Conformality from field-string duality on Abelian orbifolds

Paul H. Frampton*

Fakultät für Physik, Universität Bielefeld, D-33501 Bielefeld, Germany
and Department of Physics and Astronomy, University of North Carolina, Chapel Hill, North Carolina 27599-3255[†]

If the standard model is embedded in a conformal theory, what is the simplest possibility? We analyze all Abelian orbifolds for discrete symmetry Z_p with $p \leq 7$, and find that the simplest such theory is indeed $SU(3)^7$. Such a theory predicts the correct electroweak unification ($\sin^2 \theta \approx 0.231$). A color coupling $\alpha_C(M) \approx 0.07$ suggests a conformal scale M near to 10 TeV. [S0556-2821(99)50124-0]

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Since, in the context of field-string duality, there has been a shift regarding the relationship of gravity to the standard model of strong and electroweak interactions we shall begin by characterizing how gravity fits in, then to suggest more specifically how the standard model fits in to the string framework.

The descriptions of gravity and of the standard model are contained in the string theory. In the string picture in ten spacetime dimensions, or upon compactification to four dimensions, there is a massless spin-two graviton but the standard model is not manifest in the way we shall consider it. In the conformal field theory extension of the standard model, gravity is strikingly absent. The field-string duality does not imply that the standard model already contains gravity and, in fact, it does not.

The situation is *not* analogous to the Regge-pole–resonance duality (despite a misleading earlier version of this introduction). That quite different duality led to the origin [1] of string theory and originated from the realization [2–5] phenomenologically that adding Regge pole and resonance descriptions is double counting and that the two descriptions are *dual* in that stronger sense. The duality [6] between the field and string descriptions is not analogous because the conformal field theory (CFT) description does not contain gravity. A first step to combining gravity with the standard model would be adding the corresponding Lagrangians.

In the field theory description [7–10] used in this Rapid Communication, one will simply ignore the massless spin-two graviton. Indeed since we are using the field theory description only below the conformal scale of ~ 1 TeV (or, as suggested later in this paper, 10 TeV) and forgoing any requirement of grand unification, the hierarchy between the weak scale and theory-generated scales like M_{GUT} or M_{Planck} is resolved. Moreover, seeking the graviton in the field theory description is possibly resolvable by going to a higher dimension and restricting the range of the higher dimension. Here we are looking only at the strong and weak interactions at accessible energies below, say, 10 TeV.

Of course, if we ask questions in a different regime, for example, about scattering of particles with center-of-mass energy of the order M_{Planck} then the graviton will become crucial [11,12] and a string, rather than a field, description will be the viable one.

It is important to distinguish between the holographic description of the five-dimensional gravity in $(AdS)_5$ made by the four-dimensional CFT and the origin of the four-dimensional graviton. The latter could be described holographically only by a lower three-dimensional field theory which is not relevant to the real world. Therefore the graviton of our world can only arise by *compactification* of a higher dimensional graviton. Introduction of gravity must break conformal invariance and it is an interesting question (which I will not answer) whether this breaking is related to the mass and symmetry-breaking scales in the low-energy theory. That is all I will say about gravity in the present paper; the remainder is on the standard model and its embedding in a CFT.

An alternative to conformality, grand unification with supersymmetry, leads to an impressively accurate gauge coupling unification [13]. In particular it predicts an electroweak mixing angle at the Z pole, $\sin^2 \theta = 0.231$. This result may, however, be fortuitous, but rather than abandon gauge coupling unification, we can rederive $\sin^2 \theta = 0.231$ in a different way by embedding the electroweak $SU(2) \times U(1)$ in $SU(N) \times SU(N) \times SU(N)$ to find $\sin^2 \theta = 3/13 \approx 0.231$ [9,10]. This will be a common feature of the models in this paper.

The conformal theories will be finite without quadratic or logarithmic divergences. This requires appropriate equal number of fermions and bosons which can cancel in loops and which occur without the necessity of spacetime supersymmetry. As we shall see in one example, it is possible to combine spacetime supersymmetry with conformality but the latter is the driving principle and the former is merely an option: additional fermions and scalars are predicted by conformality in the TeV range [9,10], but in general these particles are different and distinguishable from supersymmetric partners. The boson-fermion cancellation is essential for the cancellation of infinities, and will play a central role in the calculation of the cosmological constant (not discussed here). In the field picture, the cosmological constant measures the vacuum energy density.

What is needed first for the conformal approach is a simple model and that is the subject of this paper.

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*Email address: frampton@physics.unc.edu

[†]Permanent address.

Here we shall focus on Abelian orbifolds characterized by the discrete group Z_p. Non-Abelian orbifolds will be systematically analyzed elsewhere.

The steps in building a model for the Abelian case (parallel steps hold for non-Abelian orbifolds) are the following:

(1) Choose the discrete group Γ. Here we are considering only $\Gamma = Z_p$. We define $\alpha = \exp(2\pi i/p)$.

(2) Choose the embedding of $\Gamma \subset SU(4)$ by assigning **4** $= (\alpha^{A_1}, \alpha^{A_2}, \alpha^{A_3}, \alpha^{A_4})$ such that $\Sigma_{q=1}^{q=4} A_q = 0 \pmod{p}$. To break $\mathcal{N} = 4$ supersymmetry to $\mathcal{N} = 0$ (or $\mathcal{N} = 1$) requires that none (or one) of the A_q is equal to zero (mod p).

(3) For chiral fermions one requires that $\mathbf{4} \neq \mathbf{4}^*$ for the embedding of Γ in $SU(4)$.

The chiral fermions are in the bifundamental representations of $SU(N)^p$

$$\sum_{i=1}^{i=p} \sum_{q=1}^{q=4} (N_i, \bar{N}_{i+A_q}). \tag{1}$$

If $A_q = 0$ we interpret $(N_i, \bar{N}_i)$ as a singlet plus an adjoint of $SU(N)_i$.

(4) The **6** of $SU(4)$ is real $\mathbf{6} = (a_1, a_2, a_3, -a_1, -a_2, -a_3)$ with $a_1 = A_1 + A_2$, $a_2 = A_2 + A_3$, $a_3 = A_3 + A_1$ (recall that all components are defined modulo p). The complex scalars are in the bifundamentals

$$\sum_{i=1}^{i=p} \sum_{j=1}^{j=3} (N_i, \bar{N}_{i \pm a_j}). \tag{2}$$

The condition in terms of a_j for $\mathcal{N} = 0$ is $\Sigma_{j=1}^{j=3}(\pm a_j) \neq 0 \pmod{p}$ [7].

(5) Choose the N of $\otimes_i SU(Nd_i)$ (where the d_i are the dimensions of the representations of Γ). For the Abelian case where $d_i \equiv 1$, it is natural to choose $N = 3$ the largest $SU(N)$ of the standard model (SM) gauge group. For a non-Abelian Γ with $d_i \neq 1$ the choice $N = 2$ would be indicated.

(6) The p quiver nodes are identified as color (C), weak isospin (W) or a third $SU(3)$ (H). This specifies the embedding of the gauge group $SU(3)_C \times SU(3)_W \times SU(3)_H \subset \otimes SU(N)^p$. This quiver node identification is guided by (7), (8) and (9) below.

(7) The quiver node identification is required to give three chiral families under Eq. (1). It is sufficient to make three of the $(C + A_q)$ to be W and the fourth H, given that there is only one C quiver node, so that there are three $(3, \bar{3}, 1)$. Provided that $(\bar{3}, 3, 1)$ is avoided by the $(C - A_q)$ being H, the remainder of the three family trinification will be automatic by chiral anomaly cancellation. Actually, a sufficient condition for three families has been given; it is necessary only that the difference between the number of $(3 + A_q)$ nodes and the number of $(3 - A_q)$ nodes which are W is equal to three.

(8) The complex scalars of Eq. (2) must be sufficient for their vacuum expectation values to spontaneously break $SU(3)^p \rightarrow SU(3)_C \times SU(3)_W \times SU(3)_H \rightarrow SU(3)_C \times SU(2)_W \times U(1)_Y \rightarrow SU(3)_C \times U(1)_Q$. Note that, unlike grand unified theories (GUTs) with or without supersymmetry, the Higgs scalars are here prescribed by the conformality condition.

This is more satisfactory because it implies that the Higgs sector cannot be chosen arbitrarily, but it does make model building more interesting

(9) Gauge coupling unification should apply at least to the electroweak mixing angle $\sin^2 \theta = g_Y^2/(g_2^2 + g_Y^2) \simeq 0.231$. For trinification $Y = 3^{-1/2}(-\lambda_{8W} + 2\lambda_{8H})$ so that $(3/5)^{1/2} Y$ is correctly normalized. If we make $g_Y^2 = (3/5)g_1^2$ and $g_2^2 = 2g_1^2$ then $\sin^2 \theta = 3/13 \simeq 0.231$ with sufficient accuracy.

In the remainder of this paper we answer all these steps for the choice $\Gamma = Z_p$ for successive $p = 2, 3, \ldots$ up to $p = 7$, then add some concluding remarks.

$p = 2$. In this case $\alpha = -1$ and therefore one cannot construct any complex **4** of $SU(4)$ with $\mathbf{4} \neq \mathbf{4}^*$. Chiral fermions are therefore impossible.

$p = 3$. The only possibilities are $A_q = (1,1,1,0)$ or $A_q = (1,1,-1,-1)$. The latter is real and leads to no chiral fermions. The former leaves $\mathcal{N} = 1$ supersymmetry and is a simple three-family model [14] by the quiver node identification C-W-H. The scalars $a_j = (1,1,1)$ are sufficient to spontaneously break to the SM. Gauge coupling unification is, however, missing since $\sin^2 \theta = 3/8$, in poor disagreement with experiment.

$p = 4$. The only complex $\mathcal{N} = 0$ choice is $A_q = (1,1,1,1)$. But then $a_j = (2,2,2)$ and any quiver node identification such as C-W-H-H has 4 families and the scalars are insufficient to break spontaneously the symmetry to the SM gauge group.

$p = 5$. The two inequivalent complex choices are $A_q = (1,1,1,2)$ and $A_q = (1,3,3,3)$. By drawing the quiver, however, and using the rules for three chiral families given in (7) above, one finds that the node identification and the prescription of the scalars as $a_j = (2,2,2)$ and $a_j = (1,1,1)$, respectively, does not permit spontaneous breaking to the standard model.

$p = 6$. Here we can discuss three inequivalent complex possibilities as follows:

(6A) $A_q = (1,1,1,3)$ which implies $a_j = (2,2,2)$. Requiring three families means a node identification C-W-X-H-X-H where X is either W or H. But whatever we choose for the X the scalar representations are insufficient to break $SU(3)^6$ in the desired fashion down to the SM. This illustrates the difficulty of model building when the scalars are not in arbitrary representations.

(6B) $A_q = (1,1,2,2)$ which implies $a_j = (2,3,3)$. Here the family number can be only zero, two or four as can be seen by inspection of the A_q and the related quiver diagram. So (6B) is of no phenomenological interest.

(6C) $A_q = (1,3,4,4)$ which implies $a_j = (1,1,4)$. Requiring three families needs a quiver node identification which is of the form *either* C-W-H-H-W-H *or* C-H-H-W-W-H. The scalar representations implied by $a_j = (1,1,4)$ are, however, easily seen to be insufficient to do the required spontaneous symmetry breaking (SSB) for both of these identifications.

$p = 7$. Having been stymied mainly by the rigidity of the scalar representation for all $p \leq 6$, for $p = 7$ there are the first cases which work. Six inequivalent complex embeddings of $Z_7 \subset SU(4)$ require consideration.

(7A) $A_q = (1,1,1,4) \Rightarrow a_j = (2,2,2)$. For the required nodes C-W-X-H-H-X-H the scalars are *insufficient* for SSB.

(7B) $A_q=(1,1,2,3)\Rightarrow a_j=(2,3,3)$. The node identification C-W-H-W-H-H-H leads to a *successful* model.

(7C) $A_q=(1,2,2,2)\Rightarrow a_j=(3,3,3)$. Choosing C-H-W-X-X-H-H to derive three families, the scalars *fail* in SSB.

(7D) $A_q=(1,3,5,5)\Rightarrow a_j=(1,1,3)$. The node choice C-W-H-H-H-W-H leads to a *successful* model. This is Model A of [10].

(7E) $A_q=(1,4,4,5)\Rightarrow a_j=(1,2,2)$. The nodes C-H-H-H-W-W-H are *successful*.

(7F) $A_q=(2,4,4,4)\Rightarrow a_j=(1,1,1)$. Scalars *insufficient* for SSB.

The three successful models (7B), (7D) and (7E) lead to an $\alpha_3(M)\simeq0.07$. Since $\alpha_3(1\text{ TeV})\geq0.10$ this suggest a conformal scale $M\simeq10$ TeV [10]. The above models have less generators than an $E(6)$ GUT and thus $SU(3)^7$ merits further study. It is possible, and under investigation, that non-Abelian orbifolds will lead to a simpler model.

For such field theories it is important to establish the existence of a fixed manifold with respect to the renormalization group. It could be a fixed line but more likely, in the $\mathcal{N}=0$ case, a fixed point [15]. It is known that in the $N\rightarrow\infty$ limit the theories become conformal, but although this 't Hooft limit [16] is where the field-string duality is derived we know that finiteness survives to finite N in the $\mathcal{N}=4$ case [17] and this makes it plausible that at least a conformal point occurs also for the $\mathcal{N}=0$ theories with $N=3$ derived above.

The conformal structure cannot by itself predict all the dimensionless ratios of the standard model such as mass ratios and mixing angles because these receive contributions, in general, from soft breaking of conformality. With a specific assumption about the pattern of conformal symmetry breaking, however, more work should lead to definite predictions for such quantities.

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