QFT, string temperature, and the string phase of de Sitter space-time

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The density of mass levels $\rho(m)$ and the critical temperature for strings in de Sitter space-time are found. QFT and string theory in de Sitter space are compared. A ''dual'' transform is introduced which relates classical to quantum string lengths, and more generally, QFT and string domains. Interestingly, the string temperature in de Sitter space turns out to be the dual transform of the QFT Hawking-Gibbons temperature. The back reaction problem for strings in de Sitter space is addressed self-consistently in the framework of the ''string analogue'' model (or thermodynamical approach), which is well suited to combine QFT and string study. We find de Sitter space-time is a self-consistent solution of the semiclassical Einstein equations in this framework. Two branches for the scalar curvature $R(\pm)$ show up: a classical, low curvature solution $(-)$, and a quantum high curvature solution $(+)$, entirely sustained by the strings. There is a maximal value for the curvature $R_{\max}$ due to the string back reaction. Interestingly, our dual relation manifests itself in the back reaction solutions: the $(-)$ branch is a classical phase for the geometry with the intrinsic temperature given by the QFT Hawking-Gibbons temperature. The $(+)$ is a stringy phase for the geometry with a temperature given by the intrinsic string de Sitter temperature. $2+1$ dimensions are considered, but conclusions hold generically in D dimensions. [S0556-2821(99)05120-6]

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I. INTRODUCTION AND RESULTS

In the context of quantum field theory (QFT) in curved spacetime, de Sitter spacetime has a Hawking-Gibbons temperature given by

$$T_{\rm DS}=\frac{\hbar}{2\pi K_B}H=\frac{\hbar c}{2\pi K_B}\frac{1}{L_{\rm DS}}$$

(see Ref. [1] for its appropriated interpretation), H being the Hubble constant, $L_{\rm DS}=cH^{-1}$ being the classical horizon size.

In the context of string theory in curved spacetime, strings in de Sitter spacetime have a maximal or critical temperature given by

$$T_S=\frac{c^3}{\alpha' K_B H}=\frac{\hbar c}{K_B}\left(\frac{L_{\rm DS}}{L_S^2}\right)$$

(see Sec. III in this paper for its appropriated derivation), $L_S\equiv(\alpha'\hbar/c)^{1/2}$ being a characteristic string length scale.

We introduce here an $\mathcal{R}$ or ''dual'' transformation over a length

$$\tilde{L}=\mathcal{R}L=L_{\mathcal{R}}^2L^{-1};$$

i.e., if $L\equiv L_{\rm DS}$, then

$$\tilde{L}_{\rm DS}=\frac{\alpha'\hbar}{c^2}H.$$

$\tilde{L}_{\rm DS}$ is precisely the Compton length of a particle whose mass is given by

$$m_{\max}=\left(\frac{c}{\alpha' H}\right).$$

This is the maximal mass for the spectrum of particle (oscillating or stable) string states in de Sitter spacetime Refs. [2–4].

The $\mathcal{R}$ transformation links classical lengths to quantum string lengths. [In de Sitter spacetime, it links the classical horizon size $L_{\rm DS}$ to the quantum string size in this spacetime. We are refering here to the oscillatory or stable strings (those from which the quantum particle states derive).]

Under the $\mathcal{R}$ transformation (see Sec. III):

$$T_S=2\pi\tilde{T}_{\rm DS}.$$

The string temperature in de Sitter spacetime turns out 2π times the ''dual'' ($\mathcal{R}$-transformated) of the Hawking temperature (and conversely). That is, the intrinsic QFT and string temperatures in de Sitter space are $\mathcal{R}$ dual. In fact, this has a more general validity: the $\mathcal{R}$-transform can map QFT and string domains (or regimes) and applies to other spacetimes as well. In particular, it plays a key role when applied to black holes [5].

In the context of QFT, de Sitter (as well as AdS) spacetime, is an exact solution of the semiclassical Einstein equations with back reaction included [6,7]. Semiclassical in this context means that quantum matter fields (including the graviton) are coupled to c-number gravity and the vacuum expectation value of matter energy momentum tensor acts in turn as a source of gravity (quantum back reaction effect).

In this paper we investigate the quantum back reaction effect of strings in de Sitter spacetime. In principle, this question should be properly addressed in the context of string field theory. On the lack of a tractable framework for it, we work here in the framework of the string analogue model (or thermodynamical approach): the string as a collection of fields Φ_n coupled to the classical background, and whose masses m_n are given by the degenerate string mass spectrum in the curved space considered. (The fields Φ_n are without self-interaction but are coupled to the classical geometry.) The fields Φ_n are ''repeated'' $\rho(m)$ times, the degeneracy of states being given by $\rho(m)$, the density of mass levels of the string.

In flat spacetime, the higher masses string spectrum is given by

$$\rho(\bar{m}) \simeq \bar{m}^{-a} e^{b\bar{m}}, \quad \bar{m} \equiv \sqrt{\frac{\alpha' c}{\hbar}} m$$

(a and b being constants, depending on the model, and on the number of space dimensions). In de Sitter spacetime, we find $\rho(m)$ is given by [Eq. (33b)]

$$\rho(\bar{m}) = \frac{\bar{m}}{\Gamma} \frac{4\gamma^2}{(1-\Gamma)^2} \exp\left[\frac{4\pi^2}{3\gamma}(1-\Gamma)\right]^{1/2},$$

$$\Gamma \equiv (1-\bar{m}^2\gamma)^{1/2}, \quad \gamma \equiv \frac{5\alpha'\hbar}{4c^3} H^2.$$

It satisfies the behavior

$$\rho(\bar{m}) \sim \exp \frac{2\pi}{\sqrt{6}} \sqrt{\frac{\alpha' c}{\hbar}} m \left[1 - \frac{5}{32}\left(\frac{m\alpha' H}{c}\right)^2 + 0\left(\frac{m\alpha' H}{c}\right)^3\right]$$

When $H=0$, it yields the flat spacetime asymptotic behavior.

Here we deal with $2+1$ dimensions, but the results are the same for D dimensions, only the numerical values of the constants will change. In QFT, the expectation value of the $(2+1)$-dimensional energy-momentum tensor for a quantum massive field [in the de Sitter invariant (Bunch-Davies) vacuum] [8,9] is given by Eq. (35). In the framework of the analogue model, the string vacuum expectation value $\langle \tau_\mu^\mu \rangle$ is given by

$$\langle \tau_\mu^\mu \rangle = \frac{\displaystyle\int_{m_0}^{m_{\max}} \langle T_\mu^\mu(m) \rangle_S \rho(m) dm}{\displaystyle\int_{m_0}^{m_{\max}} \rho(m) dm},$$

$\langle T_\mu^\mu(m) \rangle_S$ being the trace stress tensor vacuum expectation value for an individual quantum field with mass in the string mass spectrum. m_0 is the lowest mass from which the asymptotic expression for $\rho(m)$ is still valid.

We apply self-consistently the string $\langle \tau_\mu^\mu \rangle$ to the right-hand side (RHS) of the semiclassical Einstein equations, we study the back reaction effect in de Sitter space of the higher excited string modes. In constant curvature spaces (such as dS and AdS) the semiclassical back reaction equations yield the scalar curvature in terms of H and of the quantum matter content (the trace $\langle \tau_\mu^\mu \rangle$).

The mass domain for fields in de Sitter spacetime is given by

$$m_{\mathrm{QFT}} < \frac{\hbar H}{c^2}$$

while in string theory, the string mass in de Sitter spacetime satisfies

$$m_S < \frac{c}{\alpha' H}.$$

Under the $\mathcal{R}$ transformation we have

$$\tilde{m}_{\mathrm{QFT}} = \mathcal{R} m_{\mathrm{QFT}} = m_S,$$

$$\tilde{m}_S = \mathcal{R} m_S = m_{\mathrm{QFT}},$$

$$\mathcal{R}\langle T_\mu^\mu \rangle_{\mathrm{QFT}} = \langle \tilde{T}_\mu^\mu \rangle_{\mathrm{QFT}} \equiv \langle T_\mu^\mu \rangle_S.$$

Here $\langle T_\mu^\mu \rangle_S$ is given by [Eq. (46)] as a function of the variable $x \equiv (m/m_{\max})^2$.

We find $\langle \tau_\mu^\mu \rangle$ up to order γ (as given by [Eq. (55b)]), in terms of α' and of the scalar curvature $R = 6H^2/c^2$:

$$\langle \tau_\mu^\mu \rangle = -\frac{\hbar H^3}{3\pi^3 c^2} \sqrt{6\gamma}\left(1 + \frac{2}{\pi}\sqrt{6\gamma}\right).$$

Inserting it self-consistently in the semiclassical Einstein equations for the effective geometry, we find for the scalar curvature:

$$R_\pm = 6\Lambda_\pm = \frac{1}{2} R_{\max}\left[1 \pm \left(1 - 4\frac{R}{R_{\max}}\right)^{1/2}\right].$$

Due to the quantum string back reaction, the curvature reaches a maximum value:

$$R_{\max} = \frac{9c^4\pi^2}{4G}\left(\frac{6}{5\alpha' c\hbar^3}\right)^{1/2}.$$

Three cases show up depending on whether (i) $R < \frac{1}{4}R_{\max}$, (ii) $R = \frac{1}{4}R_{\max}$, or (iii) $R < \frac{1}{4}R_{\max}$.

Case (i) describes two semiclassical de Sitter spacetimes with constant positive curvatures $R_\pm > 0$ and well defined associated temperatures $T_{(\pm)}$.

Case (ii) describes one semiclassical de Sitter (positive curvature) space for which $R_+ = R_- = \frac{1}{2}R_{\max}$.

For (iii), no real spacetime geometries, nor temperatures are possible.

Two branches, $(+)$ and $(-)$, for the curvature show up. The leading term is R in the $(-)$ branch, while is $R_{\rm max}$ in the $(+)$ branch. In an expansion in $R/R_{\rm max}$, classical de Sitter space is recovered in the $(-)$ branch. $R(-)$ is a low curvature, classically allowed solution, while $R(+)$ is a ''quantum'' branch (it does not exist classically) and its curvature is very high. The quantum string back reaction generates this branch.

Our dual relation between classical-QFT and the string domains manifests here again in the back reaction solutions: the branch $(-)$ is a classical phase for the geometry which temperature is given by the QFT Hawking-Gibbons temperature $T(-)=T_{\rm DS}=(\hbar c/2\pi K_B)\,(R_-/6)^{1/2}$.

The branch $(+)$ is a stringy phase for the geometry which temperature is the intrinsic string de Sitter temperature $T(+)=T_S=(c^2/\alpha' K_B)\,(6/R_+)^{1/2}$. Moreover, our dual relation and the two phases: a classical-QFT phase (with the Hawking temperature) and a quantum-string phase (with the string temperature), appear to be a generic feature and are very enlighting for black holes. Our study of the string black hole temperature and quantum string back reaction for black holes is reported elsewhere Ref. [5].

The duality transform introduced here does not require the existence of any symmetry or isometry in the curved background. In this sense, it is not related to the T duality, a well established symmetry of strings in backgrounds with isometries. Our duality is more of the type of a classical-quantum (or wave-particle) duality relating classical-semiclassical and quantum behaviors or regimes, here extended and including the quantum string regime. At the stage we are using it, this duality is a consequence or a matter of fact from the results of this paper.

The QFT and string results of this paper and of Ref. [5] satisfy this duality (they do not assume it). In this sense, this is not a conjectural duality. It could be that a more general operation underlies this transform, but we do not study it in this paper.

This paper is organized as follows. In Sec. II we summarize de Sitter spacetime and the QFT Hawking-Gibbons temperature. In Sec. III, we derive the string temperature in de Sitter spacetime and its dual relation to the Hawking-Gibbons temperature. In Sec. IV we find the quantum string back reaction and its solution. In Sec. V we present the concluding remarks.

II. de SITTER SPACE-TIME

de Sitter space-time is a cosmological space with constant scalar curvature (R), and vanishing spatial curvature index (K). Its D-dimensional metric is given by

$$ds^2=-c^2dt^2+a^2(t)(dr^2+r^2d\Omega_{D-2}^2), \tag{1}$$

$$a(t)=e^{Ht},$$

t being the cosmic time and $H=d\ln a(t)/dt$ being the Hubble constant.

de Sitter space-time can be generated by a cosmological constant (Λ). The curvature $R_{\mu\nu},R,H$, and Λ are related by

$$R_{\mu\nu}-\frac{1}{2}g_{\mu\nu}R+\Lambda g_{\mu\nu}=0, \tag{2a}$$

$$R=D(D-1)\frac{H^2}{c^2}=\frac{2D}{D-2}\Lambda, \tag{2b}$$

$$\Lambda=\frac{(D-1)(D-2)}{2}\frac{H^2}{c^2}. \tag{2c}$$

The D-dimensional de Sitter metric can be also expressed in terms of the so-called static coordinates

$$ds^2=-A(r)c^2dT^2+A^{-1}(r)dr^2+r^2d\Omega_{D-2}^2, \tag{3}$$

where

$$A(r)=1-\frac{H^2r^2}{c^2} \tag{4}$$

which show the existence of a horizon at

$$r=L_{\rm DS}=cH^{-1}. \tag{5}$$

In the context of quantum field theory in curved space-time, de Sitter space-time has a Hawking-Gibbons temperature $(T_{\rm DS})$ given by Ref. [1]

$$T_{\rm DS}=\frac{H\hbar}{2\pi K_B}. \tag{6a}$$

Notice that this expression for $T_{\rm DS}$ holds in any number of space-time dimensions. In terms of R and Λ, $T_{\rm DS}$ reads

$$T_{\rm DS}=\frac{\hbar c}{2\pi K_B}\sqrt{\frac{R}{D(D-1)}} \tag{6b}$$

or

$$T_{\rm DS}=\frac{\hbar c}{2\pi K_B}\sqrt{\frac{2\Lambda}{(D-1)(D-2)}}. \tag{6c}$$

If one defines a surface gravity $\kappa_{\rm DS}$ equal to cH, $T_{\rm DS}$ [Eq. (6)] reads

$$T_{\rm DS}=\frac{\hbar\kappa_{\rm DS}}{2\pi K_B c}. \tag{7}$$

Furthermore, $T_{\rm DS}$ can be also expressed in terms of the classical length scale $L_{\rm DS}$ [Eq. (5)] as

$$T_{\rm DS}=\frac{\hbar c}{2\pi K_B}\frac{1}{L_{\rm DS}}. \tag{8}$$

III. QUANTUM STRINGS IN de SITTER SPACE-TIME

String theory in de Sitter space-time is exactly integrable in any dimension [10]. However, explicit expressions for the string solutions are not easy to write due to the complexity of the equations, and even of the solutions. Two main quantum frameworks have been studied and solved: (i) canonical quantization of generic strings in any dimension [2,3] and (ii) semiclassical quantization of exact circular strings configurations in $2+1$ space-times Ref. [4].

In this section we will consider the case (i) (canonical quantization). We now remind the reader of some of the main issues [Eqs. (2) and (3)]. In canonical quantization, one treats the de Sitter classical background exactly, and considers the string oscillations around its center of mass as perturbations. The string center of mass is an exact solution of the geodesic equation. The perturbations (dimensionless) parameter is here

$$\frac{L_S}{L_{\rm DS}}=\frac{H}{c}\sqrt{\frac{\alpha'\hbar}{c}}=\sqrt{\frac{2\Lambda\alpha'\hbar}{(D-1)(D-2)c}}\ll 1, \tag{9}$$

where $L_{\rm DS}$ (de Sitter length or horizon) is given by [Eq. (5)] and L_S (string length scale) is

$$L_S\equiv\left(\frac{\alpha'\hbar}{c}\right)^{1/2}. \tag{10}$$

Here $\alpha'\equiv c^2/2\pi T$, where T is the string tension $[(\alpha')^{-1}$: linear mass density].

In this framework, the mass spectrum formula in de Sitter space for an N-th level state is given by [2,3]

$$\alpha'\left(\frac{c}{\hbar}\right)m^2=24\sum_{n>0}\frac{2n^2-H^2m^2(\alpha'^2/c^2)}{\sqrt{n^2-H^2m^2(\alpha'^2/c^2)}}+2N\frac{2-H^2m^2(\alpha'^2/c^2)}{\sqrt{1-H^2m^2(\alpha'^2/c^2)}}. \tag{11}$$

One of the consequences of the spectrum is that the number of string oscillating states, although being very large, is finite. This maximum number is given by

$$N_{\max}\simeq \mathrm{Int}\left[0.15\left(\frac{L_{\rm DS}}{L_S}\right)^2\right]=\mathrm{Int}\left[0.15\left(\frac{c^3}{\alpha'\hbar H^2}\right)\right]. \tag{12}$$

Furthermore, there is a maximum mass ($m_{\max}$) for the corresponding real mass solutions

$$\alpha'\left(\frac{c}{\hbar}\right)m_{\max}^2\simeq\left(\frac{L_{\rm DS}}{L_S}\right)^2, \tag{13}$$

i.e.,

$$m_{\max}^2\simeq\left(\frac{c}{\alpha' H}\right)^2.$$

As [Eq. (9)] is fulfilled for oscillating string states, i.e., $1\ll(L_{DS}/L_S)^2(=c^3/\alpha' H^2\hbar)$, the number of oscillating strings and the maximum string mass are large.

The fact that there is a maximum mass implies the existence of a maximum or critical temperature for the strings in de Sitter space-time. The temperature T_S corresponding to $m_{\max}$ [Eq. (13)], is given by

$$T_S=\frac{c^3}{\alpha' H K_B}, \tag{14}$$

or, in terms of the classical and string length scales $L_{\rm DS}$ and L_S,

$$T_S=\frac{\hbar c}{K_B}\left(\frac{L_{\rm DS}}{L_S^2}\right). \tag{15}$$

If we compare this maximal or critical temperature for strings in de Sitter space-time (T_S) with the quantum field theory Hawking-Gibbons temperature for de Sitter space-time ($T_{\rm DS}$) [Eq. (6a)], we have

$$T_S=\left(\frac{c^3\hbar}{2\pi\alpha' K_B^2}\right)\frac{1}{T_{\rm DS}}. \tag{16}$$

Let us define now the following transformation $\mathcal{R}$ over a length L:

$$\tilde{L}=\mathcal{R}L=L_{\mathcal{R}}^2L^{-1}. \tag{17}$$

If $L_{\mathcal{R}}=L_S$ [Eq. (10)], and we apply this transformation to $L\equiv L_{\rm DS}$ [Eq. (5)], we obtain

$$\tilde{L}_{\rm DS}=\mathcal{R}L_{\rm DS}=\frac{\alpha'\hbar H}{c^2}. \tag{18}$$

But $\tilde{L}_{\rm DS}$ is precisely the (reduced) Compton wave length $(\lambda=\hbar/mc)$ of a particle whose mass is equal to $m_{\max}$ given by [Eq. (13)], i.e., $\tilde{L}_{\rm DS}$ is the minimal quantum length of a string in de Sitter space. Therefore, this transformation links the classical de Sitter length scale $L_{\rm DS}$ to the quantum string–de Sitter one $\tilde{L}_{\rm DS}$.

The string temperature T_S [Eq. (15)] in de Sitter space time can be rewritten in terms of $\tilde{L}_{\rm DS}$ [Eq. (18)] as

$$T_S=\frac{\hbar c}{K_B}\frac{1}{\tilde{L}_{\rm DS}}. \tag{19}$$

We see now from [Eq. (8)] and [Eq. (19)] that the following relations hold under the $\mathcal{R}$ transformation:

$$\tilde{T}_{\rm DS}=\frac{1}{2\pi}T_S \tag{20}$$

and

$$\tilde{T}_S=2\pi\, T_{\rm DS}. \tag{21}$$

From the above equations we can read as well

$$T_ST_{\rm DS}=\tilde{T}_S\tilde{T}_{\rm DS}.$$

That is, the maximal string temperature in de Sitter space-time is the dual (in the sense of the $\mathcal{R}$ transformation [Eq. (17)]) of the Hawking (QFT) temperature.

IV. QUANTUM STRING BACK REACTION IN de SITTER SPACE-TIME

When quantum matter (particle fields, strings) is present in de Sitter space-time, the relation between the scalar curvature and the cosmological constant Λ will be modified through the semiclassical Einstein equations. Semiclassical in this context means that matter, which is a q number, is coupled to c number gravity through the equations

$$R_{\mu\nu}-\frac{1}{2}R\,g_{\mu\nu}+\Lambda g_{\mu\nu}=\frac{8\pi G}{c^4}\langle\tau_{\mu\nu}(q,g_{\mu\nu})\rangle. \tag{22}$$

The space-time background metric $g_{\mu\nu}$ generates a nonzero vacuum expectation value of the energy momentum tensor $\langle\tau_{\mu\nu}\rangle$, which in turn acts as a source of curvature. (For instance, in four-dimensional quantum field theory, matter fields $\hat{\phi}$ include the graviton and $\langle T_{\mu\nu}(\hat{\phi},g_{\mu\nu})\rangle$ is calculated up to one loop order, where $\langle T_{\mu\nu}\rangle$ stands for its renormalized value [11,7].)

For maximally symmetric (constant curvature) spaces (such as de Sitter and anti-de Sitter), these equations read

$$\left(\frac{2-D}{2D}R+\Lambda\right)g_{\mu\nu}=\frac{8\pi G}{c^4}\langle\tau_{\mu\nu}\rangle \tag{23}$$

which yields the trace equation

$$R-\frac{2D}{D-2}\Lambda=-\frac{16\pi G}{c^4(D-2)}\langle\tau^{\mu}_{\mu}\rangle \tag{24a}$$

or

$$R=\frac{2D}{D-2}\Lambda_{\rm eff}, \tag{24b}$$

where

$$\Lambda_{\rm eff}=\Lambda-\frac{8\pi G}{Dc^4}\langle\tau^{\mu}_{\mu}\rangle \tag{24c}$$

which shows clearly quantum matter as a source of curvature and of temperature [Eq. (7b)].

As $\langle\tau_{\mu\nu}\rangle$ is proportional to $g_{\mu\nu}$, de Sitter (DS) [as well as anti–de Sitter ($A_{\rm DS}$)] backgrounds are exact self-consistent solutions to [Eq. (22)] with back reaction included.

In de Sitter space, there is one real parameter α family of de Sitter group invariant vacua $|\alpha>$. Here $\langle\tau_{\mu\nu}\rangle$ is the expectation value in the Bunch-Davies [8,9] (''Euclidean'' or ''inflationary'') vacuum obtained for $\alpha=0$.

In order to study the back reaction problem for string theory in de Sitter space-time, we will work in the framework of the string analogue model, and in a $2+1$ space-time, where we will use the results coming from semiclassical quantization of exact circular strings configurations Ref. [4]. However, one should have in mind that the $2+1$ string dynamics could be embedded in a higher dimensional space-time and our results generalized to higher dimensions as well.

In the spirit of the analogue model, we consider here the string as a collection of fields $\hat{\phi}_n$ coupled to the classical background, and whose masses m_n are given by the degenerate mass spectrum of the string. The fields $\hat{\phi}_n$ are free (without self-interactions) but interact here with the classical geometry. The (higher) mass spectrum is described by the density of mass levels $\rho(m)$. As it is known, in flat space-time $\rho(m)$ is given by

$$\rho(\bar{m})\sim\bar{m}^{-a}\exp b\,\bar{m}, \tag{25}$$

where we have introduced the adimensional mass variable

$$\bar{m}\equiv\sqrt{\frac{\alpha' c}{\hbar}}m \tag{26}$$

(which will prove useful later on). The constants a and b depend on the string model and on the dimensions of the space-time.

In de Sitter space-time, $\rho(m)$ has a different behavior from the one of [Eq. (25)], as it follows from the string mass spectrum in de Sitter space [Eq. (11)]. Classical string equations of motion and constraints have been solved exactly for circular string configurations $[t=t(\tau),r=r(\tau),\phi=\sigma]$ in a $2+1$ de Sitter space-time Ref. [12]. Semiclassical quantization of the time periodic (oscillating) solutions has been performed Ref. [4]. For $\alpha' H^2\hbar/c^3\ll 1$ [i.e., $(L_S/L_{\rm DS})^2\ll 1$], corresponding to the semiclassical quantization here, and which is always satisfied for oscillating strings, the results are the following. (i) The quantized mass formula is given, for large n , by

$$\alpha'\left(\frac{c}{\hbar}\right)m^2\simeq 4n(1-\gamma n), \tag{27}$$

where

$$\gamma\equiv\frac{5\alpha' H^2\hbar}{4c^3}. \tag{28}$$

(Notice that for $H=0$ one recovers the mass formula for closed strings in Minkowski space.) (ii) The number of oscillatory circular string states, although being very large, is finite

$$N_{\max}\simeq{\rm Int}\left[0.34\frac{c^3}{\alpha' H^2\hbar}\right]. \tag{29}$$

(iii) The level spacing is approximately constant, in $(\alpha' c/\hbar)^{-1}$ units (although smaller than in Minkowski space-time and slightly decreasing).

Furthermore, from [Eq. (27)] and [Eq. (28)], the maximum value for the string mass states is given by

$$m^2_{\max} \simeq \frac{4}{5}\left(\frac{c}{\alpha' H}\right)^2 \tag{30a}$$

or

$$\overline{m}^2_{\max} \simeq \gamma^{-1} \tag{30b}$$

[see Eqs. (26) and (28)]. The above results are in very good agreement with the ones corresponding to canonical quantization of generic strings [Eqs. (12) and (13)]. It must be noticed that Eq. (30) will provide a maximum string temperature similar to the one of Eq. (14).

The asymptotic degeneracy of levels d_n (in flat as well as in curved space-time) is generically $\sim n^{-(D+1)/2} e^{4\pi\sqrt{(D-2)n/6}}$ for any noncompact D-dimensional space-time. For closed string solutions and $D=3$, the asymptotic degeneracy of levels d_n reads

$$d_n \sim n^{-2} e^{4\pi\sqrt{n/6}}, \tag{31a}$$

where n has now to be expressed as a function of the quantized mass. It is through the relation $m=m(n)$ of the mass spectrum, that the differences due to the space-time curvature enter in the above formula.

The density of mass levels $\rho(m)$ and the degeneracy d_n satisfy

$$\rho(m)dm = d_n(m)dn. \tag{31b}$$

From [Eq. (27)] and [Eq. (28)] we have

$$n \simeq \mathrm{Int}\left(\frac{2c^3}{5\alpha' H^2 \hbar}\left\{1-\left[1-\frac{5}{4}\left(\frac{\alpha' H m}{c}\right)^2\right]^{1/2}\right\}\right) \tag{32a}$$

or

$$n \simeq \mathrm{Int}\left\{\frac{1}{2\gamma}[1-(1-\overline{m}^2\gamma)^{1/2}]\right\}, \tag{32b}$$

in terms of the adimensional variables $\overline{m}$ and γ [Eqs. (26) and (28)].

Therefore from Eqs. (31b) and (32b), the asymptotic string density of mass levels in de Sitter space is

$$\rho(m) \sim \left(\alpha' \frac{c}{\hbar}\right) m \frac{d_n}{1-2\gamma n} \tag{33a}$$

which for $H=0$ [$\gamma=0$, Eq. (28)] gives the flat space-time relation $\rho(m)\sim m d_n(m)$.

From Eqs. (32b), (33a), and (31a), we obtain

$$\rho(\overline{m}) \sim \overline{m}(1-\overline{m}^2\gamma)^{-1/2}\left[\frac{1}{2\gamma}[1-(1-\overline{m}^2\gamma)^{1/2}]\right]^{-2} \times \exp\left\{\frac{4\pi}{\sqrt{6}}\left[\frac{1}{2\gamma}[1-(1-\overline{m}^2\gamma)^{1/2}]\right]^{1/2}\right\}, \tag{33b}$$

where γ is given by [Eq. (28)]. Equation (33b) generalizes to de Sitter spacetime the standard flat space time behavior [Eq. (25)].

If we develop the exponent of $\rho(\overline{m})$ in powers of $\overline{m}^2\gamma = (m/m_{\max})^2 < 1$ [Eqs. (26) and (30b)], we have

$$\rho(\overline{m})\,\alpha\,\exp\frac{2\pi}{\sqrt{6}}\sqrt{\frac{\alpha' c}{\hbar}}m\left\{1-\frac{1}{8}\left(\frac{m}{m_{\max}}\right)^2+O\left[\left(\frac{m}{m_{\max}}\right)^3\right]\right\}$$

or, showing the explicit dependence on H,

$$\rho(\overline{m})\,\alpha\,\exp\frac{2\pi}{\sqrt{6}}\sqrt{\frac{\alpha' c}{\hbar}}m\left[1-\frac{5}{32}\left(\frac{\alpha' H}{c}\right)^2 m^2+\cdots\right].$$

We see that for $H=0$ one recovers the flat space time asymptotic behavior.

Now, returning to the semiclassical Einstein equations (24a), (24b), and (24c), $\langle \tau^\mu_\mu \rangle$ will be the vacuum expectation value of the trace of the stress tensor for the collection of fields (interacting with the background) which correspond to the string tower of mass states in de Sitter space. In the framework of the analogue model, the string vacuum expectation value $\langle \tau^\mu_\mu \rangle$ is given by

$$\langle \tau^\mu_\mu \rangle \simeq \frac{\int_{m_0}^{m_{\max}} \langle T^\mu_\mu(m)\rangle_S\, \rho(m)dm}{\int_{m_0}^{m_{\max}} \rho(m)dm}, \tag{34}$$

where $\langle T^\mu_\mu \rangle_S$ is the vacuum expectation value of the trace of the stress tensor for an individual quantum field. We integrate over string field masses and divide by the total mass degeneracy. In fact we should have $\langle n(m)\rangle \rho(m)$, where $\langle n(m)\rangle \sim \int_0^\infty k^{D-2}dk$, but this divergent contribution cancels out (as it appears as a multiplicative factor for both numerator and denominator).

Since in de Sitter space-time the number of particle oscillating states is finite, the sum goes up to $m_{\max}$ [Eq. (30)] (instead of up to infinity as for Minkowski space-time); m_0 is the lowest mass from which the asymptotic expression of the density of mass levels [Eq. (33)] is valid. Therefore, we are studying the back reaction effect in de Sitter space-time due to the higher excited string modes.

Here $\rho(m)$ [Eq. (33)] depends only on the mass as usual, therefore $\langle T^\mu_\mu \rangle_S$ will be chosen for our study to be the expectation value of the stress tensor for a massive scalar field (in the de Sitter invariant or Bunch-Davies vacuum).

The quantum field theory value $\langle T^\mu_\mu(m)\rangle_{\mathrm{QFT}}$, corresponding to a scalar massive field in a $2+1$ de Sitter space-time (in the de Sitter invariant vacuum), is given by Refs. [13] and [9]

$$\langle T^\mu_\mu \rangle_{\mathrm{QFT}} = \frac{\hbar H^3}{4\pi c^2}\left(\frac{mc^2}{\hbar H}\right)^2\left[(1-6\zeta)-\left(\frac{mc^2}{\hbar H}\right)^2\right]^{1/2} \times \mathrm{ctg}\,\pi\left[(1-6\zeta)-\left(\frac{mc^2}{\hbar H}\right)^2\right]^{1/2}, \tag{35}$$

where ζ, a numerical factor, is the scalar coupling ($-\frac{1}{2}\zeta R\phi$; conformal coupling : $\zeta=\frac{1}{8}$). Notice that for a massless scalar field there is no trace anomaly in $2+1$ dimensions. This happens too also for any odd dimensional de Sitter space-time Ref. [14]. In addition, for these spaces, $\langle T^{\mu}_{\mu}\rangle$ is finite and no renormalization procedure is needed, in contrast to the $D=4$ case Ref. [15].

At this point, let us analyze the mass scales in the corresponding quantum field theory (QFT) and string theory in de Sitter space-time. From Eqs. (32) and (35), one can read the following domains for the field mass in QFT and in string theory ($\zeta=0$ for simplicity) :

$$m_{\mathrm{QFT}}<\frac{\hbar H}{c^2} \tag{36a}$$

and

$$m_S<\frac{2}{\sqrt{5}}\,\frac{c}{\alpha' H} \tag{36b}$$

which can be rewritten as well as [see Eqs. (26), (30a), and (30b)]

$$\frac{m_{\mathrm{QFT}}}{M_H}<1 \tag{37a}$$

and

$$\frac{m_S}{m_{\max}}=\bar{m}_S\sqrt{\gamma}<1. \tag{37b}$$

Here $M_H\equiv\hbar H/c^2$ is the mass scale of de Sitter space. Equation (37a) just means that m_{QFT} is a test particle field in de Sitter space, and $m_{\max}$ is the maximum value for string states in the $2+1$ de Sitter semiclassical quantization.

Now we express the above inequalities in terms of L_{DS} [Eq. (5)] and $\tilde{L}_{\mathrm{DS}}$, being the later the minimal Compton wave length as before. Here, according to $m_{\max}$ given by Eq. (30a), $\tilde{L}_{\mathrm{DS}}$ is

$$\tilde{L}_{\mathrm{DS}}=\frac{\sqrt{5}}{2}\,\frac{\alpha' H\hbar}{c^2} \tag{38}$$

(semiclassical and canonical quantizations of the string just differ in the factor $\sqrt{5}/2$).

From Eqs. (36a), (36b), (5), and (38) we have

$$m_{\mathrm{QFT}}<\frac{\hbar}{c}\,\frac{1}{L_{\mathrm{DS}}} \tag{39a}$$

and

$$m_S<\frac{\hbar}{c}\,\frac{1}{\tilde{L}_{\mathrm{DS}}}. \tag{39b}$$

But these domains are going to be exchanged by the $\mathcal{R}$ transformation given by Eq. (18). In fact, if we apply the $\mathcal{R}$ transformation to both sides of Eqs. (39a) and (39b) we obtain

$$\tilde{m}_{\mathrm{QFT}}<\frac{\hbar}{c}\,\frac{1}{\tilde{L}_{\mathrm{DS}}} \tag{40a}$$

and

$$\tilde{m}_S<\frac{\hbar}{c}\,\frac{1}{L_{\mathrm{DS}}}. \tag{40b}$$

($L_{\mathcal{R}}=(\sqrt{5}\alpha'\hbar/2c)^{1/2}$ in Eq. (7). The numerical factor $\sqrt{5}/2$, that appears here and in $\tilde{L}_{\mathrm{DS}}$ as well, is just due to the slightly smaller $m_{\max}$ one obtains in semiclassical quantization [Eq. (30a)] as compared with the canonical quantization [Eq. (13)]. Obviously, the action of the $\mathcal{R}$ transformation is equal for both cases.)

We can summarize the action of the $\mathcal{R}$ transformation, on the masses and on their domains, in the following Eqs. (37a), (37b), (40a), and (40b):

$$\tilde{m}_{\mathrm{QFT}}=\mathcal{R}m_{\mathrm{QFT}}=m_S, \tag{41a}$$

$$\tilde{m}_S=\mathcal{R}m_S=m_{\mathrm{QFT}}, \tag{41b}$$

$$\mathcal{R}\left(\frac{m_{\mathrm{QFT}}c^2}{\hbar H}\right)=\frac{m_S}{m_{\max}}=\bar{m}_S\sqrt{\gamma}. \tag{41c}$$

Now we are able to write the vacuum expectation value (VEV) of the stress tensor $\langle T^{\mu}_{\mu}\rangle_S$ that appears in the RHS of Eq. (34), and which corresponds to the high masses of the string domain. From the previous mass-domain study, it is clear that $\langle T^{\mu}_{\mu}\rangle_S$ is precisely the $\mathcal{R}-$transformed object of $\langle T^{\mu}_{\mu}\rangle_{\mathrm{QFT}}$: i.e.,

$$\mathcal{R}\langle T^{\mu}_{\mu}\rangle_{\mathrm{QFT}}=\langle\tilde{T}^{\mu}_{\mu}\rangle_{\mathrm{QFT}}\equiv\langle T^{\mu}_{\mu}\rangle_S. \tag{42}$$

Applying the $\mathcal{R}$ transformation [Eqs. (41a) and (41c)] to $\langle T^{\mu}_{\mu}\rangle_{\mathrm{QFT}}$, given by Eq. (35), we obtain

$$\langle\tilde{T}^{\mu}_{\mu}\rangle=\frac{\hbar H^3}{4\pi c^2}(\overline{m}^2\gamma)[(1-6\zeta)-\overline{m}^2\gamma]^{1/2}$$
$$\times\mathrm{ctg}\,\pi[(1-6\zeta)-\overline{m}^2\gamma]^{1/2} \tag{43}$$

in terms of the adimensional variable

$$\overline{m}^2\gamma=\frac{5}{4}\left(\frac{\alpha' mH}{c}\right)^2$$

[Eqs. (26) and (28)].

In order to compute $\langle\tau^{\mu}_{\mu}\rangle$ [Eq. (34)], it is convenient to express $\rho(\overline{m})d\overline{m}$ [Eq. (33)] and $\langle\tilde{T}^{\mu}_{\mu}\rangle$ [Eq. (43)], in terms of the adimensional variable $x\equiv m^2\gamma$ (running ratio $m^2/m^2_{\max}$) :

$$\rho(\overline{m})d\overline{m}=\frac{1}{2\gamma}\rho(x)dx, \tag{44}$$

where

$$\rho(x)\equiv(1-x)^{-1/2}[1-(1-x)^{1/2}]^{-2}\times\exp\left\{\frac{2\pi}{\sqrt{3\gamma}}[1-(1-x)^{1/2}]^{1/2}\right\} \tag{45}$$

and

$$\langle\tilde{T}^{\mu}_{\mu}\rangle=-\frac{\hbar H^3}{4\pi c^2}F(x), \tag{46}$$

where

$$F(x)\equiv -x(1-x)^{1/2}\mathrm{ctg}\,\pi(1-x)^{1/2} \tag{47}$$

(we set here $\zeta=0$ for simplicity).

Finally, $\langle\tau^{\mu}_{\mu}\rangle$ will be given by

$$\langle\tau^{\mu}_{\mu}\rangle=-\left(\frac{\hbar H^3}{4\pi c^2}\right)\frac{\int_{x_1}^{x_2}F(x)\rho(x)dx}{\int_{x_1}^{x_2}\rho(x)dx}\equiv-\left(\frac{\hbar H^3}{4\pi c^2}\right)\frac{I_N}{I_D}, \tag{48}$$

where $x_1=\overline{m}_0^2\gamma$. In our case, the adimensional variable x runs in the interval $[x_1,\frac{3}{4})$. About the upper limit x_2, a word on $F(x)$ is now in order. $F(x)$ is a non-singular (monotonically) decreasing function in the interval [0,1], and $F(x)>0$ for x in $[0,\frac{3}{4})$. But this later interval is in fact the safe range for the physical validity of $\langle T^{\mu}_{\mu}\rangle_{\mathrm{QFT}}$ (the mass of the test particle m is much smaller than the mass scale M_H of a de Sitter universe), and hence for $\langle\tilde{T}^{\mu}_{\mu}\rangle$.

On the other hand, if we consider the integral I_N in the numerator of [Eq. (48)], the exponential of $\rho(x)$ [Eq. (45)] plays a leading role in the interval $[x_1,\frac{3}{4})$ from the physical point of view since $\gamma^{-1}\gg 1$. Therefore, the monotonically decreasing behavior of the function $F(x)$ can be approximated by the straight line $y=-(8/3\pi)(x-\frac{3}{4})$.

After a straightforward calculation one obtains for I_N and I_D [Eq. (48)] the following expressions:

$$I_N=-\frac{32}{3\pi}\left\{-\frac{3}{4}\left[-\frac{e^{z/\lambda}}{2z^2}-\frac{e^{z/\lambda}}{2\lambda z}+\frac{1}{2\lambda^2}E_i\left(\frac{z}{\lambda}\right)\right]-e^{z/\lambda}(z\lambda-\lambda^2)+2E_i\left(\frac{z}{\lambda}\right)\right\}_{z_1}^{z_2} \tag{49}$$

and

$$I_D=4\left\{-\frac{e^{z/\lambda}}{2z^2}-\frac{e^{z/\lambda}}{2\lambda z}+\frac{1}{2\lambda^2}E_i\left(\frac{z}{\lambda}\right)\right\}_{z_1}^{z_2}, \tag{50}$$

where

$$z=[1-(1-x)^{1/2}]^{1/2}, \tag{51}$$

$$\lambda=\frac{\sqrt{3\gamma}}{2\pi}. \tag{52}$$

Considering the $\lambda(\sqrt{\gamma})$ leading terms, we have for I_N and I_D:

$$I_N\simeq\frac{128}{3\pi}e^{2\pi/\sqrt{6\gamma}}\lambda^2(1+7\sqrt{2}\lambda) \tag{53}$$

$$I_D\simeq 16e^{2\pi/\sqrt{6\gamma}}\lambda\left(\frac{1}{\sqrt{2}}+3\lambda\right). \tag{54}$$

From Eqs. (48), (53), and (54), $\langle\tau^{\mu}_{\mu}\rangle$ reads, up to order γ [Eq. (28)]:

$$\langle\tau^{\mu}_{\mu}\rangle=-\frac{\hbar H^3}{3\pi^3c^2}\sqrt{6\gamma}\left(1+\frac{2}{\pi}\sqrt{6\gamma}\right) \tag{55a}$$

or, in terms of the scalar curvature R [Eq. (2b); $R=6H^2c^{-2}$]

$$\langle\tau^{\mu}_{\mu}\rangle=-\frac{R^2}{36\pi^3}\left(\frac{5\alpha' c\hbar^3}{6}\right)^{1/2}\left[1+\frac{2}{\pi}\left(\frac{5\alpha'\hbar}{4c}R\right)^{1/2}\right]. \tag{55b}$$

Inserting $\langle\tau^{\mu}_{\mu}\rangle$ [Eq. (55b)] into the back reaction [Eq. (24a)] for $D=3$, we have

$$R-6\Lambda=\frac{4GR^2}{9\pi^2c^4}\left(\frac{5\alpha'\hbar^3c}{6}\right)^{1/2}\left[1+\frac{2}{\pi}\left(\frac{5\alpha'\hbar}{4c}R\right)^{1/2}\right] \tag{56a}$$

(for $D=3,[G]=L^2t^{-2}M^{-1}$) or [Eqs. (24b) and (24c)]:

$$R=6\Lambda_{\mathrm{eff}}, \tag{56b}$$

where

$$\Lambda_{\mathrm{eff}}=\Lambda+\frac{2GR^2}{27\pi^2c^4}\left(\frac{5\alpha'\hbar^3c}{6}\right)^{1/2}\left[1+\frac{2}{\pi}\left(\frac{5\alpha'\hbar}{4c}R\right)^{1/2}\right]. \tag{57}$$

We are going to analyze now the physical consequences of the back reaction [Eq. (56a)]. For simplicity we consider $\langle\tau^{\mu}_{\mu}\rangle$ [Eq. (55a)] up to order $\sqrt{\gamma}$. We have

$$R-6\Lambda\simeq\hat{\alpha}R^2, \tag{58}$$

where

$$\hat{\alpha} \equiv \frac{4G}{9c^4\pi^2}\left(\frac{5\alpha' c\hbar^3}{6}\right)^{1/2} \tag{59}$$

[Eq. (58)] is a second order equation in R, similar to the one found for the back reaction of massless quantum fields (including the graviton) in four-dimensional de Sitter space-time Ref. [7]. The expression for $\hat{\alpha}$ is here different as it contains α'. (In the case of massless QFT, $\hat{\alpha}$ arise from the trace anomaly $\langle T^{\mu}_{\mu}\rangle$.)

From Eq. (58) we have two solutions [Eq. (2b)]

$$R_{\pm} = 6\Lambda_{\pm} \tag{60}$$

with

$$\Lambda_{\pm} = \frac{1}{12\hat{\alpha}}[1 \pm (1 - 24\Lambda\hat{\alpha})^{1/2}]. \tag{61}$$

$\Lambda_{\pm}$ are the effective cosmological constants.

One can distinguish three cases.

(i) If $\Lambda < 1/24\hat{\alpha} = 3c^4\pi^2/32G\hbar\,(6/5\alpha' c\,\hbar^3)^{1/2}$. We have two de Sitter space-times with curvatures [Eq. (60)]

$$R_{\pm} = \frac{9c^4\pi^2}{8G}\left(\frac{6}{5\alpha' c\hbar^3}\right)^{1/2} \times \left\{1 \pm \left[1 - \frac{32G\Lambda}{3c^4\pi^2}\left(\frac{5\alpha' c\,\hbar^3}{6}\right)^{1/2}\right]^{1/2}\right\}. \tag{62}$$

Both branches are well defined and have $R_{\pm} > 0$.

(ii) If $\Lambda = 1/24\hat{\alpha}$, there is a unique de Sitter space-time

$$R_{+} = R_{-} = \frac{1}{4\hat{\alpha}} = \frac{9c^4\pi^2}{16G}\left(\frac{6}{5\alpha'\hbar^3 c}\right)^{1/2}. \tag{63}$$

(iii) If $\Lambda > 1/24\hat{\alpha}$, there are neither physical real curvatures nor temperatures.

For small Λ,

$$\Lambda \ll \frac{1}{24\hat{\alpha}}\left(= \frac{3c^4\pi^2}{32G}\left(\frac{6}{5c\alpha'\hbar^3}\right)^{1/2}\right),$$

for which $\langle T^{\mu}_{\mu}\rangle_{\mathrm{QFT}}$, and hence $\langle \tilde{T}^{\mu}_{\mu}\rangle$ and $\langle \tau^{\mu}_{\mu}\rangle$, are not trivial, we have

$$R_{-} \simeq 6\Lambda \ll \frac{1}{4\hat{\alpha}}, \tag{64a}$$

$$R_{+} \simeq \frac{1}{\hat{\alpha}} = \frac{9c^4\pi^2}{4G}\left(\frac{6}{5\alpha' c\,\hbar^3}\right)^{1/2} \equiv R_{\max}. \tag{64b}$$

From the above equations we see that one recovers the classical space-time for the R_{-} solution. R_{-} is a small curvature solution. On the contrary, R_{+} does not represent a classical allowed configuration and its curvature is very high. The two branches of solutions are generically of different kind. We call the R_{-} branch ''classical'' as it represents solutions which are classically allowed, while the R_{+} branch will be the ''quantum'' one as the configurations do not occur classically.

From Eqs. (60) and (61), we read a maximum value for the effective cosmological constant:

$$\Lambda_{\max} \simeq \frac{1}{6\hat{\alpha}} = \frac{3c^4\pi^2}{8G}\left(\frac{6}{5\alpha' c\,\hbar^3}\right)^{1/2}. \tag{65}$$

In terms of $\Lambda_{\max}$ alternatively of $R_{\max}$, we have

$$\Lambda_{\pm} = \frac{1}{2}\Lambda_{\max}\left[1 \pm \left(1 - 4\frac{\Lambda}{\Lambda_{\max}}\right)^{1/2}\right], \tag{66}$$

$$R_{\pm} = \frac{1}{2}R_{\max}\left[1 \pm \left(1 - 4\frac{R}{R_{\max}}\right)^{1/2}\right]. \tag{67}$$

In an expansion in $R/R_{\max}$, the leading order is $R_{(-)} = R, R_{(+)} = R_{\max}$. QFT de Sitter temperature [Eq. (6b)] associated to the classical branch $R_{(-)}$ is

$$T_{-\mathrm{DS}} = \frac{\hbar c}{2\pi k_B}\left(\frac{R_{(-)}}{6}\right)^{1/2}.$$

The string quantum branch $R_{(+)}$ has a string temperature

$$T_{+\mathrm{string}} = \frac{c^2}{\alpha' k_B}\left(\frac{6}{R_{(+)}}\right)^{1/2}.$$

V. CONCLUSIONS

A combined study of QFT and string theory in curved backgrounds allowed us to go further in the understanding of quantum gravity effects. The string analogue model (or thermodynamical approach) is a suitable framework in cosmology and black holes to combine both QFT and string study, and address the problem of quantum string back reaction. The dual relationship shown here between the two domains: classical-QFT and quantum string, applies also to other space-times and plays a key role in the black hole case Ref. [5].

The string black hole temperature and quantum string back reaction for black holes is reported in another paper Ref. [5]. The two phases correspond to the evaporation from a classical black hole geometry with intrinsic temperature given by the QFT Hawking temperature to a string phase for the geometry (sustained by the quantum string back reaction) which temperature becomes the intrinsic string temperature. These studies and our dual relation between classical-QFT and string phases appear irrespective of conformal invariance. A similar study for anti–de Sitter space time is under investigation by these authors.

QFT in anti–de Sitter space time does not possess an intrinsic or Hawking temperature. Strings in AdS space-time do not possess a maximal or critical temperature [3,4]. The partition function for a gas of strings in AdS space-time is defined at any positive temperature Ref. [3].

Such results for strings in AdS space-time were also confirmed in the presence of a full conformal invariant AdS string background Wess-Zumino-Witten-Novikov (WZWN) model SL(2,R) (AdS with torsion) [16]. As shown in Ref. [16] conformal invariance *simplifies* the mathematics of the problem but the physics remain mainly *unchanged*. For low and high masses, the string mass spectra in conformal and nonconformal backgrounds are the same.

The purpose of this paper was to go further in the understanding of string theory in de Sitter space-time and motivate (and *at priory* justify) the choice of de Sitter space time: (i) the cosmological (inflationary) relevance of de Sitter space-time, (ii) the present knowledge of string dynamics in conformal and nonconformal invariant backgrounds, in particular in the conformal and nonconformal invariant AdS backgrounds mentioned above, (iii) the lack, at the present time, of a full string conformal invariant treatement involving de Sitter space-time.

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