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Detecting an association between gamma ray and gravitational wave bursts

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If γ-ray bursts (GRBs) are accompanied by gravitational wave bursts (GWBs) the correlated output of two gravitational wave detectors evaluated in the moments just prior to a GRB will differ from that evaluated at other times. We can test for this difference without prior knowledge of either the GWB wave form or the detector noise spectrum. With a model for the GRB source population and GWB spectrum we can put a limit on the in-band rms GWB signal amplitude. Laser-Interferometer Gravitational Wave Observatory I detector observations coincident with 1000 GRB observations could lead us to exclude with 95% confidence associated GWBs with $h_{\mathrm{RMS}} \gtrsim 1.7\times 10^{-22}$. [S0556-2821(99)50222-1]

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Gamma ray bursts (GRBs) are believed to arise from shocks in a relativistic fireball triggered by rapid accretion on a newly formed black hole [1]. In this scenario the γ-ray production takes place some distance from the black hole, making it difficult to test this model with conventional astronomical observations. The violent formation of a black hole is likely to produce a substantial gravitational wave burst (GWB); thus, we expect GRBs to be preceded by GWBs. Observation of GWBs associated with GRBs, made by the new detectors now under construction [2,3], may be the only means of testing directly this GRB model.

Proposed GRB progenitors include coalescing binary systems, hypernovae or collapsars [1]. Statistical evidence points to at least three different subclasses of GRBs [4]; so, the actual progenitors may include these as well as other systems. Matched filtering (MF) — the focus of most of the gravitational wave detection literature — requires detailed knowledge of the actual GWB wave form: without that detailed knowledge it cannot be used to detect a distinct GWB associated with a GRB. Additionally, since GRBs occur at cosmological distances the signal-to-noise ratio (SNR) of any individual GWB will likely be insufficient for a high confidence detection with the new gravitational wave detectors. (Reference [5] described a MF analysis but made the, now unlikely, assumption that GRBs all arise from double neutron star mergers.) Detection techniques other than MF that aim to detect distinct GWBs will perform even worse.

Here we suggest an alternative method for detecting a GWB-GRB *association.* If GWBs are associated with GRBs, the correlated output of two GW detectors will be different in the moments immediately preceding a GRB (*on-source*) than at other times not associated with a GRB (*off-source*). (While we focus on GRBs in this paper any plausible class of astronomical events can serve as a trigger.) A statistically significant difference between on- and off-source cross-correlations would support a GWB-GRB association and represent a detection of gravitational waves by the detector pair. We can measure this difference using Student's t-test without requiring any foreknowledge of the signal wave form, source or source population (though with such a model the effectiveness of the test can be improved). The measured difference can be used to establish a confidence interval (CI) or upper limit (UL) on the rms amplitude of GWBs associated with GRBs. The CI and/or UL, in turn, constrains any model for model for GRB-GWB pairs.

In the following analysis we restrict attention to the two full-length LIGO detectors (denoted $\mathcal{D}_i$, $i=1,2$). These detectors are nearly identically oriented and lie $\sim$3000 Km apart. We place *no* requirements on the detector noise except that it be quasi-stationary, exhibit no long-term trends, and that the cross-correlation is weak compared to the auto-correlation. In particular, the noise may be non-Gaussian and may exhibit small amplitude fluctuations, such as might be associated with alignment variations, on short or long time-scales. Finally, without loss of generality we assume the noise has zero mean and denote its one-sided power spectral density (PSD) by $S_i(f)$.

(a) *On-source and off-source distributions*. Suppose that a GWB, associated with a GRB, is incident from direction $\vec{n}$ on the GW detector $\mathcal{D}_i$ at time $t_a^{(i)}$. The *lag* δt, equal to $t_a^{(2)} - t_a^{(1)}$, depends only on $\vec{n}$, which we know from the GRB observation. The lag is also the same as the difference $t_\gamma^{(2)} - t_\gamma^{(1)}$, where $t_\gamma^{(i)}$ is the arrival time at detector $\mathcal{D}_i$ of the GRB.

Assuming that GWBs precede GRBs, focus attention on the output $x_i(t)$ of detector $\mathcal{D}_i$, for $0 \leqslant t_\gamma^{(i)} - t \leqslant T$. Choosing the delay T as long, but no longer, than necessary to ensure that x_i includes the possible GWB signal, compute the weighted cross-correlation

$$
\begin{aligned}
X &:= \langle x_1, x_2 \rangle \\
&:= \int \int_0^T dt\, dt'\, x_1(t_\gamma^{(1)} - t) Q(|t-t'|) x_2(t_\gamma^{(2)} - t'). \qquad (1)
\end{aligned}
$$

The filter kernel Q is at our disposal: we discuss its choice in (c) below.

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The collection of X computed for each of N_{on} GRBs forms the set $\mathcal{X}_{\mathrm{on}}$ of *on-source* events. To complement $\mathcal{X}_{\mathrm{on}}$, construct a set $\mathcal{X}_{\mathrm{off}}$ of N_{off} *off-source events*, using data segments x_i corresponding to random sky directions and arrival times not associated with any GRB.

The sample sets $\mathcal{X}_{\mathrm{off}}$ and $\mathcal{X}_{\mathrm{on}}$ are drawn from populations whose distributions we denote p_{off} and p_{on}. For T much greater than the detector noise auto- and cross-correlation times, the central limit theorem implies that p_{off} is normal with mean and variance

$$\mu_{\mathrm{off}} := \mathrm{E}[\langle n_1, n_2\rangle], \tag{2a}$$

$$\sigma^2_{\mathrm{off}} := \mathrm{E}[(\langle n_1, n_2\rangle - \mu_{\mathrm{off}})^2]. \tag{2b}$$

Here $n_i(t)$ denotes noise from detector i and $\mathrm{E}[\cdot]$ represents an ensemble average across the detector output. Note that μ_{off} is just the detector noise cross-correlation evaluated at the lag δt.

Now suppose that GRBs are preceded by GWBs. Elements of $\mathcal{X}_{\mathrm{on}}$ then take the form

$$X = \langle n_1, n_2\rangle + \langle h_1, n_2\rangle + \langle n_1, h_2\rangle + \langle h_1, h_2\rangle, \tag{3}$$

where $h_i(t)$ is detector i's response to the incident GWB. Define P_i by

$$P_i := 4\int_0^\infty df\, |\tilde{h}_i(f)|^2 / S_i(f). \tag{4}$$

When $\overline{P_i}$, the average of P_i over the source population, is much less than unity p_{on} is also a normal distribution with variance $\sigma^2_{\mathrm{on}} = \sigma^2_{\mathrm{off}}$ and mean

$$\mu_{\mathrm{on}} = \mu_{\mathrm{off}} + \overline{s}, \qquad \text{where} \tag{5a}$$

$$s := \langle h_1, h_2\rangle. \tag{5b}$$

(If $\overline{P_i}$ is large then the individual GWBs are readily detectable through other means.)

(b) *Detecting a GRB/GWB association.* Pose the null hypothesis

$$H_0:\ p_{\mathrm{off}}(X) = p_{\mathrm{on}}(X). \tag{6}$$

Rejecting H_0 supports a GWB-GRB association. Since p_{on} and p_{off} are normal and differ, if at all, only in their means, we can test H_0 using Student's t-test [6].

The t statistic is defined from $\mathcal{X}_{\mathrm{on}}$ and $\mathcal{X}_{\mathrm{off}}$ by

$$t := \frac{\hat{\mu}_{\mathrm{on}} - \hat{\mu}_{\mathrm{off}}}{\Sigma}\sqrt{\frac{N_{\mathrm{on}} N_{\mathrm{off}}}{N_{\mathrm{on}} + N_{\mathrm{off}}}}, \tag{7a}$$

$$\Sigma^2 = \frac{\hat{\sigma}^2_{\mathrm{on}}(N_{\mathrm{on}} - 1) + \hat{\sigma}^2_{\mathrm{off}}(N_{\mathrm{off}} - 1)}{N_{\mathrm{on}} + N_{\mathrm{off}} - 2}, \tag{7b}$$

where $\hat{\mu}_{\mathrm{on}}$ and $\hat{\mu}_{\mathrm{off}}$ ($\hat{\sigma}^2_{\mathrm{on}}$ and $\hat{\sigma}^2_{\mathrm{off}}$) are the *sample* means (variances) of $\mathcal{X}_{\mathrm{on}}$ and $\mathcal{X}_{\mathrm{off}}$, respectively.

The expectation value of t, averaged over the source population and across the detector noise processes, is

$$\mu_t := E[t] = \frac{\overline{s}}{\sigma}\sqrt{\frac{N_{\mathrm{on}} N_{\mathrm{off}}}{N_{\mathrm{on}} + N_{\mathrm{off}}}}. \tag{8}$$

The relative orientation of the two Laser-Interferometer Gravitational Wave Observatory (LIGO) detectors guarantees that $h_1(t)$ and $h_2(t)$ are very nearly identical. For LIGO, then, $\overline{s}$ is non-negative and μ_t is positive in presence of a GWB-GRB association and zero otherwise.

The t statistic depends only on the inter-detector cross-correlation associated with GRBs. The expectation value μ_t is unaffected by any noise that is not correlated with GRBs. Additionally, the entire effect of small variations in the detector noise, either on short or long timescales, is part of the estimated variances σ^2_{on} and σ^2_{off} and does not require special treatment.

The actual value of t given observed sets $\mathcal{X}_{\mathrm{on}}$ and $\mathcal{X}_{\mathrm{off}}$ will vary from μ_t. The distribution of t is normal for large $N = N_{\mathrm{on}} + N_{\mathrm{off}}$ and for small N is tabulated in most statistics texts [7]. We can thus find a t_0 such that, when H_0 is true ($\mu_t = 0$) t is greater than t_0 in less than a fraction α (e.g., 5%) of all observations. This is our test: if we observe t greater than t_0 we reject H_0 and conclude that we have found evidence of a GWB-GRB association with significance $1 - \alpha$ (e.g., 95%).

(c) *The filter kernel Q*. The filter kernel Q [cf. Eq. (1)] used to form the observations X is at our disposal. If we knew the signal $h_i(t)$ corresponding to each GRB trigger we could construct a Q that maximizes s:

$$Q(\tau) = \int_{-\infty}^{\infty} df\, e^{2\pi i f\tau}\frac{\tilde{h}_1(f)\tilde{h}_2^*(f)}{S_1(|f|)S_2(|f|)}, \tag{9}$$

where $\tilde{h}_i$ is the Fourier transform of h_i. For the LIGO detectors, the h_i are identical and the optimal Q depends only on their common functional form $h(t)$ through $|\tilde{h}(f)|^2$.

Any knowledge we have of the signal's expected character can be put into Q. For LIGO we can choose Q to match the signal model irrespective of the GWB wave form details if $|\tilde{h}(f)|^2$ is independent of other signal parameters. This happens, for instance, in the case of an inspiraling binary. For GWBs associated with GRBs there is no reason to believe that $|\tilde{h}(f)|^2$ will be known *a priori*, let alone that it have this special property. Lacking detailed knowledge, we recommend adopting Q given by Eq. (9) with $|\tilde{h}(f)|^2$ assumed to be unity in the detector band.

(d) *Setting upper limits*. Having specified Q we can test H_0 [cf. Eq. (6)] to rule on a GWB-GRB association. Alternatively, we can use the observed t to determine a confidence interval (CI) or upper limit (UL) on μ_t, and hence $\overline{s}$, which is related to the GWB wave strength [cf. Eqs. (8), (5b)]. If we specify a model for $|\tilde{h}(f)|^2$ and the spatial distribution of GRB sources, this becomes a constraint on the model.

To measure the effectiveness of the proposed test consider the UL *most likely* to be placed on $\overline{s}$ if H_0 is, in fact, true.

When H_0 is true the most likely *observed* t is zero. Denoting the corresponding UL on μ_t as $\mu_{t,\max}$ the most likely UL on $\overline{s}$ is thus

$$\frac{\overline{s}}{\sigma} \leqslant \mu_{t,\max} \sqrt{\frac{N_{\text{on}}+N_{\text{off}}}{N_{\text{on}}N_{\text{off}}}} \tag{10a}$$

$$= \begin{cases} \mu_{t,\max}\sqrt{2/N_\gamma} & (N_{\text{on}}=N_{\text{off}}=N_\gamma), \\ \mu_{t,\max}/\sqrt{N_{\text{on}}} & (N_{\text{off}} \gg N_{\text{on}}). \end{cases} \tag{10b}$$

Since the duty cycle of GRBs is low, N_{off} can be made much larger than N_{on}. Even if both sample sets are the same size, however, the limit obtained will be weaker by only a factor of $2^{1/2}$.

The upper limit $\mu_{t,\max}$ corresponding to an observed t of zero and different degrees of confidence is given in [8], Table X. For reference we note that $\mu_{t,\max}$ is 1.96 for 95% and 2.58 for 99% confidence; correspondingly, if the observed t is zero then a 95% UL on μ_t is 1.96.

A derived CI and/or UL on $\overline{s}$ implies, within the context of a GWB-GRB source model, a CI and/or UL on the rms GWB signal amplitude in the detector band. As an example, suppose that each GRB is accompanied by the formation of a several solar mass black hole and a corresponding millisecond timescale GWB in the source rest frame. Assume further that $|\tilde{h}(f)|^2$ is approximately constant in the corresponding KHz bandwidth B_s. (This is consistent with numerical models of supernova core collapse [9,10] and with the formation or ring-down of all but the most rapidly rotating solar mass black holes [11].) At the detector, the signal power from a source at redshift z lies in the bandwidth B_s/z', where z' is equal to $1+z$.

For simplicity, assume that the detector noise PSDs $S_i(f)$ are identical and equal to a constant S_0 in the detector bandwidth B_d, which we take to be approximately 100 Hz about a central frequency of 150 Hz. Outside the detector band we set S_i equal to infinity. (This is a rough approximation to the actual shape of the noise PSD of LIGO [12].) Finally, note that B_s is much larger than B_d, so that B_s/z' completely overlaps B_d for some large range of z'.

With these assumptions,

$$s = \int_{-\infty}^{\infty} df |\tilde{h}(f)|^2 \tilde{Q}(f) = \frac{2A^2 B_d}{S_0^2} \quad \text{and} \tag{11}$$

$$\sigma^2 = \frac{T}{4}\int_{-\infty}^{\infty} df\, S_1(|f|) S_2(|f|) |\tilde{Q}(f)|^2 = \frac{TB_d}{2S_0^2}, \tag{12}$$

where A is defined by

$$\int_{-\infty}^{\infty} df |\tilde{h}(f)|^2 = \frac{2A^2 B_s}{z'}. \tag{13}$$

From Eqs. (11), (12), and (13) it follows that

$$\frac{\overline{s}}{\sigma} = \mathrm{E}\left[\frac{2\sqrt{2}A^2 B_d}{\sqrt{TB_d}S_0}\right] \simeq \frac{2\sqrt{2}\,\overline{A^2} B_d}{\sqrt{TB_d}S_0}, \tag{14}$$

where we have replaced A^2 by its mean over the source population (a good approximation when A is sharply peaked about its mean). From Eqs. (10) and (14) and assuming that H_0 is true we find

$$\overline{A^2} \leqslant A^2_{\max} = \frac{\mu_{t,\max}}{2\sqrt{2}}\left[\frac{TB_d}{N_{\text{on}}}\right]^{1/2} \frac{S_0}{B_d}, \tag{15}$$

with $N_{\text{off}} \gg N_{\text{on}}$ and $\mu_{t,\max}$ obtained ([8], from Table X) with $x=0$ (corresponding to $t=0$).

We expect that different GWBs will have different wave forms and durations. Define the rms signal power in the detector band by

$$h^2_{\text{RMS}} := \overline{\left[\frac{2}{\tau}\int_{f\in B_d} df |\tilde{h}(f)|^2\right]}, \tag{16}$$

where $h(t)$ is the GWB wave form, τ its duration in the detector band, and the average is over the source population. In our example—broadband bursts whose bandwidth includes the detector band—we can approximate $1/\tau$ by the detector bandwidth B_d. Combining Eqs. (16), (15), and (13) we find the UL on h_{RMS}:

$$h^2_{\text{RMS}} \leqslant [1.7\times 10^{-22}]^2 \frac{\mu_{t,\max}}{1.96}\left(\frac{T}{0.5\ \text{s}}\frac{1000}{N_{\text{on}}}\right)^{1/2} \times \frac{S_0}{(3\times 10^{-23}\ \text{Hz}^{-1/2})^2}\left(\frac{B_d}{100\ \text{Hz}}\right)^{3/2}. \tag{17}$$

The reference values of B_d and S_0 are characteristic of the initial LIGO detectors [12]. For T [cf. Eq. (1)] we assume GRBs are generated by internal shocks in the fireball; then, the GRB-GWB delay is approximately 0.1 sec in the source rest frame [13]. To accommodate GRBs at redshifts $z \leqslant 4$ we take $T \sim 0.5$ sec. Finally, $\mu_{t,\max}$ equal to 1.96 corresponds to a 95% confidence UL [8].

If, on the other hand, GRBs are generated when the fireball is incident on an external medium, then [[14], Eq. (3.6)] with $n_1=1$, $\alpha=1$, $E_{51}=10$, and $\Gamma \gtrsim 100$ gives a source rest-frame delay $\lesssim 100$ sec, in which case T should be 500 s and the corresponding UL on h_{RMS} is 9.4×10^{-22}.

From Eq. (17) we see that the shorter we can make T the stricter the limit we can set. Our uncertainty in T can be approached either by choosing the longest likely T or by evaluating the test statistic for several different T's. We have described the first possibility here, which has the disadvantage that the sensitivity of the test is weakened over the ideal if the actual delay is much shorter than T. The analysis in the second case is only slightly different than that presented here, since we must take into account an appropriate trials factor when evaluating the probability of detection or the magnitude of the upper limit on h_{RMS}.

Two final notes are in order. To calculate the X [cf. Eq. (1)], which are at the heart of our analysis, we must know accurately the GRB source direction. Bright bursts in the BATSE3B catalog have positional accuracies of $\delta\theta \lesssim 1.5°$ [15]. The corresponding uncertainty in s is $\lesssim 5\%$, which does not affect significantly the UL on $\overline{s}$.

Finally, the proposed BATSE follow-on—SWIFT—is not an all-sky GRB detector. It will have greater sensitivity than BATSE, but observe only a fraction of the sky at any one time. If SWIFT pointing favors the sky normal to the LIGO detector plane, LIGO's sensitivity to GWBs from observed GRBs will be maximized, increasing the sensitivity of the test described here.

(e) *Conclusions.* If gamma-ray bursts (GRBs) are associated with the violent formation of a stellar mass black hole they are likely preceded by a gravitational wave burst (GWB). Observing the associated GWB may be the only way to test directly this GRB model.

The GWB wave form is not known *a priori*, owing both to the violent nature of the event and the uncertainty in the GRB progenitor. Nevertheless, we can still detect an association between GWBs and GRBs by comparing the correlated output of two gravitational wave detectors immediately preceding a GRB to the correlation at other times. From the magnitude of the difference we can set an upper limit (UL) or determine a confidence interval (CI) on the rms GWB amplitude in the detector waveband, averaged over the source population. This CI and/or UL constrains any GRB-GWB model we do invoke.

This analysis has several important advantages over matched filtering, the method at the focus of most of the gravitational wave detection literature. In particular, it becomes more sensitive as the number of observed GRBs increases, does not require any knowledge of the GWB wave forms, is insensitive to the presence of non-Gaussian detector noise, and does not require statistical independence of the detectors or knowledge of their correlated noise. It is also the first example of a robust analysis that does not require detailed knowledge of either the source wave form or its statistical character. It is thus a powerful addition to the growing arsenal of analysis techniques aimed at making gravitational wave detection an astronomical tool.

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