phi_\rho)^\dagger \tag{20}$$

or just consider the MCS field described by Eq. (2) minimally coupled to a CS field. Had we employed a topological mass instead of the Proca's, the Feynman rules would be the same unless for the propagator for the matter field which would become

$$\Delta^{\alpha\beta}_{\rm MCS}=\frac{-i}{p^2-M^2+i\epsilon}\left[g^{\alpha\beta}-\frac{p^\alpha p^\beta}{p^2}+iM\varepsilon^{\alpha\beta\rho}\frac{p_\rho}{p^2}\right]. \tag{21}$$

As this propagator has a better ultraviolet behavior than Eq. (11), the corresponding theory will be in principle renormalizable.

Using this propagator we found a scattering amplitude

$$\frac{4ie^2|M|}{\theta}[s+2i\cot(\alpha)], \tag{22}$$

which differs from Eq. (19) just by a phase factor. In the case of minimal coupling we get a result which contains an additional numerical factor 1/4 in the front of Eq. (22). The two possible couplings give different cross sections and, at the present, there is no way to select a preferred one. Besides that, radiative correction should produce diverse cross sections even in the cases associated to Eqs. (19) and (22) where the corresponding Lagrangians differ only by the mass terms. This is apparent from an inspection of the asymptotic behavior of the matter field vector propagators. In the case of the Lagrangian with a Proca mass, the longitudinal term in the propagator spoils renormalizability. However, a more careful analysis, to be done in the next section, shows that the degree of superficial divergence is actually lowered. Taking into consideration this fact we conclude that the effective degree of divergence for a generic one loop graph γ is

$$d(\gamma)=3-N_A-\frac{1}{2}N_\phi, \tag{23}$$

where N_A and N_ϕ are the number of external lines belonging to the CS and to the matter vector field. In spite of the improved behavior, as established by Eq. (23), the model still suffers from renormalization problems due to the divergence of graphs with N_ϕ equal to four and six. If the corresponding counterterms are added to Eq. (9) then the relations (14)–(16) will not be able to guarantee renormalizability even at one loop. Actually, higher order loops will contain nonrenormalizable divergences. The model is renormalizable only up to one loop. Of course these comments do not apply if the mass has a topological origin.

III. ONE LOOP RENORMALIZATION

As we will show now, the one loop contributions to the amplitudes for the theory defined by Eq. (9) have an effective degree of divergence as given in Eq. (23). By power counting, any one loop graph with $n_{\rm CS}$ and n_ϕ internal CS and matter field lines, and containing V_1 and V_2 trilinear and quadrilinear vertices has the degree of superficial divergence given by

$$d(\gamma)=3-n_{\rm CS}+V_1=3-\frac{N_\phi}{2}+V_1, \tag{24}$$

where we used

$$n_{\rm CS}+n_\phi=V_1+V_2, \tag{25}$$

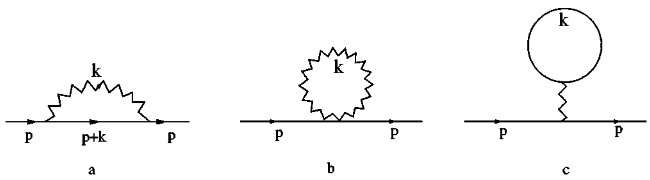

FIG. 2. Graphs contributing to the one loop correction to the vector meson propagator.

$$2n_{\phi}+N_{\phi}=2V_1+2V_2, \tag{26}$$

$$2n_{\rm CS}+N_{\rm CS}=V_1+2V_2. \tag{27}$$

Formula (23) follows now from the following observations.

(1) Equations (14) and (15) imply that the sum of graphs with $N_{\rm CS}$ external CS lines will contain the $N_{\rm CS}^{th}$ power of the external momenta and therefore in such sum the degree of divergence is effectively reduced by $N_{\rm CS}$.

(2) In addition to that, due to Eq. (16) the contraction of the longitudinal part of the matter field propagator with the vertex Γ_1 produces a result whose degree in the loop momentum is reduced by V_1.

After establishing Eq. (23) we shall now examine each case of divergence, i.e., with $d(\gamma)\geqslant 0$, as specified by Eq. (23). Notice that logarithmically divergent parts are odd in the integration variables and therefore vanish under symmetric integration. This implies that, up to one loop, the six point vertex function of the matter field and the four point function with external lines associated to two CS field and two matter fields do not generate counterterms. However, the radiative corrections to the two and to the four point vertex function of the matter field, the vacuum polarization and the trilinear vertex all have $d(\gamma)>0$. We shall examine separately each one of them. We will employ dimensional regularization which, for the cases under consideration, already acts as a renormalization. Let us begin by the self-energy contributions which in one loop correspond to the graphs shown in Fig. 2. Symmetric integration shows immediately that the contributions of the graphs 2(b) and 2(c) vanish. The amplitude associated to 2(a) is (details of this calculation are presented in the Appendix)

$$\Sigma^{\alpha\beta}(p)=\int\frac{d^3k}{(2\pi)^3}\Gamma_1^{\nu\alpha\alpha'}(p,p+k)\Delta_{\alpha'\beta'}(p+k)$$
$$\times\Gamma_1^{\mu\beta'\beta}(p+k,p)D_{\mu\nu}(k). \tag{28}$$

Although the degree of divergence indicates a quartic divergence, because of Eq. (16), the above expression diverges only quadratically. We obtain

$$\Sigma^{\alpha\beta}(p)=\frac{e^2}{8\pi\theta}F(p)\varepsilon^{\alpha\beta\rho}p_\rho, \tag{29}$$

where (from now on we assume M to be positive)

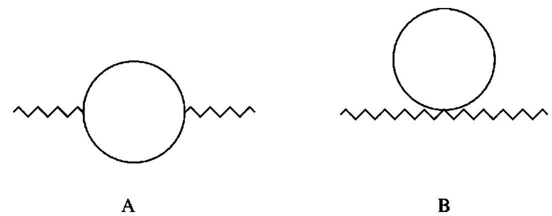

FIG. 3. One loop vacuum polarization graphs associated with the CS field.

$$F(p)=\frac{(3M^2+p^2)}{2Mp^2}\left[(M^2+p^2)-\frac{1}{2M\sqrt{p^2}}(M^2-p^2)^2\right.$$
$$\left.\times\log\left(\frac{M+\sqrt{p^2}}{M-\sqrt{p^2}}\right)\right]. \tag{30}$$

Although finite the above expression does not vanish at $p^2=M^2$. To secure that the physical mass is M one still has to proceed with a finite renormalization. Thus we define a renormalized amplitude by

$$\Sigma_R^{\alpha\beta}(p)=\Sigma^{\alpha\beta}(p)-\delta M\varepsilon^{\alpha\beta\rho}p_\rho$$
$$=\frac{e^2}{8\pi\theta}F_R(p)\varepsilon^{\alpha\beta\rho}p_\rho, \tag{31}$$

where $\delta M=e^2M/2\pi\theta$ and

$$F_R(p)=\frac{(M^2-p^2)}{2Mp^2}\left[3M^2-p^2-\frac{(M^2-p^2)(3M^2+p^2)}{2M\sqrt{p^2}}\right.$$
$$\left.\times\log\left(\frac{M+\sqrt{p^2}}{M-\sqrt{p^2}}\right)\right]. \tag{32}$$

In this situation we obtain the renormalized propagator

$$\Delta_R^{\alpha\beta}=\frac{-i}{p^2-M^2-\Sigma'(p^2)}\left[g^{\alpha\beta}-\left(1-\frac{\Sigma'(p^2)}{p^2}\right)\frac{p^\alpha p^\beta}{M^2}\right.$$
$$\left.-i\Theta(p)\varepsilon^{\alpha\beta\rho}p_\rho\right], \tag{33}$$

where

$$\Sigma'(p^2)=\frac{e^2}{8\pi\theta}\frac{p^2F_R^2(p)}{(p^2-M^2)}$$

$$\text{and}\quad \Theta(p)=\frac{e^2}{8\pi\theta}\frac{F_R(p)}{(p^2-M^2)}. \tag{34}$$

One sees that the needed counterterm corresponds to a CS term for the matter field.

Let us now focus our attention on the vacuum polarization contributions to the CS field two point function. The associated graphs have been drawn in Fig. 3. We have

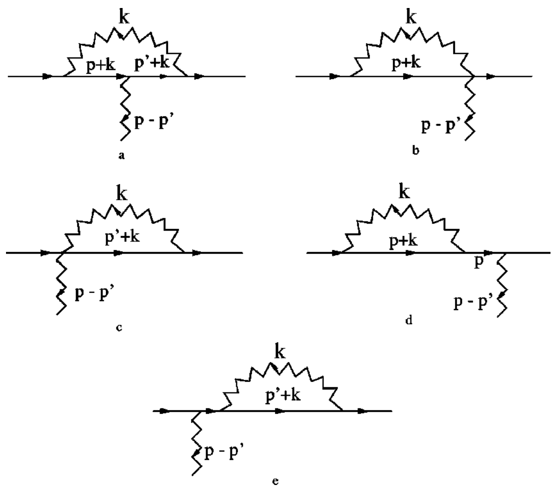

FIG. 4. Graphs contributing to the vector meson anomalous magnetic moment.

$$\Pi^{\mu\nu}=\Pi_A^{\mu\nu}+\Pi_B^{\mu\nu}, \tag{35}$$

with

$$\Pi_A^{\mu\nu}=\int\frac{d^3k}{(2\pi)^3}\Gamma_1^{\mu\beta\alpha}(p+k,k)\Delta_{\alpha\alpha'}(k)\Gamma_1^{\nu\alpha'\beta'}\times(k,p+k)\Delta_{\beta'\beta}(p+k),$$

$$\Pi_B^{\mu\nu}=\int\frac{d^3k}{(2\pi)^3}\Gamma_2^{\mu\nu\alpha\beta}\Delta_{\alpha\beta}(k). \tag{36}$$

After some calculations whose details are relegated to the Appendix, we get

$$\Pi^{\mu\nu}=\frac{ie^2}{8\pi}\Pi(p^2)\left(g^{\mu\nu}-\frac{p^\mu p^\nu}{p^2}\right), \tag{37}$$

which is transversal as required by current conservation. In the last expression

$$\Pi(p^2)=\frac{(p^2-4M^2)}{4M^2}[4M-(p^2+4M^2)I_0], \tag{38}$$

with

$$I_0=\int_0^1 dx\,\frac{1}{\sqrt{M^2-p^2x(1-x)}}. \tag{39}$$

For low momentum $\Pi^{\mu\nu}$ approaches the expression

$$\Pi^{\mu\nu}=\frac{ie^2}{6\pi M}(g^{\mu\nu}p^2-p^\mu p^\nu), \tag{40}$$

implying that the effective low momentum Lagrangian contains a Maxwell term, analogously to what happens in the spin-1/2 case.

The lowest order contributions to the trilinear vertex come from the first three graphs shown in Fig. 4. The corresponding analytic expressions are, respectively,

$$\Gamma_a^{\mu\alpha\beta}(p,p')=\int\frac{d^3k}{(2\pi)^3}[\Gamma_1^{\rho\alpha\alpha'}(p,p+k)\Delta_{\alpha'\rho'}\times(p+k)\Gamma_1^{\mu\rho'\sigma'}(p+k,p'+k)\Delta_{\sigma'\beta'}\times(p'+k)\Gamma_1^{\sigma\beta'\beta}(p'+k,p')D_{\sigma\rho}(k)], \tag{41}$$

$$\Gamma_b^{\mu\alpha\beta}(p,p')=\int\frac{d^3k}{(2\pi)^3}[\Gamma_1^{\rho\alpha\alpha'}(p,p+k)\Delta_{\alpha'\beta'}\times(p+k)\Gamma_2^{\mu\sigma\beta'\beta}D_{\sigma\rho}(k)], \tag{42}$$

$$\Gamma_c^{\mu\alpha\beta}(p,p')=\int\frac{d^3k}{(2\pi)^3}[\Gamma_2^{\mu\rho\alpha\alpha'}\Delta_{\alpha'\beta'}(p'+k)\Gamma_1^{\sigma\beta'\beta}\times(p'+k,p')D_{\sigma\rho}(k)]. \tag{43}$$

These expressions are in general very complicated but in the low momentum regime a great simplification occurs. Indeed, performing the calculations on the matter field's mass shell we get

$$\begin{aligned}\Gamma_a^{\mu\alpha\beta}(p,p')=\frac{-e^3M}{8\pi\theta}\Bigg[&\frac{5}{4M^2}g^{\beta\mu}\varepsilon^{\alpha\sigma\rho}p_\sigma p'_\rho+\frac{5}{4M^2}g^{\alpha\mu}\varepsilon^{\beta\sigma\rho}p_\sigma p'_\rho-\frac{14}{4M^2}g^{\alpha\beta}\varepsilon^{\mu\sigma\rho}p_\sigma p'_\rho\\&+p^\beta\left(-\frac{17}{24M^2}\varepsilon^{\alpha\mu\rho}p_\rho-\frac{15}{8M^2}\varepsilon^{\alpha\mu\rho}p'_\rho\right)\\&+p'^\alpha\left(\frac{15}{8M^2}\varepsilon^{\beta\mu\rho}p_\rho+\frac{17}{24M^2}\varepsilon^{\beta\mu\rho}p'_\rho\right)\\&+p^\mu\left(\frac{2}{3M^2}\varepsilon^{\alpha\beta\rho}p_\rho+\frac{1}{M^2}\varepsilon^{\alpha\beta\rho}p'_\rho\right)\\&+p'^\mu\left(\frac{1}{M^2}\varepsilon^{\alpha\beta\rho}p_\rho+\frac{2}{3M^2}\varepsilon^{\alpha\beta\rho}p'_\rho\right)\Bigg],\end{aligned} \tag{44}$$

$$\Gamma_b^{\mu\alpha\beta}(p,p')=\frac{-e^3M}{8\pi\theta}\left[-2\varepsilon^{\alpha\beta\mu}+\frac{4}{3M^2}p^\beta\varepsilon^{\alpha\mu\rho}p_\rho-\frac{2}{3M^2}p^\mu\varepsilon^{\alpha\beta\rho}p_\rho\right], \tag{45}$$

$$\Gamma_c^{\mu\alpha\beta}(p,p')=\frac{-e^3M}{8\pi\theta}\left[-2\varepsilon^{\alpha\beta\mu}-\frac{4}{3M^2}p'^\alpha\varepsilon^{\alpha\mu\rho}p'_\rho-\frac{2}{3M^2}p'^\mu\varepsilon^{\alpha\beta\rho}p'_\rho\right]. \tag{46}$$

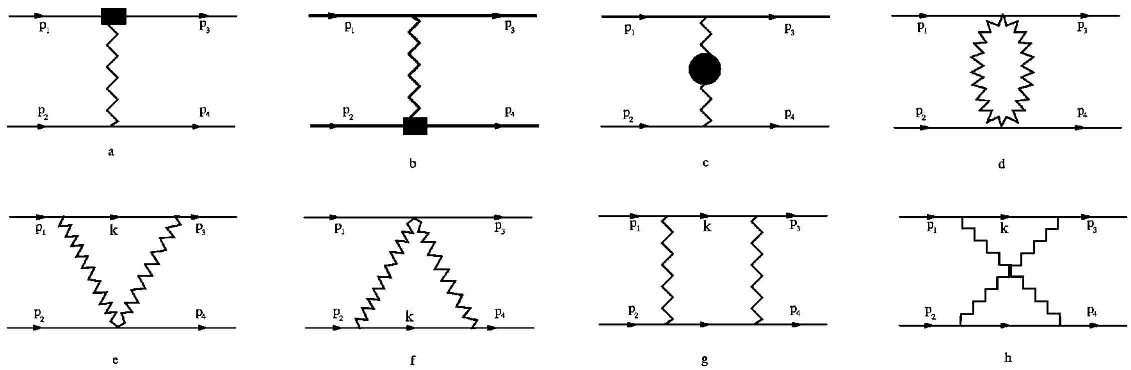

FIG. 5. One loop contributions to the vector meson scattering.

It can be easily verified that these results satisfy current conservation as expressed in the Ward identity

$$e\frac{d\Sigma^{\alpha\beta}(p)}{d(p)^{\mu}}\bigg|_{p^2=M^2}=\Gamma_a^{\mu\alpha\beta}(p,p)+\Gamma_b^{\mu\alpha\beta}(p,p)+\Gamma_c^{\mu\alpha\beta}(p,p). \tag{47}$$

We are now in a position to calculate the vector meson anomalous magnetic moment. Usually this is done by coupling the matter field to an external electromagnetic field. In addition to that, to disentangle the various contributions, here it is also convenient to do the calculations in a particular frame where $\vec{p}+\vec{p}\,'=0$, the Breit frame. It is then found that the magnetic moment is given by [14]

$$\mu=\lim_{q\to 0}\frac{1}{2M}\,\epsilon_{ij}\frac{q^j}{q^2}\Gamma^i(\vec{p},-\vec{p}), \tag{48}$$

where

$$\Gamma^i(\vec{p},-\vec{p})=\varepsilon^*_{\beta}(-\vec{p},s)\Gamma^{i\alpha\beta}(\vec{p},-\vec{p})\varepsilon_{\alpha}(\vec{p},s), \tag{49}$$

and the limit prescription singles out the term proportional to $\vec{q}$. In the tree approximation we get

$$\mu=\frac{e}{2M}gs=\pm\frac{e}{2M}, \tag{50}$$

where $s=\pm 1$ is the spin. The above result means that the gyromagnetic factor is $g=1$. The gyromagnetic factor may be changed (sometimes $g=2$ is desirable [15]) if the magnetic term [16]

$$\mathcal{L}_{\mathrm{mag}}=-ie\gamma(\partial_{\mu}A_{\nu}-\partial_{\nu}A_{\mu})\phi^{*\mu}\phi^{\nu} \tag{51}$$

is added to Eq. (9). In such case, it is easy to check that the magnetic moment for a spin one particle becomes $e(1+\gamma)/(2M)$. Nevertheless, as the new vertex (51) does not obey Eq. (16) the ultraviolet behavior of the Green functions is definitely wrecked. Thus, if one insists in having $g=2$ another approach becomes mandatory. For this reason we shall not anymore consider the possibility of adding Eq. (51). After these remarks let us proceed toward the computation of the anomalous magnetic moment. From the expressions for the three graphs given above we have

$$\Gamma^i_{a-c}(\vec{p},-\vec{p})=\varepsilon^*_{\beta}(-\vec{p},s)\Gamma^{i\alpha\beta}_{a+b+c}\varepsilon_{\alpha}(\vec{p},s)=\frac{3e^3}{8\pi\theta}\epsilon^{ij}q_j\,. \tag{52}$$

We still have to add the contributions of the graphs 4(d) and 4(e) which corresponds to the analytic expressions

$$\Gamma_d^{\mu\alpha\beta}(p,p')=[\Sigma_R^{\alpha\alpha'}(p)\Delta_{\alpha'\beta'}(p)\Gamma_1^{\mu\beta'\beta}(p,p')] \tag{53}$$

and

$$\Gamma_e^{\mu\alpha\beta}(p,p')=[\Gamma_1^{\mu\alpha\alpha'}(p,p')\Delta_{\alpha'\beta'}(p')\Sigma_R^{\beta'\beta}(p')]. \tag{54}$$

However, for small momenta and in the Breit frame,

$$\Gamma_d^{i\alpha\beta}(\vec{p},-\vec{p})=\frac{-e^3M}{8\pi\theta}\bigg[-\frac{1}{M^2}p^{\beta}\varepsilon^{\alpha i\rho}p_{\rho}+\frac{1}{M^2}g^{i\beta}\varepsilon^{\alpha\sigma\rho}p_{\sigma}p'_{\rho}\bigg], \tag{55}$$

$$\Gamma_e^{i\alpha\beta}(\vec{p},-\vec{p})=\frac{-e^3M}{8\pi\theta}\bigg[\frac{1}{M^2}p^{\alpha}\varepsilon^{\beta i\rho}p_{\rho}+\frac{1}{M^2}g^{i\alpha}\varepsilon^{\beta\sigma\rho}p_{\sigma}p'_{\rho}\bigg], \tag{56}$$

so that, saturating with external polarizations, we get

$$\varepsilon^*_{\beta}(-\vec{p},s)[\Gamma_d^{i\alpha\beta}(\vec{p},-\vec{p})+\Gamma_e^{i\alpha\beta}(\vec{p},-\vec{p})]\varepsilon_{\alpha}(\vec{p},s)=-\frac{2e^3}{8\pi\theta}\epsilon^{ij}q_j\,. \tag{57}$$

The sum of Eqs. (52) and (57) gives

$$\Gamma^i(\vec{p},-\vec{p})=\frac{e^3}{8\pi\theta}\varepsilon^{ij}q_j\,. \tag{58}$$

Thus, up to one loop, the magnetic moment for vector particles of spin ± 1 is

$$\mu=\pm\frac{e}{2M}\left[1\pm\frac{e^2}{8\pi\theta}\right]. \tag{59}$$

It should be noticed that the graphs with self-energy corrections contribute significantly to the final result. This is similar to what happens for the case of spin-1/2 particles in the Coulomb gauge and results from the fact that in both cases the interaction with the CS field modifies the free propagators in an essential way. In our case, as can be seen from Eq. (29) a CS term is produced. It is also worthwhile to remark that, as in the spin-1/2 case, the anomalous magnetic moment has the same sign for both spin up and spin down situations [10].

IV. DISCUSSION

The calculation of the radiative corrections to the propagators and vertices done in the previous section allow us to incorporate vacuum polarization and anomalous magnetic moment effects in the two body scattering amplitudes computed earlier. In fact, in the low energy regime we get

$$\mathcal{M}_a^{s_1=1,s_2=1,s_3,s_4}=\frac{ie^4M}{8\pi\theta^2}(\cos\alpha-i\sin\alpha)\left\{\frac{1+s_4}{1-\cos\alpha}[-2+(1-s_3)\cos\alpha+(1+s_3)\cos^2\alpha-i(1+s_3)\sin\alpha\cos\alpha]\right\}, \tag{60}$$

$$\mathcal{M}_a^{s_1=-1,s_2=-1,s_3,s_4}=\frac{ie^4M}{8\pi\theta^2}(\cos\alpha+i\sin\alpha)\left\{\frac{-1+s_4}{1-\cos\alpha}[2-(1+s_3)\cos\alpha-(1-s_3)\cos^2\alpha-i(1-s_3)\sin\alpha\cos\alpha]\right\}, \tag{61}$$

$$\mathcal{M}_a^{s_1=1,s_2=-1,s_3,s_4}=\frac{ie^4M}{8\pi\theta^2}\left\{\frac{-1+s_4}{1-\cos\alpha}[(1+s_3)-2s_3\cos\alpha-(1-s_3)\cos^2\alpha+2i\sin\alpha-i(1-s_3)\sin\alpha\cos\alpha]\right\}, \tag{62}$$

$$\mathcal{M}_a^{s_1=-1,s_2=1,s_3,s_4}=\frac{ie^4M}{8\pi\theta^2}\left\{\frac{1+s_4}{1-\cos\alpha}[(-1+s_3)-2s_3\cos\alpha+(1+s_3)\cos^2\alpha+2i\sin\alpha-i(1+s_3)\sin\alpha\cos\alpha]\right\}, \tag{63}$$

for the contributions from the vertex corrections indicated in Fig. 5(a). The result for Fig. 5(b) is obtained from the above expressions by exchanging s_3 and s_4. For the vacuum polarization diagram, Fig. 5(c), we found the result

$$\mathcal{M}_c^{s_1,s_2,s_3,s_4}=\frac{2ie^4M}{3\pi\theta^2}[(1+s_1s_3)\cos\alpha-i(s_1+s_3)\sin\alpha]\times[(1+s_2s_4)\cos\alpha-i(s_2+s_4)\sin\alpha]. \tag{64}$$

.

The next diagram, Fig. 5(d), does not contribute at leading order of $|\vec{p}|/M$. To complete the one loop calculation it is still necessary to compute the contributions of the triangle [Figs. 5(e) and 5(f)] and box [Figs. 5(g) and 5(h)] diagrams. These graphs present a logarithmic ultraviolet divergence (by Lorentz covariance the would be linear divergence does not appear). In addtion to that they are so intricate that even using dimensional regularization and employing an algebraic computer program the integrals did not turned out to be feasible.

To summarize, in this work we have studied some properties of spin one particles interacting through a CS field. As argued in the introduction, this is a complex system but we found some simplifications which allowed us to study its behavior up to the one loop level. We have indicated another possible scheme as the use of the complex MCS model minimally coupled to a CS term. Although this model is power counting renormalizable, the algebraic structure is very much cumbersome that no practical results are possible even at the one loop level. In such situation the scheme adopted by us seems to be the most useful although not being easily generalizable to higher orders.

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APPENDIX

In this appendix we shall present some details of the calculations of the vector meson self-energy and of the CS polarization tensor. Let us begin considering Eq. (28). There are two denominators so that, employing Feynman's trick,

$$\frac{1}{AB}=\int_0^1 dx\frac{1}{[(A-B)x+B]^2} \tag{A1}$$

and changing the integration variable, $k\to k+px$ one finds

$$\Sigma^{\alpha\beta}(p)=\frac{-ie^2}{\theta}\int_0^1 dx\int\frac{d^3k}{(2\pi)^3}\frac{L^{\alpha\beta}}{[k^2-C^2]^2}, \tag{A2}$$

where $C^2=M^2(1-x)-p^2x(1-x)$ and

$$L^{\alpha\beta}=\{[3M^2+xp^2](\varepsilon^{\sigma\beta\rho}k_\sigma k^\alpha+\varepsilon^{\alpha\sigma\rho}k_\sigma k^\beta)p_\rho+[(1-x)M^2p^2-(1-x)(p_\mu k^\mu)^2(x^2-x^3)p^4]\varepsilon^{\alpha\beta\sigma}p_\sigma +2xp^2\varepsilon^{\alpha\beta\rho}k_\rho k_\sigma p^\sigma\}/M^2. \tag{A3}$$

Using dimensional regularization one can perform the momentum integration to get

$$\Sigma^{\alpha\beta}(p)=\frac{e^2}{8\pi\theta M^2}\int_0^1 dx\,\frac{6M^4+[p^4-M^2p^2-6M^4]x+[M^2p^2-7p^4]x^2+6p^4x^3}{\sqrt{M^2(1-x)-p^2x(1-x)}}\varepsilon^{\alpha\beta\sigma}p_\sigma. \tag{A4}$$

The computation of the remaining parametric integration is straightforward and produces the result (30).

Let us now turn to the polarization tensor. From Eqs. (35) and (36) and proceeding analogously to what we have done before we arrive at

$$\Pi^{\mu\nu}=\frac{e^2}{M^2}\int\frac{d^3k}{(2\pi)^3}\frac{N_1^{\mu\nu}}{[k^2-M^2][(k+p)^2-M^2]}, \tag{A5}$$

where

$$\begin{aligned}N_1^{\mu\nu}=&[k^2(p^\mu k^\nu+k^\mu p^\nu)-2k^\mu k^\nu k^\alpha p_\alpha]+[8M^2k^\mu k^\nu\\&-(4M^2g^{\mu\nu}+p^\mu p^\nu)k^2+k^\alpha p_\alpha(2p^\mu k^\nu+2k^\mu p^\nu\\&-2g^{\mu\nu}k^\beta p_\beta)+p^2(-3k^\mu k^\nu+2g^{\mu\nu}k^2)]\\&+[3M^2(p^\mu k^\nu+k^\mu p^\nu-6g^{\mu\nu}k^\alpha p_\alpha)]\\&+[M^2(4M^2g^{\mu\nu}+p^\mu p^\nu-3p^2g^{\mu\nu})].\end{aligned} \tag{A6}$$

Employing again Feynman's formula (A1), translating the integration variable, $k\rightarrow k-px$ and deleting the terms odd in k, results

$$\Pi^{\mu\nu}=\frac{e^2}{M^2}\int_0^1 dx\int\frac{d^3k}{(2\pi)^3}\frac{N_2^{\mu\nu}}{[k^2-a^2]^2}, \tag{A7}$$

where, as before, $a^2=M^2-p^2x(1-x)$ and

$$\begin{aligned}N_2^{\mu\nu}=&M^2[4M^2g^{\mu\nu}+p^\mu p^\nu(1-6x+8x^2)\\&+p^2g^{\mu\nu}(-3+6x-4x^2)]+k^\mu k^\nu[8M^2-3p^2+2p^2x]\\&+k^2[-4M^2g^{\mu\nu}-p^\mu p^\nu(1+2x)+2p^2g^{\mu\nu}]\\&+[2k^\mu p^\nu k^\alpha p_\alpha+2p^\mu k^\nu k^\alpha p_\alpha-2g^{\mu\nu}(k^\alpha p_\alpha)^2].\end{aligned} \tag{A8}$$

With the help of dimensional regularization this becomes

$$\Pi^{\mu\nu}=\frac{ie^2}{8\pi M^2}\int_0^1 dx\,\frac{N_3^{\mu\nu}}{[M^2-p^2x(1-x)]^{1/2}}, \tag{A9}$$

with

$$\begin{aligned}N_3^{\mu\nu}=&M^2(2-12x+8x^2)[p^\mu p^\nu-p^2g^{\mu\nu}]\\&+p^2[p^\mu p^\nu(-x+7x^2-6x^3)\\&+p^2g^{\mu\nu}(-x-x^2+2x^3)].\end{aligned} \tag{A10}$$

The parametric integration can be done exactly producing the result (37).

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